

## RECIPROCAL INVISIBLE CLOAK WITH HOMOGENEOUS METAMATERIALS

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**Abstract**—Based on linear optical transformation method, a diamond shaped reciprocal cloak with perfect invisibility in a certain direction is proposed. Compared with traditional cloaks, the object hidden inside the reciprocal cloak is not blind and can receive information from the outer region. Moreover, the reciprocal cloak is constructed of nonsingular homogeneous material parameters with reduced anisotropy that is relatively easy for practical realization. Full wave simulations validate the performance of the cloak.

### 1. INTRODUCTION

About forty years ago, Veselago [1] studied a theoretical material with simultaneously negative permittivity and permeability, and predicted that it would have a negative index of refraction. This study had not attracted much attention until the fabrication of artificial materials that possess negative index of refraction was experimentally demonstrated by Shelby et al. [2] in 2001. A fascinating term, metamaterials, is thus created and continues to develop rapidly [3–6]. In 2006, the power full means of transformation optics generalized by Leonhardt and Pendry et al. [7,8] built up a bridge between the function of metamaterials and material parameter distribution, and provided a complete prescription for controlling and guiding electromagnetic wave at will [3,9–18]. One of the most popular and intriguing topics in this field is perhaps the “invisible cloak” [3], which was experimentally verified at microwave frequency band by Schurig et al. It makes the incident wave detour around the cloaked region without disturbance, thus an arbitrary object might be hidden inside. But the object has to be “blind”, since no outside electromagnetic waves can reach within the cloaked space [19].

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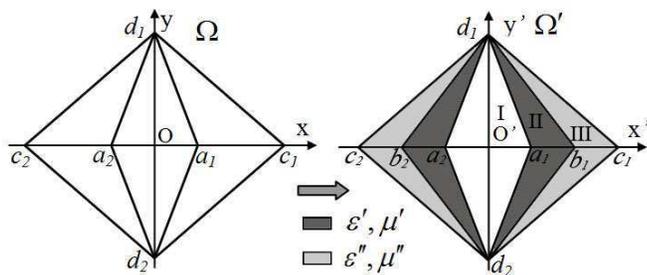
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To overcome this limitation, some new cloaking schemes were developed. The external cloak [20, 21] designed based on combining the concepts of complementary media and transformation optics is capable of hiding arbitrary object at a distance outside the cloaking shell, thus share information with the surrounding. The open cloak [22, 23] strategy provides an approach to open a window on the surface of a cloak, so that exchanging information and matter with the outside can be achieved. Recently, the idea of reciprocal cloak is proposed by our group [24]. It enables arbitrary object hidden inside the cloaked region to exchange information with the outside and may have many potential applications. If a sensor or an antenna [25–27] were hidden in such a cloak, its presence would not be detected. However, practical fabrication of the aforementioned reciprocal cloak is not so far achievable for the material parameters are highly anisotropic and even singular.

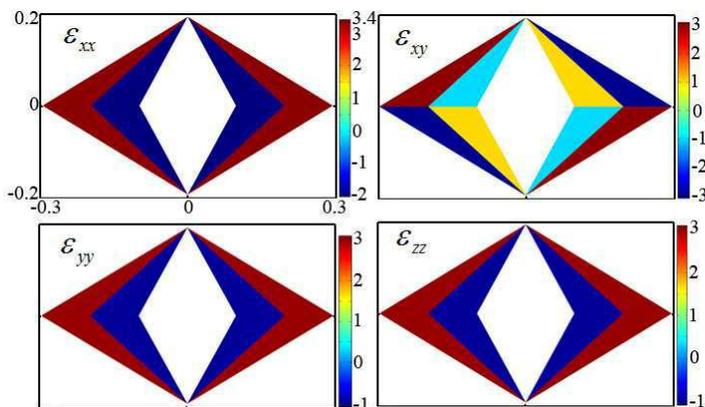
Considering the physical realization of a transformation device, anisotropy in material parameter distribution should be reduced. In this paper, we propose a diamond shaped reciprocal cloak with nonsingular and homogeneous material parameters. Results show that different from traditional cloak, the incident wave propagates through the reciprocal cloak and penetrates into the cloaked space. As a consequence, the hidden object can receive signals from the outside, but its presence will not be sensed by the surroundings. Moreover, the diamond reciprocal cloak is constructed by eight blocks of homogeneous materials with reduced anisotropy, which is possible to be realized by layered metamaterials [28] and periodic inductor-capacitor (L-C) circuits [29].

## 2. THEORETICAL MODEL

Figure 1 shows the schematic diagram for the construction of the reciprocal cloak with homogeneous material parameters. Taking the first quadrant as an example, line segment  $d_1O$  in the original space ( $\Omega$ ) is transformed into  $d_1b_1$  in the physical space ( $\Omega'$ ), while keeping the inner ( $d_1a_1$ ) and outer ( $d_1c_1$ ) boundary unchanged. Therefore, the whole space is divided into three regions. Working principle of the cloak can be described in two steps. First, the cloaked region (region I) is optically canceled by its complementary media (region II). Then, a correct optical path in the canceled space is restored by the recovering layer (region III). Coordinate transformation for the complementary layer can be written as  $x' = (a_1 - b_1)x/a_1 - b_1y/d_1 + b_1$ ,  $y' = y$ ,  $z' = z$ . Following the procedures illustrated in [30], permittivity and permeability tensors for the complementary media layer can be



**Figure 1.** Schematic diagrams for describing the coordinate transformation of the reciprocal cloak.



**Figure 2.** Material parameter distributions for the reciprocal cloak.

obtained as follows.

$$\frac{\epsilon'}{\epsilon_s} = \frac{\mu'}{\mu_s} = \begin{bmatrix} \frac{(a_1-b_1)^2}{a_1^2} + \frac{b_1^2}{d_1^2} & -\frac{b_1}{d_1} & 0 \\ -\frac{b_1}{d_1} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \frac{a_1}{a_1 - b_1} \quad (1)$$

With the coordinate transformation of  $x' = (c_1 - b_1)x/c_1 - b_1y/d_1 + b_1$ ,  $y' = y$ ,  $z' = z$ , we can obtain the permittivity and permeability tensors for the recovering layer. Material parameters in the other quadrants can be deduced by means of axial symmetry. Figure 2 shows the material parameter distribution of the cloak. Geometry parameters are chosen as  $a_1 = a_2 = 0.1$  m,  $b_1 = b_2 = 0.2$  m,  $c_1 = c_2 = 0.3$  m,  $d_1 = d_2 = 0.2$  m. It is clear that all values are finite and homogeneous in each region. Although the parameters are still anisotropic, it is

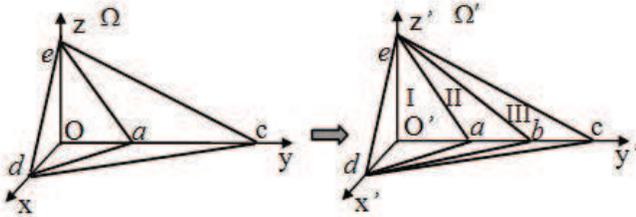
possible to be realized by layered metamaterials [28] and periodic L-C circuit [29]. The dielectric properties of the metamaterials constructed from layers of silver and silica are usually narrow-banded, and they are often difficult to implement from a practical point of view. The periodic L-C circuit approach of metamaterials leads to non-resonant structures with lower loss and wider bandwidth.

$$\varepsilon'' = \mu'' = \begin{bmatrix} \frac{(c_1-b_1)^2}{c_1^2} + \frac{b_1^2}{d_1^2} & -\frac{b_1}{d_1} & 0 \\ -\frac{b_1}{d_1} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \frac{c_1}{c_1 - b_1} \quad (2)$$

Similarly, based on the coordinate transformation in linear form, three dimensional (3D) reciprocal cloak with homogeneous material parameters can also be designed. The 3D reciprocal cloak is divided into eight parts by  $x$ ,  $y$ ,  $z$  axes. A schematic diagram of the coordinate transformation in the first quadrant is shown in Figure 3. Surface  $Ode$  in the original space ( $\Omega$ ) is transformed into  $bde$  in the physical space ( $\Omega'$ ), while keeping the inner ( $ead$ ) and outer ( $ecd$ ) boundaries unchanged. In this case, the coordinate transformation equations for the complementary layer become  $x' = x$ ,  $y' = -bx/d + (a-b)y/a - bz/e + b$ ,  $z' = z$ . Corresponding permittivity and permeability tensors can be obtained as:

$$\frac{\varepsilon'}{\varepsilon_s} = \frac{\mu'}{\mu_s} = \begin{bmatrix} 1 & \frac{-b}{d} & 0 \\ \frac{-b}{d} & \frac{b^2}{d^2} + \frac{(a-b)^2}{a^2} + \frac{b^2}{e^2} & \frac{-b}{e} \\ 0 & \frac{-b}{e} & 1 \end{bmatrix} \cdot \frac{a}{a-b} \quad (3)$$

With the coordinate transformation of  $x' = x$ ,  $y' = -bx/d + (c-b)y/c - bz/e + b$ ,  $z' = z$ , permittivity and permeability tensors for the recovering layer are deduced and shown in Equation (4). Material parameters in the other quadrants can be obtained by means of axial



**Figure 3.** Schematic diagrams for describing the coordinate transformation of the 3D reciprocal cloak in the first quadrant.

symmetry.

$$\varepsilon'' = \mu'' = \begin{bmatrix} 1 & & & 0 \\ \frac{-b}{d} & \frac{b^2}{d^2} + \frac{(c-b)^2}{c^2} + \frac{b^2}{e^2} & & \\ 0 & & \frac{-b}{e} & \\ & & & 1 \end{bmatrix} \cdot \frac{c}{c-b} \quad (4)$$

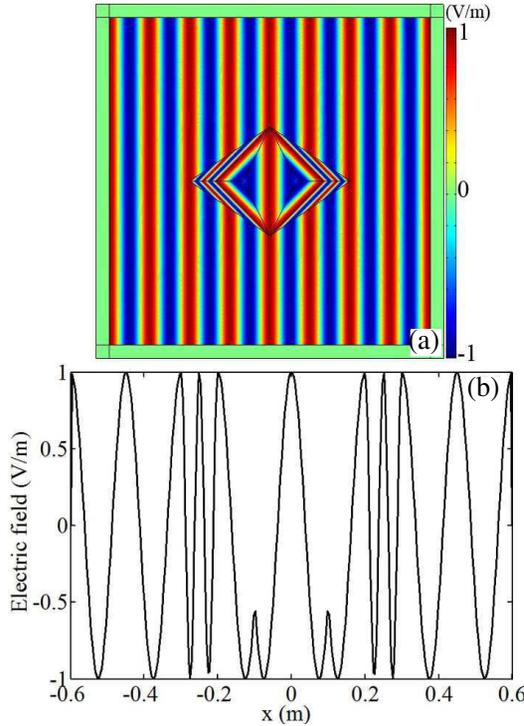
Since the coordinate transformation is performed along  $x$  or  $y$  axis of the Cartesian coordinates, perfect reciprocal invisibility can be achieved along a certain direction. In what follows, we make full wave simulation based on the commercial software COMSOL Multiphysics to confirm the derived material parameters.

### 3. SIMULATION RESULTS AND DISCUSSION

In this section, performance of the reciprocal cloak is simulated under TE wave irradiation. Frequency of the TE wave in all simulations is set to be 2 GHz. Geometry parameters of the 2D reciprocal cloak are chosen as  $a_1 = a_2 = 0.1$  m,  $b_1 = b_2 = 0.2$  m,  $c_1 = c_2 = 0.3$  m,  $d_1 = d_2 = 0.2$  m.

First, we demonstrate the scheme shown in Figure 2, i.e., the complementary and recovering layers are optically canceled, and the whole transformation region is equal to a layer of air. For a TE plane wave with unit magnitude propagating along  $x$  axis, electric field distribution is shown in Figure 4(a). In the simulation, a perfectly matched layer is applied to truncate the computational domain. From Figure 4(a), it is seen that the incident wave permeates into the cloaked region, and the fields outside the cloak stay unperturbed, that is, perfect cloaking effect can be achieved. Figure 4(b) plots the electric field distribution along  $x$  axis of the computational domain. As can be seen, electric field intensity at the center of the reciprocal cloak is equal to that outside. It indicates that an object enclosed in the reciprocal cloak can receive information from the outside, but its presence will not be detected.

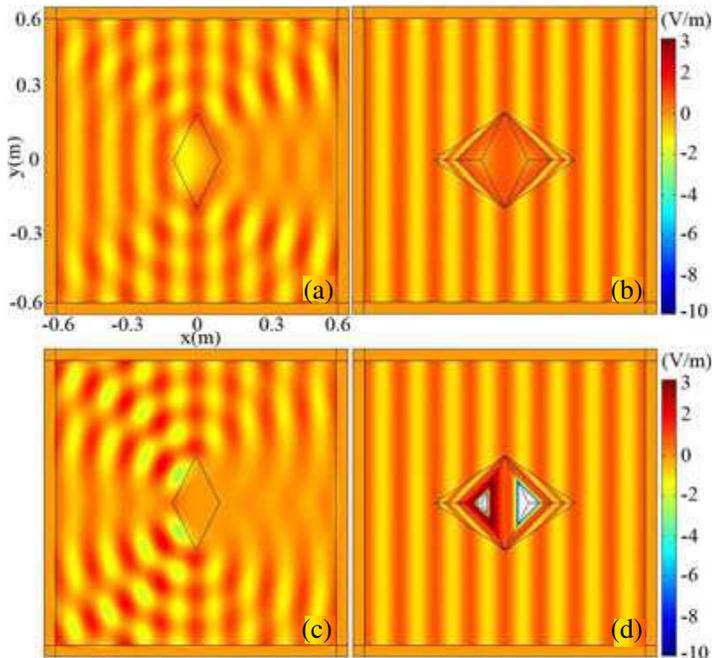
Next, we show that an arbitrary object can be cloaked as long as its complementary layer is “custom-made” according to Equation (1). For example, in Figures 5(a) and (b), we demonstrate the cloaking of an object with linear changing permittivity ( $\varepsilon_o = 0.2 - 0.1x$ ). Figure 5(a) shows the scattering of the object under plane wave irradiation. In Figure 5(b), the object is perfectly hidden when it is covered by the reciprocal cloak. As can be seen, electric field penetrates into the cloaked region. Therefore, the hidden object is not “blind”, and it can “see” the outside world. Besides, any lossy object ( $\varepsilon_o = -1 + 2j$ ) can also be cloaked as shown in Figures 5(c) and (d). In this case,



**Figure 4.** (a) Electric field distribution in the vicinity of the reciprocal cloak. (b) Electric field distribution along  $x$  axis.

according to Equation (1), the complementary media layer should also be lossy.

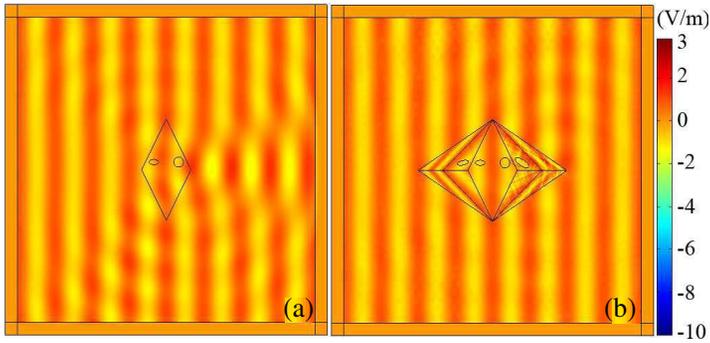
Moreover, any inhomogeneity placed inside the homogeneous filling of the cloak can be cloaked if its “antioject” is located in the complementary layer. Figure 6 shows the simulation results of the reciprocal cloak when two objects are embedded inside the homogeneous cloaked region with  $\varepsilon_o = 1.5$ . The elliptical object on the left side has permittivity of  $\varepsilon_{o1} = 2$ , while the circular object on the right side has linearly changing permittivity of  $\varepsilon_{o2} = 3 - 0.5x + 0.2y$ . In this case, two antiojects with  $\varepsilon'_{o1} = 2\varepsilon'$  and  $\varepsilon'_{o2} = (3 - 0.5x + 0.2y)\varepsilon'$  should be mapped into the complementary layer according to the coordinate transformation of  $x' = (a_1 - b_1)x/a_1 - b_1y/d_1 + b_1$ ,  $y' = y$ ,  $z' = z$ . The shape of the antioject is dependent on the shape and position of the object located in the cloaked region. The perfect recovered wave front of the impinging plane wave shown in Figure 6(b) demonstrates the effectiveness of the reciprocal cloak for cloaking



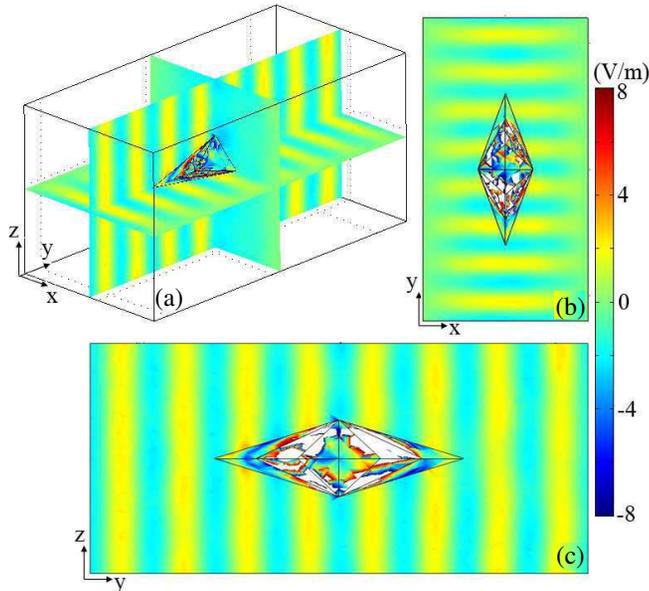
**Figure 5.** Simulation results of the reciprocal cloak under TE plane wave irradiation. (a) Scattering pattern of a rhombic object with linearly changing permittivity ( $\epsilon_o = 0.2 - 0.1x$ ). (b) The object in (a) is hidden by the reciprocal cloak. (c) Scattering pattern of a lossy object ( $\epsilon_o = -1 + 2j$ ). (d) The object in (c) is hidden by the reciprocal cloak.

arbitrary objects.

Figure 7 shows the simulation results of the 3D reciprocal cloak. Permittivity of the object is set to be  $\epsilon_o = 1.5$ . A 2 GHz uniform plane wave with electric field polarized along  $z$  axis is impinging onto the reciprocal cloak along  $y$  direction. It is seen that the electromagnetic wave propagates into the cloaked region and recovers the original status when passing through. Figures 7(b) and (c) show the electric field distribution in the planes of  $z = 0$  and  $x = 0$ , respectively. The results demonstrate the performance of the 3D reciprocal cloak. The slight fluctuation shown in the figure is believed to be induced by the numerical method due to discretization. It can be improved by fine mesh with a cost of memory consumption and computation time.



**Figure 6.** (a) Scattering pattern of the homogeneous region ( $\varepsilon_o = 1.5$ ) with an elliptical object ( $\varepsilon_{o1} = 2$ ) and a circular object ( $\varepsilon_{o2} = 3 - 0.5x + 0.2y$ ) placed inside. (b) The objects in (a) are cloaked by the reciprocal cloak with two antiobjects located in the complementary layer.



**Figure 7.** Simulation results of the three dimensional reciprocal cloak. (a) Three dimensional profile of the electric field ( $Ez$ ) distribution. (b) Electric field distribution in  $z = 0$  plane. (c) Electric field distribution in  $x = 0$  plane.

#### 4. CONCLUSION

In conclusion, a diamond reciprocal cloak is proposed and designed based on coordinate transformation theory. The cloak is constituted of eight blocks of homogeneous materials, and it performs well for a certain polarized wave. Numerical results demonstrate that the cloaked object is able to receive signals from the outside, while its presence is not detectable by electromagnetic waves. Moreover, the performance of the cloak is independent of the shape of the cloaked objects, as long as their antiojects are mapped into the complementary layer. Our results show the possibility to realize the reciprocal cloak in practice.

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