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DIRECTION OF ARRIVAL ESTIMATION FOR NONUNIFORM PLANAR ARRAY BASED ON PIECEWISE INTERPOLATION METHOD

W. S. Elshennawy[1,*], A. M. Attiya[2], E. A. Hashish[3], and I. A. Eshrah[3]

[1]Orange Business Service OBS, Egypt

[2]Electronics Research Institute, El-Tahreer St., Dokki, Giza 12622, Egypt

[3]Electrical Engineering Dept., Faculty of Engineering, Cairo University, Giza, Egypt

Abstract—The problem of direction-of-arrival (DOA) estimation by using spectral search for a non-uniform planar array is addressed. New search methods for DOA estimation based on piecewise interpolation are proposed. The relationships between these methods and Fourier-Domain (FD) root-MUSIC are discussed. The proposed methods are based on dividing the multiple signal classification (MUSIC) null-spectrum function into a number of equal subintervals. These subintervals are interpolated by using low-degree polynomials. Piecewise interpolation methods based on elementary functions are used to reduce the required computations of MUSIC null-spectrum function. This property reduces the computational complexity compared with line-search methods for DOA estimation. The Cramér Rao Lower Bound (CRB) is used as a benchmark to check the accuracy and validity of the proposed methods.

1. INTRODUCTION

Conventional spectral algorithms as MUSIC involve a spectral search step. The computational complexity of this spectral search is not preferred in real-time processing. Extensions of Root-MUSIC algorithm such as interpolated root-MUSIC [1], element-space root-MUSIC [2], manifold separation (MS) [3], FD root-MUSIC [4], and

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* Corresponding author: Wael Sabry Elshennawy (wael.elshennawy@orange.com).

modified polynomial rooting (MPR) [5] are introduced as an alternative search-free algorithm to tackle this problem for nonuniform arrays. In this case polynomial rooting is used instead of spectral domain search step. Fourier weighted least-squares (FWDLS) root-MUSIC uses the weighted least squares to approximate the MUSIC null-spectrum function [6]. This method refines FD-root MUSIC technique by evaluating the improved values of the Fourier series coefficients. Fourier-Domain (FD) line-search MUSIC does not require polynomial rooting but it uses simple line search [1]. This method applies zero-padded inverse Fourier transform to the FD root-MUSIC polynomial step [6].

The degree of FD root-MUSIC polynomial has to be sufficiently high to minimize the truncation errors. However, the computational complexity of root-finding of the polynomial is rather high and it may not be suitable for real time applications [7]. On the other hand, the root-finding DOA estimation methods suffer from that the roots may not fall exactly on the unit circle in the presence of any perturbation like the effect of finite snapshots. If the root-MUSIC function has negative values below zero in some of its minima, a different procedure is used to estimate the signal DOAs from the roots of the negative polynomial [1]. The primary difficulty for root-MUSIC is how to distinguish the signal roots from the extra roots of the MUSIC polynomial. Ideally, it is assumed that the signal roots have larger radii than the extra roots, but this may not hold for low signal-to-noise ratio (SNR). Inside the unit circle, there might be large number of roots but only few of them are the typical signal roots. If an extra root has a radius larger than a signal root, the signal root is missed and it may take the angle of the extra root as the direction for the signal. This may be estimated as a secondary detection problem [8].

To reduce the computational complexity of the FD-root MUSIC approach in these cases, FD line-search MUSIC is used. This FD line-search MUSIC method does not require polynomial rooting step. Thus, it has less computational complexity due to the simple line search procedure. The key idea of this modification follows the work in [8, 9]. However, it has a substantially reduced complexity compared with conventional MUSIC null-spectrum function. It uses a large number of zero padding values to evaluate the inverse discrete Fourier transformation (IDFT) step in order to obtain a comparable DOA estimation performance to those of spectral search-free methods. Thus, it has high complexity order. Also, the performance of FD line-search MUSIC method is worse than that of root-finding-based methods in the case of closely spaced sources because of the threshold effect [1].

This is the motivation here to propose a DOA estimation method based on hybridization between the spectral search-free DOA estimation methods [6] and line-search based DOA estimation method [1]. This hybrid DOA estimation method applies the line-search algorithm to the extended root-MUSIC method in such a way leading to a reduction in computational complexity. The nulls of the resultant spectrum correspond to the signal DOAs. Moreover, it divides the MUSIC null spectrum into several subintervals. The idea of dividing the MUSIC null spectrum is similar to the work in [8] with some differences in the modeling and the method of evaluating the signal DOAs. The response of a non-uniform planar array over each subinterval is modeled with a low-degree polynomial interpolant, permitting a MUSIC null-spectrum to be constructed for each subinterval.

Only the minima with each subinterval are considered as candidate signal DOAs. The line-search method can be applied to each subinterval spectrum in parallel. Thus, this method reduces the computational burden that may render from root-finding technique [9] with a simple line search method. The proposed DOA estimation method can be considered as computationally efficient method for interpolating the MUSIC null-spectrum function compared with the FD line-search MUSIC.

This paper is organized as follows. In Section 2, the signal model is defined, the key assumptions are stated, and a brief overview of the wavefield modeling is introduced. Then, an extension to root-MUSIC function for the case of a non-uniform planar array is derived. In the following three Sections 3, 4, and 5 the proposed DOA estimation methods are introduced by using different piecewise interpolation methods. The CRB of the proposed methods are evaluated in conjunction with the simulation results, and compared with FD line-search MUSIC and FD root-MUSIC methods. Finally Section 9 presents the concluding remarks.

2. BACKGROUND

Assume a non-uniform planar array as shown in Figure 1 composed of M stationary omnidirectional elements located in XY plane. The distance of the mth elements from the origin is r_m, and γ_m is the corresponding angular position measured from the x-axis. P non-coherent narrowband signal sources are incident in the x-y plane (where $P < M$). These sources are impinging on the array from direction $\underline{\phi} = [\phi_1, \phi_2, \ldots, \phi_n, \ldots, \phi_P]^T$, where $[\cdot]^T$ stands for matrix transpose. Furthermore, K-snapshots are observed by the array at time instants t_k, $k = 1, \ldots, K$. The array steering matrix $\underline{\underline{B}}$ is defined in terms of

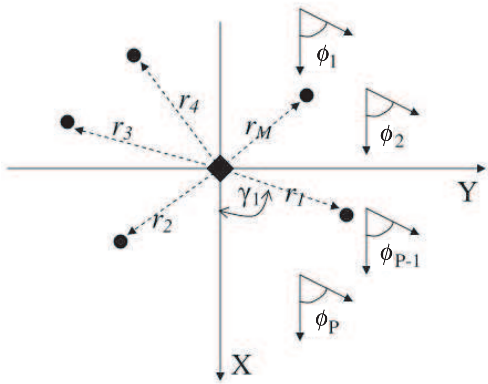

Figure 1. Location of non-uniform planar array formed of M elements which are subjected to P incident plane waves.

the steering vectors $\underline{b}(\phi_n) = [b_1(\phi_n), \ldots, b_M(\phi_n)]^T$ as [10]

$$\underline{\underline{B}}_{M\times P} = [\underline{b}(\phi_1), \underline{b}(\phi_2), \ldots, \underline{b}(\phi_P)]. \tag{1}$$

The array steering vector $\underline{b}(\phi)$ is modeled by manifold separation technique (MST) [11] as the product of a sampling matrix $\underline{\underline{G}}(r_m, \gamma_m)$ (which is independent on the wavefield) and a coefficient vector $\underline{d}(\phi)$ (which is independent on the array) as follows:

$$\underline{b}(\phi) \simeq \underline{\underline{G}}\,\underline{d}(\phi), \quad \text{for} \quad 0 \leq \phi \leq 2\pi, \tag{2}$$

where $\underline{d}(\phi)$ is an $n \times 1$ Vandermonde vector which depends on ϕ and n. The MST can perfectly model the array steering vector $\underline{b}(\phi)$ only by using an infinite number of Fourier coefficients [6]. In practice, the sampling matrix can be truncated by considering only a finite number of excitation modes (n). The parameter n characterizes the accuracy of the approximation in (2) and it becomes exact for $n \to \infty$. The received signal is given by array data matrix $\underline{\underline{U}}$ [12]

$$\underline{\underline{U}}_{M\times K} = \underline{\underline{B}}_{M\times P}\underline{\underline{S}}_{P\times K} + \underline{\underline{N}}_{M\times K}, \tag{3}$$

where $\underline{\underline{U}} = [\underline{u}(1), \ldots, \underline{u}(K)]$ and $\underline{u}(\cdot)$ is expressed as $M\times 1$ array vectors. It should be noted that $\underline{\underline{B}} \times \underline{\underline{S}}$ and $\underline{\underline{N}}$ are defined in the similar way as $\underline{\underline{U}}$. The matrix $\underline{\underline{S}}$ is the signal matrix with rank $(\underline{\underline{S}}\underline{\underline{S}}^H) = P$, where $[\cdot]^H$ is the conjugate transpose, and $\underline{\underline{N}}$ represents the observation noise. The noise is modeled as a stationary second-order ergodic zero-mean variance σ_n^2, with spatially and temporally white circular complex Gaussian process [13].

In practical situations, the exact array covariance matrix $\underline{\underline{R}}_{UU}$ is unavailable, and its sample estimate $\underline{\underline{\widehat{R}}}_{UU}$ is given as [10]:

$$\underline{\underline{\widehat{R}}}_{UU} = \frac{1}{K}\sum_{i=1}^{K}\underline{u}(i)\underline{u}(i)^H. \tag{4}$$

The eigen decomposition of the sample covariance matrix (4) yields two groups of eigenvectors. As, their associated eigenvalues $(\widehat{\lambda}_1 \geq \ldots \geq \widehat{\lambda}_P \geq \widehat{\lambda}_{P+1} \geq \ldots \geq \widehat{\lambda}_M)$ are sorted in non-ascending order, the eigenvectors can be divided into noise $\underline{\underline{\widehat{E}}}_N \triangleq [\widehat{e}_{p+1}, \ldots, \widehat{e}_M]$ and signal $\underline{\underline{\widehat{E}}}_S[\widehat{e}_1, \ldots, \widehat{e}_P]$-subspaces eigenvectors respectively [10, 13]. Thus, the DOA estimation technique uses the noise subspace eigenvectors $\underline{\underline{\widehat{E}}}_N$ in defining a function that introduces an indication of the signal DOA based upon minima versus angle.

2.1. Root-MUSIC Algorithm

Root-MUSIC is a modified version of the MUSIC algorithm where the signal DOAs are determined as the roots of a polynomial formed by noise subspace [14]. Unlike MUSIC method which is applicable to general array configurations, Root-MUSIC is restricted to uniform linear arrays. From the orthogonality condition existing between the noise subspace eigenvectors $\underline{\underline{\widehat{E}}}_N$ and array steering vectors $\underline{b}(\phi_n)$ at the signal DOAs, the Euclidean distance expression $d^2 = |\underline{b}(\phi)^H \underline{\underline{\widehat{E}}}_N \underline{\underline{\widehat{E}}}_N^H \underline{b}(\phi)| = 0$ satisfies this equality at each DOA [10]. By defining $D(\phi)$ as follows:

$$\begin{aligned} D(\phi) &= \left|\underline{b}(\phi)^H \underline{\underline{C}}\, \underline{b}(\phi)\right| = \sum_{n=1}^{M}\sum_{m=1}^{M} e^{-jkr_n\cos(\gamma_n-\phi)} C_{nm} e^{jkr_m\cos(\gamma_m-\phi)} \\ &= \sum_{l=-M+1}^{M-1} c_l e^{jkr_l\cos(\gamma_l-\phi)}, \end{aligned} \tag{5}$$

where $\underline{\underline{C}} \triangleq \underline{\underline{\widehat{E}}}_N \underline{\underline{\widehat{E}}}_N^H$ and c_l is the sum of the diagonal elements of $\underline{\underline{C}}$ along the lth diagonal such that $c_l = \sum_{m-n=l} C_{nm}$. It should be noted that the sharp minima of (5) represent the signal DOAs, which is evaluated by searching over ϕ with a fine grid. However, this method suffers from substantially-higher computational complexity associated with evaluating large number of spectral points and locating the minima of the MUSIC spectral domain. The computational complexity of the spectral search step of MUSIC method, (having order of $O(JMP)$), is typically higher than that of the eigen decomposition step, (of order $O(M^3)$), because $M \ll J$, where J is the total number of spectral points. For each spectral point, the product of $\underline{\underline{\widehat{E}}}_N$ and $\underline{b}(\phi_n)$ has to be computed.

By using MST [11], Equation (2) can be written as:

$$\underline{b}(z) \simeq \underline{\underline{G}}\,\underline{d}(z), \quad \text{for} \quad 0 \leq z \leq 1, \tag{6}$$

where $z = \exp(-ji\phi)$; $i = -n, \ldots, 0, \ldots, n$. Consequently, Equation (5) can be expressed in terms of $\underline{b}(z)$ [15].

$$D(\phi) \simeq \left|\underline{d}(\phi)^H \underline{\underline{G}}^H \underline{\underline{C}}\,\underline{\underline{G}}\,\underline{d}(\phi)\right| = \left|\underline{d}(1/z)\,\underbrace{\underline{\underline{G}}^H \underline{\underline{C}}\,\underline{\underline{G}}}_{\triangleq \underline{\underline{C'}}}\,\underline{d}(z)\right| \triangleq D(z). \tag{7}$$

Here, $D(z)$ is a polynomial of degree $2n-2$ whose roots appear in conjugate reciprocal pairs such that if z_0 is a root of $D(z)$ then $1/z_0^*$ is another root, where $(\cdot)^*$ is the complex conjugate [1, 4].

$$\begin{aligned} D(z) &= \sum_{l=-n+1}^{n-1} c_l' z^l \simeq \prod_{l=1}^{n} (1 - z_0 z^{-1})(1 - z_0^* z) \\ &= d_o + d_1 z^{-1} + d_2 z^{-2} + \ldots + d_{n-1} z^{-(n-1)}, \end{aligned} \tag{8}$$

where the coefficient c_l' is the sum of elements of $\underline{\underline{C'}}$ along the lth diagonal. The methods which are based on root-finding require estimating number of roots equal to the degree of $D(z)$. This $D(z)$ is composed of the vectors $d = [d_0, d_1, \ldots, d_{n-1}]^T$ which lie in the noise subspace. These roots correspond to the actual incident signals, and the other roots which do not lie on the unit circle do not correspond to the signals. These roots are known as spurious roots. Thus, the DOA signal estimation problem is reduced to finding roots of a polynomial as opposed to merely plotting the MUSIC null-spectrum or searching for peaks in the MUSIC null-spectrum.

The error in locating the correct roots locations are due to many reasons. Among these reasons are the facts that the incoming signals are partially correlated, the array covariance matrix is approximated by time averaging, and SNR is relatively low. This is the reason why it has to exert care in exercising the use of root-MUSIC by knowing the assumptions and conditions under which the calculations are made. It is reported in many studies that Root-MUSIC shows better performance than MUSIC especially in environments where the signals are located closer and/or they have low SNR [16]. In the following sections, (7) is represented by using piecewise interpolation methods into a low-degree polynomial interpolant of elementary functions. Thus, it reduces the computational burden. It can be noted that the only difference with those of root-finding based methods is the way the polynomial $D(z)$ is formulated and the evaluation of signal DOAs by using simple line-search method rather than root-finding technique.

2.2. Piecewise Interpolation Methods

Equation (7) can be efficiently approximated by using piecewise interpolation method into an equivalent polynomial functions that takes the form of a series of elementary functions. Therefore it provides simplicity in evaluating the signal DOAs of low-degree polynomials by searching its corresponding domain for minima (encircled with dashed lines) using simple line-search based method as shown in Figure 2. Assume that $D(\phi)$ is sampled on the interval $0 \leq \phi \leq 2\pi$. The domain is then divided into h-equal distant subintervals such that each subinterval contains q data samples. Thus, the total number of samples $J = h \cdot q$, and the samples of the mth subinterval $D_m(\phi)$ start with φ_1 and end with φ_q with a subinterval width $T_\phi = q\Delta\phi$, and $\Delta\phi = 2\pi/(J-1)$.

Obviously, the accurate interpolation within any of these subintervals does not require interpolant of high degrees. Moreover, the edge spectral point of the subinterval is multiplied by a Hann function which has a zero at the end of the small interval ΔT_ϕ with p-additional data samples, where the Hann function is scaled so that its peak value equals to the edge spectral point value.

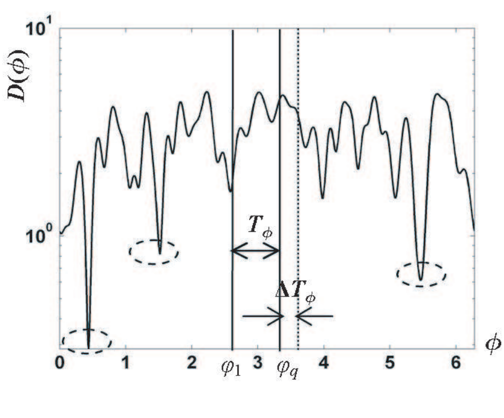

Figure 2. Spatial spectrum of $D(\phi)$ versus ϕ.

Clearly, this modified piecewise interpolation method does not require the evaluation of different kinds of derivatives. Moreover, it has the effect of suppressing the ripples due to Gibbs phenomenon [17]. As the subinterval width is lengthened by ΔT_ϕ, this results in more resolution of the resultant domain of the interpolant. The polynomial interpolants are then invoked in order to evaluate the signal DOAs by searching the whole domain which consists of a w data samples by using simple line-search method. Recalling that the piecewise polynomial interpolation has better performance in regards to continuous polynomial interpolation [18] and introduces an additional degree of freedom in choosing the polynomial interpolating functions.

This is further exploited to obtain smooth polynomial interpolant despite its piecewise nature. Therefore, it overcomes the practical and theoretical difficulties associated with high-degree polynomial interpolant and avoids round off errors. The Gibbs phenomenon appears in piecewise interpolation methods as a result of an accuracy reduction to the first order away from the edge spectral points, and causes oscillations in the neighborhoods of jumps. Gibbs phenomenon [19] occurs for almost all series expansions. This phenomenon does not occur only in trigonometric Fourier series, but also, it exists in other classical orthogonal series such as spline expansions and wavelet series [17]. The proposed piecewise interpolation methods have to fulfill the lowest approximation errors throughout all possible values within the consecutive subintervals. The differentiability property of the polynomial interpolant should be also preserved. The subsequent section addresses a brief review for three piecewise interpolation methods in line with the proposed DOA signal estimation method.

3. PIECEWISE TRIGONOMETRIC INTERPOLATION (PTI) LINE-SEARCH MUSIC

Piecewise trigonometric interpolation line-search MUSIC method is an alternative to root-MUSIC type approach to DOA estimation method which can be used for non-uniform planar array. In this method, the $D_m(\phi)$ spectrum is expressed as trigonometric series expansions by using trigonometric piecewise interpolation method discussed in [20] as:

$$D_m(\phi) \simeq \sum_{l=0}^{t/2} \delta_l \cos(2\pi l\phi/(T_\phi + \Delta T_\phi)) + \beta_l \sin(2\pi l\phi/(T_\phi + \Delta T_\phi)) \triangleq f_1(\phi), \tag{9}$$

where the complex coefficients δ_l and β_l are independent and normally distributed. Each of these coefficients has its mean values $(\mu_{\delta_l},\ \mu_{\beta_l})$, and variances $(\sigma_{\delta_l},\ \sigma_{\beta_l})$ respectively. These coefficients are evaluated by solving a linear system of equations, which is obtained by equating spectral data samples of (7) to a matrix system of (9). The resultant matrix of coefficients has a size $t \times t$ and is non-singular and diagonal. Hence this interpolation method is well defined [20]. Moreover, the computational complexity associated with the solution of this system involves $O(t^2)$ operation. The expected number of signal DOAs per subinterval is given by [21]:

$$EN_F = 2\sqrt{\sum_{l=0}^{t/2} l^2 \left\{\sigma_{\delta_l}^2 + \sigma_{\beta_l}^2\right\} \Big/ \sum_{l=0}^{t/2} \left\{\sigma_{\delta_l}^2 + \sigma_{\beta_l}^2\right\}}. \tag{10}$$

The exponential form of (9) is related to that of root-MUSIC representation (5) where both are similar with the difference of interpolant degree:

$$F_1(\phi) = \sum_{l=-t/2}^{l=t/2} \alpha_l \exp(j2\pi l\phi/(T_\phi + \Delta T_\phi)). \tag{11}$$

The following formulas define the transformation between the two representation of (9) and (11); $\alpha_0 = \delta_0$, $\alpha_l = (\delta_l - j\beta_l)/2$, and $\alpha_l = (\delta_l + j\beta_l)/2$, thus the polynomials are Hermitian. It can be noted that the sum and the product of symmetric (Hermitian) polynomials are also symmetric (Hermitian) polynomials which means that the polynomials are rings [22]. Equation (11) can be written as follows:

$$f_1(z') = \sum_{\ell=-t/2}^{t/2} \alpha_\ell z'^\ell, \tag{12}$$

where z' denotes $\exp(j2\pi\phi/(T_\phi + \Delta T_\phi))$ and it follows the so-called conjugate reciprocity property, i.e., $z'_0 = (1/z'_0)^*$. In the absence of noise (12) has zeros which precisely lie on the unit circle and their angles are functions of the signal DOAs in similar way to the root-MUSIC method. Clearly, the degree of (8) is usually higher than that of (12). Thus, it results in lowering the computation burden. As the structures of (8) and (12) are related, it is expected the result of asymptomatic performance analysis [16] of (8) can be used as well for this method. However, the signals DOAs are evaluated by simple line search method.

4. PIECEWISE PRONY INTERPOLATION (PPI) LINE-SEARCH MUSIC

This piecewise interpolation method is devoted to approximating the subintervals $D_m(\phi)$ of (7) by using direct Prony method [23]. Interpolating $F_2(\phi)$ by using $q+p$ equidistant data samples within the domain $T_\phi + \Delta T_\phi$ using PPI method leads to $F_2'(\phi)$, where $F_2(\phi)$ is expressed as

$$F_2(\phi_l) = \left\{ \begin{array}{ll} D_m(\phi_l); & l = 1, \ldots, q \\ w(l); & l = q+1, \ldots, q+p \end{array} \right\}, \quad w(l) = D_m(\phi_q) \cdot \sin^2(l\pi/p - 1). \tag{13}$$

The polynomial interpolant $F_2'(\phi)$ is approximated by t exponential functions [23] which equals to $F_2(\phi)$ at $\phi_l = \phi_1, \ldots, \phi_{q+p}$ as

$$F_2(\phi_l) \simeq \sum_{i=1}^{t} A_i e^{(\alpha_i + j\omega_i)\cdot(l-1)(T_\phi + \Delta T_\phi) + j\psi_i} \triangleq F_2'(\phi_l) = \sum_{i=1}^{t} h_i g_i''^{\,l-1}, \tag{14}$$

where $h_i = A_i e^{j\psi_i}$, and $g_i'' = e^{(\alpha_i + j\omega_i)(T_\phi + \Delta T_\phi)}$, provided that $(T_\phi + \Delta T_\phi)$, A_i, α_i, ω_i, ψ_i are the sampling period, amplitude, damping factor, angular velocity, and initial phase respectively. Prony method is a technique for extracting sinusoid or exponential signals from data series by solving a set of linear equations [24]. The estimation problem is based on the minimizing the squared error δ over the $q+p$-data values as follows [23]:

$$\delta = \sum_{l=1}^{q+p} |\varepsilon[l]|^2, \quad \varepsilon[l] = F_2(\phi_l) - \sum_{i=1}^{t} h_i g_i''^{\,l-1}. \tag{15}$$

Let us consider the linear prediction polynomial $F_2'(z'')$ [24] that has the roots g_i'', then the expression for Prony interpolant can be expressed as:

$$F_2'(z'') \triangleq \prod_{i=1}^{t} (z'' - g_i''^{\,-1}). \tag{16}$$

It should be noted here that the definition of (16) is similar to that which has been used in (8) but they are different in the degree of the polynomials and the locations of the roots. In this example, the number of data samples equals twice the polynomial interpolant order, i.e., $q+p=2t$, thus the basic Prony method is used [24]. Prony method is classified as a high-resolution spectrum estimation method. The accuracy of the signals DOAs estimation depends on the SNR, the number of subintervals (h), and the number of data samples ($q+p$).

5. PIECEWISE CHEBYSHEV INTERPOLATION (PCI) LINE-SEARCH MUSIC

The subinterval $D_m(\phi)$ is represented by using the discrete Chebyshev expansions. This piecewise interpolation method implies the use of non equidistant sampling points that satisfies the uniform convergence condition, efficient, and stable interpolation method [25]. It involves calculation of the Chebyshev nodes ϕ_k, which are defined as Cheybshev-Gauss-Lobatto (CGL) [25];

$$\phi_l = \cos((l-0.5)\pi/(q+1)); \quad l = 0, \ldots, q-1. \tag{17}$$

The extrema of the Chebyshev polynomials are not evenly distributed and clustered around the subinterval edges. Thus it is not necessary to use Hann function for this DOA estimation method. This piecewise interpolation method benefits from the advantages of Chebyshev polynomials, i.e., the least deviation from zero property, and the recurrence relationship for efficient computation [26]. In this case, the Chebyshev polynomials of the first kind are used to approximate the subintervals of the null spectrum, and the series expansion takes the form of truncated Chebyshev expansion at degree t as:

$$D_m(\phi) \simeq \sum_{i=0}^{t-1} c_i'' T_i(\phi) \triangleq f_3^t(\phi) = \sum_{i=0}^{t-1} d_i z'''^i \triangleq f_3(z'''), \tag{18}$$

where d_i are the coefficients of the $f_3(z''')$, and $T_i(\phi)$ and c_i'' are first kind Chebyshev polynomials and their associated coefficients, respectively. The coefficients are efficiently evaluated by using fast cosine transform (FCT), and their computations are of order $O(t \log_2 t)$ operation. Note that $f_3(z''')$ in (18) has a polynomial of degree $(t-1)$ which is lower than that of $D(z)$ in (8). The Chebyshev approximation error can be defined as follows [25]:

$$|f_3^t(\phi) - D_m(\phi)| \leq \frac{2(\varphi_q - \varphi_1)^{t+1}}{4^{t+1}(t+1)!} \max_{\varphi_1 \leq \phi \leq \varphi_q} |f_3^{t+1}(\phi)|. \tag{19}$$

Equation (19) is used to determine the degree of polynomial function (t) that ensures the interpolation method can perfectly approximate the subintervals $D_m(\phi)$.

6. CRAMÉR-RAO BOUND

The analysis of ultimate performance of an unbiased estimator can be obtained by using the assumptions of unconditional data signal model, large number of snapshots (K), and Gaussian distribution of noise with zero mean. This is generally conducted by using the CRB. Assume that $\theta = [\phi^T \rho^T \sigma_n]^T$ denotes the unknown parameter vector, where ρ is the $P^2 \times 1$-vector made from signal covariance matrix $\widehat{\underline{\underline{R}}}_{ss}$. Under the assumptions of data signal model discussed in Section 2 and the Gaussian hypothesis, the Fisher information matrix (FIM) for the parameter vector θ is given by [27]:

$$\begin{aligned}\mathrm{FIM}_{i,l} &= K \cdot \mathrm{Tr}\left(\frac{d\underline{\underline{R}}_{UU}}{d\theta_i}\underline{\underline{R}}_{UU}^{-1}\frac{d\underline{\underline{R}}_{UU}}{d\theta_l}\underline{\underline{R}}_{UU}^{-1}\right), \\ &\quad \text{for} \quad i,l = 1,\ldots,P^2+P+1,\end{aligned} \tag{20}$$

where $\mathrm{Tr}(\cdot)$ stands for trace. Both ρ and σ_n are nuisance parameters. The parameter of interest in this study is only in ϕ — block of CRB $= \mathrm{FIM}^{-1}$, which it is denoted by CRB (ϕ) on the signal covariance matrix of an unbiased estimate of ϕ and is given by [28];

$$\begin{aligned}&\mathrm{CRB}(\phi) = \frac{\sigma_n}{2K}\left\{\mathrm{Re}(\underline{\underline{Y}}^H \underline{\underline{\Pi}}_B^{\perp} \underline{\underline{Y}}) \odot (\underline{\underline{R}}_{SS}\underline{\underline{B}}^H \underline{\underline{R}}_{UU}^{-1}\underline{\underline{B}}\underline{\underline{R}}_{SS})^T\right\}^{-1}, \\ &\underline{\underline{Y}}_{M\times P} = [\underline{y}_1,\ldots,\underline{y}_P]; \quad \underline{y}_i = (d\underline{b}(\phi_i)/d\phi_i), \\ &\quad \underline{\underline{\Pi}}_B^{\perp} = I - \underline{\underline{\Pi}}_B; \quad \underline{\underline{\Pi}}_B = \underline{\underline{B}}(\underline{\underline{B}}^H\underline{\underline{B}})^{-1}\underline{\underline{B}}^H,\end{aligned} \tag{21}$$

where $\odot$ is the Hadamard-Schur product. The diagonal elements of $\mathrm{CRB}(\phi)$ represent the variances of estimation errors. It should be noted that the $[\mathrm{CRB}(\phi)_{ii}]$ values are dependent on K, M, σ_n, $\widehat{\underline{\underline{R}}}_{ss}$ and SNR values, and become asymptotically equal to the exact CRB for large K, and M values [29].

7. COMPUTATIONAL COMPLEXITY ANALYSIS

The complexity orders of the PTI, PPI, and PCI line-search MUSIC methods are presented in Table 1 compared with those of the FD line-search MUSIC and FD root-MUSIC methods. It can be noted that all of the methods in Table 1 include the eigen decomposition step which has a computational complexity of order $O(M^3)$, the computation of J samples of the MUSIC null spectrum requires computational complexity of order $O(JMP)$, and the complexity to compute the spectrum domain of the polynomial interpolant is of order $O(wt)$.

It is clear from Table 1 that the three-proposed DOA estimation methods, i.e., PTI, PPI, PCI line-search MUSIC methods have

Table 1. The orders of computational complexities for the proposed DOA estimation methods.

Algorithm	**Computational Complexity**	**Order level**
PCI line-search MUSIC	$O(M^3 + JMP + h \cdot (t \log_2 t) + wt)$	10^4
PTI line-search MUSIC	$O(M^3 + JMP + ht^2 + wt)$	10^4
PPI line-search MUSIC	$O(M^3 + JMP + 2h \cdot (t^2 + t^3) + wt)$	10^3
FD Line-Search MUSIC [1]	$O(M^3 + nMP + J \log_2 J)$	10^7
FD root-MUSIC [1]	$O(M^3 + nMP + n \log_2 n + \text{degree-}n\,\text{rooting})$	10^3

Table 2. Non-uniform planar array elements locations within XY-plane.

Element number	**1**	**2**	**3**	**4**	**5**
X/λ	-0.83	-0.16	0.17	0.22	0.81
Y/λ	-0.15	-0.11	0.31	0.63	-0.62

computational complexity levels of much lower orders than the FD line-search MUSIC and comparable to that of the FD root-MUSIC.

8. SIMULATION RESULTS

In this section, the performances of the proposed DOA estimation methods are presented, and illustrative comparisons between the performances of the three methods are given for the array of omni-directional elements as shown in Figure 1. The XY locations of the non-uniform planar array in terms of the operating wavelength are given in Table 2. The array is illuminated by two uncorrelated sources of equally power levels. In the first and second examples, the DOA estimation performances of the PCI, PPI, and PTI line-search MUSIC are tested versus CRB and compared to that of FD line-search MUSIC and FD root-MUSIC respectively in the case of $n = 19$. The incident fields are assumed to impinge from the direction angles $\phi_1 = 20°$ and $\phi_2 = -10°$ for the case of first example, and from the direction $\phi_1 = 15°$ and $\phi_2 = 20°$ for the second and third examples respectively in the azimuth plane $(-\pi < \phi < \pi)$. Throughout the simulations, 1000-

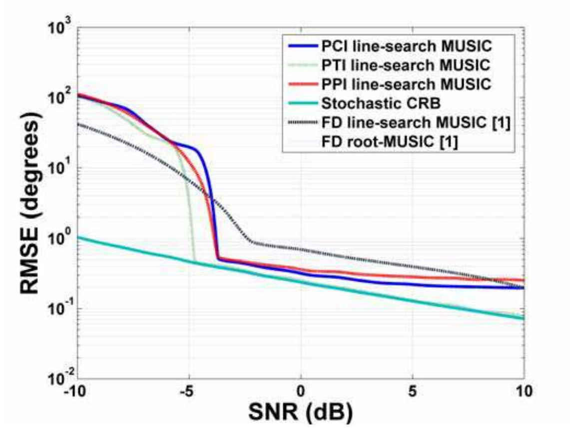

Figure 3. DOA estimation RMSEs versus SNR for $K = 100$, $\phi_1 = 20°$, $\phi_2 = -10°$, $n = 19$. The array of Figure 1 is used, FD line-search MUSIC and FD root-MUSIC are overlapped.

independent Monte Carlo runs have been conducted in each example. The number of snapshots used to estimate the covariance matrices are $K = 100$.

For a diagonally-signal covariance matrix $\underline{\underline{R}}_{SS}$, Figure 3 shows that the DOA estimation RMSEs of the proposed methods are tested versus the SNR. The proposed DOA estimation methods asymptomatically exhibit the same performance as the stochastic CRB for SNR > -4 dB. Thus, these methods are considered statistically efficient for SNR > -4 dB. Note that in this figure, the PTI line-search MUSIC and CRB curve are nearly equal above -4 dB (i.e., they merge into one curve). The performances of PTI, PCI, and PPI line-search MUSIC, and FD line-search MUSIC and FD root-MUSIC are compared for the case of $n = 19$ as shown in Figure 3. It can be observed that the proposed DOA estimation methods substantially outperform the FD line-search MUSIC and FD root-MUSIC for SNR $>$ -4 dB and close to both of them for SNR < -4 dB with preference to the proposed DOA estimation methods in achieving low computations compared to FD line-search MUSIC method and nearly equal to that of FD-root-MUSIC. The performance of PTI line-search is the highest among the proposed DOA estimation methods, and it has analytic performance asymptomatically to that of FD root-MUSIC. The performance of PPI line-search MUSIC is close to that of PCI line-search MUSIC except for -6 dB $<$ SNR < -4 dB, with the exception that the RMSEs of PCI line-search MUSIC shows faster decrease rate for SNR > 2 dB. This asserts that the choice of the

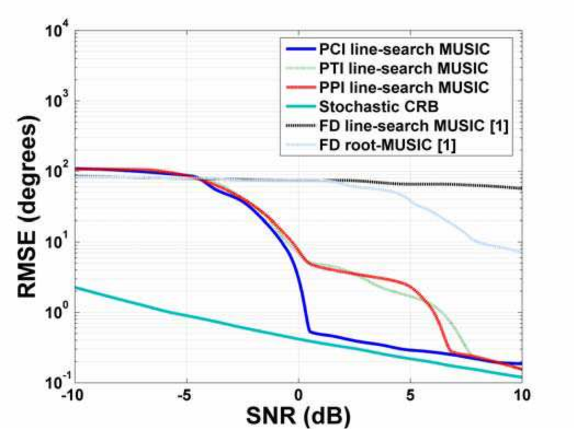

Figure 4. DOA estimation RMSEs versus SNR for $K = 100$, $\phi_1 = 15°$, $\phi_2 = 20°$, $n = 19$. The array of Figure 1 is used.

subinterval representation model plays important role in controlling the performance of the DOA estimation method.

In the second example, the performances of the proposed methods are examined for the case of closely spaced signal sources. It is assumed that two sources impinge on the array used in Figure 1 from the directions $\phi_1 = 15°$ and $\phi_2 = 20°$ in the azimuth plane. All other parameters are same as the first example.

Figure 4 displays the DOA estimation RMSEs of the proposed methods versus the SNR compared to the FD line-search MUSIC, and FD root-MUSIC [1]. For SNR $> -4.5\,\text{dB}$, the performance of PCI line-search MUSIC is better than those of the PTI and PPI line search MUSIC in the case of closely spaced signal sources. This property has an important role in achieving pronounced performance improvements at relatively low SNR and closely angular spacing between sources. From the shown figure, it is clear that the performance of the line-search based methods can be enhanced by using proper subinterval basis functions such as the trigonometric, Prony, and Chebyshev polynomials. It is shown also the PCI line-search MUSIC has the best performance than others.

In Figure 5, the impact of the number of snapshots K on the proposed methods compared to those of the FD root-MUSIC and FD line-search MUSIC is shown. The performance of the proposed methods outperforms those of FD root-MUSIC and FD line-search MUSIC for $K > 10$ and SNR $= 20\,\text{dB}$. It can be noted that the behavior of the RMSEs in finding the signal DOAs versus K is similar to those curves of signal DOA estimation RMSEs versus SNR values.

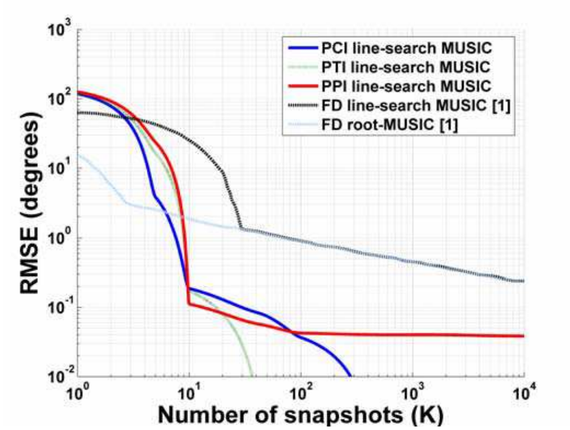

Figure 5. DOA estimation RMSEs versus SNR for $\phi_1 = 15°$, $\phi_2 = 20°$, $n = 19$, SNR $= 20\,\text{dB}$. The array of Figure 1 is used.

9. CONCLUSIONS

The signal DOAs are estimated by using hybridization between spectral search-free methods and line-search based DOA estimation method. It is shown that the proposed line-search based methods, i.e., PCI, PPI, PTI line-search MUSIC, have better performance than FD root-MUSIC and FD line-search MUSIC especially for relatively low SNR values in terms of complexity order and performance. An important property of these methods is that they have computationally efficient implementation because they are based on piecewise interpolation method which can be calculated in a parallel scheme. The PTI line-search MUSIC exhibits the same analytic performance as the FD root-MUSIC. The proposed DOA estimation methods offer attractive alternative to the existing DOA estimation line-search based methods like FD line-search MUSIC. On the other hand, the proposed methods are more accurate at relatively low SNR and faster when implemented using parallel processors. For DOA signals estimation of closely angular separation between sources, the PCI line-search MUSIC outperforms other proposed methods as well as the FD root-MUSIC and FD line-search MUSIC at relatively low SNR, due to the appropriate choice of interpolant representation model.

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