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AN EFFECTIVE WIDEBAND SPECTRUM SENSING METHOD BASED ON SPARSE SIGNAL RECONSTRUCTION FOR COGNITIVE RADIO NETWORKS

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Abstract—Wideband spectrum sensing is an essential functionality for cognitive radio networks. It enables cognitive radios to detect spectral holes over a wideband channel and to opportunistically use under-utilized frequency bands without causing harmful interference to primary networks. However, most of the work on wideband spectrum sensing presented in the literature employ the Nyquist sampling which requires very high sampling rates and acquisition costs. In this paper, a new wideband spectrum sensing algorithm based on compressed sensing theory is presented. The proposed method gives an effective sparse signal representation method for the wideband spectrum sensing problem. Thus, the presented method can effectively detect all spectral holes by finding the sparse coefficients. At the same time, the signal sampling rate and acquisition costs can be substantially reduced by using the compressive sampling technique. Simulation results testify the effectiveness of the proposed approach even in low signal-to-noise (SNR) cases.

1. INTRODUCTION

Due to the rapid development of wireless communication, the need for an efficient utilization of wireless spectrum is growing. Recent

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measurements witness that most of the assigned frequency bands are unused and available for other possible applications [1]. Therefore, cognitive radio (CR) technology has been proposed as a promising technique to improve the spectrum utilization efficiency [2, 3]. CR networks offer improved service quality, allowing a secondary user (SU) to search and exploit the spectrum opportunities without causing interference to the primary user (PU). Spectrum sensing is a key enabling functionality of CR networks, monitoring the PU activities and detecting spectrum holes for dynamic spectrum access.

Wideband CR networks has received significant attention recently. In wideband CR networks, on one hand, secondary users attain more transmission opportunities in the wide frequency range, in which primary users co-exist. On the other hand, the task of wideband spectrum sensing remains an open challenge due to wideband spectrum acquisition implementation. Various wideband spectrum sensing methods have been proposed in the literature [4–7]. An early scheme for wideband spectrum sensing exploits a narrowband bandpass filter on the secondary user radio frequency (RF) front-end which is tuned to a specific band in order to sequentially sense one channel at a time [4]. A novel multiband joint detection (MJD) framework for wideband sensing is proposed in [5]. The authors formulate the spectrum sensing problem as a class of optimization problems, which maximize the aggregated opportunistic throughput of a cognitive radio system under some constraints on the interference to the PUs. An efficient implementation of the MJD framework is given in [6]. Taking the spectrum sensing time into consideration, Paysarvi-Hoseini and Beaulieu propose a multiband sensing-time-adaptive joint detection (MSJD) framework proposed in [7]. The MSJD framework can dynamically balance the quality and speed of sensing through an adaptive selection of the sensing time, which is assumed to be fixed in MJD.

Typically, only a small fraction of the wideband spectrum is occupied by primary users, i.e., the signal is sparse in the frequency domain. The compressed sensing (CS) [8, 9] theory provides a novel method to sense sparse or compressible signals efficiently using a sub-Nyquist sampling rate. Several frameworks [10–13] based on the CS theory are proposed for the wideband spectrum sensing problem. In [10], a compressed sensing algorithm is derived for the coarse sensing task of spectrum band classification. Sub-Nyquist rate samples are utilized to detect and classify frequency bands via a wavelet-based edge detector. A mixed-signal parallel segmented compressive sensing architecture is introduced in [11] for wideband spectrum sensing. In the proposed method, high analog-to-digital conversion is avoided by carrying out an analog basis expansion in parallel before sampling. The wideband spectrum sensing reliability may be compromised due to the wireless channel fluctuations and fading effects. It is possible to improve the sensing accuracy through cooperative approaches significantly. In [12], a cooperative approach to wideband spectrum sensing is developed. To collect spatial diversity against wireless fading, multiple CRs collaborate during the sensing task by enforcing consensus among local spectral estimates. Another framework of cooperative spectrum sensing is presented in [13]. It exploits the belief propagation algorithm to effectively fuse the prior information from neighboring nodes for local spectrum sensing.

This paper presents a new wideband compressed spectrum sensing algorithm that permeates the benefits of sparse signal representation and compressive sampling. With the help of sparse signal representation, the wideband channel occupancy state can be detected through the reconstruction of received signal amplitude vector. The use of compressive sampling technique can achieve reduced signal sampling rate and acquisition costs. The rest of the paper is organized as follows. The wideband spectrum sensing model is described in Section 2. In Section 3, we introduced our wideband compressed spectrum sensing framework. Section 4 shows some simulation results. Section 5 provides a concluding remark to summarize the paper.

2. DATA MODEL

Suppose that a total of B Hz in the frequency range $[f_L,\ f_H]$ is available for a wideband CR network. The entire wideband channel can be divided into K non-overlapping narrowband subbands. In a particular geographical region and within a particular time interval, some of the K subbands might not be used by the primary users and are available for opportunistic spectrum access. We model the detection problem on subband k as one of choices between a hypothesis "0", which represents the absence of primary signals, and an alternative hypothesis "1", which represents the presence of primary signals. An example where only some of the subbands are occupied by primary users is depicted in Fig. 1 [5]. The underlying hypothesis vector is a binary representation of the subbands that are allowed for or prohibited from opportunistic spectrum access.

Suppose that Q ($Q \ll K$) subbands are occupied by primary users. For simplicity, all cognitive radios are assumed to stay silent during the detection interval. Then the only spectral power remaining in the air is emitted by the primary users. Thus, the received signal of a secondary

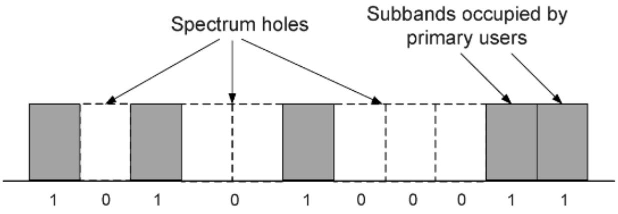

Figure 1. Schematic illustration of the occupancy of a wideband channel.

user can be given by [14]

$$s(t) = \sum_{q=1}^{Q} a_q s_q(t) e^{j2\pi f_q t} + w(t)$$

where t is the time index. a_q stands for the amplitude of the qth primary signal. $s_q(t)$ is the baseband representation. f_q represents the carrier frequency of the qth primary signal and is the center frequency of one of the occupied subbands. The item $w(t)$ is the complex additive white Gaussian noise.

Assume that f_s is the sampling rate, which is much higher than the data rate of each source. The data sample at the CR receiver is

$$s\left(\frac{n}{f_s}\right) = \sum_{q=1}^{Q} a_q s_q\left(\frac{n}{f_s}\right) e^{j(2\pi/f_s) f_q n} + w\left(\frac{n}{f_s}\right), \ (n = 1, \dots, N) \quad (1)$$

where N is the number of samples. The main task of wideband spectrum sensing is to detect channel occupancy state based on the received data sample.

3. WIDEBAND COMPRESSED SPECTRUM SENSING FRAMEWORK

3.1. Overcomplete Representation

In matrix form, (1) can be written as

$$\mathbf{s} = \mathbf{B}\mathbf{a} + \mathbf{w} \quad (2)$$

where $\mathbf{s} = [s(\frac{1}{f_s}), \dots, s(\frac{N}{f_s}))]^T$, is the N-sample received vector. $\mathbf{B} = [\mathbf{b}_1, \dots, \mathbf{b}_Q]$, with $\mathbf{b}_q = [s_q(\frac{1}{f_s})e^{j(2\pi/f_s)f_q}, \dots, s_q(\frac{N}{f_s})e^{j(2\pi/f_s)f_q N}]^T$ for $q = 1, \dots, Q$, which includes the frequency information of the incident received signal. $\mathbf{a} = [a_1, \ldots, a_Q]^T$ is the signal amplitude vector, $\mathbf{w} = [w(\frac{1}{f_s}), \ldots, w(\frac{N}{f_s})]^T$ represents the complex additive white Gaussian noise vector, and the superscripts $(\cdot)^T$ denotes the transpose operation.

To cast the wideband spectrum sensing problem as a sparse representation problem, we introduce an overcomplete representation $\mathbf{\Psi}$ in terms of all possible channel occupancy states. Let $\{\hat{f}_1, \ldots, \hat{f}_K\}$ be a sampling grid of the center frequencies of K non-overlapping narrowband subbands. If the ith ($i \in \{1, \ldots, K\}$) subband is occupied, $\hat{f}_i$ would be contained in the frequency components of the received signal. We construct an $N \times K$ matrix composed of steering vectors corresponding to the center frequency of each subband as its columns

$$\mathbf{\Psi} = [\mathbf{b}(\hat{f}_1), \ldots, \mathbf{b}(\hat{f}_K)] \tag{3}$$

where $\mathbf{b}(\hat{f}_i) = [s_q(\frac{1}{f_s})e^{j(2\pi/f_s)\hat{f}_i}, \ldots, s_q(\frac{N}{f_s})e^{j(2\pi/f_s)\hat{f}_i N}]^T$, $(i = 1, \ldots, K)$. In this framework $\mathbf{\Psi}$ is known and does not depend on the actual channel occupancy state. We represent the signal amplitude by a $K \times 1$ vector $\mathbf{x}$, where the element $\mathbf{x}_i$ is nonzero and equals to a_q if the qth subband is occupied for some q and zero otherwise. Thus, the signal model can be reduced to

$$\mathbf{s} = \mathbf{\Psi}\mathbf{x} + \mathbf{w}.$$

Because the number of occupied subbands is small, the unknown amplitude vector $\mathbf{x}$ is sparse, with only small number of nonzero elements. Its sparsity can be measured by the ℓ norm $\|\mathbf{x}\|_\ell$, for $\ell \in [0, 2)$, where $\ell = 0$ yields the exact sparsity order.

3.2. Compressive Sampling

According to the CS theory, the sparse signal can be reconstructed successfully with high probability from M measurements, usually $M \ll N$ [15]. To this end, we adopt an $M \times N$ compressive sampling matrix $\mathbf{\Phi}$ at the receive transmitter to collect a compressed signal sample vector $\mathbf{y}$ from $\mathbf{s}$, as follows

$$\mathbf{y} = \mathbf{\Phi}\mathbf{s} = \mathbf{\Phi}(\mathbf{\Psi}\mathbf{x} + \mathbf{w}) = \mathbf{A}\mathbf{x} + \mathbf{n} \tag{4}$$

where $\mathbf{A} = \mathbf{\Phi}\mathbf{\Psi}$ is an $M \times K$ projection matrix. $\mathbf{n} = \mathbf{\Phi}\mathbf{w}$ represents the compressed noise. $\mathbf{s}$ is the discrete-time representation of received signal at Nyquist sampling rate f_{Nyq}. The average sampling rate is now $(M/N)f_{Nyq}$, where $M \ll N$ is chosen to induce strong compression, but for accurate recovery, M should be larger than spectrum sparsity order determined by the occupied channel bandwidth with respect to the total bandwidth. There are many options for $\mathbf{\Phi}$, an easy example construction of $\mathbf{\Phi}$ is by choosing elements that are drawn independently from a random distribution such as Gaussian [9].

3.3. Sparse Signal Reconstruction

In fact, the wideband spectrum sensing problem can be transformed to the problem of estimation of signal amplitude vector $\mathbf{x}$ through overcomplete representation of the received signal. The approach forms an estimate of the signal energy as a function of hypothesized channel occupancy state, which ideally contains dominant peaks at the occupied subbands. The central assumptions are that each subband is occupied by at most one primary transmitter and that the number of primary users is small. With this assumption the underlying amplitude vector is sparse, i.e., $\mathbf{x}$ has only a few nonzero elements. Now the focus is to reconstruct $\mathbf{x}$ with high probability. This inverse problem can be solved when it is regularized to favor sparse signal fields by the ℓ_1 methodology. The appropriate objective function for the problem is expressed as [18]

$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1. \tag{5}$$

The ℓ_2 term forces the residual $\mathbf{y} - \mathbf{A}\mathbf{x}$ to be small, whereas the ℓ_1 term enforces sparsity of the representation. The parameter λ controls the tradeoff between the sparsity of $\mathbf{x}$ and the residual norm. Once $\mathbf{x}$ is found, the occupied subbanbs locations correspond to the locations of the peaks in $\mathbf{x}$. Thus, the channel occupancy state can be detected by searching peaks in the reconstructed signal amplitude vector.

The data for the model are complex-valued, hence, neither linear nor quadratic programming can be used for numerical optimization. Instead, a second-order cone programming (SOCP) [16] framework is adopted to solve the problem. Equation (4) can be calculated by SOCP software packages such as CVX [17].

The procedure of the proposed wideband spectrum sensing method can be summarized as follows

Step 1: Collect received data sample $\mathbf{s}$ and construct the overcomplete basis matrix $\mathbf{\Psi}$ by (3).

Step 2: Generate a random compressive sampling matrix $\mathbf{\Phi}$.

Step 3: Obtain the projection matrix $\mathbf{A}$ and compressed signal sample vector $\mathbf{y}$ by (4).

Step 4: Reconstruct the signal amplitude vector $\mathbf{x}$ from (5).

Step 5: Plot $\mathbf{x}$ versus frequency, find peaks in $\mathbf{x}$ and obtain the occupied subbands locations.

3.4. Cooperative Wideband Spectrum Sensing

It is possible to improve the detection performance through cooperative approaches. Consider the case in which two SUs cooperatively sense the K subbands by exchanging their sensed data in a fusion center. According to (2), the received data of the two SUs can be written as

$$\mathbf{s_1} = \mathbf{Ba_1} + \mathbf{w_1}$$
$$\mathbf{s_2} = \mathbf{Ba_2} + \mathbf{w_2}$$

where $\mathbf{a_1}$ and $\mathbf{a_2}$ represent the received signal amplitude vector of two SUs, respectively. Then the signal model in the fusion center is given as

$$\mathbf{S} = \mathbf{Bu} + \mathbf{W}$$

where $\mathbf{S} = [\mathbf{s_1}, \mathbf{s_2}]$, $\mathbf{u} = [\mathbf{a_1}, \mathbf{a_2}]$, $\mathbf{W} = [\mathbf{w_1}, \mathbf{w_2}]$. Similarly, $\mathbf{S}$ can be represented by the overcomplete basis matrix $\mathbf{\Psi}$ as follows

$$\mathbf{S} = \mathbf{\Psi U} + \mathbf{W}$$

where $\mathbf{U}$ is composed of two sparse column vectors corresponding to the received signal amplitudes of the two SUs. After compressive sampling, compressed signal sample matrix can be obtained as

$$\mathbf{Y} = \mathbf{\Phi S} = \mathbf{\Phi}(\mathbf{\Psi U} + \mathbf{W}) = \mathbf{AU} + \mathbf{W_0}$$

where $\mathbf{W_0}$ is the compressed noise matrix. Just like (5), $\mathbf{U}$ can be reconstructed by solving the following equation [18]

$$\min_{\mathbf{U}} \|\mathbf{Y} - \mathbf{AU}\|_2^2 + \lambda \|\mathbf{U_0}\|_1 \tag{6}$$

where each element in $\mathbf{U_0}$ is the mean of a row of $\mathbf{U}$. The key of the cooperative wideband sensing method is to obtain $\mathbf{U}$ and $\mathbf{U_0}$. Then the channel occupancy state can be detected by searching peaks in $\mathbf{U_0}$.

4. SIMULATION RESULTS

This section provides computer simulation results to testify the proposed wideband spectrum sensing algorithm. Consider a wideband partitioned into $K = 40$ non-overlapping subbands with equal bandwidth. The center frequency of each subband is from 1 MHz to 40 MHz with 1 MHz intervals. The wideband channel is occupied by Q PUs. Each user employs Binary Phase Shift Keying (BPSK), and the carrier frequency is randomly generated between 1–40 MHz. Some cognitive nodes are randomly distributed in the region covered by the primary transmitters. They will sense the corresponding subbands occupied by the PUs individually or cooperatively. The Nyquist sampling rate f_s is set at 80 MHz. N samples are used to collect transmitted signal at Nyquist sampling rate. M is the number of samples after compression. The $M \times N$ compressive sampling matrix $\mathbf{\Phi}$ is generated by using the Matlab function randn (M, N). The SNR of a CR receiver is defined as the ratio of the average received signal power to noise power over the entire wideband.

To evaluate the performance of the proposed algorithm, we define a channel occupancy state decision vector $\hat{\mathbf{d}}$, $\{\hat{\mathbf{d}}(k)\}_{k=1}^{K} \in (0, 1)$, $\hat{\mathbf{d}}(k) = 1$ or 0 indicating whether the subband is occupied by a PU or not. In the detection process, we search Q peaks in the reconstructed signal amplitude vector $\mathbf{x}$ to get the corresponding subbands locations $k_1, k_2, \ldots, k_Q$. Then we let $\hat{\mathbf{d}}(k)$ $(k \in \{k_1, k_2, \ldots, k_Q\})$ to be 1 and other elements in $\hat{\mathbf{d}}$ to be 0 to get the channel occupancy state decision vector. The true channel occupancy state is $\mathbf{d}$, which has the same definition with $\hat{\mathbf{d}}$. The probability of detection P_d is evaluated by comparing the channel occupancy state decision with the true state over all subbands occupied by PUs, as follows [12]

$$P_d = E\left\{\frac{\mathbf{d}^T\left(\mathbf{d} = \hat{\mathbf{d}}\right)}{\mathbf{1}^T\mathbf{d}}\right\}.$$

In the first simulation, the wideband channel is occupied by three

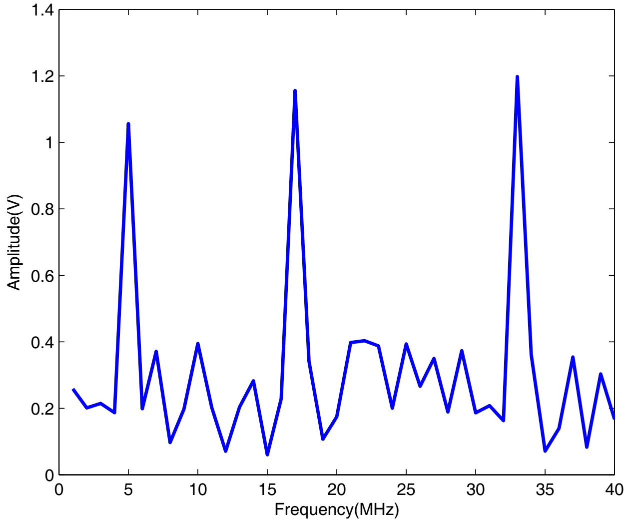

Figure 2. Reconstructed signal amplitude versus frequency, for $\text{SNR} = -10\,\text{dB}$, $Q = 3$, $N = 500$, $M = 200$.

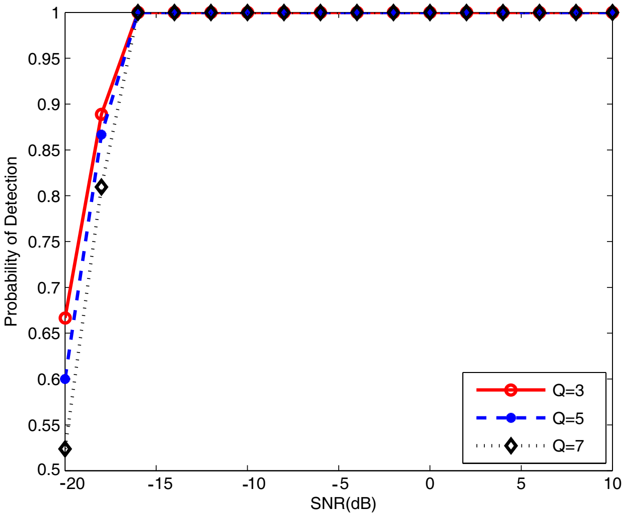

Figure 3. Probability of detection curves of the proposed algorithm for different numbers of PUs, $N = 500$, $M = 200$.

PUs, and the carrier frequencies of PU signals are set at 5 MHz, 17 MHz, and 33 MHz, respectively. Fig. 2 depicts the reconstructed signal amplitude vector $\mathbf{x}$ by solving the Equation (5), for $N = 500$, $M = 200$, and SNR $= -10$ dB. It can be seen that the proposed method can reconstruct the signal amplitude successfully. It is easy to find three peaks in $\mathbf{x}$. Since the occupied subbands locations correspond to the locations of the peaks in $\mathbf{x}$, three subbands can be decided to be occupied. The center frequencies of the occupied subbands are 5 MHz, 17 MHz and 33 MHz, respectively. In this way, spectral holes over a wideband channel can be detected effectively.

In the second simulation, we demonstrate the detection performance of the proposed algorithm with the number of active PUs changed. Fig. 3 shows the probability of detection curves for different numbers of PUs with SNRs varying from -20 dB to 10 dB. N and M are set to be 500 and 200, respectively. It is shown that the probability of detection drops as the number of PUs increases, for the same SNR.

In the third simulation, the proposed method is compared with two common spectrum sensing methods: one is energy detection [4], and the other is maximum-minimum eigenvalue (MME) detection [19]. Fig. 4 depicts the probability of detection curves of the three methods with SNRs varying from -20 dB to 10 dB. The parameters of the proposed method are set as: $Q = 3$, $N = 500$ and $M = 200$. It is shown that the performance of energy detection and MME detection

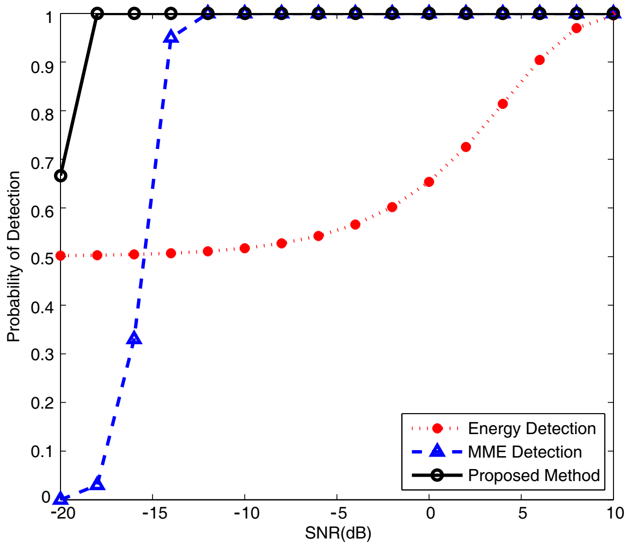

Figure 4. Probability of detection curves of the three methods, for $Q = 3$, $N = 500$, $M = 200$.

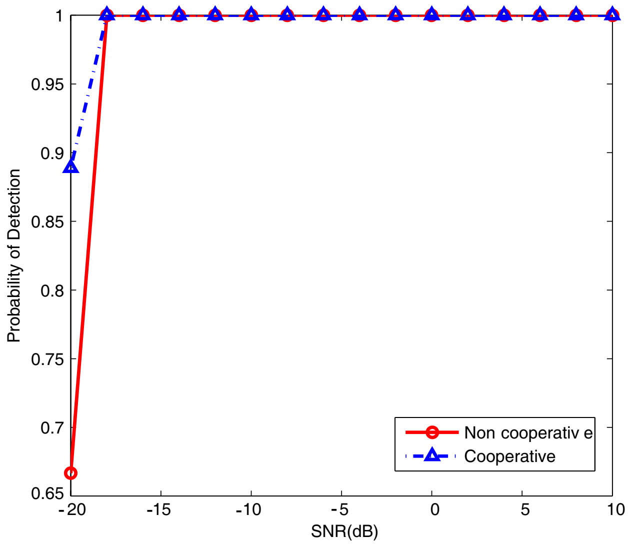

Figure 5. Probability of detection curves of the cooperative and non-cooperative method, for $Q = 3$, $N = 500$, $M = 200$.

are not good in low SNR cases, but the proposed method can achieve high probability of detection even when the SNR is -18 dB. Simulation results testify the effectiveness of the presented wideband spectrum sensing approach even in low SNR cases.

In the final simulation, the proposed non-cooperative sensing method is compared with the cooperative approach in terms of detection performance. In the cooperative approach, two SUs sense the K subbands and detect the channel occupancy state cooperatively. The probability of detection of the cooperative approach can be evaluated through $\mathbf{U_0}$ obtained by solving the Equation (6). For both methods, $Q = 3$, $N = 500$, $M = 200$, and the SNR is varying from -20 dB to 10 dB. Fig. 5 depicts the probability of detection curves of both methods. It can be seen that the detection performance of the two methods are almost the same. Only when the SNR is equal to -20 dB, through cooperative approach, the probability of detection is increased by about 20%.

5. CONCLUSIONS

This paper presents a new wideband spectrum sensing algorithm based on sparse signal representation and compressive sampling. The wideband spectrum sensing problem can be transformed to the problem of estimation of signal amplitude vector $\mathbf{x}$ through sparse signal representation. Then spectrum holes can be detected effectively over a wideband channel by searching peaks in the reconstructed signal amplitude. The use of compressive sampling effectively reduces the sampling rate requirements, making it possible to simultaneously monitor a very wide band of spectrum opportunities. Simulation results show that the proposed algorithm can achieve high probability of detection in low SNR cases.

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