

## MUTUAL COUPLING ANALYSIS USING FDTD FOR DIELECTRIC RESONATOR ANTENNA REFLECTARRAY RADIATION PREDICTION

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**Abstract**—A simulation technique based on Finite-Difference Time-Domain (FDTD) is used to analyze mutual coupling effects in reflectarray environment. The neighbouring element method has the ability to analyze actual non-identical reflectarray unit-cell accurately compared to the traditional Floquet simulation which assumes all unit-cell is identical. It is also found that the nearest neighbouring unit-cell located in  $E$ -plane has a larger mutual coupling effects compared to the neighbouring unit-cell in  $H$ -plane. A good agreement is shown between simulation and measurement results. This technique presents a new prediction method for the radiation pattern of reflectarray antenna.

### 1. INTRODUCTION

The reflectarray antenna was first introduced by Berry and Malech in 1963 [1]. Since then, many advance structures have been designed to further enhance the capability of reflectarray antenna especially in satellite communication [2–14]. A reflectarray antenna integrates the advantages of reflector type antenna and phased array antennas [14]. This antenna consists of an array of unit-cells illuminated by a primary feed, typically horn antenna. Each unit-cells will re-radiate the power

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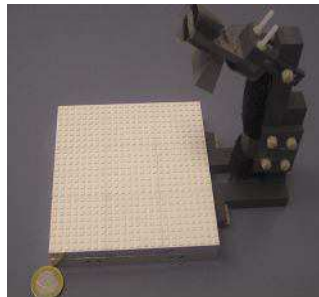
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received from the horn for a given phase shift. Such technique requires modifying the geometrical and electrical parameters of the unit-cells in order to steer the main beam in a specific direction.

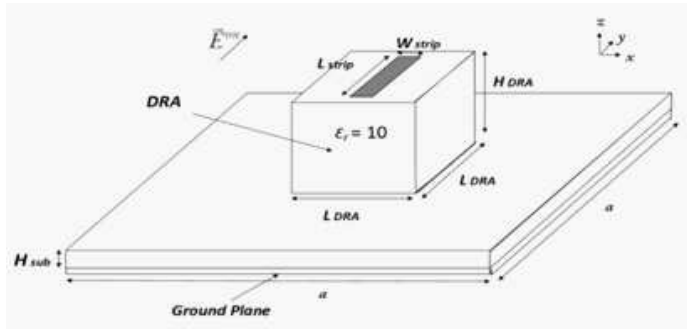
Currently, it is difficult to analyze the whole reflectarray structure including the primary feed and a large array of unit-cells using available commercial software. Such simulation needs a lot of computer memory and it could only be done for a printed reflectarray with a size less than  $15.5 \times 10.8$  wavelengths at its resonant frequency [15]. For a large size of reflectarray, a simulation based on unit-cell multiplied by array factor can be used to predict the radiation patterns of dielectric resonator antenna (DRA) reflectarrays [16]. Such technique gives a very fast result, however large discrepancy between simulation and measurement is obtained for sidelobe-level (SLL). This is due to the fact that mutual coupling effects are accounted from the identical unit-cells, which is not true in the real reflectarray environment.

Investigations of mutual coupling in reflectarrays are not easy due to two main factors: unit-cells are not identical and no port can be defined since the unit-cells are illuminated by primary feed [17]. Thus, this forbids the calculation of the standard mutual coupling coefficient. Due to the different phase shift in the unit-cells, each unit-cell in the array should not be identical. Unfortunately, current unit-cell analysis is based on infinite array method [18], which assumes that all the unit-cell in the array is identical. Another approach to analyze unit-cell is introduced in [19] which are based on unitary isolated unit-cell. However, since it is based on single unit-cell, no mutual coupling effect is accounted.

In order to include actual mutual coupling-effects in reflectarray simulation, a simulation technique to analyze mutual coupling effect in reflectarray environment is introduced. It is based on FDTD that includes the coupling effects from the actual neighbouring unit-cell.



**Figure 1.** Benchmark  $24 \times 24$  DRA reflectarray in Ka-band [16].



**Figure 2.** DRA unit-cell [20].

This can be performed by evaluating the field perturbation factor, which accounted for all coupling effects.

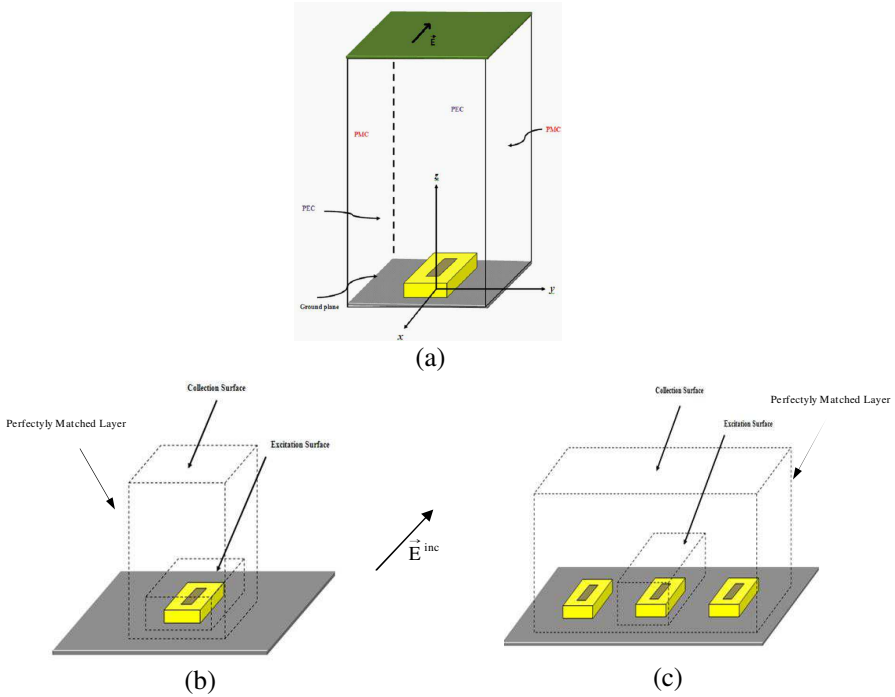
In this paper, a unit-cell based on DRA is first re-introduced. Then, the mutual coupling effect of the neighbouring unit-cell to the phase variation is investigated. In addition, the effects on the  $E$ - and  $H$ -plane of the neighbouring unit-cells are also investigated. After that, the actual  $24 \times 24$  unit-cells in the benchmark DRA (Figure 1) are simulated and the results are compared with the measurement results. At the end, conclusion is finally drawn.

## 2. UNIT-CELL

The standard procedure to design a reflectarray antenna is based on the unit-cell approach. This can be done by adjusting the geometry and electrical parameter of unit-cell, which can result in phase shift. The strip-loaded DRA has been introduced previously in Figure 2 and has been validated experimentally in [20]. Here, again we will use the same structure, to assess mutual coupling effects between unit-cells in terms of phase variation. This follow with the study on the coupling effect that brought by the nearest neighbour in  $E$ - and  $H$ -plane.

### 2.1. DRA Unit-cell

The unit-cell consists of a square DRA ( $L_{dra} \times L_{dra} \times H_{dra} = (2.7 \times 2.7 \times 0.6) \text{ mm}^3$ ) made from Duroid ( $\epsilon_r = 10$ ) with a metallic strip ( $W_{strip} \times L_{strip}$ ) etched on top of it. The  $W_{strip}$  is fixed at 0.3 mm while  $L_{strip}$  act as a tuning parameter in order to have a phase variation. The DRA stands on the top of a flat Duroid structure with thickness ( $H_{sub}$ ) of 0.2 mm and inter-element spacing of  $a$  equivalent to 5 mm. Ground plane is added at the back of the structure for the purpose of blocking the back radiation.



**Figure 3.** Simulation method to analyze unit-cell, (a) Floquet, (b) isolated-element, (c) neighbouring-element.

A simulation tool based on FDTD method has been developed to assess the reflection phase of the structure in the waveguide environment. Three different methods, namely Floquet, Isolated-element and neighbouring-elements [17] have been used to simulate the same DRA unit-cell as shown in Figure 3. Details of these methods are explained in the following sections.

## 2.2. FDTD Methods in Waveguide Simulation

In this section, three different methods to analyze reflectarray's unit-cell are introduced. The first method is a well-known Floquet simulation [21–26]. It is based on infinite-periodic elements, which assume all the coupling effects are coming from the identical neighbouring unit-cells. This technique is well used in designing reflectarray's unit-cell especially to determine the actual parameter of unit-cell in the array. However, it is not good to predict the radiation pattern of the overall reflectarray when mutual coupling effect that comes from identical neighbouring unit-cell doesn't give actual effect in reflectarray antenna [16].

As an alternative, FDTD based on field perturbation method has been developed as shown in Figures 3(b) and 3(c). The former (Figure 3(b)) is called isolated-element method. In this case, only a unit-cell of the original array is illuminated by a plane wave excitation. The near-field response from a unit-cell is calculated at the collection surface. Thus, no mutual coupling effect is accounted. The later (Figure 3(c)) is called the neighbouring-elements method. It evolves from the isolated-element method, which a unit-cell is now added with the neighbouring elements. Here, the plane wave excitation is still the same; however the collection surface is difference. Therefore, the calculated near-field is represented by the response from the unit-cell with the neighbouring elements. This method is on our interest since it can account the mutual coupling effects by calculating the field perturbation brought by the actual neighbouring unit-cell. Such technique is related directly to the reflectarray since it can be used to simulate unit-cells surrounded by non-identical unit-cell.

These three methods are then compared in term of the reflection coefficient phase for the same case of  $L_{strip}$  equal to 1.2 mm which resonates around 30.5 GHz (Figure 4). For the case of Neighbouring-element method, two additional neighbouring unit-cells are added in  $E$ -plane of the same structure with the same  $L_{strip}$  value of 1.2 mm. From the graph, it can be seen that the resonance frequency of the isolated-element has been shifted from 29.8 GHz to 30.5 GHz for Floquet and neighbouring-element methods. This phenomenon is strictly due to the coupling effects from the identical unit-cells. Although both of the Floquet and neighbouring-element methods resonate at the same frequency, the phase variation is slightly different. The Floquet method's phase variation is slightly steeper than neighbouring-element method. The difference in the gradient is due to the fact that the coupling effect is accounted from the infinite identical

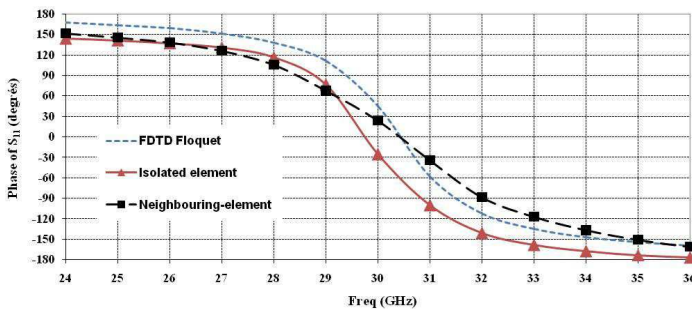


Figure 4. Reflection coefficient of  $S_{11}$  (phase).

neighbouring unit-cells for Floquet method and not for neighbouring-element method (two identical neighbouring unit-cells only). Here, we can conclude that mutual coupling effect can really effect the phase variation and could also degrade the design performance of the overall radiation pattern of the antenna. In order to verify the existence of the coupling effect, further investigation on the phase variation is done on the central unit-cell surrounded by many more neighbouring unit-cells. This will be our interest in the next sub-sections.

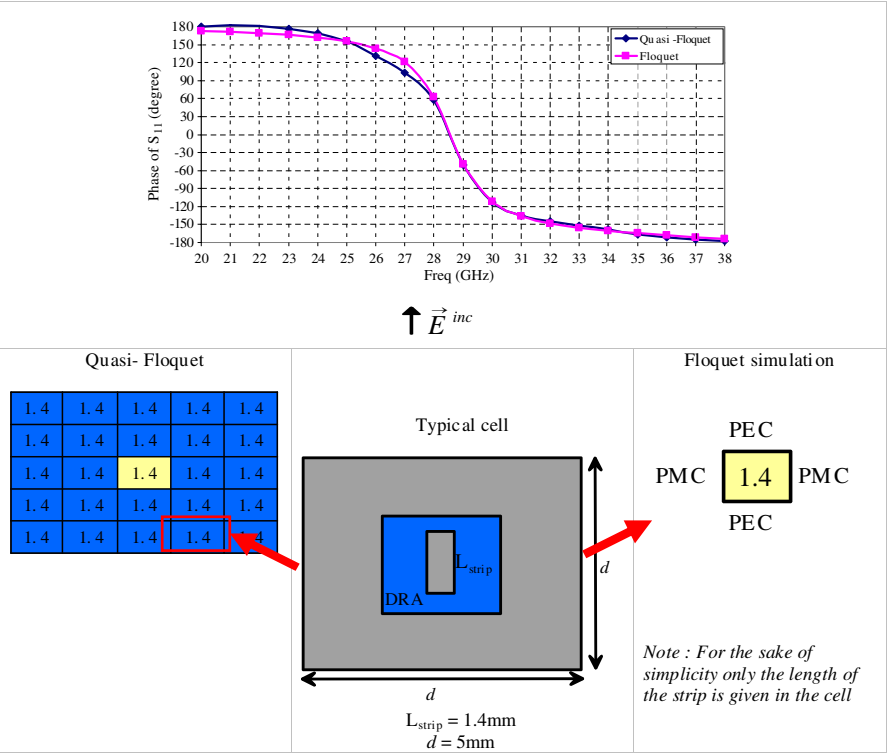
### *2.2.1. Coupling Effects from Several Neighbouring Unit-cells on the Phase Variation*

In this section, the coupling effects from several more neighbouring unit-cells on the phase variations are studied. The main difference with the previous section is that we will now consider the effect of up to 24 neighbouring cells. By doing so, more accurate representation of coupling effects brought by neighbouring unit-cells to the illuminated central unit cells are compared to the coupling brought by two neighbouring cells. In these cases, two FDTD simulations ('neighbouring-element' approach and 'quasi-Floquet' approach) will be needed. 'Neighbouring element' approach will represent a unit-cell surrounded by neighbouring elements (non-identical unit-cells) while 'quasi-Floquet' approach will represent the coupling effect from identical-unit cells. This 'quasi-Floquet' approach can be seen as an approximation of the actual 'Floquet' approach where a finite number of identical neighbours is used. In this study, the unit-cell of Figure 2 ( $L_{strip}$  varies) will be used for comparison. The objective of this section is to evaluate the coupling effects from the unit-cells of reflectarray environment where the unit-cells are non-identical.

As the 'quasi-Floquet' approach is not yet assessed, we first compare this technique with the well known 'Floquet' approach. In this comparison, the DRA unit-cell using  $L_{strip} = 1.4$  mm is selected. The graph comparison is shown in Figure 5 along with the corresponding strip lengths layout. Good agreement between the two curves is achieved, except a slight glitch between 20 and 28 GHz. This minor difference can be linked to the limited number of neighbours. Since the agreement using windowing truncation of 5 by 5 unit-cells is quite good and to save simulation time, this 'Quasi-Floquet' approach will be used for comparison in the next sections.

### *2.2.2. Identical Unit-cells*

As the 'quasi-Floquet' approach has been validated, we will use this approach as the reference to study the modification of coupling when

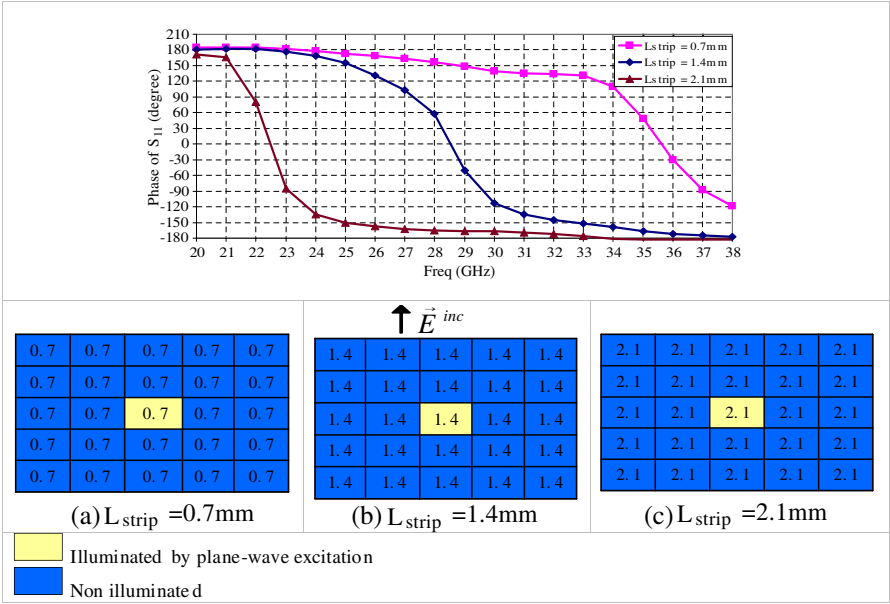


**Figure 5.** Phase variation comparison between ‘Quasi-Floquet’ and ‘Floquet’ approaches ( $L_{\text{strip}} = 1.4\text{ mm}$ ).

different neighbours are introduced surrounding the illuminated unit-cell. Different values of strip length ( $L_{\text{strip}} = 0.7, 1.4$  and  $2.1\text{ mm}$ ) will be considered. Figure 6 presents the phase variation using ‘quasi-Floquet’ approach for the three different strip lengths. It shows that the resonant frequency is different for each:  $22.5, 28.5$  and  $35.5\text{ GHz}$  respectively. Such information on the resonance frequencies will be useful when we make a comparison with non-identical unit-cells as will be discussed in the next section.

2.2.3. Non Identical Unit-cells

In this section, the ‘neighbouring-element’ method is used. A  $5 \times 5$  environment is considered with an illuminated cell whose strip length is  $1.4\text{ mm}$ . All of the 24 neighbours are identical except one or two cells whose strip length is changed to  $0.7\text{ mm}$  or  $2.1\text{ mm}$ . By doing so,



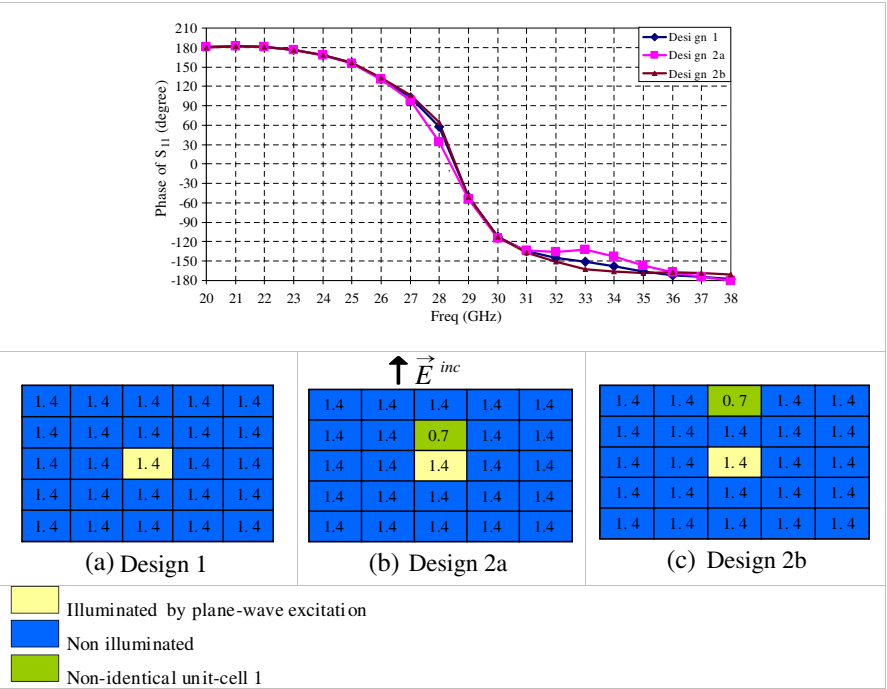
**Figure 6.** Phase variation of different unit-cells using ‘quasi-Floquet’ approach.

the influence of the non-identical cells on the phase variation of the illuminated cell can be evaluated. Six configurations are considered (Design 2a to 2d in Figure 7 and Figure 8, respectively, Design 3 in Figure 9 and Design 4 in Figure 10).

For Design 2a and 2b, a unique non-identical unit-cell is located in the  $E$ -plane. Two tests are done by placing the non-identical unit-cell one (Design 2a (Figure 7(b))) or two cells (Design 2b (Figure 7(c))) apart from the central illuminated unit-cell. The phase variations graphs for these two cases are then compared with the phase variation produced by ‘quasi-Floquet approach’ and this is given in Figure 7.

For both designs, a disturbance on the phase variation occurs around 33 GHz, i.e., close to the resonant frequency of the inserted strip ( $L_{strip} = 0.7\text{mm}$ ). In theory, for strip length equal to 0.7 mm, the resonant frequency should be 35.5 GHz and not 33 GHz (Figure 7). However, as the environment is modified, this difference can be explained. Also, the effect on the phase variation for Design 2a is higher than for Design 2b. It shows that the coupling effect is higher when the unit-cell is closer to the illuminated unit-cell. Coupling effect also slightly shifts the resonant frequency of the illuminated unit-cell as can be observed in the graph. This is consistent with the modification





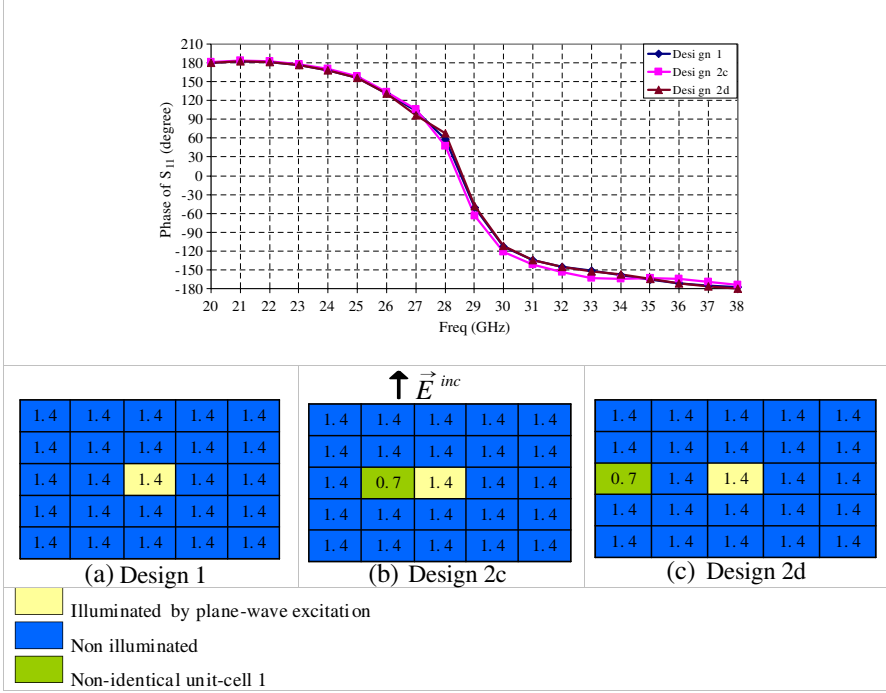
**Figure 7.** Phase variation comparison of non-identical and non-illuminated unit-cells in  $E$ -plane.

of the resonant frequency of the ‘disturbing’ cell that has already been reported in the text.

Design 2c and 2d repeat the study in  $H$ -plane (Figures 8(b) and (8c)). The graph comparison is shown in Figure 8. From the graph, no major variation can be seen compared to ‘quasi-Floquet’ which confirms the coupling effect is very low in the  $H$ -plane. However, we can guess a slight perturbation around 33 GHz, i.e., close to the resonance of the ‘disturbing’ cell.

From the study of these two cases, we can conclude again that the coupling effects in  $E$ -plane is higher than  $H$ -plane. This is basically due to the fields calculated at the neighbouring unit-cells in  $E$ -plane are higher than in  $H$ -plane. Such in  $E$ -plane, will directly give high field disturbance to the central unit-cell and therefore, we decided to investigate the next couple of tests only in  $E$ -plane.

For Design 3, we change the strip length of the non-identical unit-cell from 0.7 mm to 2.1 mm (Figure 9(b)). In Figure 9, we observe a disturbance at 22 GHz which corresponds to the resonant frequency of a cell with  $L_{strip} = 2.1$  mm (Figure 6). In addition, the resonant

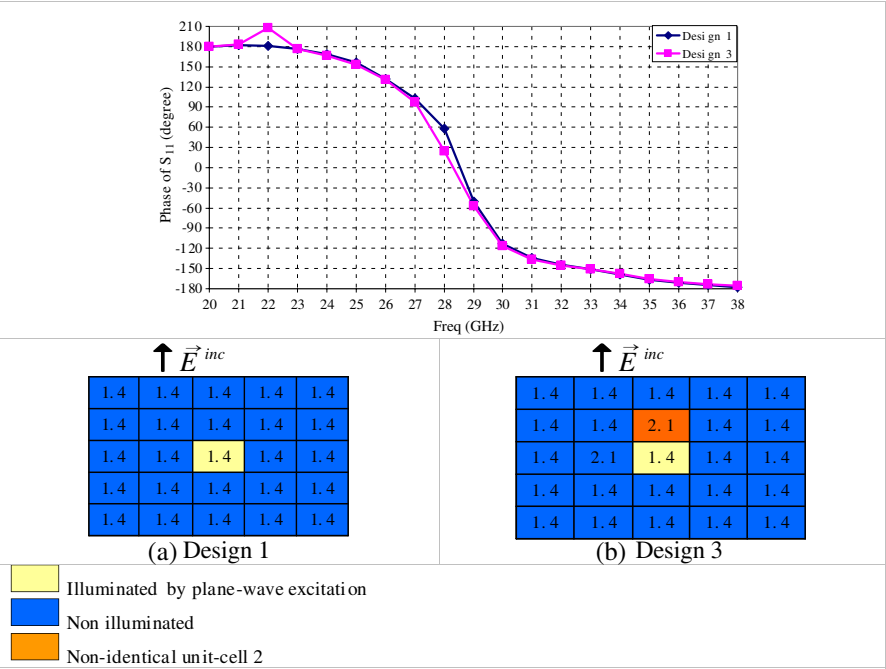


**Figure 8.** Phase variation comparison of non-identical and non-illuminated unit-cells in  $H$ -plane.

frequency of the illuminated unit-cell is also shifted. This confirms the observed effects for the previous design.

Finally, for Design 4, we include two non-identical unit-cells in the  $E$ -plane (Figure 10(b)). We observe that the phase variation becomes worst as it combines the effects of both ‘disturbing’ cells. Firstly, a disturbance on the phase variation is found at 33 GHz, due to the non-identical unit-cell with 0.7 mm strip length. Then, another disturbance on the phase variation is found at 22 GHz due to the unit-cell with 2.1 mm strip length. Finally, quite a large shift in the resonant frequency of the illuminated cell is also observed.

In a reflectarray design, it is important to use the actual phase variation of the used unit-cells to choose the appropriate layout. Here, we have demonstrated that the mutual coupling effect can change the phase variation. In practise, this is a real issue, as the actual environment of a cell is not known before the entire reflectarray layout has been designed. This means an accurate computation of coupling can only be done at the end of the synthesis process.

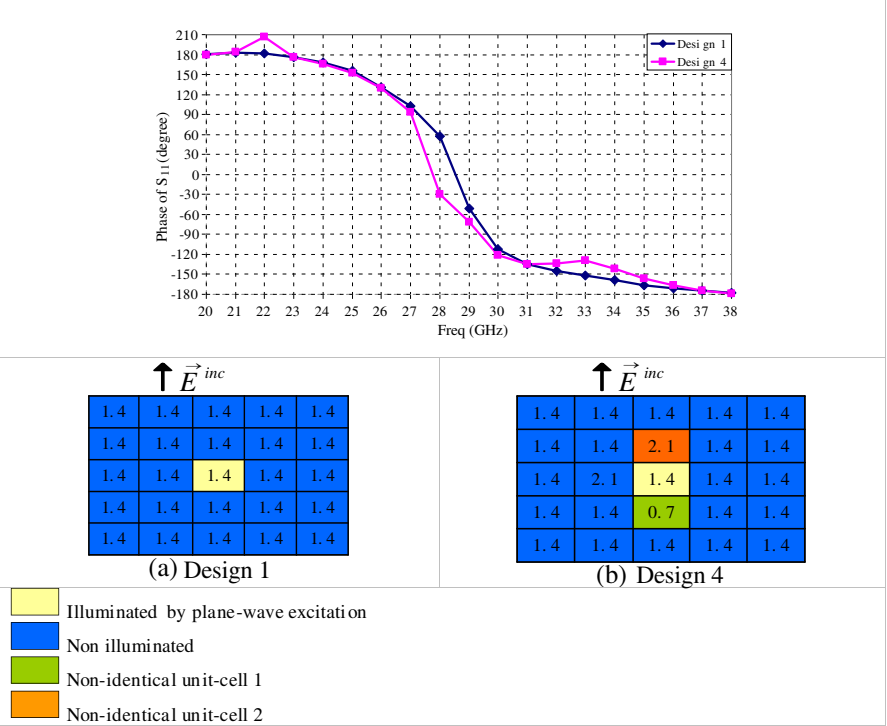


**Figure 9.** Phase variation comparison of non-identical and non-illuminated unit-cells in  $E$ -plane.

3. DRA REFLECTARRAY

Based on information of the unit-cell investigated in the previous section, all the three methods are used to predict the radiation pattern of the DRA reflectarray. The  $24 \times 24$  unit-cells of DRA reflectarray that is shown in Figure 1 will be our benchmark antenna. In order to predict the simulated radiation pattern related to the mutual coupling effects, all the three methods described in the previous section are simulated.

Previously, the Floquet simulation is used to predict radiation pattern using a simple array factor. The array theory is based on the standard array factor multiplied by a radiation pattern of a DRA unit-cell. Unfortunately, this technique cannot predict the radiation pattern very well when compared with measurement results as shown in Figure 11. Although, it can predict the main beam quite well, however it cannot predict accurately the side-lobe level (SLL). There are large discrepancies in  $E$ - and  $H$ -plane of the SLL obtained between measurement and calculation which is due to the fact that coupling

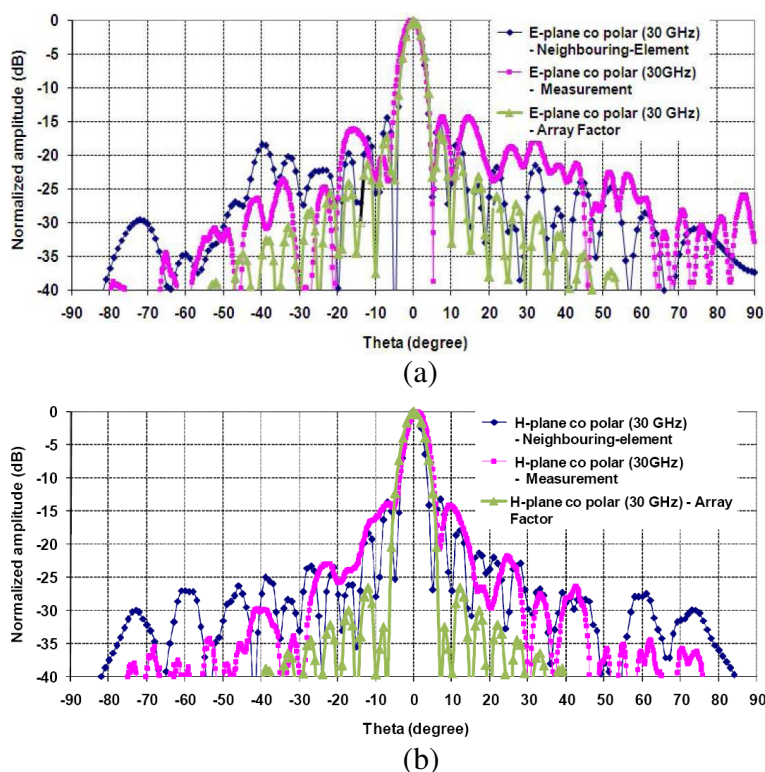


**Figure 10.** Phase variation comparison of non-identical and non-illuminated unit-cells in  $E$ -plane.

effects and unitary radiation pattern is come from the same identical unit-cell which is not true in the real reflectarray environment. In order to have a good prediction, actual neighbouring unit-cell should be included using neighbouring-element method. To facilitate simulation setup, two actual nearest neighbour located in  $E$ -plane (as shown in Figure 3(c)) will be included in the calculation. In order to have the overall radiation pattern of the reflectarray, the radiation pattern for each setup (with the correct position in the array) is first being calculated. Then, all the individual setup for radiation patterns will be added together to have the overall radiation patterns of the reflectarray. By doing so, the actual coupling effect from the nearest neighbour to the actual unit-cell can be calculated. Thus, it can give better prediction on the overall radiation pattern. This is confirmed by the measurement results as shown in Figure 11, which give a much better prediction in the main lobe and SLL.

The measured gain of the reflectarray is compared with the calculated directivity (Table 1). The directivity is extracted from the

3-D radiation patterns for Floquet, isolated-unit-cell and neighbouring-unit cell methods. The maximum directivity of 31.5 dB, 30 dB and 29 dB are obtained respectively for Floquet, isolated and neighbouring-unit-cell method. However, the maximum gain (27 dBi) is achieved at 30 GHz from the measurement. A large loss is obtained compared to Floquet simulation due to the fact that mutual coupling is considered in this calculation. A slightly better prediction is obtained for the case of isolated unit-cell although no mutual coupling effects are considered. Smaller loss is obtained when compared with neighbouring unit-cell method. This loss could be due to the spill over loss, material loss and other losses. This result also confirms that this technique is can be used to predict radiation pattern of reflectarray antenna which include actual mutual coupling effects from non-identical cells.



**Figure 11.** Measured vs. simulated radiation pattern (array factor concept and neighbouring-element method); (a) *E*-plane co-polar, (b) *H*-plane co-polar.

**Table 1.** Gain and directivity of DRA reflectarray at 30 GHz.

Method	Gain/Directivity
a) Measurement	27 dBi
b) Floquet (Array factor)	31.5 dB
c) Isolated-element	30 dB
d) Neighbouring-element	29 dB

#### 4. CONCLUSIONS

A perturbation technique based on the field disturbance brought by neighbouring unit-cells has been introduced. This technique shows the capability to analyze mutual coupling effects effectively in reflectarray environment compared to the traditional Floquet simulation. The effects on the phase variation show that the coupling effect has much more influence on  $E$ -plane than  $H$ -plane. Finally, this technique is applied to predict radiation pattern of the overall DRA reflectarray. A very good agreement is achieved when compared with the benchmarked DRA reflectarray antenna.

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