NOVEL WIDEBAND MULTILAYER BUTLER MATRIX USING CB-CPW TECHNOLOGY

M. Ben Kilani¹, M. Nedil^{1, *}, N. Kandil¹, M. C. E. Yagoub², and T. A. Denidni³

¹Underground Communications Research Laboratory (LRCS), UQAT-LRCS, Université de Québec en Abitibi-Témiscamingue, 450, 3e Avenue, Val d'or, QC J9P 1S2, Canada

 $^2\mathrm{EECS},$ University of Ottawa, 800 KING Edward, Ottawa ON K1N 6N5, Canada

³INRS-EMT, Université de Québec, Place Bonaventure, 800, de la Gauchetière Ouest, Bureau 6900, Montréal, Québec H5A 1K6, Canada

Abstract—This paper presents a novel design of a 4×4 twolayer wideband Butler matrix using a CB-CPW wideband multilayer directional coupler as well as a novel CB-CPW wideband elliptic directional coupler. With this configuration, the proposed matrix was designed to avoid crossovers as in conventional Butler matrices, thus reducing its size, losses and the design complexity while leading to a broad bandwidth of 3 GHz. To evaluate the performance of the proposed matrix, experimental prototypes of the directional elliptic coupler, the CB-CPW multilayer directional coupler and Butler matrix were fabricated and measured. Furthermore, a 4-element antenna array was connected to the matrix to form a beamforming antenna system at 5.8 GHz. As a result, four orthogonal beams were realized in the band 4.5–7.5 GHz.

1. INTRODUCTION

With the growth of wireless communication users, the need for more advanced solutions to reduce the co-channel fading and improve the capacity increases significantly. Consequently, many directions have been investigated. For instance, in antenna issue, wireless communication systems use beamforming antenna arrays to reduce

Received 8 May 2012, Accepted 21 June 2012, Scheduled 3 July 2012

^{*} Corresponding author: Mourad Nedil (mourad.nedil@uqat.ca).

interferences and improve their capacity [1]. Indeed, by using narrow beams available from a multi-beam antenna array, it is possible to improve the gain in desired signal direction while reducing it toward interference directions [1]. In this perspective, the Butler matrix circuit has been used in various beam antenna array systems to produce multiple beams [2]. A standard $N \times N$ Butler matrix has N input and N output ports. For each input port, the network will produce signals with progressive phase shifts at the output ports with equal power. Feeding an N-element antenna array using the Butler matrix will produces N orthogonal beams at its outputs [2].

Compared to microstrip technology, the CPW technology exhibits lower loss, ease of making shunt and series connections, ease of controlling the characteristics of CPW lines by changing the slot and strip widths, and ability to be used in millimeter-wave applications [3]. However CPW line dimensions are greater than those of microstrip lines [3]. A solution is the use of conductor-backed coplanar waveguide (CB-CPW) technology which consists of adding a ground plane on the bottom of the substrate. This technique allows reducing the dimensions of microwave structures and eliminates air bridges for top ground plane connection [4].

Line crossovers have been one of the main drawbacks of Butler matrices since they take more space and may add undesired effects, such as increased insertion loss and additional line cross couplings [5]. Many architectures of Butler matrix using wideband crossovers have been proposed [6-8]. Some other configurations have been investigated allowing to reduce dimensions without offering very wide bandwidth [9, 10]. Few authors have suggested different matrix configurations that does not require crossing lines [11–14]. In [11], a 4×4 CPW Multilaver Butler matrix based on slot-coupled directional couplers is reported. This matrix offers a bandwidth of only 1.5 GHz due to the use of conventional hybrid couplers. A 4×4 microstrip Butler matrix without crossing is reported in [12], which is not valid for an 8×8 configuration. Suspended strip lines with multilayer structure have also been studied in [13]. Main drawbacks of this structure are its narrow bandwidth in addition to the complexity of the design. In [14], a multilayered SIW 4×4 Butler matrix is introduced. Based on the combination of hybrids with broad-wall slot coupling and changes of layers at places at the *E*-plane couplers, the structure offers wide bandwidth and needs a very high degree of precision related to the fabrication process.

In this paper, the design of a new wideband CPW beamforming network based on the 4×4 Butler matrix is described. This matrix is advantageously implemented to avoid using any crossing lines. In addition, an elliptic configuration is used to offer wider bandwidth compared to structures based on conventional CPW branch-line couplers. This is achieved by controlling different design parameters like eccentricity [15]. To validate the proposed design, an experimental prototype of the proposed 4×4 Butler matrix was designed, fabricated and measured. Simulation and experimental results including return loss and insertion loss are presented and discussed. Furthermore, to examine the performance of the proposed matrix in terms of beamforming, a 4 antenna array was built and connected to the matrix to form a multiple beam antenna system. Simulations and measurements were carried out on this beamforming system and the obtained results discussed.

2. DESIGN PROCEDURE

The performance of the Butler matrix depends critically on the bandwidth of directional couplers. Furthermore, increasing the order of a conventional Butler matrix requires more couplers and crossings, thus, more space and losses. In fact, a conventional $N \times N$ Butler matrix uses Nn/2 hybrids with N/2(n-1) phase shifters. For planar structures, the number of crossovers needed may be calculated as reported in [5].

$$C_n = 2C_{n-1} + 2^{n-2}(2^{n-2} - 1) \tag{1}$$

where n is the matrix order, which is related to the number of ports by $N = 2^n$. In (1) n should be equal or greater than 2, and $C_1 = 1$.

For instance, the number of crossovers needed to implement a matrix with 32 ports is 416, which is huge and could introduce a lot of losses. For this reason, configurations without crossovers are more suitable. In this perspective, a new Butler design is proposed.

Figure 1 shows the proposed 4×4 Butler matrix that uses 3 dB CB-CPW directional elliptic couplers, 3 dB CB-CPW directional slotcoupled couplers, and phase shift transmission lines. Using the 3D HFSS electromagnetic (EM) software [16], the design procedure of the directional elliptic coupler, the directional slot-coupled coupler and the 4×4 Butler matrix was carried out. In the following subsections, the design and the performance of each component of this matrix are presented and described in more details.

2.1. CB-CPW Slot-coupled Directional Coupler

Directional couplers are indispensable components in monolithic microwave integrated circuits (MMICs). CB-CPW broadside-coupled couplers have recently found applications in the design of many



Figure 1. Block diagram of the proposed two-layer Butler matrix.



Figure 2. Layout of the slot-coupled directional coupler.

important microwave and millimeter-wave circuits owing to their broadband characteristic and tight-coupling properties [14, 18].

To avoid crossovers in the Butler matrix circuit, a slot-coupled directional coupler is proposed (Fig. 2). It ensures coupling between two CPW lines through a rectangular slot etched on the common ground plane. For this type of couplers, the numerical analysis is based on the odd and even modes of propagation [17]. The even mode propagates when equal currents, in amplitude and phase, flow on the two coupled lines, while the odd mode is obtained when currents have equal amplitudes, but opposite phases [14]. The coupled region is characterized by two transmission lines with characteristic impedances, Z_{0o} (Odd) and Z_{0e} (Even), and two effective relative dielectric constants namely, ε_{reffo} and ε_{reffe} . The characteristic impedance in

Progress In Electromagnetics Research C, Vol. 31, 2012

the coupling section is given by combining the impedance of the two above modes.

The coupler was designed and optimized to reach values of $3 \, dB$ and 90° for the power and the phase, respectively. It was noted that the coupling mechanism is controlled by the thickness of the substrate, dimensions of transmission lines, and the dimension of the rectangular slot size located on the common ground plane.

Simulations and measurements results of this coupler are in good agreement as shown in Figs. 3 and 4, respectively. The coupler's parameters were optimized in order to obtain tighter coupling between the output ports (required condition to implement the Butler matrix with better performance in terms of magnitude characteristics). The phase difference between the two output signals is of 90° in the band of interest (4.5–7.5 GHz).



Figure 3. Scattering parameters of directional slot-coupled coupler: (a) simulated, (b) measured.



Figure 4. Simulated and measured phase difference.

2.2. CB-CPW Elliptic Directional Coupler

The quadrature hybrid coupler, or 3 dB branch-line coupler has been one of the mainstays in microwave systems having uses in power splitters and mixers [15]. The main drawback of such couplers is their narrow bandwidth. As a solution, different configurations of quadrature couplers have been proposed [19–21]. The most interesting configuration of 3 dB branch-line coupler using open-circuited quarterwavelength series-stubs added at each port as a matching network has been reported in [19]. However, this configuration does not offer a very wide bandwidth in addition to the problem of complexity related to the structure. In this work, we designed a CB-CPW wideband quadrature coupler based on the approach reported in [15]. The chosen elliptic configuration allows widening the bandwidth by controlling different design parameters like eccentricity.

The layout of the proposed wideband 3 dB CB-CPW directional elliptic coupler is shown in Fig. 5. This component is symmetrical and has the following property: if port 1 is fed, then the signal travels to port 2 (direct) and consequently, port 3 is coupled while port 4 is isolated. The structure was designed using a duroid substrate (RT/Duroid 5880) having a dielectric constant of $\varepsilon_r = 2.2$ and thickness of 0.508 mm. The input power is split equally (~ -3 dB) between the two output ports with 90° of phase difference.

Using Green's function, a theoretical analysis of the Z-matrix for an elliptic disk with ports at the periphery was proposed in [15]. It was concluded that even and odd elements of the impedance matrix



Figure 5. Layout of CPW directional elliptic coupler.

Progress In Electromagnetics Research C, Vol. 31, 2012

Z depends on several parameters especially the eccentricity of the elliptic network, the operational center frequency, the characteristic impedance and the location of the ports in addition to the substrate characteristics [15]. Using the Z-matrix of the elliptic network, S-parameters of the quadrature hybrid coupler were evaluated using the expression [15]:

$$S = (Z - Z_0)(Z + Z_0)^{-1}$$
(2)

where Z_0 is the characteristic impedance of the transmission lines connected to the device.

For a four-port quadrature hybrid, it is possible to evaluate the S-parameters using eigenimpedances without involving any matrix inversion. This is allowed by maintaining the symmetry along x and y axes. In this case, the four eigenvalues are given as [15]:

The eigenreflections are then related to the eigenimpedances by:

$$Z_{e1} = Z_{11} + Z_{12} + Z_{13} + Z_{14}$$

$$Z_{e2} = Z_{11} - Z_{12} + Z_{13} - Z_{14}$$

$$Z_{e3} = Z_{11} + Z_{12} - Z_{13} - Z_{14}$$

$$Z_{e4} = Z_{11} - Z_{12} - Z_{13} + Z_{14}$$
(3)

The eigenreflections are then related to the eigenimpedances by:

$$S_{ei} = \frac{Z_{ei} - 1}{Z_{ei} + 1}, \quad i = 1, 2, 3, 4.$$
(4)

Finally, the S-parameters are obtained by:

$$S_{11} = \frac{1}{4} \left(S_{e1} + S_{e2} + S_{e3} + S_{e4} \right)$$

$$S_{12} = \frac{1}{4} \left(S_{e1} - S_{e2} + S_{e3} - S_{e4} \right)$$

$$S_{13} = \frac{1}{3} \left(S_{e1} + S_{e2} - S_{e3} - S_{e4} \right)$$

$$S_{14} = \frac{1}{3} \left(S_{e1} - S_{e2} - S_{e3} + S_{e4} \right)$$
(5)

Initially, we connected the elliptic component directly to the 50Ω ports without using matching networks. In this case, for a given substrate, the design variables are: eccentricity e, major axis a and port angle v. Taking into consideration that the operational center frequency is essentially determined by the major axis of the elliptic structure, we used the scaling expressions presented in [15] to determine an initial value of the major axis corresponding to our center frequency, which is 6 GHz. The optimized value of the major axis was found to be equal to 24 mm. Then, by tuning the eccentricity and the

port angle using HFSS [16], a flat-coupling response was obtained for the coupled ports with $v = 51^{\circ}$ and b = 18.48 m (e = 0.77).

Thus, we improved the matching by simply adding a section of transmission line to the ports. The eigenimpedances $Z_{ei,0}$ at the input of the matching network can be obtained by transforming the eigenimpedances at the periphery of the elliptic patch through [15]:

$$Z_{ei,0} = \frac{Z_1(Z_{ei} + jZ_1 \tan \beta S)}{Z_1 + jZ_{ei} \tan \beta S}, \quad i = 1, 2, 3, 4$$
(6)

where Z_{ei} is the eigenimpedance at the periphery of the elliptical patch, β is the wavenumber of the line, Z_1 is the line characteristic impedance, and S is the line length.

These two last parameters were optimized to reach $Z_1 = 26.03 \Omega$ corresponding to $W_1 = 3.21 \,\mathrm{mm}$ and $S = 6.51 \,\mathrm{mm}$. Finally, the matching network was connected to the external ports having characteristic impedance $Z_2 = 50 \Omega$ corresponding to port width $W_2 = 1.07 \,\mathrm{mm}$. Also, the port slot width G was chosen equal to 0.15 mm to reduce the structure size. In addition, the slot width in the elliptic part G' was tuned to 0.3 mm to optimize the coupling factor.

Based on this design, an experimental prototype was fabricated and tested. Fig. 6 shows the photograph of the fabricated coupler. The simulated and measured return loss and insertion loss are shown in Fig. 7. From these results, it can be noted that a bandwidth of 3 GHz (~ 50%) is achieved which is clearly wider than the bandwidth of conventional CPW branch-line couplers (25%) [11].

The average value of coupling for the direct port and the coupled port is $-3.5 \,\mathrm{dB}$. The return and insertion losses are greater than 10 dB within the operating band. The simulated and measured phase shifts between the two ports are plotted in Fig. 8. The phase difference between the direct and coupled ports is approximately 90° across the



Figure 6. Photograph of fabricated circuit prototype.



Figure 7. Scattering parameters of directional elliptic coupler: (a) simulated, (b) measured.



Figure 8. Simulated and measured phase difference.

operating band, which confirms the proposed approach. Furthermore, the comparison between simulated and experimental data shows good agreement.

2.3. 4×4 Butler Matrix

The layout of the proposed Butler matrix is presented in Fig. 9. Combining the components presented in previous sections, the circuit was designed using a two-layer substrates. The top layer is coupled to the bottom layer through the slot-coupled directional couplers. The Butler matrix circuit was designed and fabricated using the same duroid substrate (RT/Duroid 5880). Fig. 10 shows the photograph of the fabricated Butler matrix. It's important to mention that the





Figure 9. Layout of the proposed 4×4 Butler matrix.

Figure 10. Photograph of fabricated circuit prototype.

proposed two-layer approach is suitable for a 4×4 design without crossings. But it cannot be extended to larger matrices. Either additional layers or crossovers are needed in this case.

CB-CPWs are known to support unwanted bulk modes [22]. In many cases, parallel-plate waveguide is formed between the top and the bottom ground planes. The energy will leak along a particular angle once the wave is launched [22, 23]. The ground planes in CB-CPW behave like parallel plate resonators. Considering the dimensions of the proposed Butler matrix, it was verified that the lowest order mode resonance frequency has been shifted to a higher frequency regime. In this case, it can be noted that the leaky wave phenomenon does not affect the performance of the proposed Butler matrix, thus avoiding use of vias [24].

To examine the performance of the proposed prototype, experimental measurements were performed. With this matrix, signals incident at input ports (#1, #2, #3, #4) are divided into four output ports (#5, #6, #7, #8) with equal amplitude and specified relative phase differences.

This Butler matrix was designed for frequency range from 4.5 to 7.5 GHz. Figs. 11 and 12 show simulation and experimental results of the insertion and return losses for ports 1 and 2 when the other ports are matched. It is clearly shown that the proposed matrix has good performance, In fact, the return loss is better than 10 dB and the



Figure 11. Simulated results of the 4×4 Butler matrix when the signal is input at (a) port 1, (b) port 2.



Figure 12. Measured results of the 4×4 Butler matrix when the signal is input at (a) port 1, (b) port 2.

coupling is almost equalized between the output ports $(7.5 \,\mathrm{dB})$.

Figures 13 and 14 show a good agreement between simulated and experimental results of the phase shift of adjacent output ports, where

Phase difference
$$1 = \text{Phase}(S(5,1)) - \text{Phase}(S(6,1))$$
 (7)

Phase difference
$$2 = \text{Phase}(S(6,1)) - \text{Phase}(S(7,1))$$
 (8)

Phase difference
$$3 = \text{Phase}(S(7,1)) - \text{Phase}(S(8,1))$$
 (9)

Phase difference
$$4 = \text{Phase}(S(5,2)) - \text{Phase}(S(6,2))$$
 (10)

Phase difference
$$5 = \text{Phase}(S(6,2)) - \text{Phase}(S(7,2))$$
 (11)

Phase difference
$$6 = Phase(S(7,2)) - Phase(S(8,2))$$
 (12)

It can be seen that the phase differences are not perfectly flat in



Figure 13. Simulated results of phase difference at adjacent output ports of Butler matrix when the signal is input at (a) port 1, (b) port 2.



Figure 14. Measured results of phase difference at adjacent output ports of Butler matrix when the signal is input at (a) port 1, (b) port 2.

the entire bandwidth which the theoretical values are in 45° and -135° (Fig. 13) respectively. The phase errors are going from 7° (4.5 GHz) to 10° (7.5 GHz) in all the bandwidth compared to desired values of 45° and -135° . These errors are due to the use of the narrowband transmission line phase shifters and transitions phases errors.

To demonstrate the performance of the designed matrix in terms of beamforming, we connect this matrix to an antenna array designed at 5.8 GHz. The photograph of the entire beamforming system is shown in Fig. 15.

As reported in [11], the bandwidth of the element antenna is about 8% and the mutual coupling is better than $-20 \,\mathrm{dB}$. The



Figure 15. Photograph of the beamforming system.



Figure 16. *H*-plane radiation pattern (a) simulated, (b) measured.

spacing between elements is kept $0.5\lambda_0$, where λ_0 is the wavelength at 5.8 GHz. This spacing is necessary to obtain the minimum level side lobes between elements at 5.8 GHz and to preserve the orthogonality condition between different beams.

However, this array has a narrow band (8%), which is not enough to test the matrix in its entire bandwidth. For this issue, we designed two antenna arrays, one is operating at 4.5 GHz while the second at 7.5 GHz. Thus, two cases (4.5 GHz, 7.5 GHz) are considered for beamforming design, when the phase errors are important. The bandwidth of the element antenna is about 8% for each frequency (4.5, 7.5 GHz) and mutual coupling is better than -20 dB. These characteristics allow testing the proposed Butler matrix in terms of beamforming at these worst cases.

Figure 16 shows the simulated and measured results of radiation patterns in the H-plane of the beamforming antenna array system at



Figure 17. Simulated H-plane radiation pattern (a) at 4.5 GHz (b) at 7.5 GHz.

5.8 GHz. With this matrix, four beams are produced for each array at -45° , -15° , 15° and 45° . It can be shown that the comparison between simulated and experimental results indicates a good agreement, which validate the proposed concept. As can be concluded, the phase errors and imbalanced coupling obtained have a minor effect on the side lobe levels of the produced beams.

Figure 17 shows the simulated results of radiation patterns in the H-plane of the beamforming antenna array system at 4.5 GHz and 7.5 GHz. With this matrix, four beams are produced for each array at -45° , -15° , 15° and 45° . As can be concluded, the obtained phase errors have a minor effect on the side lobe levels of the produced beams. These features make the proposed matrix suitable for broadband beamforming applications.

3. CONCLUSION

A novel 4×4 Butler matrix was designed, fabricated and tested using a bi-layer structure. Also, a novel elliptic directional coupler was designed allowing wider bandwidth than conventional CPW branch-line couplers. In addition, a slot-coupled coupler was implemented in order to avoid the use of line crossovers. Simulated and experimental results demonstrate that the proposed approach offers good performance in terms of bandwidth and constant phases. Furthermore, the proposed matrix was successfully connected to an antenna array to form a beamforming system, resulting in four orthogonal beams at 5.8 GHz.

REFERENCES

- Ho, M.-J., G. L. Stuber, and M. Austin, "Performance of switchedbeam smart antennas, for cellular radio systems," *IEEE Trans. Veh. Technol.*, Vol. 47, No. 1, 10–19, Feb. 1998.
- Butler, J. and R. Lowe, "Beamforming matrix simplifies design of electronically scanned antennas," *Electronic Design*, Vol. 9, 170– 173, 1961.
- Jackson, R. W., "Considerations in the use of coplanar waveguide for millimeter wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, Vol. 34, 1450–1456, 1986.
- 4. Ghione, G. and C. Naldi, "Parameters of coplanar waveguides with lower ground plane," *Electron. Lett.*, Vol. 19, 734–735, 1983.
- Corona, A. and M. J. Lancaster, "A high-temperature superconducting butler matrix," *IEEE Trans. on Applied Superconductivity*, Vol. 13, No. 4, Dec. 2003.
- He, J., B.-Z. Wang, Q.-Q. He, Y.-X. Xing, and Z.-L. Yin, "Wideband X-band microstrip Butler matrix," *Progress In Electromagnetics Research*, Vol. 74, 131–140, 2007.
- Traii, M., M. Nedil., A. Gharsallah, and T. A. Denidni, "A novel wideband Butler matrix using multi-layer technology," *Microwave* and Optical Technol. Lett., Vol. 51, No. 3, 659–663, 2009.
- Nedil, M., A. M. Habib, and T. A. Denidni, "Novel ultra-wideband CB-CBW Butler matrix," *IEEE Antennas and Propagation* Society International Symposium, 1–4, 2008.
- Traii, M., M. Nedil, A. Gharsallah, and T. A. Denidni, "A new design of compact 4 × 4 Butler matrix for ISM applications," *International Journal of Microwave Science and Technology*, Vol. 2008, 2008.
- Denidni, T.-A. and M. Nedil, "Experimental investigation of a new butler matrix using slotline technology for beamforming antenna arrays," *Institution of Engineering and Technology (IET) Microwaves, Antennas & Propagation*, Vol. 2, No. 7, 641–649, Oct. 2008.
- 11. Nedil, M., T. A. Denidni, and L. Talbi, "Novel Butler matrix using CPW multilayer technology," *IEEE Trans. Microwave Theory Tech.*, Vol. 54, 499–507, 2006.
- Dall'Omo, C., T. Monediere, B. Jeko, F. Lamour, I. Wolk, and M. Elkael, "Design and realisation of a 4 × 4 microstrip butler matrix without any crossing in millimeter-wave," *Microwave and Optical Tech. Lett.*, Vol. 38, No. 6, Sep. 2003.

- 13. Bona, M., L. Manholm, J. P. Satarski, and B. Svensson, "Low-loss compact butler matrix for a microstrip antenna," *IEEE Trans. on Microwave Theory and Tech.*, Vol. 50, No. 9, Sep. 2002.
- Djerafi, T. and K. Wu, "Multilayered substrate integrated waveguide 4 × 4 Butler matrix," *International Journal of RF and Microwave Computer-aided Engineering*, Vol. 22, No. 3, 336–344, May 2012.
- Chan, K.-L., F. A. Alhargan, and S. R. Judah, "A quadrature hybrid design using a four-port elliptic patch," *IEEE Trans. Microwave Theory Tech.*, Vol. 45, 307–310, 1997.
- 16. HFSS 12.1, ANSYS, Inc. Canonsburg, Pennsylvania, USA.
- Tefiku, F., E. Yamashita, and J. Funada, "Novel directional couplers using broadside-coupled coplanar waveguides for doublesided printed antennas," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, 275–281, Feb. 1996.
- Chang, C.-P., J.-C. Chiu., H.-Y. Chiu., and Y.-H. Wang, "A 3-dB Quadrature coupler using broadside-coupled coplanar waveguides," *IEEE Microwave and Wireless Components Lett.*, Vol. 18, No. 3, Mar. 2008.
- Johnosono, S., T. Fujii, and I. Ohta, "Design of broadband CPW branch-line 3 dB couplers," *Proceedings of the 36th European Microwave Conf.*, 36–39, 2006.
- Wong, Y. S., S. Y. Zheng, and W. S. Chan, "Multifolded bandwidth branch line coupler with filtering characteristic using coupled port feeding," *Progress In Electromagnetics Research*, Vol. 118, 17–35, 2011.
- 21. Li, B., X. Wu, and W. Wu, "A miniaturized branch-line coupler with wideband harmonics suppression," *Progress In Electromagnetics Research*, Vol. 17, 181–189, 2010.
- Riaziat, M., R. Majidi-Ahy, and I. J. Feng, "Propagation modes and dispersion characteristics of coplanar waveguides," *IEEE Trans. Microwave Theory Tech.*, Vol. 38, 245–251, 1990.
- 23. Haydl, W. H., "Resonance phenomena and power loss in conductor-backed coplanar structures," *IEEE Microwave Guided Wave Lett.*, Vol. 20, 514–516, 2000.
- Haydl, W. H., "On the use of vias in conductor-backed coplanar circuits," *IEEE Trans. Microwave Theory Tech.*, Vol. 50, 1571– 1577, 2002.