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STUDIES ON THE DYNAMICS OF A SYSTEM OF BILATERALLY COUPLED CHAOTIC GUNN OSCILLATORS

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Abstract—The dynamics of a system of two bilaterally coupled chaotically oscillating X-band Gunn oscillators (GOs) has been studied by numerical simulation and by hardware experiment. The effect of variation of the coupling strengths between two oscillators in two paths has been explored. The chaotic oscillations in two GOs have become synchronized in most of the cases when coupling factors (CFs) are around 20% or more. However, the transformation of chaotic states of the GOs to quasi-periodic ones has been observed for some values of CFs. A detailed numerical analysis on the instantaneous error parameters of the GO state variables is presented to identify different steady state dynamical conditions of the system. Experimental observations of the GO output frequency power spectra and the averaged product of the two GO outputs in the coupled mode confirm the occurrence of synchronization as well as quenching of chaotic oscillations for different values of CFs.

1. INTRODUCTION

The coupled mode of oscillation of two or more periodic oscillators has been extensively studied in the literature. In this respect the oscillators considered are, electronic [1–4], optical [5], mechanical [6] or biological [7, 8] in nature and the coupling mode taken into account is unilateral (master-slave type) or bilateral (one influencing the other) [9, 10]. Since any real oscillator has inherent nonlinearity, the coupled mode of operation of more than one oscillator leads to several interesting dynamical phenomena like frequency locking, quasi periodicity, chaos, intermittency etc. [10]. In practice,

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coupled electronic oscillators have important applications in spectrally pure signal generation, power combining, coherent modulation and detection, lock in amplification and filtering, frequency synthesis etc. [11–13]. Recently, the dynamics of a system of bilaterally coupled periodic Gunn oscillators (BCPGO) has been studied [14]. There it has been shown that, depending on the magnitude of the coupling factors (CF) between two oscillators, the system could operate in a synchronized state, a quasi periodic state or a chaotic state.

In recent years, chaotic mode of oscillations of physical and biological systems has attracted the interest of several researchers. As such different techniques of chaos generation in physical systems have been invented. Depending on the frequency band of operation, RC oscillators, Colpitts oscillators, PLL based circuits, negative resistance circuits are used in them [15–18]. Also rapid self de-correlation, interference rejection, multipath fading tolerant properties of chaotic signals have made it very useful in communication systems. Further, the synchronization possibility of chaotic signals, first discussed by Pecora and Caroll [19], has increased the interest of studies on coupled chaotic oscillations. As a logical extension of the studies on the BCPGO [14], one may question on the modification of the dynamics of the system if the GOs become chaotic. Adopting the simple method reported in [20] a BCPGO can be converted into a BCCGO system. Here, the GOs would be operated in the under bias condition and a weak external RF field would be injected in the cavities. The motivation of the present study is to examine the effect of the mutual interaction between two chaotic oscillators. It may result into the synchronization between two CGOs or the quenching of chaotic oscillations in one or both. The strength of coupling between the oscillators would have a significant role in the steady state dynamics of the system. We have explored the BCCGO system dynamics through numerical simulation and experimental studies. In practice, a BCCGO represents a model of a bidirectional chaos based microwave communication systems having forward and feedback paths between the transmitter and receiver.

The paper has been organised in the following way. In Section 2 after describing the structure of the BCGO system, its circuit theoretic model is proposed and corresponding differential equations of the system are derived. Section 3 systematically describes the steps of the numerical simulation. The parameter values required for the periodic and the chaotic mode of oscillations of single GO have been obtained. Then the response of a coupled system for various values of CFs is studied. The time evaluations of the error samples between the equivalent state variables of two GOs are examined for verification of

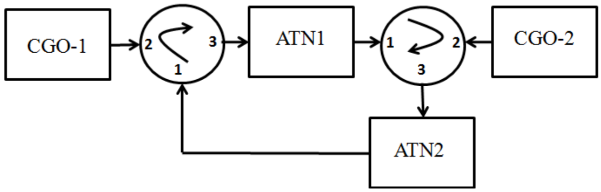


Figure 1. Simplified functional structure of BCCGO.

synchronization between two GOs in the coupled system. The range of values of CFs leading to synchronization is obtained in this section. The experimental study on the dynamics of a BCCGO system has been reported in Section 4. It describes the way of design of the coupled system, the method of variation of the coupling factors between the GOs and procedure of studying the relation between the GOs in frequency domain and also in time domain. The main observations of the study and the potential application of the BCCGO system are discussed in the concluding Section 5.

2. DESCRIPTION OF THE BCCGO SYSTEM AND ITS CIRCUIT THEORETIC MODEL

Figure 1 shows the functional block diagram of the coupled oscillator system under study. As GO is itself a one port system so for their bidirectional coupling, circulators and attenuators are required in the hardware arrangement. At first consider both the GOs are identical in their free running condition and are operating in chaotic mode. Then output of CGO1 (CGO2) is suitably attenuated and coupled with CGO2 (CGO1) through circulators.

Figure 2 shows the circuit theoretic model of the BCCGO System under study. Here, a single periodic GO is modelled [14] as a series network of the device impedance and the cavity impedance. The device impedance comprises of a nonlinear voltage dependant resistance (r) and a nonlinear voltage dependent capacitance (c). v_r and v_c represent respectively the voltage drop across r and c. The cavity impedance consists of a series combination of inductor L, capacitor C and the resistor R. This R takes care of the cavity loss and the load resistor. Applying Kirchhoff's mesh law, the differential equation describing the dynamics of the single GO can be obtained as follows [14]:

$$\frac{d^2q}{d\tau^2} = aq - bq^3 - c\frac{dq}{d\tau} - d\left(\frac{dq}{d\tau}\right)^3 \quad (1)$$

Here $\frac{dq}{dt} = i$ is the instantaneous circulating current in the equivalent

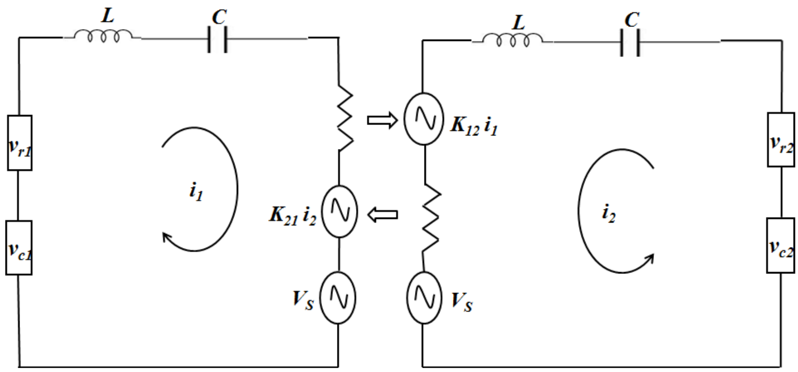


Figure 2. Equivalent circuit theoretic model of the BCCGO system.

mesh, q the instantaneous charge, and $\tau(\omega_r t)$ the time normalized to the cavity resonant frequency ω_r $(= \frac{1}{\sqrt{LC}})$. The coefficients a, b, c, d are related with the device and the cavity parameters and implicitly depend on the magnitude of the applied dc bias (V_B) across the Gunn diode [20].

A periodic GO can be made to operate in the chaotic mode if the applied dc bias be less than the threshold value necessary for negative differential resistance (NDR) mode of operation and a weak RF field of frequency ω_r be injected in the cavity. Mathematically it can be taken into account by considering an additional voltage source in the equivalent circuit and taking the value of c suitable for under bias of operation of the diode. In this condition, system equation will be modified as:

$$\frac{d^2q}{d\tau^2} = aq - bq^3 - c\frac{dq}{d\tau} - d\left(\frac{dq}{d\tau}\right)^3 + q_s\cos(\Omega\tau) \qquad (2)$$

Here q_s and Ω are respectively the charge equivalent to the amplitude and the normalized angular frequency of the external field present within the cavity.

When two such chaotically operated GOs are bidirectionally coupled, the effect of coupling is taken into account by considering two voltage sources, one in each oscillator of the BCCGO system. This is shown in Figure 2. The additional voltage source is proportional to the difference of current flowing through the two oscillators. The constant of proportionality represents the coupling factor (CF) of one oscillator with the other. Thus the differential equations of the bilaterally coupled chaotic oscillator system become

$$\frac{d^2q_1}{d\tau^2} = a_1q_1 - b_1q_1^3 - c_1\frac{dq_1}{d\tau} - d_1\left(\frac{dq_1}{d\tau}\right)^3 + k_{21}\left(\frac{dq_2}{d\tau} - \frac{dq_1}{d\tau}\right) + q_s\cos(\Omega\tau) \quad (3)$$

$$\frac{d^2q_2}{d\tau^2} = a_2q_2 - b_2q_2^3 - c_2\frac{dq_2}{d\tau} - d_2\left(\frac{dq_2}{d\tau}\right)^3 + k_{12}\left(\frac{dq_1}{d\tau} - \frac{dq_2}{d\tau}\right) + q_s\cos(\Omega\tau) \quad (4)$$

Here $(\frac{dq_1}{dt})$, $(\frac{dq_2}{dt})$ represent the instantaneous circulating current through the equivalent loop of CGO1 and CGO2 respectively. k_{12} and k_{21} are the CFs of the two paths by which two CGOs are connected. k_{21} (k_{12}) gives effect of CGO2 (CGO1) to CGO1 (CGO2). Obviously, putting one of the CFs equal to zero in (3) and (4) one can obtain the system equations of the Unilaterally Coupled Chaotic Gunn Oscillator (UCCGO).

3. NUMERICAL ANALYSIS

A complete analytical solution of (3) and (4) is difficult to obtain, if not impossible. Here, these equations are studied numerically. To this end, (3) and (4) are decomposed into two pairs of first order differential equations as follows:

$$\frac{dq_1}{d\tau} = p_1 \quad (5)$$

$$\frac{dq_2}{d\tau} = p_2 \quad (6)$$

$$\frac{dp_1}{d\tau} = a_1q_1 - b_1q_1^3 - c_1p_1 - d_1p_1^3 + k_{21}(p_2 - p_1) + q_s\cos(\Omega\tau) \quad (7)$$

$$\frac{dp_2}{d\tau} = a_2q_2 - b_2q_2^3 - c_2p_2 - d_2p_2^3 + k_{12}(p_1 - p_2) + q_s\cos(\Omega\tau) \quad (8)$$

Then we modelled (5) to (8) using MATLAB based Simulink Software as shown in Figure 3 and perform the numerical experiment.

Two GOs taken in the BCCGO system studied in the present paper are identical in structure. This is ensured in the numerical experiment by choosing identical values of a, b, c and d in two oscillators. The time series data of q's and q's are recorded in different simulation runs. However, the steady state dynamics is observed by disregarding sufficient number of values of state variables q and p from the initial stages of each run. The record of time development of q, the estimation of the components present in the time series data of

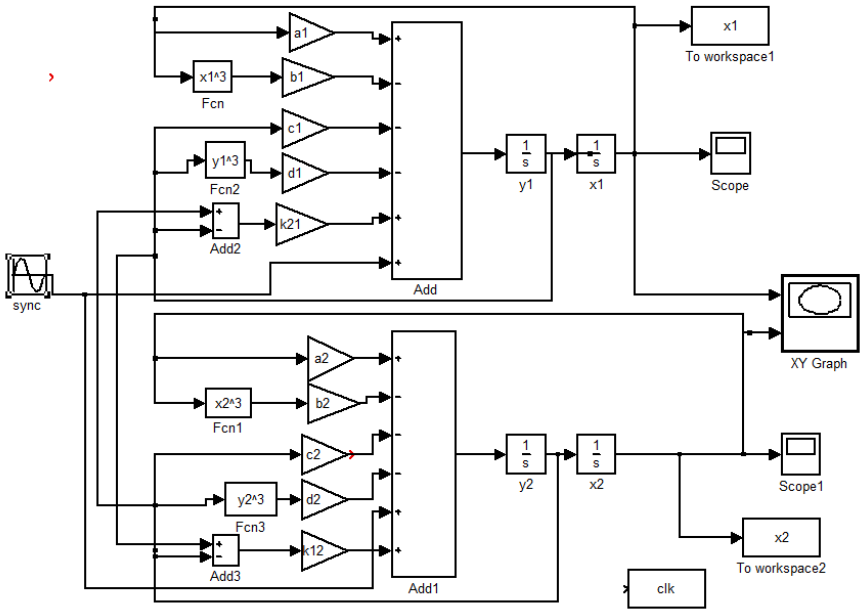
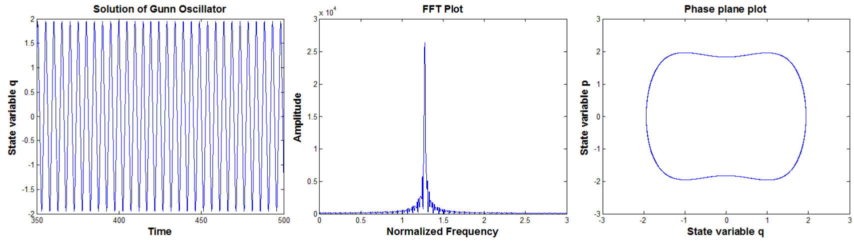


Figure 3. Simulink model of the BCCGO system for numerical experiment.

q, the phase space representation of q and corresponding p etc. are used to study the dynamics of the system under different conditions of oscillations and bilateral coupling of the GOs.

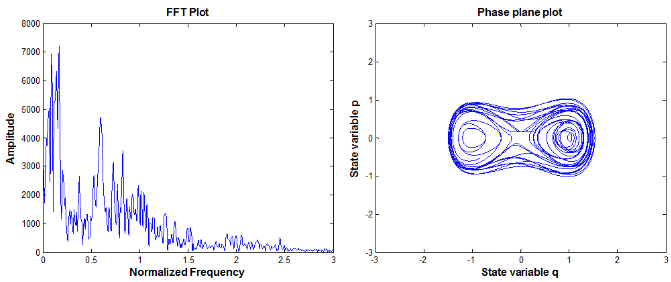
3.1. Free Running Dynamics of a GO in Periodic Mode

With $k_{12} = k_{21} = 0$ and $q_s = 0$ in the system equation, one gets the situation of free running operation of the GO. In this condition, we vary the parameter c, keeping a, b and d fixed at 1.0, 1.0 and 0.015 respectively. In a real system, this represents the variation of the dc bias voltage from low value. The value of the parameter c reduces and becomes negative as the Gunn diode enters into the negative differential resistance region from a normal positive condition. When the value of c is -0.05, the time series data of q indicates a steady periodic oscillation with a steady state amplitude $q_0 = 2.0$ and a normalized frequency of oscillation, $\omega_0 = 1.27$. The time domain curve of q, the frequency spectrum of time series data of q and the phase space representation of q-p data are shown in Figure 4. To operate the GO in an under bias condition [20], the value of c should be less negative compared to -0.06, while other parameters are kept fixed at aforementioned values.



(a) (b) (c)

Figure 4. Numerically obtained results of BCCGO system for $k_{12} = 0$ and $k_{21} = 0$, $a = 1$, $b = 1$, $c = -0.05$, $d = 0.015$, $q_s = 0$, $\Omega_s = 1.27$. (a) Time domain plot of q, (b) frequency spectrum of q, (c) phase space plot of q and p.



(a) (b)

Figure 5. Numerically obtained results of BCCGO system for $k_{12} = 0$ and $k_{21} = 0$, $a = 1$, $b = 1$, $c = +0.002$, $d = 0.015$, $q_s = 0.15$, $\Omega_s = 1.27$. (a) Frequency spectrum of q, (b) phase space plot of q and p.

3.2. Generation of Chaotic Oscillations in GOs

The value of c is taken as 0.002 with $a = 1$, $b = 1$ and $d = 0.015$ to ensure the under bias operation of the GO. The effect of injected RF field in the cavity is taken in the numerical experiment by putting $q_s = 0.15$ and $\Omega_s = 1.27$. The oscillators are kept in the isolated condition by putting the values of k_{12} and k_{21} as zero. In this condition, we obtain the frequency spectrum of the time series data of q and the phase space plot of q-p as shown in Figures 5(a) and 5(b). It indicates that the GOs are in chaotic mode of oscillation. The estimation of the maximum Lyapunov exponent (MLE) and the correlation dimension (CD) of the time series data $q(t)$ obtained in this situation has been done using the commercial Chaos Data Analyser (CDA) software [21]. The obtained values of MLE and CD are 0.02 and 1.795 respectively and this confirms the chaotic state of the GO.

3.3. Studies of the Coupled Dynamics of the BCCGO System

Keeping the individual GO in the chaotic state, we choose non zero values of k_{12} and k_{21} to obtain the bilaterally coupled system. In the numerical experiment, the initial values of state variables q and p of the two GOs are taken independently and the time evolutions

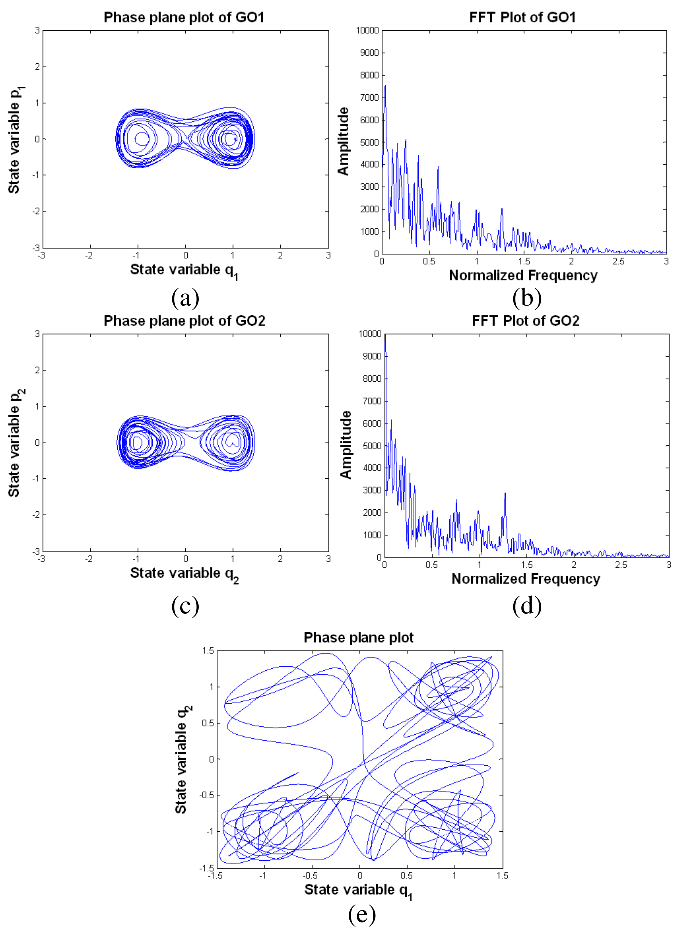


Figure 6. Numerically obtained results of BCCGO system for $k_{12} = 0.15$ and $k_{21} = 0.06$, $a = 1$, $b = 1$, $c = +0.002$, $d = 0.015$, $q_s = 0.15$, $\Omega_s = 1.27$. For CGO1, (a) phase space plot of q_1 and p_1, (b) frequency spectrum of q_1. For CGO2, (c) phase space plot of q_2 and p_2, (d) frequency spectrum of q_2. (e) Phase space plot of q_1 and q_2.

of these variables are examined in the steady state. Our interest here is to find the relation between the outputs of the two GO for different values of the coupling strengths (k_{12} and k_{21}). Intuitively it is anticipated that the coupled dynamics of the GOs could lead to any of the following states: (i) uncorrelated chaotic states (UCS), (ii) completely synchronized chaotic states (CSCS), (iii) uncorrelated quasi

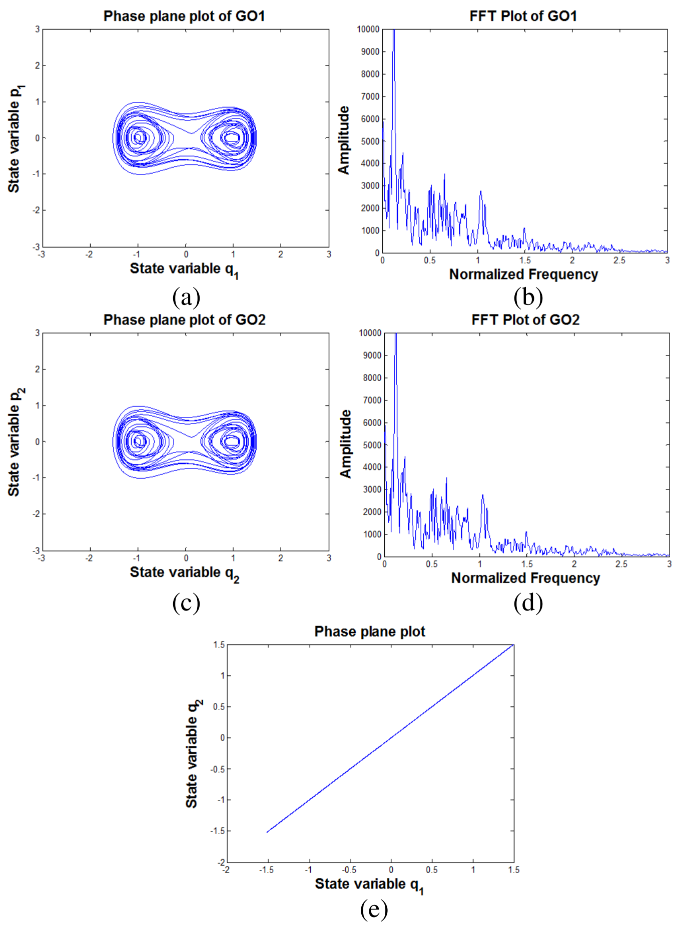


Figure 7. Numerically obtained results of BCCGO system for $k_{12} = 0.15$ and $k_{21} = 0.4$, $a = 1$, $b = 1$, $c = +0.002$, $d = 0.015$, $q_s = 0.15$, $\Omega_s = 1.27$. For CGO1, (a) phase space plot of q_1 and p_1, (b) frequency spectrum of q_1. For CGO2, (c) phase space plot of q_2 and p_2, (d) frequency spectrum of q_2. (e) Phase space plot of q_1 and q_2.

periodic states (UQPS), (iv) synchronized quasi periodic states (SQPS) of two CGOs in the coupled system. Occurrences of these states are identified in the simulation experiment by noting the time evolution of state variables q_1 and q_2 in two dimensional plane. The numerical solution of the system equations has been obtained with device and system parameters $a = 1$, $b = 1$, $c = 0.002$, $d = 0.015$. For these values, the GOs operate in chaotic modes. The values of CFs have been chosen accordingly in each simulation run.

Some results have been shown in Figures 6, 7. The apparent similarity in the spectral characteristics of q_1 and q_2 as well as a linear curve having a slope of approximately 45° in the q_1-q_2 plane give a strong indication of synchronization between the two CGOs for certain values of k_{12} and k_{21} (Case-CSCS). Two CGOs, when bilaterally coupled for certain small values of CFs, remain chaotic in nature but their outputs have no correlation (Case-UCS). This is observed in the structure-less curve in the q_1-q_2 plane. Interestingly it is also observed that in the coupled condition the CGO outputs become quasi-periodic having multiple discrete components. For some values of k_{12} and k_{21} these outputs are synchronized (Case-SQPS) and for some other values they are uncorrelated (Case-UQPS). Figure 8 shows exhaustive results of simulation experiments finding the effects of CFs on the BCCGO dynamics. Here aforementioned four states are indicated by different colours in the two-dimensional plane, where x

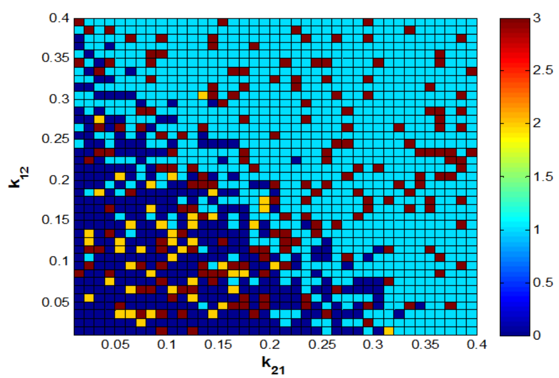


Figure 8. Numerically obtained results finding the effects of CFs on the BCCGO dynamics. Here four states are indicated by different colours in the two-dimensional plane: (i) uncorrelated chaotic states (0), (ii) completely synchronized chaotic states (1), (iii) uncorrelated quasi periodic states (2), (iv) synchronized quasi periodic states (3).

and y represent values of k_{21} and k_{12} respectively. A general remark regarding the results obtained from the simulation is as follows:

1. For small values of CFs (< 0.2) the oscillators behave predominantly in uncorrelated manner even in the coupled condition. The states of the oscillators could be both chaotic and quasi-periodic in this situation (UCS and UQPS).
2. With the increase in the value of one of the CFs (> 0.2) or both chaotic oscillations are synchronized in most of the cases (CSCS).
3. However for some values of CFs in this region, chaotic oscillations are modified to quasi-periodic nature, but the output of the GOs still remain in the synchronized state (SQPS).

The results in the numerical experiment [shown in Figure 8] are consistent with the dynamics of coupled chaotic systems reported elsewhere [22–26]. The CFs between two CGOs are the bifurcation parameters for the transition of the system state from isolated chaos to synchronized chaos or non chaotic state. It is observed that for higher values of CFs (> 0.2), the chaotic oscillations of two GOs become synchronized in about 90% of the simulation runs. The transition of chaotic states to non chaotic quasi-periodic states due to interaction of two CGOs is observed for some simulation runs (about 10%) with large values of CFs. Similar dynamics has been reported for several other coupled chaotic systems (like Rossler system [22], Lorenz system [24], semiconductor laser [23, 25], one dimensional map [26] etc.).

3.4. Studies on Error Samples of State Variables of Two GOs

The difference of corresponding state variables (q or p) of the two CGOs at a particular instant of time can be a good indicator of synchronization between two CGOs [27, 28]. We define e_1 and e_2 as $(q_2 - q_1)$ and $(p_2 - p_1)$ respectively. Then the evolution of e_1 and e_2 in time can be obtained from the time-derivatives of e_1 and e_2. Using Equations (5) to (8) one gets:

$$\dot{e_1} = e_2 \tag{9}$$

$$\dot{e_2} = ae_1 - ce_2 - (k_{12} + k_{21})e_2 - be_1^3 - de_2^3 - 3de_2p_2(p_2 - e_2) - 3be_1q_1(q_1 + e_1) \tag{10}$$

Performing the numerical solution of these equations using fourth order Runge-Kutta technique in Matlab platform one gets the error variables as functions of time. The device and system parameters are chosen as before for operating the GOs in chaotic mode. For a particular simulation run with fixed values of k_{12} and k_{21} one finds

e_1 and e_2. The quantity $\sqrt{(e_1^2 + e_2^2)}$ obtained at each time instant represents the distance between the phase-space trajectories of the two systems described by state-variable pairs (q_1, p_1) and (q_2, p_2). We obtain the time-series data of $\sqrt{(e_1^2 + e_2^2)}$ and find the Maximum Lyapunov exponent λ_d of the samples. It is observed that for small values of k_{12} and k_{21} (< 0.1) in most cases the values of λ_d is positive, indicating the increase of separation between the state-space trajectories of two GOs. This means that the CGOs are in the UCS

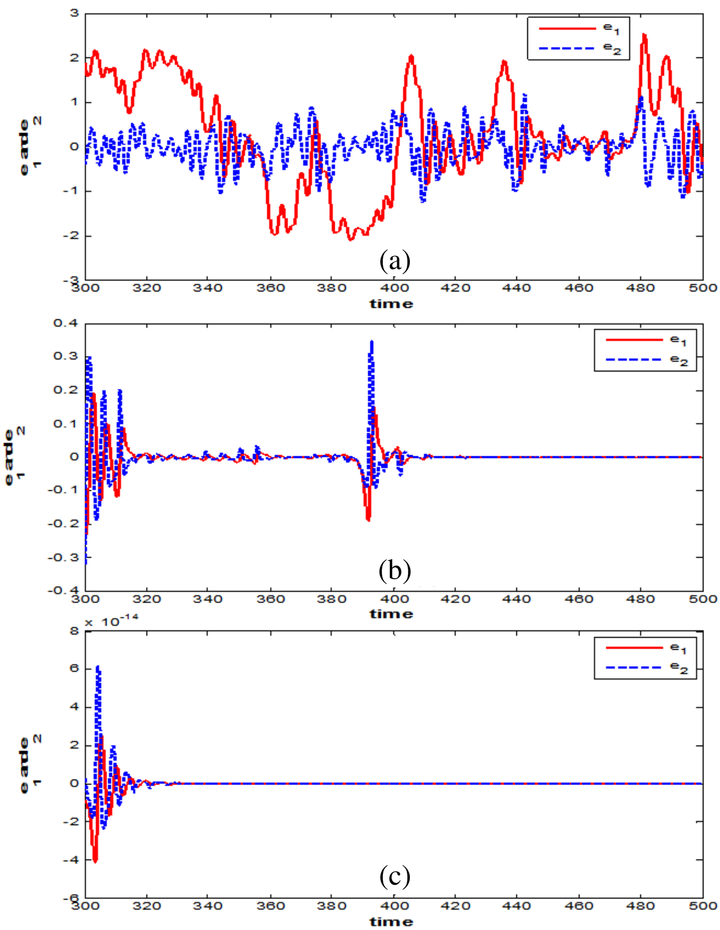


Figure 9. Time evolution of the error dynamics of e_1 and e_2 of the BCCGO system with $k_{12} = 0.15$. (a) $k_{21} = 0.06$, (b) $k_{21} = 0.1$, (c) $k_{21} = 0.3$.

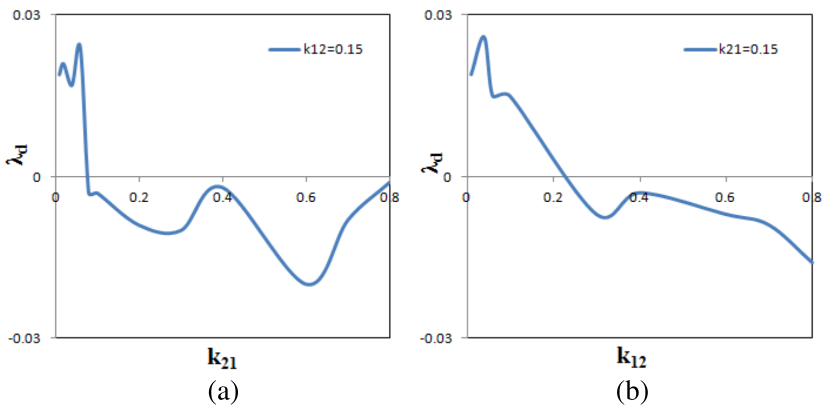


Figure 10. Variation of λ_d with CFs of the BCCGO system. (a) λ_d vs k_{21} for $k_{12} = 0.15$, (b) λ_d vs k_{12} for $k_{21} = 0.15$.

state for lower values of CFs. However for higher values of CFs (> 0.1) in most simulation runs the values of λ_d is obtained negative. This means that the CGOs are in the CSCS state. Figures 9, 10 show some results of the variation of e_1 and e_2 with time as well as the variation of λ_d with CFs.

4. EXPERIMENTAL STUDIES

The experimental study on the BCCGO dynamics is done using a hardware circuit comprising of two wave guide based GOs (VJU, Model No. X2152, Serial Nos. 1031 and 1288), two X-band attenuators (ECIL, Model No. X321, Serial Nos. 0815 and 0818) and two X-band circulators (SICO, Model Nos. XC621 and XC622, Serial Nos. 439 and 440). The hardware arrangement is shown in Figure 11. The coupling coefficients (CFs) are determined by measuring the ratio of the output power to input power at an attenuator for a particular micrometer screw (attached to the attenuator) reading. For convenience of the experiment, the calibration curves showing the attenuation factor as a function of micrometer screw reading of an attenuator is obtained beforehand.

The GOs are operated in the under biased condition with a weak injected RF field in the oscillator cavities to produce chaotic oscillations. The wave guide cavities of the GOs are so adjusted that the GOs oscillate at a frequency 10.000 GHz in the periodic mode with

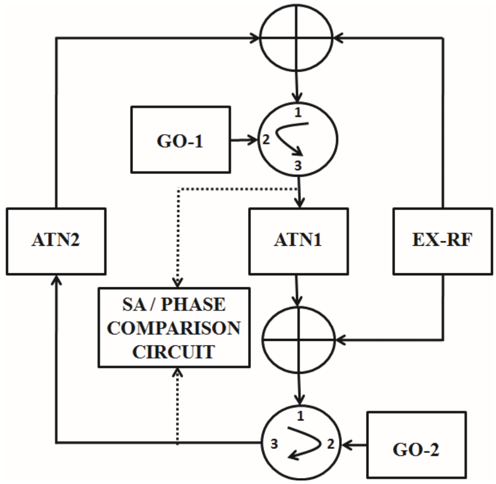


Figure 11. Simplified block diagram of the hardware arrangement of BCCGO system.

the dc bias 9.5 Volts. The output power of the GOs is found to be 14.5 dBm. The frequency and the power of the GOs are measured using a spectrum analyzer (Rhode and Schwarz, FSL SA, 9 kHz–18 GHz). For chaotic oscillations, the dc bias of the GOs are fixed at 5.08 Volts and injected external RF field to the GO cavities is taken from a microwave signal generator (Agilent N5183A, 100 kHz–20 GHz). The frequency (f_{RF}) and the power (P_{RF}) of the injected field are fixed at 10.025 GHz and 0 dBm respectively. These insures the requirements for chaotic oscillation of the GOs, namely, an injected weak RF field of frequency close to the cavity resonant frequency with the dc bias below the threshold dc voltage [20]. The choice of identical cavity dimensions and equal dc bias voltages ensures the coupling between two identical CGOs in our experiment.

The output spectra of two GOs in the coupled condition are recorded to study the dynamics of the system. The influence of the CFs in the dynamics is examined by repeating the experimental observations for several sets of k_{12} and k_{21}. Some representative observations are given in Figure 12. Here the value of k_{21} is kept at 0.18 and that of k_{12} is varied. It is observed that for low value of k_{12}, the GO outputs remains chaotic, but their spectral nature are different as observed in the SA. However for some relatively higher values of k_{12} (~ 0.18 or more), the spectral nature of the outputs of the GOs are almost similar as observed in the SA record. Qualitatively, these observations may be interpreted as the synchronization of two CGOs in

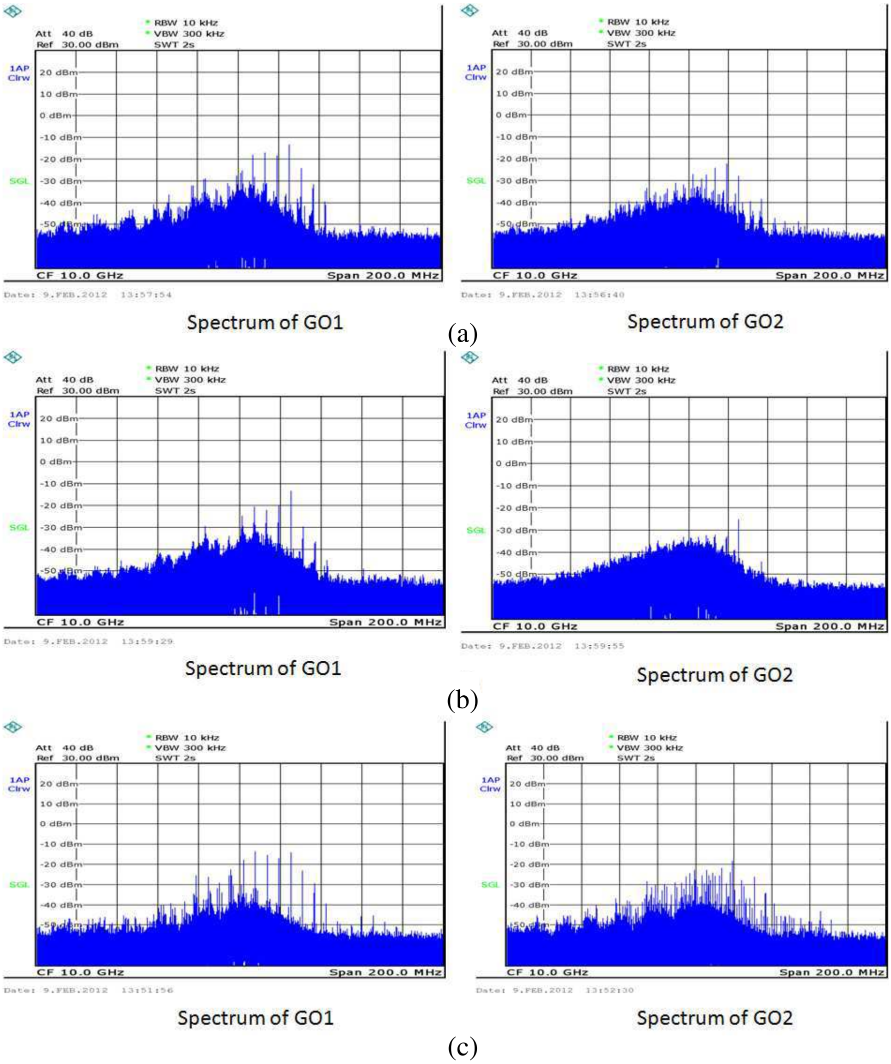


Figure 12. Experimentally obtained output spectra of two GOs in the BCCGO system for different CFs, $f_{\mathrm{RF}} = 10.025\,\mathrm{GHz}$, $P_{\mathrm{RF}} = 0\,\mathrm{dBm}$, $v_{B1} = v_{B2} = 5.08\,\mathrm{Volts}$, $k_{21} = 0.18$. (a) $k_{12} = 0.17$, (b) $k_{12} = 0.07$, (c) $k_{12} = 0.56$.

the bilaterally coupled mode. For still higher values of k_{12} one gets the modification of chaotic output of the GOs. Specifically, the continuous broadband spectrum of the CGOs output is converted into a spectrum

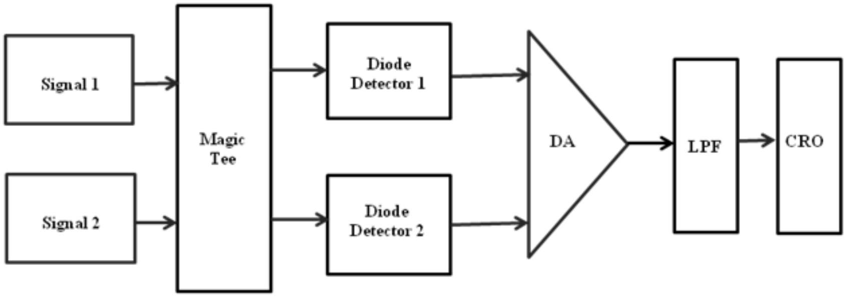
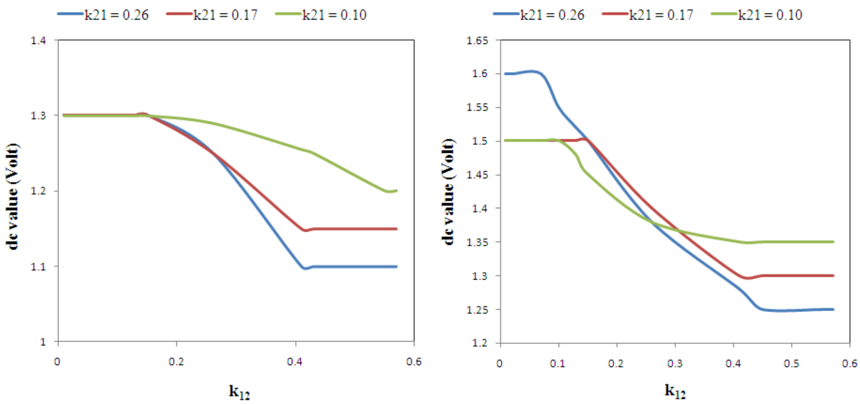


Figure 13. Simplified structure of the phase comparator circuit.

having discrete lines. The frequencies of the component signals are incommensurately related. As such we get quasi-periodic output from the CGOs. Thus the nature of coupling between two CGOs changes the dynamics of the system from chaotic to quasi-periodic type.

However, the frequency domain measurement is not able to convincingly prove the synchronization between two chaotic oscillations. Also to compare the nature of two chaotic signals at 10 GHz frequency region in the time domain, one requires real time oscilloscope in the X-band. In the absence of such infrastructure, we derived an indirect method of comparing the time domain nature of two CGOs outputs. A microwave frequency phase comparator circuit (PCC) is realized using a magic tee (SICO, Model No. XE-350, Serial No. 764), two diode detectors (SICO, Model No. XD451, Serial Nos. 2723 and 2724), a difference amplifier [29]. The structure of the PCC is shown in Figure 13. Some elementary experimental works have been reported in this regard [30].

First we operate the GOs of the bilaterally coupled system in periodic mode (applying proper dc bias) and adjust their frequencies at slightly different values (about 20 MHz off tuned). The outputs of the GOs are applied to the input ports of the magic tee in the PCC circuit. Keeping k_{21} (= 0.1) fixed and varying k_{12} we note in the SA that GOs become synchronized for k_{12} around 0.2. At that condition, the dc voltage obtained at the output of the difference amplifier of the PCC gives a reduced value compared to the value obtained for lower k_{12} with GOs in the not synchronized condition. A further increased in k_{12} gives further reduction in the dc value. Taking the other value of k_{21} (0.17 and 0.26) and repeating the experiment by changing k_{12}, we observed similar nature in the value of output dc voltage. The reason for these observations is as follows: the dc voltage at the PCC is proportional to the phase difference (PD) between two signals applied at the PCC



(a) (b)

Figure 14. (a) Variation of dc voltage as obtained experimentally from the PCC with k_{12} when two GOs are periodic and of different frequencies (GO1 10.0 GHz, GO2 9.98 GHz), (b) variation of dc voltage as obtained experimentally from the PCC with the coupling factor k_{12} when both the GOs are in chaotic state.

input. The decrease in the values of the steady state PD with the increase of k_{12} reduces the value of the dc voltage. Figure 14(a) shows the results of one such experiment. Thus, the increase of the CFs between the two GOs brings them phase synchronized condition and this is evident from the reduced value of the PCC output. Next we operate the GOs in the chaotic mode by applying under biased dc voltage and injecting weak RF signal in the cavity. The experiment is then repeated by varying k_{12} keeping k_{21} fixed. The recorded values of the PCC output dc are shown in Figure 14(b) for different cases. Qualitatively, one can interpret the observations as synchronization of chaotic outputs of the GOs for a range of values of k_{12} and k_{21}.

However one has to remember the broad band spectral nature of the chaotic signal in interpreting the above mentioned experimental observations. The output of PCC circuit is time averaged version the product of $E_1(t)$ and $E_2(t)$, where $E_1(t)$ and $E_2(t)$ are the inputs to the magic tee of PCC. When these input signals are periodic and are synchronized, the PCC output is a dc voltage, proportional to PD between $E_1(t)$ and $E_2(t)$. But when $E_1(t)$ and $E_2(t)$ are chaotic, they would have large number of component signals. Thus, even the corresponding component signals of same frequencies of $E_1(t)$ and $E_2(t)$ are at constant PD, the low pass version of the product of $E_1(t)$ and $E_2(t)$ would be a sum of a dc and ac component. The ac voltage part is due to the cross product of different components having unequal frequencies. So, one could not get only a pure dc output even in the synchronized state of the GO outputs. However the bandwidth of the chaotic signals at GO outputs is 100 MHz to 120 MHz. This is only about one to two percent of the centre frequency (about 10 GHz). Hence, the ac part of PCC output would be averaged by the tuned detectors used in the experiment. Our experimental observations confirmed the synchronization of CGOs for a range of CFs and these are consistent with the observations made with the SA in the frequency domain.

5. CONCLUSION

In the paper, we report a detailed investigation on the dynamics of the BCCGO system through numerical simulation as well as experimental studies. It is observed that two chaotically oscillating GOs when bilaterally coupled can enter into different states depending on the values of the coupling strengths. When both the CFs are small, i.e., when two CGOs interact weakly, their dynamics although affected but remains uncorrelated. However, the interaction of CGOs for higher values of CFs (one or both) leads to qualitative synchronization of the chaotic oscillations of the GOs. Numerical simulation of the mathematical model of the BCCGO system reveals the influence of the CFs on the dynamics of the system. The synchronization of chaotic GOs is established by showing the stability of the error dynamics of the state variables of the two oscillators. Experimental evidence of chaos synchronization is obtained through the observation of the structural similarity between the frequency-power spectra of two CGO outputs. Further, we have examined the time domain average of the product (dc voltage) of GO outputs using a phase comparator circuit, first with periodically oscillating GOs and then with chaotic GOs in coupled condition. The dc value is found to decrease with increasing CF when periodic GOs are in the synchronized state. With chaotic GOs, similar type of decreasing dc value is obtained with increasing CFs, which qualitatively proves the occurrence of synchronization between two GOs. Realisation of mutually synchronized chaotic GOs in the bilaterally coupled condition is very useful in the direction of implementing chaos based microwave communication system. By varying the CFs between two CGOs in the synchronized state, it would be possible to get deterministically phase shifted versions of chaotic oscillations, capable of carrying information bits. Further, it is observed that the chaotic oscillations of the GOs may be modified in the bilateral coupling mode and quasi periodic state of the GOs may result for some values of the CFs. Hence, it is concluded that by properly designing the bilateral coupling structure, the chaotic oscillations of two oscillators could be controlled.

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