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EVALUATION OF LIGHTNING CURRENT AND VELOCITY PROFILES ALONG LIGHTNING CHANNEL USING MEASURED MAGNETIC FLUX DENSITY

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Abstract—In this paper, an inverse procedure algorithm is proposed in the time domain to evaluate lightning return stroke currents along a lightning channel using measured magnetic flux density at an observation point while the current velocity along a lightning channel is assumed to be a height dependent variable. The proposed method considers all field components and it can evaluate the full shape of currents and the current velocity at different heights along a lightning channel. Moreover, a sample of measured magnetic flux density from a triggered lightning experiment is applied to the proposed algorithm and the evaluated currents and current velocities are validated using a measured channel base current and magnetic flux density at another observation point.

1. INTRODUCTION

Lightning current is an important parameter for considering the direct and indirect effects of lightning on power lines, for which the direct effects are considered for lightning strikes on power conductors, shielding wires and towers. On the other hand, the indirect effects of lightning are considered when investigating the coupling between the electromagnetic fields due to a lightning channel and power lines

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when lightning strikes the ground or any object near a line. Several methods are used to evaluate the lightning current and these can be classified into two main groups, i.e., the methods based on direct measurement of current and the methods based on using inverse procedure algorithms. In the direct measurement of lightning current, current coils can be installed at the top of towers or on the ground surface in the case of triggered lightning experiments [1]. Therefore, in order to use measured currents in the lightning studies, simulation using current functions can be done where the function parameters are determined based on measured current and using numerical methods [2, 3]. Likewise, the lightning current can be evaluated based on measured electromagnetic fields at a number of observation points at different distances with respect to a lightning channel using inverse procedure algorithms [4–8]. The comparison between the direct measurement method and the inverse procedure algorithms shows that the direct measurement methods can consider only a limited number of lightning occurrences while the inverse procedure algorithms can cover a wide range of lightning occurrences based on recorded electromagnetic fields that arise from these events. Furthermore, the direct measurement methods only consider the channel base current while the inverse procedure algorithms can evaluate the current behaviour at different heights along a lightning channel. In this study, the inverse procedure algorithm is considered to evaluate the lightning currents using measured fields due to a lightning channel.

Several studies have been undertaken by previous researchers to evaluate lightning current using measured electromagnetic fields at observation points. A number of inverse procedure algorithms only consider the radiation component of the electromagnetic fields in order to simplify the field expressions [9–13]. This means such algorithms can only be used at far distances from a lightning channel at which point the radiation component of the electromagnetic fields has more effect on the total field. Furthermore, in these algorithms a current model is a necessary basic assumption and the return stroke velocity is assumed to be a constant parameter equal to the average value of the velocity along a lightning channel. However, experimental measurements show that the return stroke velocity is actually a height dependent variable that has an increasing trend over the first few metres of height along a lightning channel and a decreasing trend after passing a peak value [14, 15]. On the other hand, some inverse procedure algorithms consider all the field components in the frequency domain while they can evaluate current values at a number of sample frequencies by settling on a known current model and assuming the velocity as a constant value along a lightning channel [4–6, 16]. These algorithms have some limitations on the number of sensors and the radial distances from a lightning channel while the synchronization of the recorded data over a large number of field sensors at different distances from the lightning channel is complicated. Also, the limitation on the radial distance from a lightning channel is another problem of these algorithms while the striking point of the lightning with respect to the field sensors is not predictable.

In this study, an inverse procedure algorithm is proposed in the time domain to evaluate the lightning return stroke current using measured magnetic flux density on the surface of the ground with a certain radial distance with respect to a lightning channel that is usually estimated using a Lightning Location Systems (LLS). The proposed method considers all the field components and also the variation of velocity at different heights along a lightning channel. Moreover, it can support different current models based on the general form of engineering current model that is proposed by Rakov [17–19]. Furthermore, it evaluates the full shape of the lightning current at different heights along a lightning channel based on the measured magnetic flux density. Therefore, after expressing the proposed method in the next part of this paper, a sample of the measured magnetic flux density is applied to the proposed method and the evaluated channel base current is compared to the corresponding measured channel base current. Moreover, the evaluated currents and the velocity profile are used for the simulation of magnetic flux density at another observation point to validate the evaluated currents and velocity profile by comparison between simulated and the corresponding magnetic flux densities at another observation point. It should be mentioned that the measured data has been obtained from a triggered lightning experiment in Florida, USA. The basic assumptions in this study are as follows:

1 — The lightning channel is assumed to be a vertical channel without any branches.

2 — The ground conductivity is assumed to be infinite.

3 — The corona current is neglected in this work.

2. LIGHTNING RETURN STROKE CURRENT

The return stroke current can be classified in two cases, i.e., the channel base current and current at different heights along lightning channel. The channel base current can be simulated by current functions whilst the latter can be modelled by current models. The widely used current functions are represented by:

1 — The Bruce and Gold function (BG) [20].

2 — The improvement of Uman and McLain on the BG function [21].

3 — The improvement of Jones on the BG function [22].

4 — The Pierce and Ciones function [23, 24].

5 — The Heidler function [25].

6 — The improvement of Diendorfer and Uman on the Heidler function (DU) [26].

7 — The improvement of Nucci on the Heidler function [27].

In this study, the DU function is used to express the evaluated channel base current that will be obtained from the proposed inverse procedure algorithm as given by Equation (1). This is because the previous studies have shown that the DU function is in good agreement with the corresponding measured channel base current. It is important to mention that the first four current functions have a problem with the value of di/dt as an effective part of the electromagnetic field expressions at $t = 0$. In the case of lightning, when $t = 0$ then di/dt must also be equal to zero, but this is not the case for these current functions.

$$i(0,t)=\left[\frac{i_{01}}{\eta_1}\frac{\left(\frac{t}{\Gamma_{11}}\right)^{n_1}}{1+\left(\frac{t}{\Gamma_{11}}\right)^{n_1}}\exp\left(\frac{-t}{\Gamma_{12}}\right)+\frac{i_{02}}{\eta_2}\frac{\left(\frac{t}{\Gamma_{21}}\right)^{n_2}}{1+\left(\frac{t}{\Gamma_{21}}\right)^{n_2}}\exp\left(\frac{-t}{\Gamma_{22}}\right)\right] \quad (1)$$

where i_{01}, i_{02} are the amplitudes of the channel base current, Γ_{11}, Γ_{12} are the front time constants, Γ_{21}, Γ_{22} are the decay-time constants, n_1, n_2 are the exponents $(2 \sim 10)$, η_1, η_2 are the amplitude correction factors.

On the other hand, different current models are presented to consider on the behaviour of current at different heights along lightning channel. These models can be classified into four main groups as follows [17, 18]:

1 — The Gas dynamic current models [28, 29].

2 — The Electromagnetic current models [30, 31].

3 — The Distributed circuits models [32, 33].

4 — The Engineering current models [34, 35], including the Bruce-Golde model (BG) [20], transmission-line model (TL) [21, 36], the modified transmission line model with linear current decay with height (MTLL) [37], the modified transmission line model with exponential current decay with height (MTLE) [38], the modified transmission line model with linear current decay and dispersion with height (MTLD) [39] and the travelling current source model (TCS) [40] as widely used current models.

Table 1. $P(z')$ and v for five widely used engineering models based on Equation (2) [41].

Model	$P(z')$	v
BG	1	∞
TCS	1	$-c$
TL	1	v_f
MTLL	$\left(1-\frac{z'}{H}\right)$	v_f
MTLE	$\exp\left(-\frac{z'}{\lambda}\right)$	v_f

In this study, the Engineering current models are considered, as the number of unknown variables is lower for these models than for the other current models. In the Engineering current models, the current at different heights along a lightning channel can be expressed by a function based on the channel base current while this function can be linear or nonlinear. Moreover, some Engineering current models can be considered under a general form proposed by Rakov as expressed by Equation (2) [17].

$$I\left(z',t\right)=I\left(0,t-\frac{z'}{v}\right)\times P(z')\times u\left(t-\frac{z'}{v_f}\right) \tag{2}$$

where: z' is the temporary charge height along lightning channel, $I(z',t)$ is current distribution along lightning channel at any height z' and any time t, $I(0,t)$ is channel base current, $P(z')$ is the attenuation height dependent factor, v is the current-wave propagation velocity, v_f is the upward propagating front velocity, u is the Heaviside function defended as

$$u\left(t-\frac{z'}{v_f}\right)=\begin{cases}1 & \text{for } t\geq\frac{z'}{v_f}\\ 0 & \text{for } t<\frac{z'}{v_f}\end{cases}$$

Equation (2) shows the currents at different heights along lightning channel can be expressed by the current values at channel base and the attenuation height dependent factor. Table 1 gives the attenuation height dependent factor and also the return stroke current velocity for a number of engineering current models based on the variable parameters in Equation (2), where H is the cloud height and λ is the decay factor.

3. RETURN STROKE VELOCITY

The return stroke velocity is an effective parameter for modelling the lightning current at different heights along a lightning channel and also for the simulation of electromagnetic fields at an observation point. Several studies have been undertaken to measure the return stroke velocity at different height levels of a lightning channel in which they show the return stroke velocity is lower than the speed of light in free space (c) and it is a height dependent variable [15, 42, 43]. It is important to mention that the return stroke velocity is usually entered into a model of the current and also the field calculations by a constant value equal to the average of velocities at different heights along a channel that is typically equal to $c/3$ to $2c/3$ [14]. However, a general function has been proposed by Cooray to consider the profile of the velocities along a lightning channel as expressed by Equation (3) [15] where v_1, v_2, v_3, v_4, λ_1, λ_2, λ_3 and λ_4 are constant factors.

$$v\left(z'\right)=\begin{cases} v_1+\left(\frac{v_2}{2}\right)\left\{2-\exp\left(\frac{-(z'-1)}{\lambda_1}\right)-\exp\left(\frac{-(z'-1)}{\lambda_2}\right)\right\}, & 1\leq z'\leq 50 \\ v_3\exp\left(\frac{-z'}{\lambda_3}\right)-v_4\exp\left(\frac{-z'}{\lambda_4}\right), & z'\geq 50 \end{cases} \tag{3}$$

In this study, the return stroke velocity is assumed to be a height dependent function based on Equation (3).

4. THE PROPOSED INVERSE PROCEDURE ALGORITHM

The magnetic flux density due to a vertical lightning channel with variable values of velocities along the lightning channel at an observation point on the surface of the ground can be evaluated by Equation (4) based on the dipole and trapezoid methods [44, 45]

$$B_\varphi^n = \begin{cases} -10^{-7}\times\Delta h\sum_{j=1}^{n} F_1(j), & n>0 \\ 0, & n=0 \end{cases} \tag{4}$$

where: Δh is the channel height step,

$$z' = n\Delta h,$$

n is the number of height steps along lightning channel $(1, 2, \ldots, n_{\max})$, v_n is the return stroke velocity in each per unit of lightning channel,

$$t_n = \sum_{i=1}^{n}\frac{\Delta h}{v_i}+\frac{\sqrt{(n\Delta h)^2+r^2}}{c},$$

$$R\left(z'=n\Delta h\right)=\sqrt{(n\Delta h)^2+r^2},$$

$$i\left(z', t_n - \frac{R(z')}{c}\right) = P(z' = n\Delta h) \times i\left(0, t_n - \sum_{i=1}^{n} \frac{\Delta h}{v_i} - \frac{\sqrt{(n\Delta h)^2 + r^2}}{c}\right),$$

$$F_1(j) = \begin{cases} k \times \left[\dfrac{P(\Delta h)}{\sqrt{\Delta h^2 + r^2}} \times \dfrac{d\left\{ i\left(0, t_n - \dfrac{\Delta h}{v_1} - \dfrac{\sqrt{\Delta h^2 + r^2}}{c}\right)\right\}}{dr} - \dfrac{r \times P(\Delta h) \times i\left(0, t_n - \dfrac{\Delta h}{v_1} - \dfrac{\sqrt{\Delta h^2 + r^2}}{c}\right)}{[\Delta h^2 + r^2]^{3/2}} \right] \\ + \left[\frac{1}{r} \times \dfrac{d\left\{ i\left(0, t_n - \frac{r}{c}\right)\right\}}{dr} - \dfrac{i\left(0, t_n - \dfrac{r}{c}\right)}{r^2} \right] & \text{if } j = 1 \\ k \times \left[\dfrac{P(j\Delta h)}{\sqrt{(j\Delta h)^2 + r^2}} \times \dfrac{d\left\{ i\left(0, t_n - \sum_{i=1}^{j} \dfrac{\Delta h}{v_i} - \dfrac{\sqrt{(j\Delta h)^2 + r^2}}{c}\right)\right\}}{dr} - \dfrac{r \times P(j\Delta h) \times i\left(0, t_n - \sum_{i=1}^{j} \frac{\Delta h}{v_i} - \dfrac{\sqrt{(j\Delta h)^2 + r^2}}{c}\right)}{[(j\Delta h)^2 + r^2]^{3/2}} \right] & \text{if } j > 1 \end{cases}$$

$$k = \begin{cases} 1, & j = n \\ 2 & j \neq n \end{cases}$$

Therefore, Equation (4) can be expressed by Equation (5) as a nonlinear equations system for different time steps as follows:

$$\begin{cases} -10^{-7} \times \Delta h \sum_{j=1}^{1} F_1(j) = B_\varphi^1 \\ -10^{-7} \times \Delta h \sum_{j=1}^{2} F_1(j) = B_\varphi^2 \\ \vdots \\ -10^{-7} \times \Delta h \sum_{j=1}^{n_{\max}} F_1(j) = B_\varphi^{n_{\max}} \end{cases} \tag{5}$$

By substituting the measured magnetic flux densities (B_m^n) at different time steps in the right hand side of Equation (5) and also transferring each of the measured values to the left hand side of the corresponding expression in Equation (5), the nonlinear equation system for the inverse procedure algorithm can be expressed by Equation (6) as follows:

$$\begin{cases} -10^{-7} \times \Delta h \sum_{j=1}^{1} F_1(j) - B_m^1 = 0 \\ -10^{-7} \times \Delta h \sum_{j=1}^{2} F_1(j) - B_m^2 = 0 \\ \vdots \\ -10^{-7} \times \Delta h \sum_{j=1}^{n_{\max}} F_1(j) - B_m^{n_{\max}} = 0 \end{cases} \tag{6}$$

Equation (6) shows that the proposed nonlinear equation system is dependent on the $F_1(j)$ expression while it is also dependent on the channel base current function, current model and return stroke velocity at different heights along a lightning channel. Therefore, by setting the DU current function with unknown parameters as a general form of the channel base current function and the MTLE (or MTLL) model as a current model and using Equation (3) with unknown parameters for considering the velocity behaviour of the $F_1(j)$ expression and entering the appropriate value of Δh in the calculations, the number of unknown parameters will be 19 parameters (10 for the channel base current, 1 for the current model and 8 for the velocity function). Noted that the accuracy of the results and also the processing time have the inverse relationship with the Δh parameter. In this paper, the value of Δh is set at 4 m. Therefore, after inserting the measured magnetic flux densities at different time steps (t_n) and geometrical parameters into Equation (6), the created non-linear equation system can be solved using different numerical methods.

In this study, the particle swarm optimization algorithm (PSO) [46–54] is applied to solve Equation (6) where the minimization of each expression in Equation (6) is considered in a multi objective function. Therefore, the value of each expression in Equation (6) can be minimized at the roots of the system whereas the roots of Equation (6) are the constant parameters of the current function, current model and velocity function. Noted that the values of factors in the DU current function are the positive integer values between 2 to 10 which are determined by PSO algorithm.

The proposed method can be applied for constant velocity case while the values of v_i parameters in Equation (4) can be substituted by an unknown parameter (v). Therefore, the number of unknown parameters in Equation (6) can be reduced to 11. In this study, the number of population and iteration are set on 200 and 100, respectively [46, 47].

The proposed method was applied on a sample of return stroke current that was obtained from the triggered lightning at Camp Blanding Florida based on the geometry of the lightning channel and the field sensors as illustrated in Figure 1. The first magnetic flux

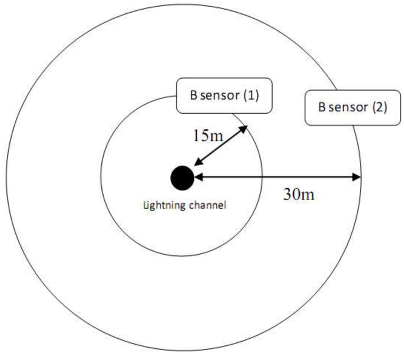

Figure 1. The geometry of field sensors.

density sensor ($r = 15\,\mathrm{m}$) is used for the feeding of the proposed inverse procedure algorithm located on the surface of the ground. Also, in order to validate the evaluated currents and return stroke velocity the evaluated channel base current is compared to the corresponding measured current. Moreover, the evaluated current at different heights and also the evaluated voltage profile along the lightning channel are used for the simulation of magnetic flux density at another observation point ($r = 30\,\mathrm{m}$) and the simulated field is compared to the corresponding measured field (at the second field sensor in Figure 1). In this study, the MTLE and MTLL current models are applied into the proposed method and the results are compared accordingly.

Figure 2 shows the measured magnetic flux density at $r = 15\,\mathrm{m}$ based on Figure 1 that is used for feeding of the proposed inverse procedure algorithm. Therefore, by setting the MTLE model as a current model, Figures 3 and 4 illustrate the evaluated channel base current and the simulated magnetic flux density at $r = 30\,\mathrm{m}$ based on the evaluated currents and voltage profile, respectively and these values are compared to the corresponding measured values. Figure 3 illustrates the evaluated current based on constant velocity, which is compared to the corresponding measured current and similar current based on velocity changes along lightning channel. Result demonstrates that the evaluated current based on velocity changes along lightning channel is in good agreement to the corresponding measured current compared to similar current based on constant velocity along lightning channel.

Figures 3 and 4 show the evaluated current and also the simulated field at another observation point are in a good agreement with respect to the corresponding measured current and field, respectively. Moreover, the measured magnetic flux density is applied to the

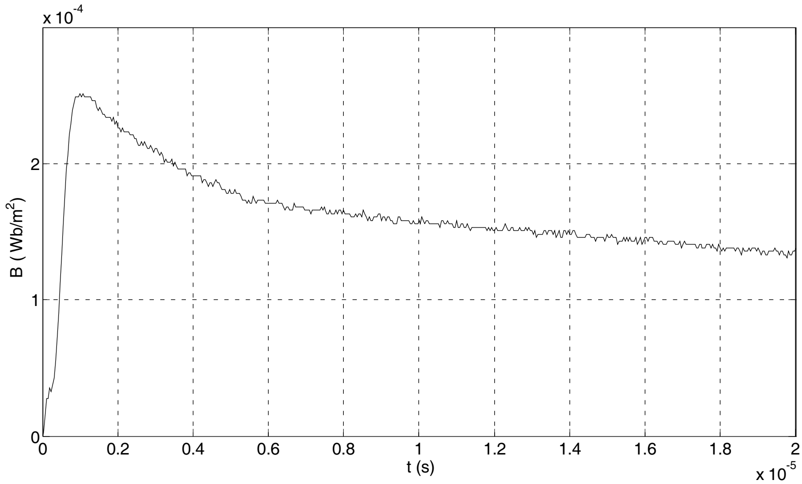

Figure 2. The measured magnetic flux density at $r = 15\,\mathrm{m}$ distance from lightning channel.

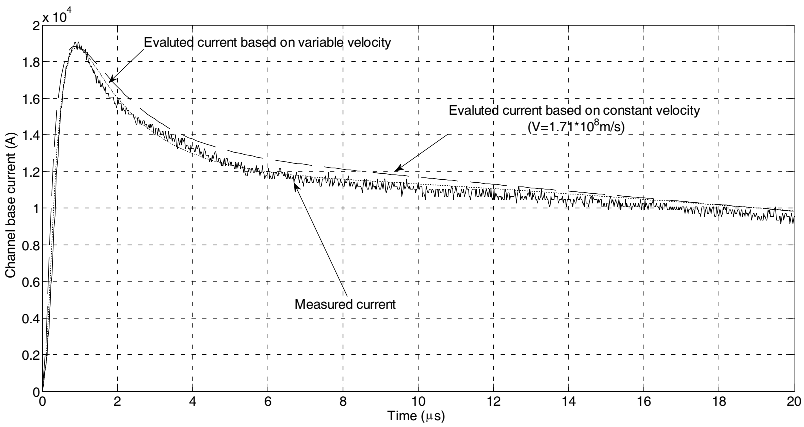

Figure 3. Comparison between the evaluated channel base currents and the corresponding measured current when the MTLE current model is applied to proposed method.

proposed inverse procedure algorithm using the MTLL model as a current model. Figures 5 and 6 show the evaluated channel base current and the simulated magnetic flux density at another observation point, respectively and they are compared to the corresponding measured data. Likewise, the current on the ground surface is

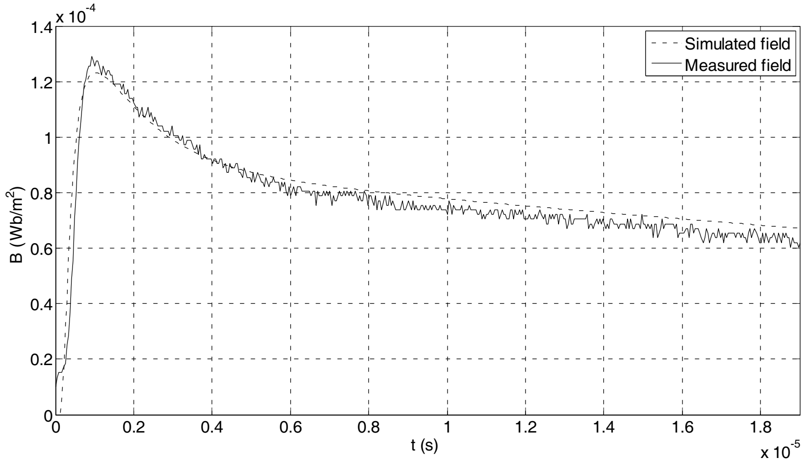

Figure 4. Comparison between the simulated magnetic flux density and the corresponding measured field at $r = 30\,\mathrm{m}$ when the MTLE current model is applied to proposed method.

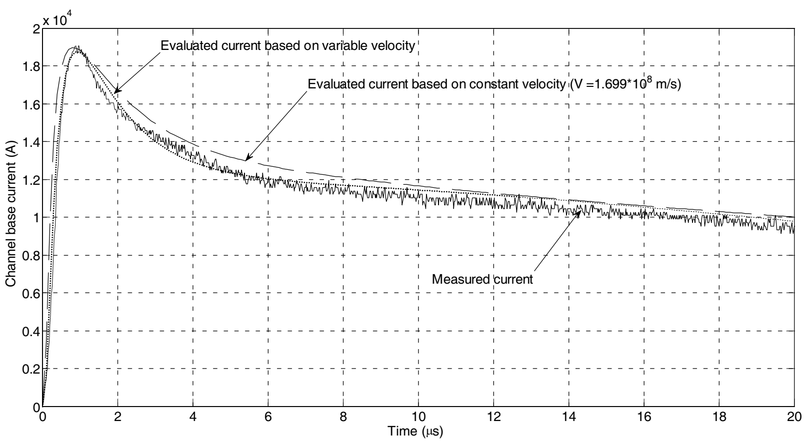

Figure 5. Comparison between the evaluated channel base current and the corresponding measured current when the MTLL current model is applied to proposed method.

evaluated based on constant value of velocity along lightning channel and it is compared to the corresponding measured current and similar current based on velocity changes along lightning channe. Figure 5 illustrates that the evaluated current based on variation of velocity along lightning channel is in a better agreement with measured current compared to similar current based on constant value of velocity.

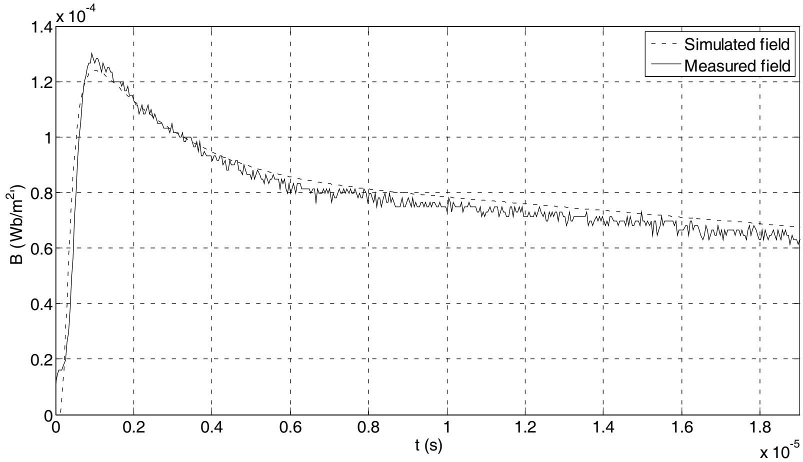

Figure 6. Comparison between the simulated magnetic flux density and the corresponding measured field at $r = 30\,\mathrm{m}$ when the MTLL current model is applied to proposed method.

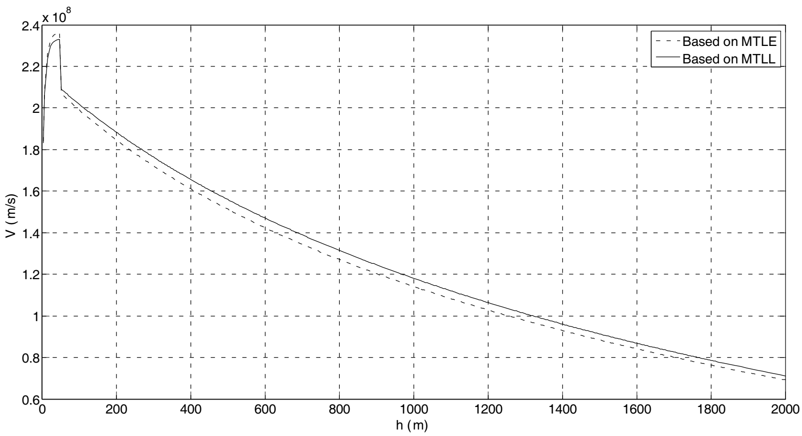

Figure 7. The velocity profiles that are obtained from inverse procedure algorithm.

The results demonstrate that the evaluated current and fields are in a good agreement with the corresponding measured current and field. The evaluated voltage profiles for both the MTLE and MTLL current model cases are shown in Figure 7 which illustrates an increasing trend of the velocity value at a height of a few meters along the lightning channel that then reduces at greater heights.

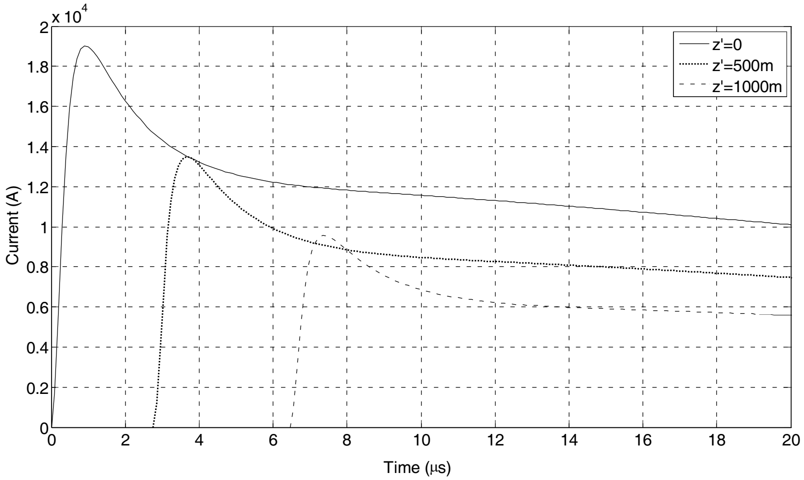

Figure 8. The behavior of evaluated currents at different heights along lightning channel.

The evaluated velocity profiles for both the MTLE and MTLL current models are very close and they are in a good agreement with the velocity value that was expressed by Cooray for this range of current peak [15].

In addition, the behaviour of evaluated currents at three different heights along lightning channel (for MTLE current model case) is shown in Figure 8 where the initial delay times ($= \sum_{i=1}^{n} \frac{\Delta h}{v_i}$) are depending on the variation of velocity along lightning channel.

On the other hand, the effect of Δh parameter on the processing time is given in Table 2 where it shows that the processing time has an inverse relationship with Δh value. Figure 9 illustrates the simulated magnetic flux densities based on different values of Δh in Table 2, which are compared to the corresponding measured field at $r = 15\,\mathrm{m}$. Noted that this computational time is obtained based on the following hardware specifications, i.e., CPU: Intel i3 processor at 2.93 GHz and Memory: 4 GB.

As shown in Figure 9, the simulated fields based on different values of Δh are generally in good agreement with measured field. Result illustrates that the simulated fields from lower values of Δh are closer to the measured field as compared to the simulated fields obtained from higher ones due to the inherent error of trapezoid method that is increased with the increase of Δh step. Therefore, in order to increase the accuracy of simulated field and also to reduce the processing time,

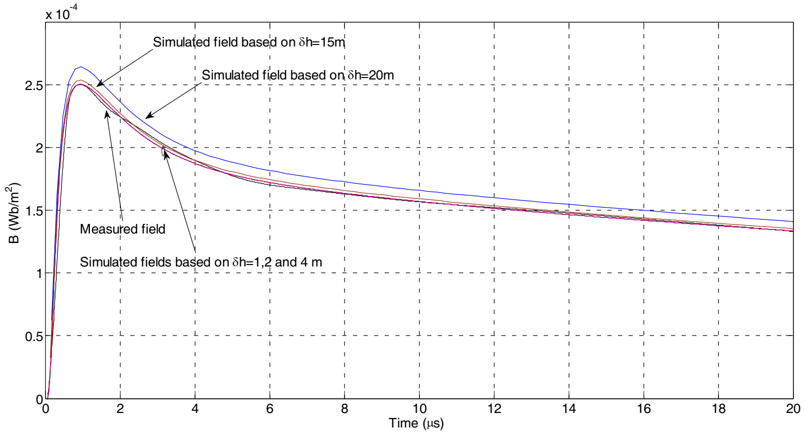

Figure 9. Comparison between simulated magnetic flux densities based on different values of Δh and the corresponding measured field at $r = 15\,\text{m}$.

Table 2. The effect of Δh parameter on the processing time of magnetic flux density.

Δh (m)	Processing time (s)
1	34.896
2	8.706
4	2.205
15	0.160
20	0.093

the value of Δh is set at $4\,\text{m}$. Noted that, the channel angle can affect on the values of electromagnetic fields and this effect can be neglected on the magnetic flux density at low values of channel angle [55, 56]. In this study, the measured magnetic flux densities are associated with the artificial triggered lightning channel with an angle of about 15°. As shown in the results, the comparison between simulated magnetic flux density based on evaluated current and velocity parameters and corresponding measured field at another observation point illustrates a good agreement between them.

The results show that the proposed method can evaluate lightning return stroke currents at different heights along a lightning channel using the measured magnetic flux density at an observation point by setting the velocity along the lightning channel as a height dependent variable. This is in contrast with other previous methods which assume the current velocity along a lightning channel is a constant parameter. Moreover, the proposed method considers all the field components while some previous methods just consider only the radiation component of the field. Furthermore, the proposed method is directly set in the time domain and it can evaluate the full shape of currents while some previous methods are set in the frequency domain and they can evaluate the current values only at a number of frequency samples. In addition, the proposed method can support the general form of different current functions and also the Engineering current model directly in the time domain.

5. CONCLUSION

In this study, a numerical algorithm is proposed in the time domain to evaluate lightning return stroke currents. The proposed algorithm considers the current velocity along lightning channel as a height dependent parameter while all the previous studies have treated this value as a constant parameter. Also, the proposed method utilized all field components for the inverse procedure algorithm directly in the time domain while some previous methods have only considered the radiation component of the field. Moreover, the proposed method is validated using measured data from a triggered lightning experiment, and the results show a good agreement between the evaluated current and fields with the corresponding measured data.

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