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DESCRIPTION OF MULTIPLY CONNECTED REGIONS WITH INDUCED CURRENTS USING T-T_0 METHOD

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Abstract—The paper presents the description of multiply connected conducting regions (MCCR) in the finite elements space. In order to define induced currents distribution in multiply connected regions, an innovative method of combined vector potentials $\mathbf{T}$ and $\mathbf{T}_0$ has been suggested. The equations of $\mathbf{T}$-$\mathbf{T}_0$ method have been presented. The relations describing sources for the field of induced currents in the discussed regions have been given. The proposed method has been applied to solve Problem No. 7 of the International TEAM Workshops. The selected results of calculation have been compared with the measurement results.

1. INTRODUCTION

In recent years, owing to progress in numerical methods and dynamic development of IT equipment in designing and analysis of electrical machines and devices, algorithms enabling calculations of electromagnetic field distribution with induced and eddy currents are used more often. In most typical structures of machines, the conducting regions are superficially multiply connected, e.g., squirrel-cage winding of an induction motor or "multi-turn" winding. The literature provides than among the methods meant to define induced currents in these types of structures the most popular is the method of scalar potential V [10, 13]. The main reasons for such popularity of the method is simplicity of formulating algorithm of the FE equations describing distribution of conduction currents, both in simply and multiply connected conducting regions (MCCR) [4]. A disadvantage related to this method is that it is rather time-consuming when it comes

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to calculations [2]. In order to determine induced currents distribution the **T** method may be also employed. This method can be only used for the analysis of induced currents in simply connected regions [9] and, though for this method algorithms concurrent to V [1, 4] method algorithms can be formed faster than, owing to being unadjusted to the currents analysis on multiply connected regions, the **T** method is not popular. Unadjustedeness of the **T** method to analysis of systems with multiply connected conducting regions can easily be explained using the circuit theory. The equations of **T** method refer to loops with eddy currents around element edges, i.e., loops of electric facet network with eddy currents i_{mi}, which are presented in Fig. 1. Although the number of these loops is usually higher than the number of independent loops, it has been founded that for MCCR it is impossible to create a set of fundamental loops necessary for achieving the induced currents that flow around the "holes". This means that using a classic **T** formulation the induced currents by fluxes Ψ_{ok} passing through the holes are not taken into account, see Fig. 1. In order to avoid unadjustedeness of the **T** method to analyse systems with MCCR, in [7] it is suggested to fill in the "holes" with a conducting material of high resistivity ρ. After the holes have been filled with the conducting material one can get a "quasi-single connected conductor" in which for purposes of determining currents distribution the classic **T** formulation can be easily employed. Researches on deploying this method have been presented in [1, 2]. Authors of these papers have emphasized

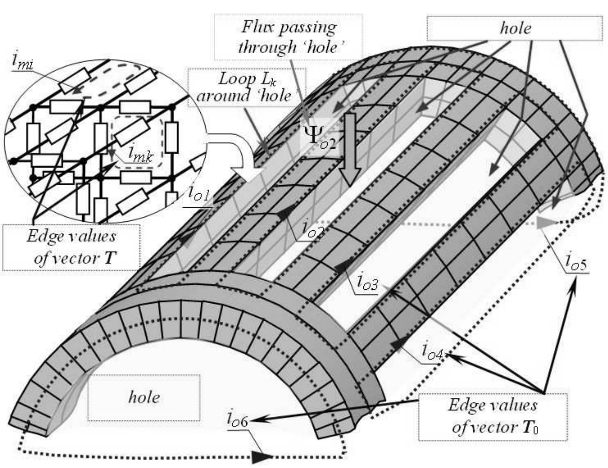


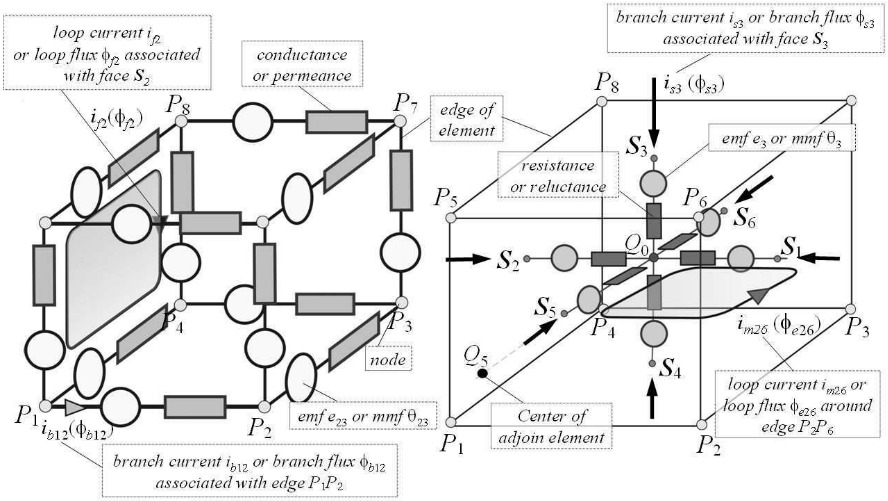
Figure 1. Squirrel cage of an induction motor as a multiply connected conductor and its facet model [5].

superiority of the $\mathbf{T}$ method over V method. They have shown that by using the classical $\mathbf{T}$ formulation the number of iterations in the solving process can be reduced and the computational time can be significantly shortened.

The $\mathbf{T}$ method suggested in [7], i.e., filling the holes with a high-resistivity conducting material, has also been discussed in [5]. The authors of this paper have noticed that when analysing the systems with multiply connected regions the computational time depends on the selected values of resistivity ρ of the material filling the holes. Increasing the resistivity ρ has a positive impact on the accuracy yet causes that the computational time is lengthened. As a result of decreasing the value ρ the calculation time is shortened but the accuracy of obtained solution is questionable.

To determine the conduction currents distribution in multiply connected regions with induced currents the authors suggest deployment of method using vector potentials $\mathbf{T}$ and $\mathbf{T}_0$. The equations of $\mathbf{T}$-$\mathbf{T}_0$ method are obtained by jointing the equations describing the distribution of edge values $\mathbf{i}_m$ of vector $\mathbf{T}$ with equations describing the edge values $\mathbf{i}_o$ of vector potential $\mathbf{T}_0$ [3, 14]. It is known that, in order to determine the induced currents distribution in conducting regions, the equations of $\mathbf{T}$-$\mathbf{T}_0$ method should be solved with equations describing the magnetic field distribution. An advantage of this method is that the equations describing the currents distribution can be easily coupled with equations describing the magnetic field distribution. Moreover both formulations: scalar potential Ω — method Ω-$\mathbf{T}$-$\mathbf{T}_0$ [6] — as well as the vector potential $\mathbf{A}$ — method $\mathbf{A}$-$\mathbf{T}$-$\mathbf{T}_0$ [14] — are allowed.

In the paper the description of multiply connected conducting regions in the finite elements space has been discussed. In order to determine the induced currents distribution in the considered regions the authors suggest a method using the $\mathbf{T}$-$\mathbf{T}_0$ formulation. Ways of formulating equations of $\mathbf{T}$-$\mathbf{T}_0$ method have been presented. The relations describing sources for the field of induced currents in the discussed regions have been given. The usefulness of $\mathbf{T}$-$\mathbf{T}_0$ method has been proved in the example of TEAM Workshops Problem No. 7 [12]. The finite element equations have been set up following [4, 11] and described using the language of circuit theory. The finite element equations arising from the scalar potential formulation and nodal elements are equivalent to nodal equations of an edge network (EN) constructed from branches associated with element edges (Fig. 2(a)); whereas the equations for the vector potentials for the system described by edge elements are represented by loop equations of the facet network (FN) made of branches joining element mid-points (Fig. 2(b)). The



(a) (b)

Figure 2. (a) Edge and (b) facet models of hexahedron.

method using the edge elements and nodal values for scalar potential formulations is called as the nodal elements method (NEM) [4, 15], while the method using the facet elements and edge values for vector potential formulations is called the edge elements method (EEM) [3, 15].

2. EDGE ELEMENT EQUATION OF T-T_0 METHOD

In the paper the way of formulating FE equations of the $\mathbf{T}$-$\mathbf{T}_0$ method has been explained on the base of the simple multiply connected region, i.e., doubly connected region — a massive conducting element with a hole (Fig. 3). Commencing formulation of the equations, after discretisation the region with facet elements, first of all an additional loop L_k is formed, with $\mathbf{i}_o$ current representing current induced by flux Ψ_o passing through the hole. The current $\mathbf{i}_o$ in the loop L_k represents the edge value of the potential $\mathbf{T}_0$ [15]. For such formed model of the region containing an additional loop L_k, in next step the loop equations of the $\mathbf{T}$-$\mathbf{T}_0$ method have been formulated.

The EEM equations can be easily formulated from the branch equations for the FN composed of facet models of elements. Those equations describe the distribution of node to node voltages and can

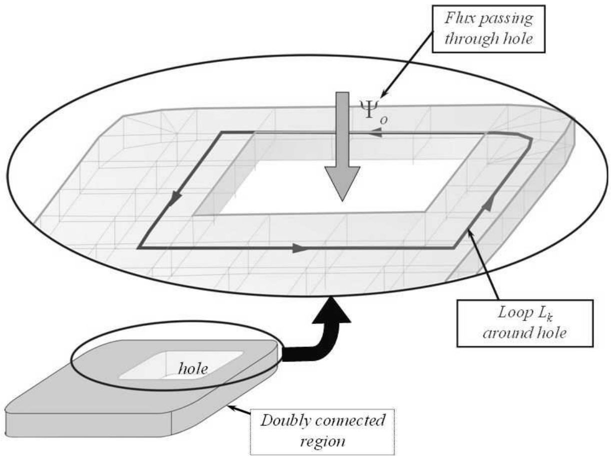


Figure 3. Doubly connected region with a shown loop L_k.

be expressed in the following form

$$\mathbf{u}_V = \mathbf{R}_\rho \mathbf{i}_s - \mathbf{e}, \tag{1}$$

where, $\mathbf{u}_V$ is the vector of branch electric potential differences, e.g., difference of potentials between nodes Q_0 and Q_5 in Fig. 2(b), $\mathbf{R}_\rho$ is the matrix of branch resistances derived from the interpolating functions of the facet element, $\mathbf{i}_s$ is the vector of branch currents associated with faces of elements, e.g., current $\mathbf{i}_{s3}$ in Fig. 2(b), and $\mathbf{e}$ represents the vector of branch electromotive forces (emfs).

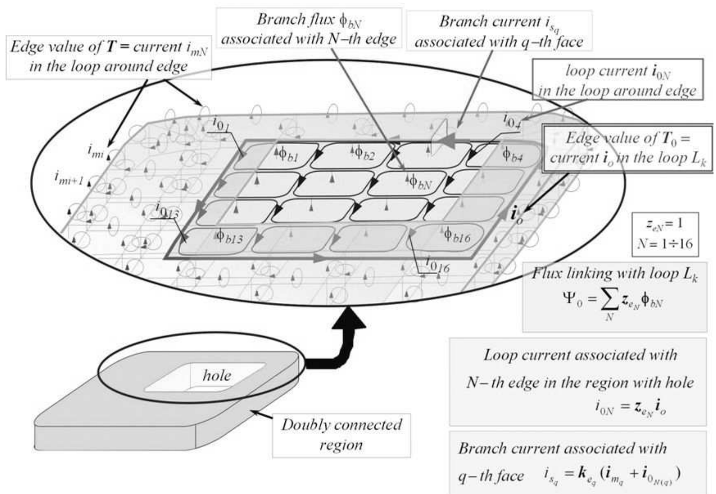
In the discussed example the vector of branch currents $\mathbf{i}_s$ is the sum of two vectors: (a) vector $\mathbf{i}_{sw}$ representing facet values of eddy currents densities $\mathbf{J}_w$ ($\mathbf{J}_w = \mathrm{rot}\mathbf{T}$) in the region with massive elements, i.e., the vector of eddy currents passing through the element facets and (b) vector $\mathbf{i}_{s0}$ of facet values of induced currents densities $\mathbf{J}_0$ ($\mathbf{J}_0 = \mathrm{rot}\mathbf{T}_0$) around the hole, i.e., the vector of induced current defined on the basis of loop current $\mathbf{i}_o$ around the hole passing through the element faces. Having considered that:

$$\mathbf{i}_s = \mathbf{i}_{sw} + \mathbf{i}_{s0} \tag{2}$$

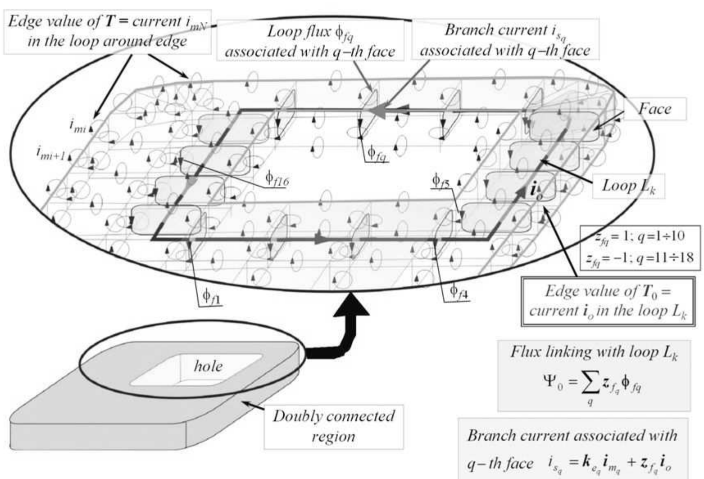
and inserted (2) to (1), the branch equations describing the node to node voltages distribution for the multiply connected region can be written in the following form

$$\mathbf{u}_V = \mathbf{R}_\rho(\mathbf{i}_{sw} + \mathbf{i}_{s0}) - \mathbf{e}. \tag{3}$$

Moreover, when formulating the equation of $\mathbf{T}$-$\mathbf{T}_0$ method one of two formulations can be used: (a) formulation using matrix $\mathbf{z}_e$ describing distribution of additional loops around holes in the edge space (in the discussed systems the loop L_k — Fig. 3) and (b) formulation that applies matrix $\mathbf{z}_f$ describing the loops distribution around holes in the facet space. Details of how matrices $\mathbf{z}_e$ and $\mathbf{z}_f$ are formed have been described in [4, 15]. Fig. 4 illustrates the description of loop L_k around the hole in the space of edges and faces elements.



(a)



(b)

Figure 4. Loop L_k around the hole: (a) the edge element space and (b) the facet element space.

First, we consider approach with matrix $\mathbf{z}_f$. In this approach the vector $\mathbf{i}_{s0}$ of induced currents passing through facets of the elements is determined on the basis of the following relations

$$\mathbf{i}_{s0} = \mathbf{z}_f \mathbf{i}_o, \tag{4}$$

and the vector $\mathbf{i}_{sw}$ of eddy currents is expressed by means of loop currents $\mathbf{i}_m$ around the edges of elements [15]

$$\mathbf{i}_{sw} = \mathbf{k}_e \mathbf{i}_m \tag{5}$$

where $\mathbf{k}_e$ is loop matrix for the loops around the edges like loop in Fig. 2(b).

Having taken into account the interrelations (4) and (5) in (3) we get the following branch equations

$$\mathbf{u}_V = \mathbf{R}_\rho \mathbf{k}_e \mathbf{i}_m + \mathbf{R}_\rho \mathbf{z}_f \mathbf{i}_o - \mathbf{e}, \tag{6}$$

on the basis of which we get the loop equations for loops with eddy currents $\mathbf{i}_m$ around edges as well as to equations for loop with current $\mathbf{i}_o$ around the hole.

Now, the equations describing loop currents around the holes, i.e., the equations that define the edge values of $\mathbf{T}_{0,}$ will be considered. The equations for loops around the holes are obtained by multiplying (6) by the matrix $\mathbf{z}_f^T$

$$\mathbf{z}_f^T \mathbf{u}_V = \mathbf{z}_f^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{i}_m + \mathbf{z}_f^T \mathbf{R}_\rho \mathbf{z}_f \mathbf{i}_o - \mathbf{z}_f^T \mathbf{e}. \tag{7}$$

The matrix product $\mathbf{z}_f^T \mathbf{u}_V$ is a vector of the sum of node to node voltages in the loop around the holes [15]. According to Kirchhoff's principle the sum of node to node voltages in the loop is equal to zero. Thus

$$\mathbf{z}_f^T \mathbf{u}_V = 0. \tag{8}$$

Having considered (7) and (8) the loop equations for the loop with currents $\mathbf{i}_o$ around the holes have been formed as

$$\mathbf{z}_f^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{i}_m + \mathbf{z}_f^T \mathbf{R}_\rho \mathbf{z}_f \mathbf{i}_o = \mathbf{z}_f^T \mathbf{e}. \tag{9}$$

The matrix Equation (9) can be conveniently presented in following form

$$\mathbf{R}_w \mathbf{i}_m + \mathbf{R}_{oc} \mathbf{i}_o = \mathbf{e}_{oc}, \tag{10}$$

where, $\mathbf{R}_{oc}$ is the matrix of loop resistances for loop around the holes

$$\mathbf{R}_{oc} = \mathbf{z}_f^T \mathbf{R}_\rho \mathbf{z}_f, \tag{11}$$

matrix $\mathbf{R}_w$ represents mutual resistances between loops with eddy currents $\mathbf{i}_m$ and loops with induced currents $\mathbf{i}_o$

$$\mathbf{R}_w = \mathbf{z}_f^T \mathbf{R}_\rho \mathbf{k}_e, \tag{12}$$

and $\mathbf{e}_{oc}$ is the vector of loop *emf* for loops around the holes

$$\mathbf{e}_{oc} = \mathbf{z}_f^T \mathbf{e}. \tag{13}$$

Basing on the Equation (6) the EE equations for loops around element edges, i.e., for loops with eddy currents, can also be formed. Multiplying the Equation (6) by the loop matrix $\mathbf{k}_e^T$ we get a vector of the sum of node to node voltages around edges of elements,

$$\mathbf{k}_e^T \mathbf{u}_V = \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{i}_m + \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{z}_f \mathbf{i}_o - \mathbf{k}_e^T \mathbf{e}. \tag{14}$$

Pursuant to Kirchhoff's voltage law for circuits the sum of voltages in a loop equals zero. Thus $\mathbf{k}_e^T \mathbf{u}_V = 0$ and

$$\mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{i}_m + \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{z}_f \mathbf{i}_o = \mathbf{k}_e^T \mathbf{e}. \tag{15}$$

The aforesaid equations are loop ones for loops with eddy currents for electric facet network obtained as a result of a discretization of the multiply connected region. The matrix expression preceding the vector $\mathbf{i}_m$ is a loop resistance matrix $\mathbf{R}_{\rho o}$ for loop with eddy currents [4], and the expression that precedes vector $\mathbf{i}_o$ represents the transposed matrix $\mathbf{R}_w$ describing mutual resistances, see (12). The product $\mathbf{k}_e^T \mathbf{e}$ represents a vector $\mathbf{e}_o$ of loop *emfs* for loops with eddy currents. By the application of these matrices the Equation (15) can be presented in the form

$$\mathbf{R}_{\rho o} \mathbf{i}_m + \mathbf{R}_w^T \mathbf{i}_o = \mathbf{e}_o. \tag{16}$$

The equations of $\mathbf{T}$-$\mathbf{T}_0$ method for conducting multiply connected regions in the facet elements space are obtained as a result of jointing the Equations (10) and (16)

$$\begin{bmatrix} \mathbf{R}_{\rho o} & \mathbf{R}_w^T \\ \mathbf{R}_w & \mathbf{R}_{oc} \end{bmatrix} \begin{bmatrix} \mathbf{i}_m \\ \mathbf{i}_o \end{bmatrix} = \begin{bmatrix} \mathbf{e}_o \\ \mathbf{e}_{oc} \end{bmatrix}. \tag{17}$$

In a similar way, the equations of $\mathbf{T}$-$\mathbf{T}_0$ method for a formulation using matrix $\mathbf{z}_e$, i.e., that matrix describing the paths around the holes in the edge elements space, can be formed. Using matrix $\mathbf{z}_e$ when formulating loop equations around the holes give the same values of vector $\mathbf{e}_{oc}$ and matrixes $\mathbf{R}_{oc}$, $\mathbf{R}_w$ for circuits with additional loops [4, 15] on condition that the loop distribution has been correctly mapped. It means when the following identity is provided

$$\mathbf{z}_f = \mathbf{k}_e \mathbf{z}_e. \tag{18}$$

Having substituted the identity (18) to (11), (12) and (13) we get formulas describing, adequately:

(a) the matrix of loop resistances $\mathbf{R}_{oc}$ for loop around the holes

$$\mathbf{R}_{oc} = \mathbf{z}_e^T \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{z}_e, \tag{19}$$

(b) the matrix of mutual resistances $\mathbf{R}_w$

$$\mathbf{R}_w = \mathbf{z}_e^T \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e, \tag{20}$$

(c) the vector of loop *emfs* $\mathbf{e}_{oc}$ for loops around the holes

$$\mathbf{e}_{oc} = \mathbf{z}_e^T \mathbf{k}_e^T \mathbf{e}. \tag{21}$$

The above formulas should be considered as referring to systems with conducting multiply connected regions described in the edge space.

Solving the Equation (17) we obtain eddy currents distribution $\mathbf{i}_m$ in loop around the edges and currents $\mathbf{i}_o$ in additional loop around the holes.

3. DESCRIPTION OF THE LOOP ELECTROMOTIVE FORCES

In the case of procedures meant to determine conduction currents distribution in multiply connected regions using $\mathbf{T}$-$\mathbf{T}_0$ method it is important to formulate proper expressions describing loop *emfs* in loops around the edges and loops around the holes. The way that the loop *emfs* are calculated depends on the deployed way of describing the location and shape of loops around the holes. It has been already mentioned that in $\mathbf{T}$-$\mathbf{T}_0$ method two formulations can be used: (a) formulation using matrix $\mathbf{z}_e$ and (b) formulation using matrix $\mathbf{z}_f$. The approach (a) is more universal and can be applied both for the magnetic scalar Ω and magnetic vector potential $\mathbf{A}$ formulations, whereas approach (b) is restricted to cases where $\mathbf{A}$ is used as the solution potential. Below, the way of formulating loop *emfs* in the facet model of multiply connected regions for systems of known fluxes distribution has been presented.

For the formulation using matrix $\mathbf{z}_e$ the Equation (17) describing induced currents distribution can be written in the following form:

$$\begin{bmatrix} \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e & \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{z}_e \\ \mathbf{z}_e^T \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e & \mathbf{z}_e^T \mathbf{k}_e^T \mathbf{R}_\rho \mathbf{k}_e \mathbf{z}_e \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_m \\ \mathbf{i}_o \end{bmatrix} = \begin{bmatrix} \mathbf{e}_o \\ \mathbf{e}_{oc} \end{bmatrix}. \tag{22}$$

The right side of Equation (22) represents sources, i.e., *emfs* $\mathbf{e}_o$ in loops around the element edges and *emfs* $\mathbf{e}_{oc}$ in loops around the holes. The field sources can be defined in terms of [4, 5, 15]: (a) branch fluxes $\boldsymbol{\phi}_b$ associated with edges of elements (Fig. 5(a)); (b) branch fluxes $\boldsymbol{\phi}_s$ penetrating faces of elements (Fig. 5(b)) or (c) loop fluxes $\boldsymbol{\phi}_e$ around edges of elements (Fig. 5(c)).

In the case (a), that is applied for Ω-$\mathbf{T}$-$\mathbf{T}_0$ formulation, the electromotive forces $\mathbf{e}_o$ in loops of FN are represented by a time

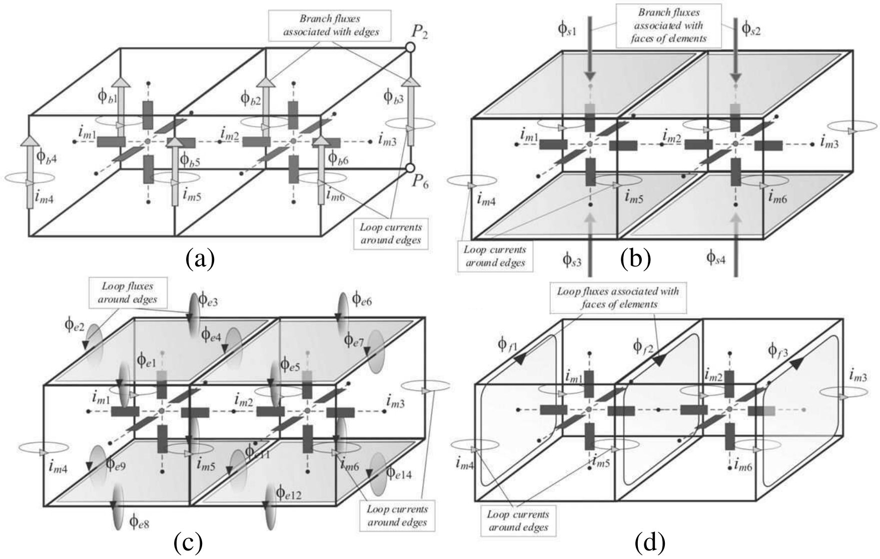


Figure 5. Fluxes associated with electric FN: (a) branch fluxes ϕ_b associated with edges of elements, (b) branch fluxes ϕ_s associated with faces of elements, (c) loop fluxes ϕ_e around edges of elements, (d) loop fluxes ϕ_f associated with faces of elements [5].

derivative of the branch fluxes $\boldsymbol{\phi}_b$ of the magnetic edges network (Fig. 6).

$$\mathbf{e}_o = -\frac{\partial}{\partial t}\phi_b. \tag{23}$$

Multiplying the vector $\mathbf{e}_o$ by the transposed matrix $\mathbf{z}_e$ the vector $\mathbf{e}_{oc}$ of loop *emfs* for loops around the holes can be calculated from

$$\mathbf{e}_{oc} = \mathbf{z}_e^T\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{z}_e^T\phi_b. \tag{24}$$

For the case (b) in which electromotive forces are defined on the basis of a known branch fluxes $\boldsymbol{\phi}_s$, i.e., the facet values of magnetic flux density $\mathbf{B}$, the expressions describing *emfs* $\mathbf{e}_o$ and $\mathbf{e}_{oc}$ may be written as

$$\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{K}^T\phi_s, \tag{25}$$

$$\mathbf{e}_{oc} = \mathbf{z}_e^T\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{z}_e^T\mathbf{K}^T\phi_s, \tag{26}$$

where $\mathbf{K}$ is a matrix transposing the branch values of the facet network into the values related to branches of the edge network [4].

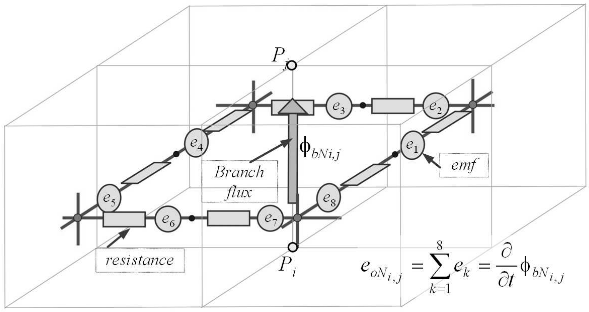


Figure 6. Branch flux $\phi_{bNi,j}$ passing through loop of electric facet network.

The field sources can be also calculated from loop fluxes $\boldsymbol{\phi}_e$ around edges, i.e., edge values of potential $\mathbf{A}$ ($\mathbf{A}$-$\mathbf{T}$-$\mathbf{T}_0$ formulation) — that is for case (c). In this case the following formulas are applied

$$\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{K}^T\mathbf{k}_e\boldsymbol{\phi}_e, \tag{27}$$

$$\mathbf{e}_{oc} = \mathbf{z}_e^T\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{z}_e^T\mathbf{K}^T\mathbf{k}_e\boldsymbol{\phi}_e. \tag{28}$$

Taking into account that $\mathbf{K}^T\mathbf{k}_e = \mathbf{k}_e^T\mathbf{K}$ [15] the loop *emfs* can be also defined as follows

$$\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{k}_e^T\mathbf{K}\boldsymbol{\phi}_e, \tag{29}$$

$$\mathbf{e}_{oc} = \mathbf{z}_e^T\mathbf{e}_o = -\frac{\partial}{\partial t}\mathbf{z}_e^T\mathbf{k}_e^T\mathbf{K}\boldsymbol{\phi}_e. \tag{30}$$

It was already pointed that in order to define the loop L_k around the holes, the formulation using matrix $\mathbf{z}_f$ can be applied. For this case, the equations describing the distribution of induced currents can be written as

$$\begin{bmatrix} \mathbf{k}_e^T\mathbf{R}_\rho\mathbf{k}_e & \mathbf{k}_e^T\mathbf{R}_\rho\mathbf{z}_f \\ \mathbf{z}_f^T\mathbf{R}_\rho\mathbf{k}_f & \mathbf{z}_f^T\mathbf{R}_\rho\mathbf{z}_f \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_m \\ \mathbf{i}_o \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial t}\mathbf{k}_e^T\phi_f \\ -\frac{\partial}{\partial t}\mathbf{z}_f^T\phi_f \end{bmatrix}. \tag{31}$$

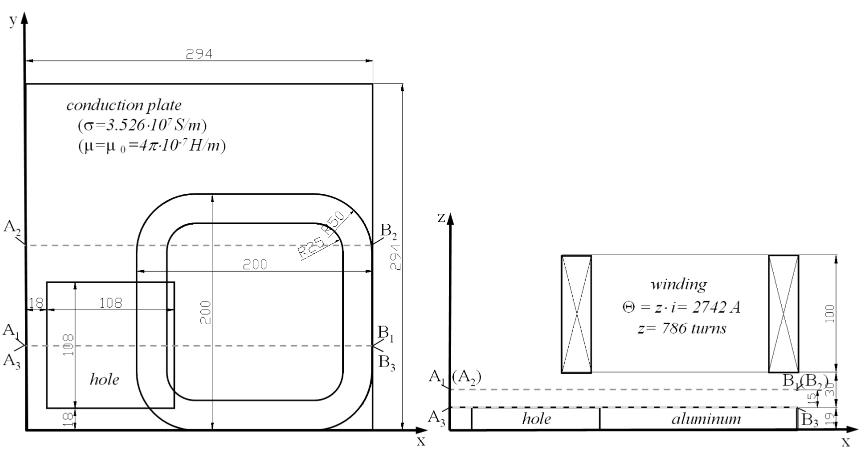
In Equation (31), the sources are formulated from a given loop fluxes $\boldsymbol{\phi}_f$ associated with element faces, i.e., loops of the EN (Fig. 5(d)). The expression $-\frac{\partial}{\partial t}\mathbf{k}_e^T\boldsymbol{\phi}_f$ describes vector $\mathbf{e}_o$ of *emfs* in loops around edges with eddy currents, while the product $-\frac{\partial}{\partial t}\mathbf{z}_f^T\boldsymbol{\phi}_f$ represents vector $\mathbf{e}_{oc}$ of electromotive forces in loops around the holes. The vector $\boldsymbol{\phi}_f$ is a vector of loop fluxes associated with faces of the elements, i.e., fluxes in loop of the EN [15]. The loop flux $\boldsymbol{\phi}_{fi}$ is a numerical representation of the integral from the vector $\mathbf{A}$ along the line connecting centres of elements of a common face S_i [15].

4. NUMERICAL EXAMPLE

Based on the presented above formulation, an algorithm and dedicated software has been developed for calculation of induced current distributions in multiply connected conducting regions. In the elaborated software the equations of $\mathbf{T}$-$\mathbf{T}_0$ method, i.e., equations describing the distribution of current flow field, have been coupled with the equations of the $\mathbf{A}$ method [3, 15], i.e., equations describing the magnetic field distribution. As result of coupling of equations the following system of $\mathbf{A}$-$\mathbf{T}$-$\mathbf{T}_0$ equations has been obtained [4]:

$$\begin{bmatrix} \mathbf{k}_e^T\mathbf{R}_\mu\mathbf{k}_e & -\mathbf{K}^T\mathbf{k}_e & -\mathbf{K}^T\mathbf{k}_e\mathbf{z}_e \\ \frac{\partial}{\partial t}\mathbf{K}^T\mathbf{k}_e & \mathbf{k}_e^T\mathbf{R}_\rho\mathbf{k}_e & \mathbf{k}_e^T\mathbf{R}_\rho\mathbf{k}_e\mathbf{z}_e \\ \frac{\partial}{\partial t}\mathbf{z}_e^T\mathbf{K}^T\mathbf{k}_e & \mathbf{z}_e^T\mathbf{k}_e^T\mathbf{R}_\rho\mathbf{k}_e & \mathbf{z}_e^T\mathbf{k}_e^T\mathbf{R}_\rho\mathbf{k}_e\mathbf{z}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}_e \\ \mathbf{i}_m \\ \mathbf{i}_o \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Theta}_z \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (32)$$

where, $\mathbf{R}_\mu$ is the matrix of branch reluctances of the FN [3] and vector $\boldsymbol{\Theta}_z$ represents the loop *mmf*s set up by the flow of current through the windings. As an example, the test problem No. 7 of the International TEAM Workshops has been examined (Fig. 7). Parallelepiped elements have been used resulting in about half a



(a) plane view (b) front view

Figure 7. The TEAM workshop problem No. 7 — Asymmetrical conductor with a hole [12].

million equations for the edge values ϕ_e of the vector potential $\mathbf{A}$ and 63 thousand equations for the edge values of $\mathbf{T}$-$\mathbf{T}_0$. The complex potentials has been used throughout and complex ICCG method has been applied for solving obtained matrix equations. The obtained simulation results have been compared with measurements published in [8]. Figs. 8, 9 and 10 present the computed and measured distributions: (a) of the magnetic flux density B_z along the line A_1-B_1,

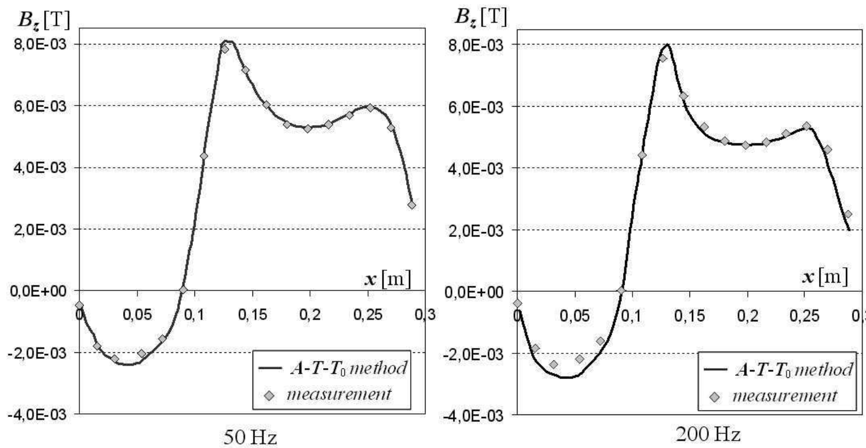


Figure 8. Magnitude of B_z along the line A_1-B_1 as shown in Fig. 7.

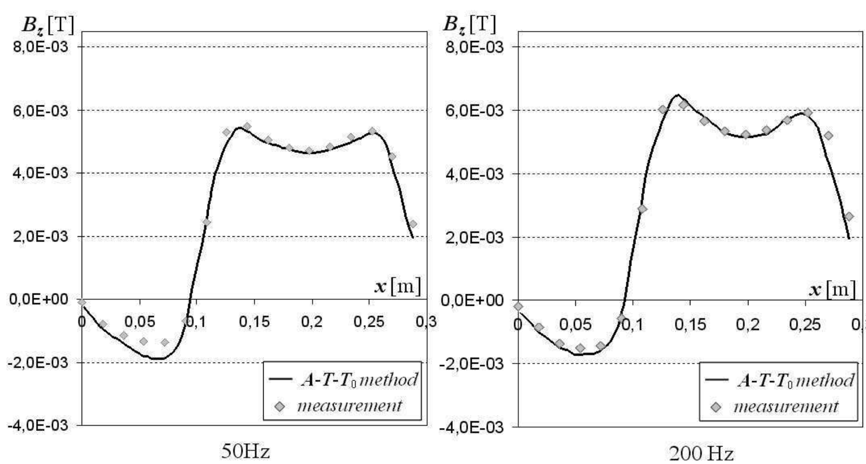


Figure 9. Magnitude of B_z along the line A_2-B_2 as shown in Fig. 1.

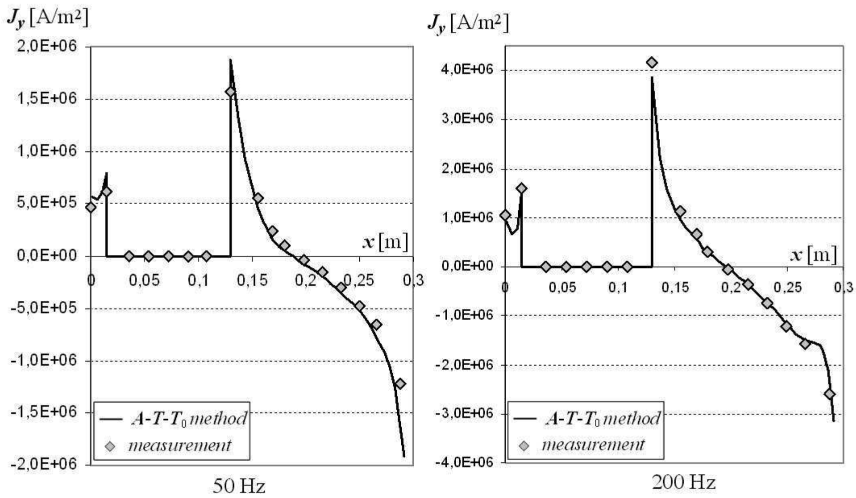


Figure 10. Magnitude of J_y along the line A_3-B_3 as shown in Fig. 1.

(b) of the magnetic flux density B_z along the line A_2-B_2, and (c) of the current density J_y along the line A_3-B_3, for two frequencies (50 Hz and 200 Hz). The positions of the lines A_1-B1, A_2-B_2 and A_3-B_3 are depicted in Fig. 7.

A comparison of simulation results with the measurements published by Nakata and Fujiwara in [8], reveals very close agreement and thus good accuracy of the used formulation. It is important to notice that the total computational time for the case of frequency of 50 Hz and imposed error threshold of 10^{-6} was typically about 20 minutes, which should be compared with 6.5 hours needed to achieve the same accuracy using a $\mathbf{A}$-V method described in [16].

5. CONCLUSION

The paper presents the method of jointed electric potentials $\mathbf{T}$-$\mathbf{T}_0$ applied to determine conducting currents distribution in multiply connected regions. The representation of the MCCR in the finite elements space has been demonstrated and discussed. The fundamental loop equations and additional loop equations around the holes for a facet model of discretized multiply connected regions have been presented. By the formulating equations for loops around the holes, the matrices $\mathbf{z}_e$ and $\mathbf{z}_f$ describing additional loops in the space of edges and faces of finite elements have been used. As a result of jointing loop equations of the electric facet network describing distribution of eddy currents $\mathbf{i}_m$ with equations of additional loops that describe the distribution of induced currents $\mathbf{i}_o$ around the holes the $\mathbf{T}$-$\mathbf{T}_0$ method equations have been obtained. The ways of formulating loop *emfs* $\mathbf{e}_o$ and $\mathbf{e}_{oc}$ for the considered facet model of a multiply connected region have been described. The usefulness of proposed method has been confirmed the example.

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