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REGIONAL VARIABILITY OF RAIN DROP SIZE DISTRIBUTION MODEL IN INDIA

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Abstract—Present study shows the development of integrated rain drop size distribution (DSD) model and gives a comparative study with DSDs of different regions in India. This work is useful for estimation of rain induced attenuation. Rain data of five different regions (Ahmedabad, Shillong, Thiruvananthapuram, Kharagpur and Hasan) was used for this work. Attenuation characteristics are different for different regions because DSD varies according to the climatic conditions. Development of DSD model for each location is not feasible. It is a demand to develop a integrated DSD model which gives the tolerable error in DSD for different regions, so that, it can be adjusted in fade margin of the communication system. The result of this work shows the good correlation between the proposed integrated DSD model and DSDs of different regions.

1. INTRODUCTION

The rapid growth in demand for wider bandwidth in communication systems has put pressure on communication engineers to develop systems operating at higher frequencies. However, the reliability and performance of radar and space communication links are reduced by atmospheric particles [1] such as oxygen, ice crystals, rain, fog, and snow [2]. At frequencies above 10 GHz the rainfall can cause several decibels of attenuation resulting into a long outage time, thus affecting reliability of communication link. Rain induced attenuation

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at higher frequencies is generally considered to be solely a function of frequency and rain fall intensity, with observed variability attributable to differences in rain drop size distribution (DSD), temperature differences and non-spherical shape of the rain drops. The variability due to rain drop size distribution alone (holding all other parameter constant) ranges between 5% to 33% standard deviation about the mean [3]. Therefore, worldwide measurements have been carried out on DSD using disdrometer [4–7]. From the various studies done on DSD, it is now well established that the distribution of the drop sizes falling at particular rain intensity varies from place to place depending on meteorological conditions [8–10]. Das et al. [8] developed separate DSD models for each region as mentioned in present study and show significant differences between ITU-R model and DSD models derived attenuation values. In present study a integrated DSD model was developed for estimation of specific attenuation (dB/km) by using eleven station year data. It is based on calculation of Mie scattering coefficients and extinction cross section of water drops of different diameters as suggested in [11] and can be used for different climatic regions at different rain rates and frequencies.

Various rain drop-size distributions like exponential, gamma and lognormal have been reported in the literature by many authors like Marshall and Palmer [12], Joss and Waldvogel [13], Joss and Gori [14], etc. It is found that, in tropical regions, the DSD represented by gamma [15–17] and lognormal [18–20] functions are better suited than the exponential function. Some measurements have been carried out in tropical region like Singapore [6], Brazil [21], Malaysia [22], Israel [18] and Nigeria [23] where rain characteristics are different from those of temperate climate. So these countries have developed their own DSD and rain induced attenuation model for the calculation of accurate fade margin. However, it is found that the lognormal distribution is suitable for tropical regions like India [24–26]. Therefore, in the present study lognormal distribution is used for developing DSD model.

2. DATA COLLECTION AT DIFFERENT SITES

The disdrometer data of different stations as mentioned in Table 1 are analyzed in the present study. A Joss-Waldvogel impact type disdrometer (RD-80) (Joss and Waldvogel, 1967), manufactured by M/s Distromet Ltd., Switzerland, has been used for data collection. The measurements of DSDs were taken with a temporal variation of 30 seconds for better resolution of rain rates. The sampling area of the outdoor unit measures $50\,\text{cm}^2$ and the indoor unit consists of an analyzer ADA-90. It converts vertical mechanical moment of the drops

Table 1. Details of 11 station year data.

No.	Station	Lat.(N), Long.(E)	Topology Monsoon Climate	Period Years	Samples	Rain Time (Min)
1.	Ahmedabad	23°04', 72°38'	Plane, SW	2	52,483	26,241.5
2.	Hasan	13°, 76°09'	Plane, SW	1	79,264	39,632
3.	Kharagpur	22°32', 88°20'	Coastal Plane, SW& NE	2	75,186	37,593
4.	Shillong	25°34', 91°53'	Hilly, SW & NE	3	2,41,849	1,20,924.5
5.	Thiruventha-puram	08°293', 76°57'	Coastal Plane, SW & NE	3	1,24,428	62,214

into electric pulses. The fall velocities of the drops with diameter corresponding to each drop class, ΔD_i, have been taken as suggested by Gunn and Kinzer [27]. The specifications of disdrometer are mentioned in Table 2. The disdrometer has certain limitations as far as counting the smaller drops are concerned, this is because of dead time error but the effects of these smaller drops are less on rain attenuation and are within 5% error limit [28]. The disdrometer was installed at the roof top of a building (5 m high) to minimize the known sources of error like acoustic noises [8]. The location of Space Applications Centre (SAC) is such that the area is free from any industrial or acoustic noise.

3. DATA ANALYSIS

3.1. Data Formatting

Rate of rain fall (mm/hr) was evaluated by calculating the total volume of water collected by the sensor of disdrometer during 30 sec. interval [29]. There are some events when rain rate is less than 1 mm/hr but those are not accounted here because they are not of much significance from the attenuation point of view. Many authors [19, 20] have suggested exponential increase in the width of the rain classes, as the rainfall duration is maximum at the low rain rates and decreases exponentially with increasing rain rate. The minimum and maximum rain rates are taken as less than 0.005 and 100 mm/hr respectively. Tokay and Short [28] has categorized rain as very light (less than 1 mm/hr), light (1–2 mm/hr), moderate (2–5 mm/hr), heavy

(5–10 mm/hr), very heavy (10 to 20 mm/hr) and extreme (more than 20 mm/hr) for classification of precipitation regimes, convective and stratiform, based on drop size distribution data collected with RD-69 disdrometer. However in the present analysis, the limits are taken as 1 mm/hr and 140 mm/hr and divided into 14 ranges of 10 mm/hr duration each. This is because it is the higher rain rate occurring for say 0.01% of the time which is of interest to communication engineer for rain induced fade margin calculations. Average number of drops falling in each of the 20 drop size classes $(N_1, N_2, N_3, \ldots, N_{20})$ were calculated at each averaged rain rate. The numbers of drops, per m^3

Table 2. Drop size classes and fall velocities.

Drop size class	Output code of processor RD-80	Average diameter of drops in class i, D_i (mm)	Fall velocity of a drop with diameter D_i, v_i (ms^{-1})	Diameter interval of drop size class i, ΔD_i (mm)
1	1–13	0.359	1.435	0.092
2	14–23	0.455	1.862	0.100
3	24–31	0.551	2.267	0.091
4	32–38	0.656	2.692	0.119
5	39–44	0.771	3.154	0.112
6	45–54	0.913	3.717	0.172
7	55–62	1.116	4.382	0.233
8	63–69	1.331	4.986	0.197
9	70–75	1.506	5.423	0.153
10	76–81	1.665	5.793	0.166
11	82–87	1.912	6.315	0.329
12	88–93	2.259	7.009	0.364
13	94–98	2.584	7.546	0.286
14	99–103	2.869	7.903	0.284
15	104–108	3.198	8.258	0.374
16	109–112	3.544	8.556	0.319
17	113–117	3.916	8.784	0.423
18	118–121	4.350	8.965	0.446
19	122–126	4.859	9.076	0.572
20	127	5.373	9.137	0.455

per unit diameter, $N(D_i)$, in each drop-size class with average diameter D_i is given [30] as:

$$N\left(D_i\right) = 10^4 \frac{N_i}{AT v_i \Delta D_i} \tag{1}$$

where N_i is the number of drops with averaged diameter D_i (mm); A is the surface area of sensor ($50\,\mathrm{cm}^2$ in present case); T is the sample time interval (0.5 minute); v_i is the terminal velocity of the rain drops with diameter D_i, as mentioned in Table 2; and ΔD_i is the channel width of the disdrometer with averaged diameter D_i. The values of $N(D_i)$, for all the twenty drop diameter classes were computed for all the fourteen averaged rain rates.

3.2. Modeling of DSD

The expression for lognormal distribution is:

$$N\left(D\right) = \frac{N_T}{\sqrt{2\pi}\ln\sigma\cdot D}\exp\left[-\frac{\ln^2\left(D/D_g\right)}{2\ln^2\sigma}\right] \tag{2}$$

with parameter D_g, σ and $N_T \cdot D_g$ is the geometric mean of drop diameters, σ represents the standard geometric deviation, which is a measure of the breadth of the spectrum and N_T is the total number of drops/m^3. Expression for lognormal distribution (2) is reproduced as:

$$N\left(D\right) = \frac{\exp A}{D}\exp\left[-0.5\left\{\frac{\left(\ln D - B\right)}{C}\right\}^2\right] \tag{3}$$

where,

$$A = \ln\left[\frac{N_T}{\sqrt{2\pi}\ln\sigma}\right] \tag{4}$$

$$B = \ln D_g \tag{5}$$

$$C = \ln\sigma \tag{6}$$

A, B and C in (3) are fit parameters of the lognormal distribution, which can be deduced, as per their definitions, from the disdrometer data by using following relations [18]:

$$\ln D_g = \frac{1}{N_T}\sum_{i=1}^{20} N_i \ln D_i \tag{7}$$

$$\ln^2\sigma = \frac{1}{N_T}\sum_{i=1}^{20} N_i\left(\ln D_i - \ln D_g\right)^2 \tag{8}$$

Table 3. Parameters of averaged DSD for eleven station year data.

Rain rate bin	Class of rain rate (mm/hr)	No. of DSDs	Avg. rain rate (mm/hr)	N_T (m^{-3})	$\ln D_g$	$\ln \sigma$
1	$1 \leq R < 10$	1,22,333	3.257025	435	−0.3526	0.422528
2	$10 \leq R < 20$	15,386	14.09698	668	−0.02977	0.44536
3	$20 \leq R < 30$	6,224	24.41529	681	0.116051	0.451312
4	$30 \leq R < 40$	3,491	34.66148	682	0.215567	0.444543
5	$40 \leq R < 50$	2,175	44.60609	718	0.281632	0.432851
6	$50 \leq R < 60$	1,385	54.62799	748	0.338451	0.421076
7	$60 \leq R < 70$	878	64.64916	788	0.376875	0.411418
8	$70 \leq R < 80$	584	74.45231	867	0.41022	0.397387
9	$80 \leq R < 90$	375	84.53734	867	0.430168	0.396382
10	$90 \leq R < 100$	268	94.74856	929	0.466845	0.384354
11	$100 \leq R < 110$	153	104.4346	956	0.485955	0.377654
12	$110 \leq R < 120$	88	114.8522	981	0.511251	0.368806
13	$120 \leq R < 130$	55	124.6033	1056	0.531333	0.361733
14	$130 \leq R < 140$	15	134.0267	1005	0.547635	0.35159

where N_T is the number concentration (m^{-3}) in the observed spectrum and N_i the number of drops in size category D_i.

The values of N_T, $\ln D_g$ and $\ln \sigma$ of eleven station year data of lognormal distribution for different rain rate ranges are given in Table 3. The numbers of rain samples falling in each rain class are also shown there in. The values of the fit parameters A, B and C as mentioned in (4), (5) and (6) were estimated for each rain rate range and have been plotted in Figs. 1(a), (b) and (c). The values of fit parameters A, and B are found to vary almost linearly with exponential increase in rain rate where as the parameter C is almost constant [19]. The variation of fit parameters A, B and C with rain rate has been studied using the Table Curve software. The best fit equations have been found as:

$$A = A_0 + A_1 (\ln R)^2 \tag{9}$$

$$B = B_0 + B_1 (\ln R)^2 \tag{10}$$

$$C = 0.4 \tag{11}$$

where A_0, A_1, B_0, B_1, and C are the fit parameters, and R is the rain rate. Insertion of (9), (10) and (11) in (3) gives the required DSD

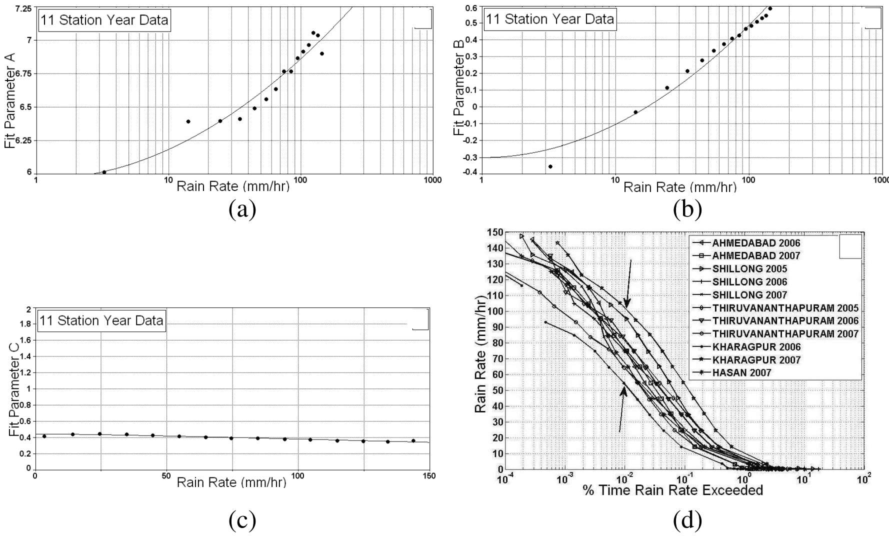


Figure 1. Variation of the fit parameter A, B and C with rain rate and cumulative distribution of rain rate.

Table 4. Values of fit parameters.

Fit Parameter	A	B
Coefficients	A_0: 5.9607263 A_1: 0.042569907	B_0: -0.30083725 B_1: 0.037124901
Correlation Coefficient	0.9405	0.9726
Fit Std. Error	0.0751	0.0437

model. The values of coefficients A_0, A_1, B_0, and B_1, their correlation coefficients and fit standard error are given as under Table 4. The value of parameter C ($\ln\sigma$) has been taken as 0.4 for DSD modeling though the experimental values change with rain rate as shown in Table 3. No perceptible variation in DSD model could be noticed by considering the variation. For example change in correlation coefficient from 0.98899 (Table 4) to 0.98292 was observed for Kharagpur 2007 data when value of C was changed from 0.4 to actual experimental value of 0.377654.

4. CUMULATIVE DISTRIBUTION OF RAIN RATE STATISTICS

Analysis of rain rate exceedance characteristics gives the information regarding the rain rate of 0.01% time of the year to achieve 0.01% link

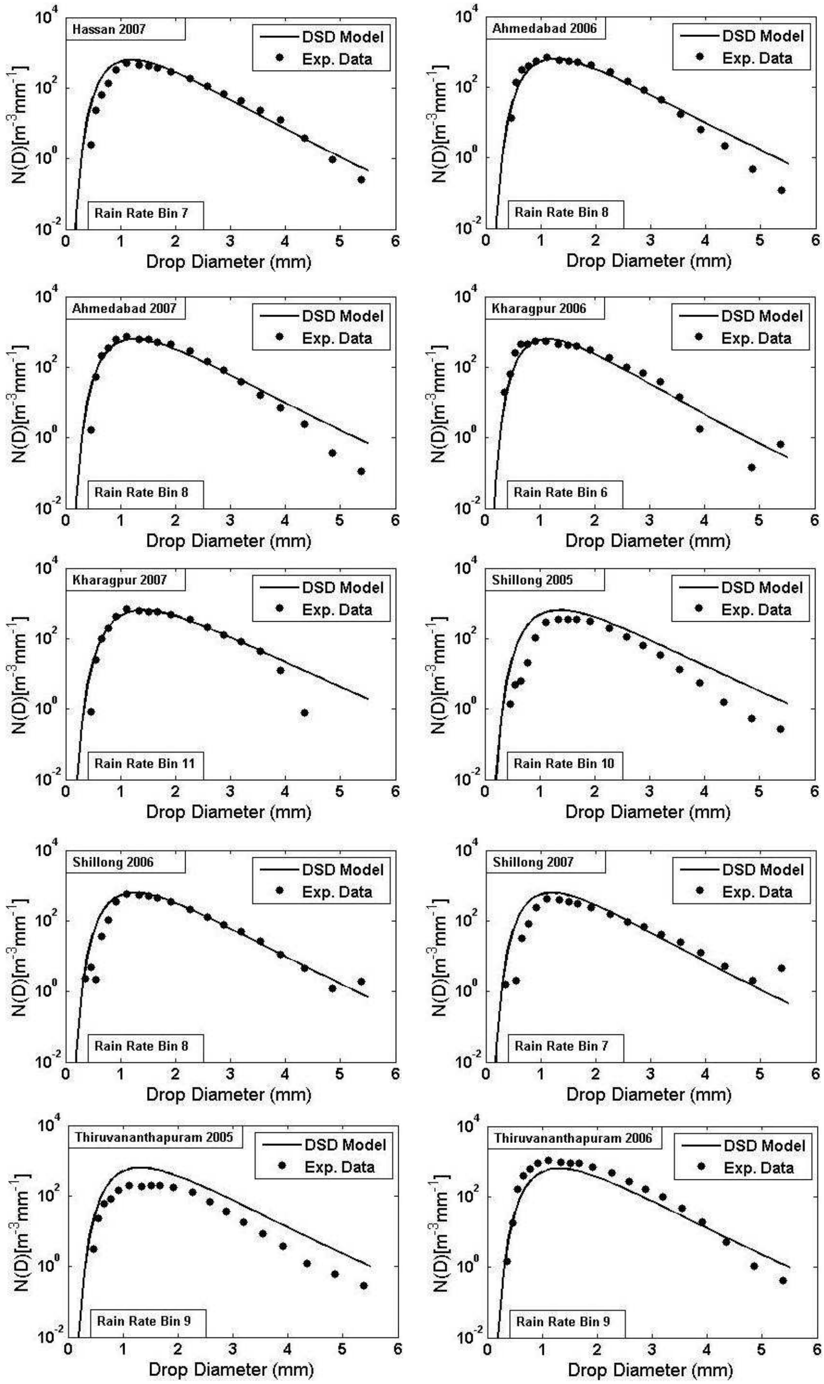
Table 5. Yearly variability of rain rate exceeding 0.01% of time and correlation of experimental DSD with integrated DSD model.

Stations	2005		2006		2007	
	Rain Rate (mm/hr)	Correlation with Integrated DSD Model	Rain Rate (mm/hr)	Correlation with Integrated DSD Model	Rain Rate (mm/hr)	Correlation with Integrated DSD Model
Hasan	x	x	x	x	65	0.96774
Ahmedabad	x	x	75	0.98132	75	0.990958
Kharagpur	x	x	55	0.95060	102	0.98899
Shillong	95	0.95162	74	0.97310	65	0.94868
Thiruvananthapuram	81	0.98190	82	0.98344	64	0.97058

reliability [8, 26]. The annual rain rate exceedance curves have been plotted in Fig. 1(d). From the Fig. 1(d) it is observed that the rain rate exceeded for 0.01% of the time varies from 50 mm/hr to 100 mm/hr for different locations. This inference is very important for designing the millimeter wave links in different regions of the country. Table 5 shows large variations in the yearly rain rate exceeding 0.01% of the time. In case of station Kharagpur, the variation is too high 55 mm/hr in 2006 and 102 mm/hr in 2007. It is therefore necessary to have data of many years at each station to develop reliable drop size distribution (DSD) Model as well as for the study of rain attenuation statistics. However there is good correlation between experimental data and integrated DSD model. Therefore integrated DSD model can be used for different stations in the country with fair amount of accuracy.

5. VALIDATION OF DSD MODEL

To validate the integrated DSD model, the experimental data of rain rate bins corresponding to the 0.01% of the time rain rate as shown in Table 5, have been plotted over the integrated DSD model in Fig. 2 using A, B and C values obtained from Equations (9), (10) and (11). The correlation coefficients, between integrated DSD model and experimental values of $N(D)$, have been shown in Table 5. The values of correlation coefficients have been found to lie between 0.94868 and 0.99096 which shows very good correlation between the proposed integrated DSD model and the experimental data.


Hassan 2007
DSD Model
Exp. Data
Rain Rate Bin 7
Ahmedabad 2006
Rain Rate Bin 8
Ahmedabad 2007
Rain Rate Bin 8
Kharagpur 2006
Rain Rate Bin 6
Kharagpur 2007
Rain Rate Bin 11
Shillong 2005
Rain Rate Bin 10
Shillong 2006
Rain Rate Bin 8
Shillong 2007
Rain Rate Bin 7
Thiruvananthapuram 2005
Rain Rate Bin 9
Thiruvananthapuram 2006
Rain Rate Bin 9
N(D)[m^-3mm^-1]
Drop Diameter (mm)

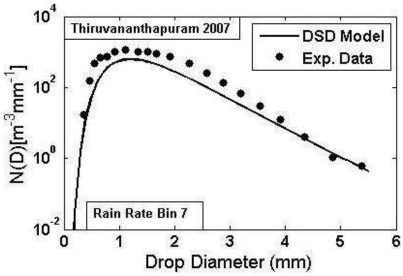


Figure 2. Validation of integrated DSD model with experimental data of different regions.

6. CONCLUSIONS

Good correlation between proposed integrated DSD model and DSD of different regions at rain rate of 0.01% time of the year has been observed. This indicates that integrated DSD model can be used for the estimation of rain induced attenuation at millimeter wave frequencies and there is no need to develop regional DSD models. Large temporal variations have been observed in the rain rate statistics at the same place. It is proposed further that, similar study should also be carried out for central and northern regions of India. Such data is essential for designing the reliable millimeter wave communication links, meeting the desired percentage reliability criterion, for different meteorological regions of the country.

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