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CONTROLLING THE ELECTROMAGNETIC FIELD BY INDEFINITE MEDIA WITH EXTREMELY STRONG ANISOTROPY

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Abstract—In this work, we show how to manipulate the electromagnetic wave at will by using an indefinite medium with extremely strong anisotropy. The negative element in the indefinite permittivity tensor goes to the negative infinity while the positive element is equal to 1, which stretches the hyperbolic equifrequency contour into a straight line type. The direction of the Poynting vector and the wave vector is aligned by the straight line type equifrequency contour along the orientation of the extremely negative permittivity, thus control the wave propagation. The other permittivity of 1 makes the indefinite medium matched with the air. Moreover, because of the hyperbolic equifrequency contour, evanescent wave can also transmit in the indefinite medium under the propagation mode, implying the possibility of controlling an evanescent wave by this special indefinite medium. Simulations are performed to demonstrate the controlling performance and a potential design to realize such a medium by metamaterial with multilayered metal/dielectric structure. This work may supply a shortcut for those former devices based on the Transformation optics.

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1. INTRODUCTION

By using transformation optics, an EM (electromagnetic) wave can be directed and controlled according to human's will, which renders a well-known innovation of cloaking an object invisible to the incident rays by guiding them around the object and returning them to their original trajectory [1–5]. After that, there has been a gold rush of studies in designing various optic devices [6–11] allowing the electromagnetic wave to be split, distorted, or squeezed. With transformation optics, optical design is largely the matter of computing the complex material parameters by coordinate transformation. However, the material design in transformation optics is usually inhomogeneous with complex anisotropy, thus, making the material rather difficult to realize even by metamaterials [12, 13].

Recently, a new type of electromagnetic medium is under study: indefinite medium, enabling an all angle negative refraction of the EM wave by its hyperbolic EFC (equi-frequency contour) [14–19]. Physically, the all angle negative refraction is brought by its strong anisotropy by contrast to the birefringence crystal with rather moderate anisotropy. Accordingly, if the anisotropy of an indefinite medium is magnified to extremity, e.g., the negative value is much larger than the positive one in the dielectric tensor, the new property may be obtained by such medium. In this work, we find a capability of guiding the EM wave propagation arbitrarily in the indefinite medium with extremely strong anisotropy.

2. THEORY

We start with the demonstration of the arbitrary guiding ability of an indefinite medium with an infinite negative permittivity by its linear type EFC. A uniaxial indefinite medium with its optic axis along tangential orientation can be described by a dielectric tensor in a polar coordinate:

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \tag{1}$$

where ε_r, ε_t and ε_{z} are permittivities in the radial, tangential and axial directions respectively, as shown in Fig. 1. The electric field and magnetic field of an incident plane wave can be expressed by the general format below:

$$\vec{E}(r) = E_0 e^{j\left(\vec{k}\cdot\vec{r}-wt\right)}, \quad \vec{H}(r) = H_0 e^{j\left(\vec{k}\cdot\vec{r}-wt\right)}$$

where k is the wave vector and ω is the frequency.

For a TM (transverse-magnetic) wave with its wave vector lying in the r-t plane and $\mathbf{H}$ field polarized along y axis, the electromagnetic field in the indefinite medium can be obtained by Maxwell's equation $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \varepsilon_0 \bar{\varepsilon} \frac{\partial \mathbf{E}}{\partial t}$ [17]:

$$\mathbf{H} = H_0 \hat{u}_z e^{j(k_r \cdot r + k_t \cdot \theta - \omega t)} \tag{2}$$

$$\begin{aligned} \mathbf{E} &= -\frac{k \times \mathbf{H}}{\omega \varepsilon_0} (\bar{\varepsilon})^{-1} \\ &= \frac{1}{\omega \varepsilon_0} \left(\frac{k_r \cdot H_0}{\varepsilon_t} \hat{u}_t - \frac{k_t \cdot H_0}{\varepsilon_r} \hat{u}_r \right) e^{j(k_r \cdot r + k_t \cdot \theta - \omega t)} \end{aligned} \tag{3}$$

Here, $\hat{u}_r$, $\hat{u}_t$, and $\hat{u}_z$ are the unit vectors along the r, t and z-axis, respectively. Then, substitute Eqs. (2) and (3) into Maxwell's equation $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_0 \mu \frac{\partial \mathbf{H}}{\partial t}$, the dispersion of the electromagnetic wave in such an indefinite medium is derived [17, 20]:

$$\begin{aligned} \left(\frac{k_r^2}{\varepsilon_t} + \frac{k_t^2}{\varepsilon_r} \right) H_0 \hat{u}_z &= \mu \mu_0 \varepsilon_0 \omega^2 H_0 \hat{u}_z \\ \frac{k_r^2}{\varepsilon_t} + \frac{k_t^2}{\varepsilon_r} &= \frac{\omega^2}{c^2} \end{aligned} \tag{4}$$

The time-averaged Poynting vector in the indefinite medium is written as:

$$\mathbf{S}_R = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* = \frac{1}{2} \frac{1}{\omega \varepsilon_0} \left(\frac{k_r H_0^2}{\varepsilon_t} \hat{u}_r + \frac{k_t H_0^2}{\varepsilon_r} \hat{u}_t \right) \tag{5}$$

In the case of $-\varepsilon_t \gg \varepsilon_r$ (or $-\varepsilon_t \to \infty$), the imaginary axis of the hyperbola also increases, much larger than the real axis, and then the hyperbola is stretched into a straight line, as shown in Fig. 1(a). Consequently, the EFC is expressed by:

$$\begin{aligned} k_t^2 &= \lim_{-\varepsilon_t \to \infty} \varepsilon_r \left(\frac{\omega^2}{c^2} - \frac{k_r^2}{\varepsilon_t} \right) \\ k_t^2 &= \frac{\omega^2}{c^2} \varepsilon_r, \end{aligned} \tag{6}$$

The normal of such a linear EFC directs the Poynting vector:

$$\begin{aligned} \mathbf{S}_R &= \lim_{-\varepsilon_t \to \infty} \frac{1}{2} \frac{1}{\omega \varepsilon_0} \left(\frac{k_r H_0^2}{\varepsilon_t} \hat{u}_r + \frac{k_t H_0^2}{\varepsilon_r} \hat{u}_t \right) \\ \mathbf{S}_R &= \frac{1}{2} \frac{1}{\omega \varepsilon_0} \frac{k_t H_0^2}{\varepsilon_r} \hat{u}_t \end{aligned} \tag{7}$$

From Eqs. (6) and (7), we will find that EM wave from any incident direction will propagate along the orientation with negative permittivity (ε_t direction), which also directs the normal of the EFC, like the oblique incidence case in Fig. 1(b). Therefore, the orientation of ε_t can be used to control the energy propagation.

If a TM wave is incident on the indefinite medium along its tangential orientation, both the wave vector k_R and Poynting vector

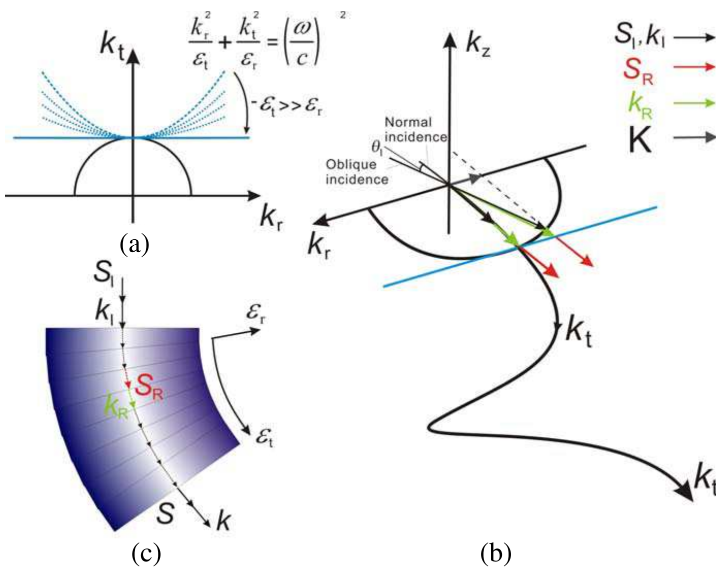

Figure 1. Refraction scheme of the TM wave in an indefinite medium with a straight line type EFC. The black circle is the EFC of the air and the blue curve is the EFC of the indefinite medium. k_I and $\mathbf{S}_I$ are the incident wave vector and the Poynting vector, respectively. k_R and $\mathbf{S}_R$ are the refractive wave vector and the Poynting vector, respectively. If $\varepsilon_r = 1$, the EFCs of the air and the indefinite medium are intersected at $k_R = 0$. (a) Initially, the blue curve is hyperbolic for an indefinite medium with comparable ε_t and ε_r. With the increase of the $|\varepsilon_t|$, the hyperbolic EFC is stretched into a straight line along the z-axis. Thus, the refracted Poynting vector $\mathbf{S}_R$ for any incident light is aligned with the orientation of the ε_t. (b) When a TM wave is incident on an indefinite medium with $-\varepsilon_t \to \infty$ with any incident angle θ_I, the refracted Poynting vector keeps perpendicular to the EFC, i.e., tangential orientation (t). For the normal incidence, both the refracted wave vector k_R and the Poynting vector $\mathbf{S}_R$ are guided by the tangential orientation (t) even if the t-axis may change. (c) If $\varepsilon_r = 1$, no reflection occurs at the interfaces of the indefinite medium because of the perfect match with the air.

$\mathbf{S}_R$ is aligned with the tangential direction of the indefinite medium, like the perpendicularly incident case shown in Fig. 1(b). As a result, the energy propagation of the TM wave is controlled by the indefinite medium with a required ε_t orientation in which the TM wave propagates following the guiding material without any reflection at the boundary (Fig. 1(c)).

According to Eq. (6), with $\varepsilon_r > 0$, the value of k_t is always real even for the evanescent mode of $k_r > (\omega/\mathrm{c})$ due to the extremely large $-\varepsilon_t$ [21].

According to Eq. (7), the Poynting vector in such an indefinite medium is aligned with tangential direction, i.e., the direction with the negative infinite permittivity, despite of any incident angle.

Next, let us consider the reflection at the interface where the TM wave is incident on the indefinite medium from the air. According to the Fresnel's equation, the reflection coefficient of a TM wave at the interface between the indefinite medium and air is written as [22]:

$$r = \frac{\varepsilon_r \cos(\theta_\mathrm{I}) - \eta}{\varepsilon_r \cos(\theta_\mathrm{I}) + \eta}, \tag{8}$$

where θ_I is the incident angle, and $\eta = \sqrt{\frac{\varepsilon_r(\varepsilon_t - \sin^2\theta_\mathrm{I})}{\varepsilon_t}}$.

In the case of a normal incidence ($\theta_I = 0$) and an extremely large value of ε_t, the limit of η is:

$$\lim_{-\varepsilon_t \to \infty} \eta = \lim_{-\varepsilon_t \to \infty} \sqrt{\frac{\varepsilon_r(\varepsilon_t - \sin^2\theta_\mathrm{I})}{\varepsilon_t}} = \sqrt{\varepsilon_r},$$

With $\varepsilon_r = 1$, put η into Eq. (8), then we will arrive at:

$$r = 0 \tag{9}$$

which shows a perfect matched interface between the air and the indefinite medium. As a result, the reflection loss is rather low when the TM wave transmits from the air to the indefinite medium or vice versa.

3. SIMULATIONS

Based on the discussion above, the wave propagation can be controlled to any required direction by designing the orientation of the indefinite medium. To demonstrate the EM wave control performance, we use COMSOL Multiphysics 4.0 to simulate the wave propagation in indefinite media of different shapes. The ideal model of the indefinite medium with ε_t being a negative infinite value and $\varepsilon_r = 1$ can be approximately realized by $-\varepsilon_t \gg 1$, which can ensure a good

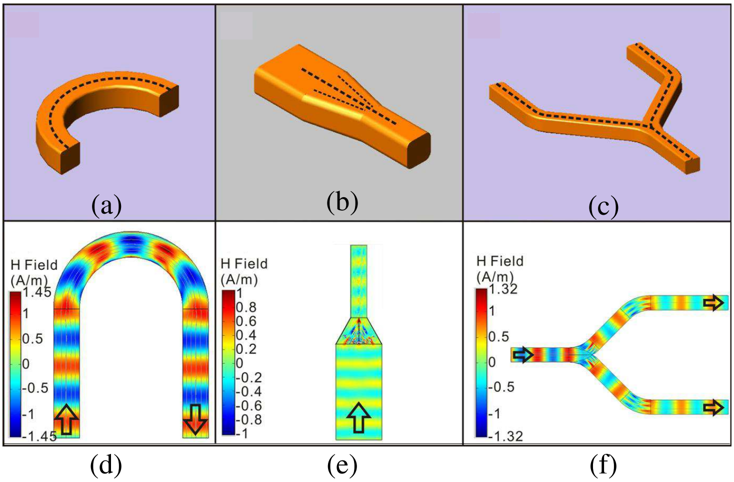

Figure 2. The three basic functional elements: (a) bending, (b) squeezing and (c) splitting and the simulation results of the **H** field distribution in the indefinite media of the three shape: (d) semi-circle with outer radius of 30 cm and inner radius of 20 cm, (e) trapezium with an input width of 30 cm and a output width of 10 cm and (f) the Y-intersection with a splitting angle of 90°. The dash lines in the (a)–(c) indicate the orientation of ε_t in each the indefinite medium. Scattering boundary condition is assigned along the indefinite medium.

guiding performance. The basic type of functional elements: bending, squeezing and splitting are realized by the indefinite media ($\varepsilon_r = 1$, $\varepsilon_t = -100$) with corresponding shapes, as shown in Figs. 2(a)–(c). Meanwhile, the negative permittivity is along the tangential direction of wave propagation. A TEM (transverse-electro-magnetic) mode wave of 13 GHz with **E** field polarized in the r-t plane is incident into the indefinite medium perpendicularly. Figs. 2(d)–(f) show the simulation results of the three elements. Wave propagation constricted in the indefinite medium is guided by the ε_t's orientation of the medium, without any reflection on the boundary. The wave-front at the output side still keeps plane with no distortion.

Due to the simple anisotropy, the medium mentioned above can be realized by using the metamaterial with simple structure, for example, the multilayered structure which is composed by alternating thin layers of a metal and a dielectric. Since metals tend to have $\varepsilon_m < 0$ and dielectrics have $\varepsilon_d > 0$ in the most frequency range, a metamaterial which contains both metal and dielectric with proper ratio can be designed to exhibit an EFC of straight line type. The thickness of each layer must be much smaller than the wavelength in the free space, yielding a reasonable effective medium. In our simulation, assume that $\varepsilon_d = 1$ and $\varepsilon_m = -500$. The indefinite medium contains 40 curved periodic stacks of metal/dielectric, whose thickness is $\lambda/90$, which enables the total structure an effective uniform medium.

The effective parameters of the designed metamaterial were calculated by:

$$\varepsilon_t = p\varepsilon_m + (1-p)\varepsilon_d, \quad \varepsilon_r = \frac{\varepsilon_m \varepsilon_d}{p\varepsilon_d + (1-p)\varepsilon_m}, \tag{10}$$

where p is the filling ratio of the metal. Under $p = 0.1$ in Eq. (10), $\varepsilon_t = -49.9$, $\varepsilon_r = 1.1$, the hyperbolic EFC is almost linear, as plotted in Fig. 3(a). The metal/dielectric slabs are arranged to form the three basic elements, with the tangent direction of the area along the wave propagation, as shown in Figs. 3(c)–(d).

Here we use HFSS, a commercial software based on the finite element method, to demonstrate the manipulation property of the 3D indefinite metamaterial. An EM wave of 13 GHz is used as a probe wave. In Fig. 4(a), the EM wave propagates in an arbitrary path by the indefinite metamaterial. Moreover, by manipulating the

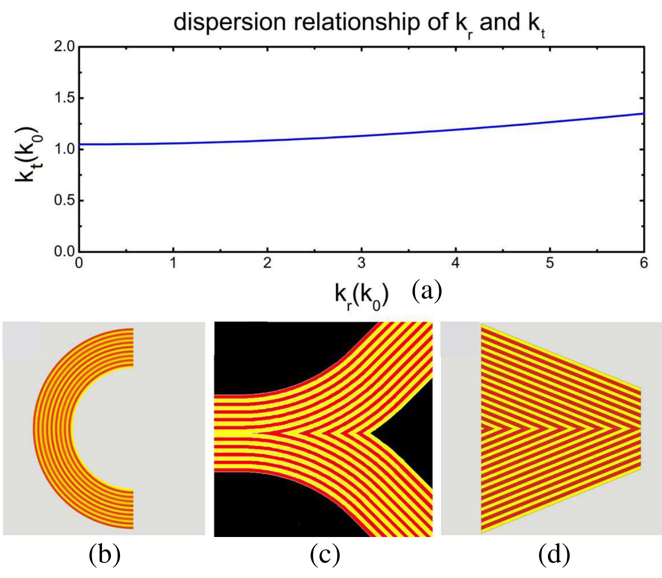

Figure 3. (a) The EFC of the indefinite metamaterial composed by multilayered metal/dielectric slab whose effective permittivities are $\varepsilon_t = -49.9$, $\varepsilon_r = 1.1$. (b) A bent path. (c) A Y shape splitter and (d) a trapezium constrictor by the indefinite metamaterial.

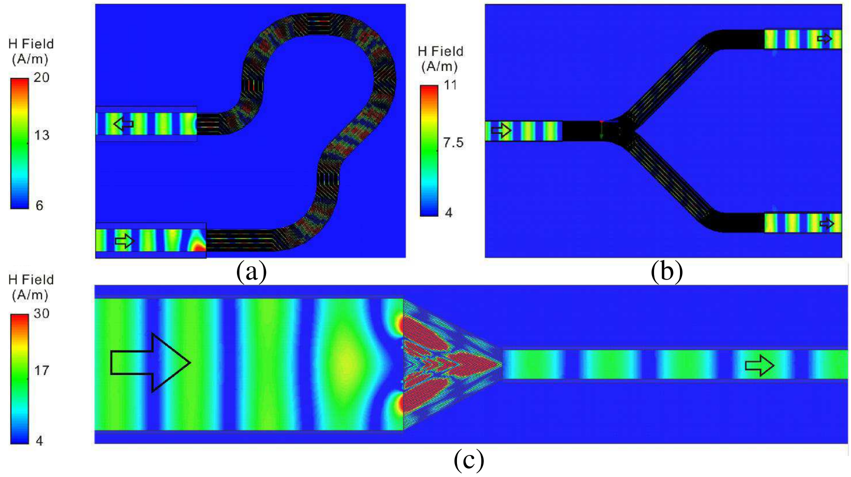

Figure 4. Simulation results of the **H** field in indefinite metamaterial composed by multilayered structure. (a) An arbitrary path. (b) Y shape splitter and (c) trapezium constrictor. Radiation boundary condition is assigned along the indefinite medium in the HFSS.

orientation of the indefinite medium, one beam can be split into two, or squeezed to a narrow path, as illustrated in Figs. 4(b), (c). Because of the extremely large $-\varepsilon_t$, the energy of the EM wave is well restricted in the metamaterial, guided by the ε_t's orientation of the indefinite medium before exiting at the other side. There is hardly any reflection at the interface between the air and the indefinite medium at the input and the output side, which can be attributed to the perfect match by $\varepsilon_r = 1.1$. The output wave keep its original wavefront as same as the incident one. These results are in good agreement with those in Figs. 2(d), (e), (f).

The control of the EM wave by transformation optics is to transform the field distribution from the original Cartesian space to a desired space in order to change the path of the wave propagation [1]. By associating the optical parameters of a material with the coordinate parameters, the desired arm space can be realized by the material with values accomplished by the coordinate transformations. Usually, the transformation is anisotropic and nonlinear, resulting in an anisotropic and inhomogeneous material [23–31], which is rather hard to realize and need to be simplified [27]. By contrast, the indefinite medium is homogenous and anisotropic along two orthogonal directions. The manipulation of EM wave by indefinite medium is to change the direction of the Poynting vector through its extremely large permittivity along the propagation direction. The permittivity at the other direction is 1, which makes the indefinite medium matched with the air. Moreover, because of the hyperbolic EFC, evanescent wave can also transmit in the indefinite medium under the propagation mode, implying the possibility of controlling an evanescent wave by the indefinite medium.

4. CONCLUSION

In conclusion, we have shown how electromagnetic fields can be dragged into almost any desired configuration by an indefinite medium with extremely strong anisotropy. The distortion of the fields is realized by controlling the direction of the Poynting vector through a straight line type EFC. The negative infinite permittivity may greatly restrict the wave energy in the indefinite medium and the positive permittivity of 1 matches with the air without the reflection loss. In the present study, indefinite media with different shapes are used to control an EM wave, which can be easily fabricated, for example, by metamaterial with multilayered structure. Unlike using the conventional method by transformation optics, our method eliminates the need of complex computation of coordinate transformation and the use of an inhomogeneous material. Additionally, the manipulation is also feasible for controlling evanescent waves. We believe this kind of indefinite media may shed light on the light propagation at nano scales.

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