

A NOVEL LPI METHOD OF RADAR'S ENERGY CONTROL

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Abstract—A novel radar energy control strategy based on an improved Interacting Multiple Model Particle Filter (IMMPF) tracking method is presented in this paper. Firstly, the IMMPF tracking method is improved by increasing the weight of the particle which is close to the system state and updating the model probability of every particle. Based on this improved IMMPF method, an energy control method for Low Probability of Intercept (LPI) is then presented, which controls the emission time and power of radar according to the target's range and radar cross section (RCS), under the condition of constant detection probability. The tracking accuracy and LPI performance are demonstrated in the Monte Carlo simulations. The results are validated through the comparisons with other methods.

1. INTRODUCTION

In order to achieve important tactical requirement of LPI, dynamically controlling the emission of a radar during sensor management is very necessary. As we know, less radar emission means more excellent performance of the LPI. An LPI management algorithm for multiple sensors is proposed in [1], by formulating the problem as a partially observed Markov decision process (POMDP) with an on-going multi-armed bandit structure. A circular equivalent vulnerable radius [2], as a function of three sets of parameters — the interceptor performance, emitter antenna pattern, and geometric/link parameters, is used to quantify the LPI capability of a waveform. Many LPI waveforms [3, 4]

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have also been designed to minimize the probability of intercept by an enemy receiver. LPI performance factor is derived and applied in [5]. The work in [6] analyses the effect of the LPI strategies and considers whether there are any fundamental limits to the ability to detect radar emissions. Those works only concern the problem of LPI ability for the radar. However, the radar should have not only good LPI capability, but also excellent detection and tracking performance.

Target tracking is one of the critical problems in the airborne surveillance systems. As we know, for targets with fixed kinematic behavior, a single model state estimator is sufficient for tracking targets. However, for targets with varying or multiple kinematic behaviors, the interacting multiple model (IMM) [7–9] is often employed. The IMM estimator is widely accepted as one of the most cost-effective dynamic multiple model methods and has been shown to achieve high performance with relatively low complexity.

The models of the target dynamics and the observations often exhibit nonlinearities; the filtering should take into account of or at least be robust against these characteristics. Particle filters [10–12] have been introduced to deal with these nonlinearities in the dynamics and measurements. Particle filter is based on Bayesian estimation theory which gives an optimal solution [13] when the dynamic behavior of the object is uncertain and the measurement of the object nondeterministic. As the maneuvering target tracking could be formulated as a multiple model nonlinear filtering problem [14], a new method combining the interacting multiple model approach with a particle filter approach is presented in [15], which is able to deal with nonlinearities and non-Gaussian noise in a mode.

In this paper, we improve the IMMPF algorithm in [15] for more accurate estimates for tracking targets. We first use the similarity between the particles and the system state to modify the weight of the particle. And the estimate of the state is obtained by computing the mode probability of every particle. Based on the improved IMMPF algorithm, a novel algorithm of energy control for LPI is proposed. Under the condition of constant detection probability, the radar's energy is adaptively designed according to different ranges and RCSs of the target.

The remainder of this paper is organized as follows. Sections 2 and 3 describe the improved IMMPF algorithm of target tracking and the energy control methods for LPI in details, respectively. Simulations of the proposed algorithms and comparison results with other methods are provided in Section 4. The conclusions are presented in Section 5.

2. IMPROVED IMMPF ALGORITHM OF TARGET TRACKING

The IMMPF tracking method is improved by increasing the weight of the particle which is close to the system state and updating the model probability of every particle in Sections 2.4 and 2.5, respectively.

2.1. System Setup

Given the system at time k

$$\mathbf{X}_k = \mathbf{F}\mathbf{X}_{k-1} + \mathbf{U}_{k-1} \quad (1)$$

$$\mathbf{Z}_k = \mathbf{H}(\mathbf{X}_k) + \mathbf{V}_k \quad (2)$$

where, $\mathbf{X}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]$ is the dynamical state of the system, and $(x_k, \dot{x}_k)$ and $(y_k, \dot{y}_k)$ are respectively the range, velocity of the direction of X and Y . $\mathbf{U}_{k-1}$ and $\mathbf{V}_k$ are the process noise matrix and measurement noise matrix. $\mathbf{F}$ is the dynamic matrix of the system and $\mathbf{Z}_k$ the measurement vector. As we know, the signal to noise ratio of the measurement vector is decided by the emitted energy. Radar is a typical active sensor which provides both range and angle measurements for target tracking. For concise description, the elevation angle is supposed to be zero here. The measurement vector of the radar is composed of $\mathbf{Z}_{rRk}$ and $\mathbf{Z}_{\theta Rk}$ which are the measurement of the range r and azimuth angle θ .

$$\mathbf{Z}_{rRk} = \sqrt{x_k^2 + y_k^2} + \mathbf{V}_{rRk} \quad (3)$$

$$\mathbf{Z}_{\theta Rk} = \arctan \frac{y_k}{x_k} + \mathbf{V}_{\theta Rk} \quad (4)$$

where $(\mathbf{Z}_{rRk})^2/(\mathbf{V}_{rRk})^2 = S_{NR}^k$ and $(\mathbf{Z}_{\theta Rk})^2/(\mathbf{V}_{\theta Rk})^2 = S_{NR}^k$. S_{NR}^k is the signal to noise ratio of the radar's echo at time k , which is decided by the emitted energy. It also has an impact on the tracking accuracy.

There are M kinematics models and N particles for every model in the system.

2.2. Interaction

Compute mixing probabilities:

$$\mu_{k-1}^l(m_k|m_{k-1}) = \frac{p_{ij}\mu_{k-1}^l(m_{k-1})}{\mathbf{b}_{k-1}^l(m_k)} \quad (5)$$

where the model m_k is set at time k . p_{ij} is the Markov transition probability from model m_{k-1} to model m_k and $\mu_k^l(m_k)$ the mixing

probability of the l th particle for model $m_k, l = 1, 2, \dots, N$. $\mathbf{b}_{k-1}^l(m_k)$ is the normalizing factor:

$$\mathbf{b}_{k-1}^l(m_k) = \sum_{m_{k-1} \in M} p_{ij} \boldsymbol{\mu}_{k-1}^l(m_{k-1}) \quad (6)$$

$\bar{\mathbf{x}}_{k-1}^l(m_k)$ is obtained by the interaction with the other models:

$$\begin{aligned} & \bar{\mathbf{x}}_{k-1}^l(m_k) \\ = & \sum_{m_{k-1} \neq m_k}^M \left[\hat{\mathbf{x}}_{k-1}^l(m_{k-1}) \boldsymbol{\mu}_{k-1}^l(m_{k-1}|m_k) + \hat{\mathbf{x}}_{k-1}^l(m_k) \boldsymbol{\mu}_{k-1}^l(m_k|m_k) \right] \quad (7) \end{aligned}$$

2.3. Prediction Stage

Predicted sample of every particle at time k :

$$\bar{\mathbf{x}}_k^l(m_k) = \mathbf{F}(m_k) \bar{\mathbf{x}}_{k-1}^l(m_k) + \mathbf{u}_{k-1}^l(m_k) \quad (8)$$

Predicted output:

$$\bar{\mathbf{z}}_k^l(m_k) = \mathbf{H} \left(\bar{\mathbf{x}}_k^l, m_k \right) \quad (9)$$

2.4. Modification of the Weight for the Particle

If the measurement vector of a particle is close to the measurement vector of the true state, its vector will be close to the vector of the true state. So it is helpful to increase the influence of those particles which are closer to the true state, for obtaining more accurate tracking results.

$\bar{\mathbf{z}}_k^l(m_k)$ is the estimation of azimuth angle $\bar{\boldsymbol{\theta}}_R^l(m_k)$ and range $\bar{\mathbf{r}}_R^l(m_k)$ from the l th particle at time k . $\mathbf{PCC}_{\theta_R, k}^l(m_k)$ and $\mathbf{PCC}_{r_R, k}^l(m_k)$ are respectively the Pearson correlation coefficient of the azimuth angle and range, which are used to measure the similarity of the two vectors.

$$\begin{aligned} & \mathbf{PCC}_{\theta_R, k}^l(m_k) \\ = & \frac{\sum_{i=k-P+1}^k \left(\bar{\boldsymbol{\theta}}_R^l(m_i) - (\bar{\boldsymbol{\theta}}_R^l(m_i))^* \right) (\mathbf{Z}_{\theta_R i} - (\mathbf{Z}_{\theta_R i})^*)}{\sqrt{\sum_{i=k-P+1}^k \left(\bar{\boldsymbol{\theta}}_R^l(m_i) - (\bar{\boldsymbol{\theta}}_R^l(m_i))^* \right)^2 \sum_{i=k-P+1}^k \left(\mathbf{Z}_{\theta_R i} - (\mathbf{Z}_{\theta_R i})^* \right)^2}} \quad (10) \end{aligned}$$

$$\begin{aligned} & \mathbf{PCC}_{r_R,k}^l(m_k) \\ = & \frac{\sum_{i=k-P+1}^k (\bar{\mathbf{r}}_R^l(m_i) - \bar{\mathbf{r}}_R^l(m_i)^*) (\mathbf{Z}_{r_R i} - (\mathbf{Z}_{r_R i})^*)}{\sqrt{\sum_{i=k-P+1}^k (\bar{\mathbf{r}}_R^l(m_i) - (\bar{\mathbf{r}}_R^l(m_i))^*)^2 \sum_{i=k-P+1}^k (\mathbf{Z}_{r_R i} - (\mathbf{Z}_{r_R i})^*)^2}} \quad (11) \end{aligned}$$

where $(\cdot)^*$ means the average value of the past P numbers.

Then $\mathbf{PCC}^l(m_k)$ is given by:

$$\mathbf{PCC}^l(m_k) = \frac{1}{2} \left(\mathbf{PCC}_{\theta_R,k}^l(m_k) + \mathbf{PCC}_{r_R,k}^l(m_k) \right) \quad (12)$$

The weight update equation can be given by:

$$\bar{\mathbf{w}}_k^l(m_k) = \mathbf{PCC}^l(m_k) \mathbf{p}_{v_k(m_k)} \left(\mathbf{z}_k - \bar{\mathbf{z}}_k^l(m_k), 0 \right) \quad (13)$$

where $\mathbf{p}_{v_k(m_k)}(\cdot)$ is the probability density of the measurement noise $\mathbf{v}_k(m_k)$.

The normalized weight can be given:

$$\bar{\mathbf{w}}_k^l(m_k) = \bar{\mathbf{w}}_k^l(m_k) / \sum_{l=1}^N \bar{\mathbf{w}}_k^l(m_k) \quad (14)$$

The new weight after the importance sampling is set as follows:

$$\bar{\mathbf{w}}_k^l(m_k) = 1/N \quad (15)$$

2.5. Update of the Model Probability

The common methods often update the model probability by the average residual matrix of all the particles and ignore the model character of single particle. Therefore, the model probability of every particle is updated here.

Residual covariance matrix of the l th particle is obtained as:

$$\mathbf{S}_k^l(m_k) = \mathbf{r}_k^l(m_k) \mathbf{r}_k^l(m_k)^T \quad (16)$$

where $\mathbf{r}_k^l(m_k)$ represents the residual vector and can be given by:

$$\mathbf{r}_k^l(m_k) = \mathbf{z}_k - \mathbf{H} \left(\bar{\mathbf{x}}_k^l, m_k \right) \quad (17)$$

The likelihood function of the model is:

$$\Lambda_k^l(m_k) = \sqrt{|2\pi \mathbf{S}_k^l(m_k)|} \exp \left(-\frac{1}{2} \left(\mathbf{r}_k^l(m_k) \right)^T \left(\mathbf{S}_k^l(m_k) \right)^{-1} \mathbf{r}_k^l(m_k) \right) \quad (18)$$

The updated model probability can be given by:

$$\boldsymbol{\mu}_k^l(m_k) = \frac{\boldsymbol{\Lambda}_k^l(m_k)\mathbf{b}_{k-1}^l(m_k)}{B_k^l} \quad (19)$$

where $B_k^l = \sum_{m_k \in M} \boldsymbol{\Lambda}_k^l(m_k)\mathbf{b}_{k-1}^l(m_k)$.

2.6. Combination Stage

The estimate of the state is obtained as:

$$\hat{\mathbf{x}}_k = \sum_{m_k=1}^M \sum_{l=1}^N \bar{\mathbf{w}}_k^l \bar{\mathbf{x}}_k^l(m_k) \boldsymbol{\mu}_k^l(m_k) \quad (20)$$

3. STRATEGY OF ENERGY CONTROL

Phased array radars [16] have the capability to switch the direction of the radar beam very quickly without inertia. So it is very free to decide the beam position and emitted energy to update the established track. The energy control methods for LPI of the phased array radar are proposed in this section.

3.1. Intercept Probability

The intercept probability [17] P_I is defined as

$$P_I = MF(2P_R/P_{SI})^{C_0} D_I T_{OT}/T_I \quad (21)$$

where MF is the mainlobe footprint (3 dB), P_{SI} the required power at intercept receiver for detection, D_I the density of intercept receivers per km^2 , and C_0 the sensitivity scaling factor. These parameters in the later simulation are drawn from [17]. Table 1 shows the parameters' values.

T_{OT} and T_I denote the illumination time by emitter and the interceptor search time, respectively. T_I is assumed to be the total tracking time. P_R is the received power at intercept receiver, which can be defined as

$$P_R = \frac{P_T G_{TI} G_I \lambda^2 G_{IP} L_I}{(4\pi)^2 R_I^2} \quad (22)$$

Table 1. Values of the parameters.

Parameter	MF	P_{SI}	C_0	D_I
Value	11.2	5×10^{-12}	0.477	0.001

where P_T is the emitted power of the radar, G_{TI} the antenna gain of the radar in the direction of the intercept receiver, G_I the antenna gain of the intercept receiver, G_{IP} the processing gain of the intercept receiver, R_I the interception range, λ the wavelength, and L_I the interceptor system loss.

As we know, it is difficult to design MF , P_{SI} , D_I and T_I as they are decided by the performance of the interceptor instead of the radar, so we have to reduce the radar's emission time T_{OT} or power P_T to lower the probability of intercept of the radar system. According to [17], P_R is equal to $P_T \times 9.003 \times 10^{-9}$ here.

During the tracking process, the cumulative probability of intercept P_{Icum} is defined as:

$$P_{Icum} = 1 - \prod_{i=1}^I (1 - P_I^i) \quad (23)$$

P_I^i is the i th intercept probability and I the intercept times.

3.2. Design of Emission Energy

Radar equation at time k is as follows:

$$R_k^4 = t_B^k \frac{P_{av}^k G_T G_R \lambda^2 \sigma_k}{(4\pi)^3 K T_R S_{NR}^k L} \quad (24)$$

where t_B^k is the single dwelling time of the beam from the normal direction at time k , P_{av}^k the average radiated power, G_R the receiver gain, σ_k the radar cross section (RCS) of the target, K Boltzmann constant, T_R and L respectively effective noise temperature and radar system loss, R_k the detection range, G_T the transmit gain, and S_{NR}^k the signal to noise ratio of the radar's echo at time k . Detection probability [18] P_d^k at time k can be given as:

$$P_d^k = p_{fa}^{1/(1+S_{NR}^k)} \quad (25)$$

where p_{fa} is the false alarm probability of the radar.

Suppose that when the target whose range and RCS are respectively $R_{\max}$ and $\sigma_{\min}$, the radar has to emit the maximum power $P_{av \max}$ with maximum dwelling time $t_{B \max}$. Radar equation and the detection probability respectively become:

$$R_{\max}^4 = t_{B \max} \frac{P_{av \max} G_T G_R \lambda^2 \sigma_{\min}}{(4\pi)^3 K T_R S_{NR \max} L} \quad (26)$$

$$P_d = p_{fa}^{1/(1+S_{NR \max})} \quad (27)$$

The detection probability is supposed to meet the requirement of constant P_d during the whole tracking process. So S_{NR}^k at time k should be equal to $S_{NR\max}$. Combining (24) with (26), the emitting energy at time k can be written as:

$$P_{av}^k t_B^k = \frac{P_{av\max} t_{B\max} \sigma_{\min}}{R_{\max}^4} \frac{R_k^4}{\sigma_k} \quad (28)$$

However, during the tracking process, R_k and σ_k are unknown before radar detection. R_k is replaced by R_k^{pre} which is predicted by R_{k-1} and v_{k-1} . R_k^{pre} is presented as

$$R_k^{pre} = R_{k-1} + v_{k-1}T \quad (29)$$

where R_{k-1} and v_{k-1} are the target's range and velocity estimated by the tracking algorithm at time $k-1$, and T is the tracking interval.

As we know, RCS is the ratio of the scattered power to the incident power in the direction of an observer at infinity [17] and can be computed by the fundamental RCS equation once the radar echo is received. The RCS σ_{k-1} at time $k-1$ is supposed to be known here. It is difficult to predict a target's RCS, so σ_k is replaced by σ_{k-1} .

So (28) becomes

$$P_{av}^k t_B^k = \frac{P_{av\max} t_{B\max} \sigma_{\min}}{R_{\max}^4} \frac{(R_k^{pre})^4}{\sigma_{k-1}} \quad (30)$$

In addition, the emitted energy methods of adaptive power (I-IMMPF-A-P) and adaptive time (I-IMMPF-A-T) are presented based on the improved IMMPF algorithm.

I-IMMPF-A-P: $t_B^k = t_{B\max}$, and P_{av}^k is designed according to change of the target range and target RCS.

$$P_{av}^k = \frac{P_{av\max} \sigma_{\min}}{R_{\max}^4} \frac{(R_k^{pre})^4}{\sigma_{k-1}} \quad (31)$$

I-IMMPF-A-T: $P_{av}^k = P_{av\max}$, and t_B^k is designed according to change of the target range and target RCS too.

$$t_B^k = \frac{t_{B\max} \sigma_{\min}}{R_{\max}^4} \frac{(R_k^{pre})^4}{\sigma_{k-1}} \quad (32)$$

After the emission time and power are designed, the radar equation can be given:

$$S_{NR}^k = t_B^k \frac{P_{av}^k G_T G_R \lambda^2 \sigma_k}{(4\pi)^3 K T_R L R_k^4} \quad (33)$$

Combined with (26), S_{NR}^k can be written as:

$$S_{NR}^k = \frac{P_{av}^k}{P_{av \max}} \frac{t_B^k}{t_{B \max}} \frac{R_{\max}^4}{R_k^4} \frac{\sigma_k}{\sigma_{\min}} S_{NR \max} \quad (34)$$

We can see that the emitted energy and signal to noise ratio influence each other during the tracking process.

4. SIMULATION RESULTS

In this section, Monte Carlo simulations are performed to analyze the performance of the improved IMMPF and the proposed energy control method. The IMM filter here is composed of Constant Velocity model (CV) F_{CV} and Coordinated Turn rate model (CT) F_{CT} .

$$F_{CV} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (35)$$

$$F_{CT} = \begin{bmatrix} 1 & \frac{\sin \omega T}{\omega} & 0 & \frac{1 - \cos \omega T}{\omega} \\ 0 & \cos \omega T & 0 & \frac{\sin \omega T}{\omega} \\ 0 & \frac{1 - \cos \omega T}{\omega} & 1 & \frac{\sin \omega T}{\omega} \\ 0 & \sin(\omega T) & 0 & \cos \omega T \end{bmatrix} \quad (36)$$

where T is the sampling interval, ω the turn factor, $T = 1$ s, and $\omega = 0.1$. In the simulation, M_c is the number of the Monte-Carlo simulation, K the total tracking time in every simulation, $M_c = 500$, and $K = 100$.

4.1. Trajectory Design

Table 2 is the description of the trajectory in detail.

Figure 1 shows the target trajectory in 100 s. The RCS of the target is designed in Fig. 2.

Table 2. Model description of the trajectory.

Time (s)	2–20	41–65	66–100
Target model	CV	CT	CV

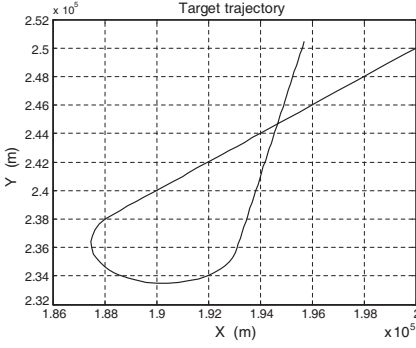


Figure 1. Trajectory of the target.

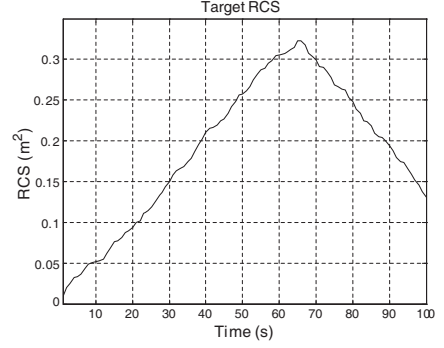


Figure 2. RCS of the target.

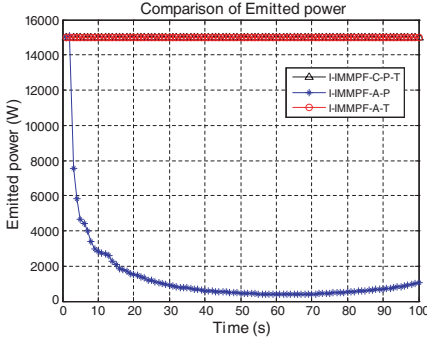


Figure 3. Comparison of Emitted power.

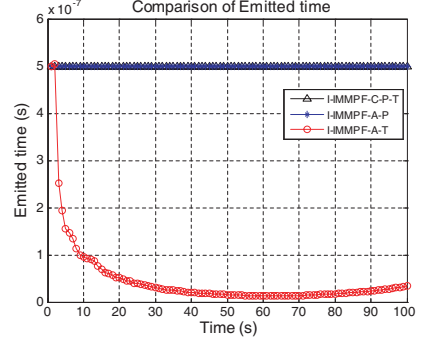


Figure 4. Comparison of Emitted time.

4.2. Comparison of LPI Performance

Based on the proposed improved IMMPF algorithm, the emitted energy methods of constant power and time (I-IMMPF-C-P-T), I-IMMPF-A-P and I-IMMPF-A-T are realized in the simulation. The three methods are described in Table 3. Emitted power of I-IMMPF-A-P and emission time of I-IMMPF-A-T are designed adaptively according to (31) and (32), respectively, which are also shown in Figs. 3 and 4.

Intercept probability is used to evaluate the LPI performance of the three methods. Fig. 5 illustrates their LPI performance. We can see that the I-IMMPF-A-T method has the best LPI ability as the emission time plays a more important role in the intercept probability computation. So the reduction of emission time is more effective.

Table 3. Energy control of the 3 methods.

Method	P_{av}^k (W)	t_B^k (s)
I-IMMPF-C-P-T	15000	$0.5 * 10^{-6}$
I-IMMPF-A-P	Adaptive	$0.5 * 10^{-6}$
I-IMMPF-A-T	15000	Adaptive

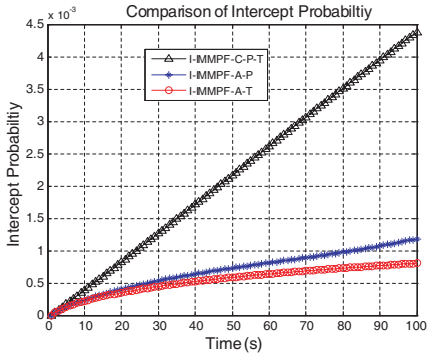


Figure 5. Comparison of intercept probability.

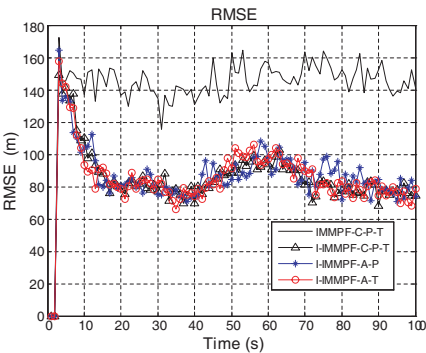


Figure 6. Comparison of RMSE.

4.3. Comparison of Tracking Performance

The root-mean-square error (RMSE) of time k and the average root-mean-square error (ARMSE) of the whole tracking process can be formulated as (37) and (38), respectively:

$$\text{RMSE}(k) = \sqrt{\frac{1}{M_c} \sum_{m=1}^{M_c} (x_k - \hat{x}_k^m)^2} \quad (37)$$

$$\text{ARMSE} = \frac{1}{K} \sum_{k=1}^K \text{RMSE}(k) \quad (38)$$

where x_k is the true state of the system and $\hat{x}_k^m$ the estimated vector at the m th simulation.

The standard IMMPF method [15] with constant power and time (IMMPF-C-P-T) is also realized to compare the tracking accuracy with the improved one. RMSE and ARMSE of all methods with 250 particles are shown in Fig. 6 and Table 4, respectively, which show that the improved IMMPF method is more accurate than the standard IMMPF. However, the improved IMMPF method will spend

Table 4. Comparison of ARMSE.

Method	ARMSE (m)
IMMPF-C-P-T	142
I-IMMPF-C-P-T	85.0
I-IMMPF-A-P	86.6
I-IMMPF-A-T	85.5

a little more time on the computation, as it has to compute the Pearson correlation coefficient to modify the weight of the particles.

Comparing with I-IMMPF-C-P-T, we can see that the proposed method I-IMMPF-A-T and I-IMMPF-A-P not only reduce more emitted energy, but also present almost the same excellent tracking accuracy. Obviously, I-IMMPF-A-T has the best LPI ability with better tracking performance.

4.4. Simulation Analysis

The improved IMMPF method achieves better tracking performance in the simulation, because it increases the weight of the particle which is close to the system state and updates the model probability of every particle. In addition, it involves the filtering for the current state and smoothing for the previous state.

Moreover, the proposed two energy control methods are more efficient than the common method which always emits constant power and time, because both the target's range and RCS are used to adaptively design the emitted time and power here. The LPI method depends on tracking algorithm as the target range has to be predicted every time.

5. CONCLUSIONS

In this paper, we have presented a new strategy of energy control for LPI based on an improved IMMPF approach. This method employs both the target's range and RCS to design the radar emitted energy with excellent tracking accuracy.

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