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UNCERTAINTY MINIMIZATION IN PERMITTIVITY MEASUREMENTS IN SHIELDED DIELECTRIC RESONATORS

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Abstract—In this paper, we present a novel general methodology which ensure a minimum uncertainty in the measurement of the real part of the permittivity of a material measured using cylindrical shielded dielectric resonators. The method is based on the fact that for any given value of the dielectric permittivity there is an optimal radius of the cylindrical dielectric rod sample. When the dielectric rod sample has the optimum radius, the width of the coverage interval associated to the real part of the dielectric permittivity measurement result — for a given confidence level — is reduced due to a lower sensitivity of the dielectric permittivity to be measured versus the variations in the resonant frequency. The appropriated radius of a given sample under test is calculated using Monte Carlo simulations for a specific mode and a specific resonant frequency. The results show that the confidence interval could be reduced by one order of magnitude with respect to its maximum width predicted by the uncertainty estimation performed using the Monte Carlo method (MCM) as established by the supplement 1 of the Guide to the Expression of Uncertainty in Measurement (GUM). The optimum radius of the sample under examination is fundamentally determined by the electromagnetic equations that describe the measurement and does not depend specifically of the sources of uncertainty considered.

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1. INTRODUCTION

Continuing with our previous work [1] in this paper we present an effective method that minimizes the uncertainty in the measurement of the real dielectric permittivity of a material. As we indicated previously, the method is valid only for low loss materials in view of the nature of the exact relations between permittivity, sample and cavity dimensions, measured resonant frequency and the unloaded Q-factor for the resonant structures. Resonant methods are the preferred technique in dielectric permittivity measurements over non-resonant measurements [2], in view to their higher accuracy and sensitivity. Although, recent papers have discussed the problems associated to the estimation of uncertainty in the measurement of dielectric permittivity of materials, but only a few have proposed a systematic methodology for the reduction of the uncertainty associated to the measurement of the dielectric permittivity using resonant cavities [3].

In order to calculate the confidence level of the results associated to transmission/reflexion methods in coaxial lines, NIST has evaluated the results obtained by different laboratories [4]. Permittivity values obtained with an electromagnetic resonator system were used as reference value and were found to be within $\pm 10\%$ of the coaxial line results. The value of the air gap between the inner and the outer conductor was the principal source of uncertainty in these coaxial line measurements. Some algorithms have been used for the correction of permittivity measurement based on the capacitor model assuming that the air-gap is uniform between the sample and the outer and inner conductor of the coaxial line. The system is treated as three capacitor in series. In other cases the reference measurement was provided by a stripline resonator [5]. This reference describes a method that minimizes the inherent uncertainty in these cavity based results. The method is based on the appropriate selection of the parameters used in the conformal mapping modeling procedure used. The expected uncertainty in the permittivity of the samples was found to be within $\pm 10\%$ when compared with results obtained using the NIST stripline resonator.

The technique most commonly used for the estimation of uncertainty in permittivity measurements is the differential analysis (partial derivative technique) [6–8] where the sensitivity coefficients are computed as the derivatives of the permittivity (real or complex) with respect to the scattering parameter and dimensional variables among others [2, 9]. It is important to notice that, there is a lack of consistency between the way in which the uncertainty is estimated in the aforementioned works and the guidelines given by the Bureau

International des Poids et Mesures (BIPM) through the Guide to the Expression of Uncertainty in Measurement (GUM) [10]. In order to estimate the uncertainty associated with measurements in a shielded dielectric resonator we recently proposed the use of a Monte Carlo method [1], as established by the supplement 1 of the GUM [11]. This method provides the bases of the present paper in where we propose a novel method to minimize the uncertainty in permittivity measurements. The minimization is achieved since we can show that an optimal value for the radius of the sample can be selected in the interval values where, the computed permittivity exhibits a low sensibility with respect to variations in the measured resonant frequency. These variations are due to all the sources of uncertainty in the measurement.

In this paper, we will first discuss the bases of the uncertainty estimation in real permittivity measurement using Monte Carlo Method. We will then show how to determine the usable frequency range for TE or TM modes used in the measurement of permittivity in a shielded dielectric resonator. Finally, we will discuss the methodology required for the minimization of the uncertainty and we will include a numerical example.

2. UNCERTAINTY ESTIMATION IN COMPLEX PERMITTIVITY MEASUREMENT USING THE MONTE CARLO METHOD

In our previous work [1], we discussed the required concepts for the uncertainty estimation using MCM in permittivity measurement. This powerful tool will now be employed to find the optimum dimension of a dielectric rod sample in order to minimize the uncertainty in the measurement of the real part of the permittivity.

For the shielded cylindrical dielectric resonator shown in Figure 1

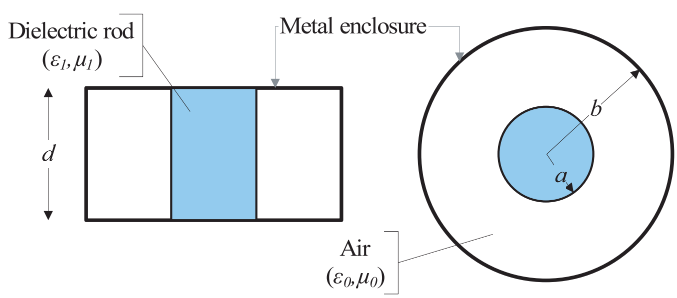

Figure 1. Shielded dielectric resonator.

the description of the measurement is summarized by Equations (1) and (3).

$$\frac{n{k_{c0}}^2\mu_1}{a{k_{c1}}^2\mu_0}-\frac{{k_{c0}}^2\mu_1}{k_{c1}\mu_0}\frac{J_{n+1}\left(k_{c1}a\right)}{J_n\left(k_{c1}a\right)}=\frac{n\alpha}{a}-k_{c0}\frac{\alpha J_{n+1}\left(k_{c0}a\right)+Y_{n+1}\left(k_{c0}a\right)}{\alpha J_n\left(k_{c0}a\right)+Y_n\left(k_{c0}a\right)}, \quad (1)$$

where,

$$\alpha=-\frac{nY_n\left(k_{c0}b\right)-bk_{c0}Y_{n+1}\left(k_{c0}b\right)}{nJ_n\left(k_{c0}b\right)-bk_{c0}J_{n+1}\left(k_{c0}b\right)}, \quad (2)$$

and,

$$\frac{k_{c1}^2+\beta^2}{k_{c0}^2+\beta^2}=\frac{\mu_1\varepsilon_1}{\mu_0\varepsilon_0}, \quad (3)$$

where b is the cavity resonator radius, a is the dielectric rod radius and d is the height of the cavity. $J_n\left(k_c\rho\right)$ is the Bessel function of the first kind and $Y_n\left(k_c\rho\right)$ is the Bessel function of the second kind, both of order n.

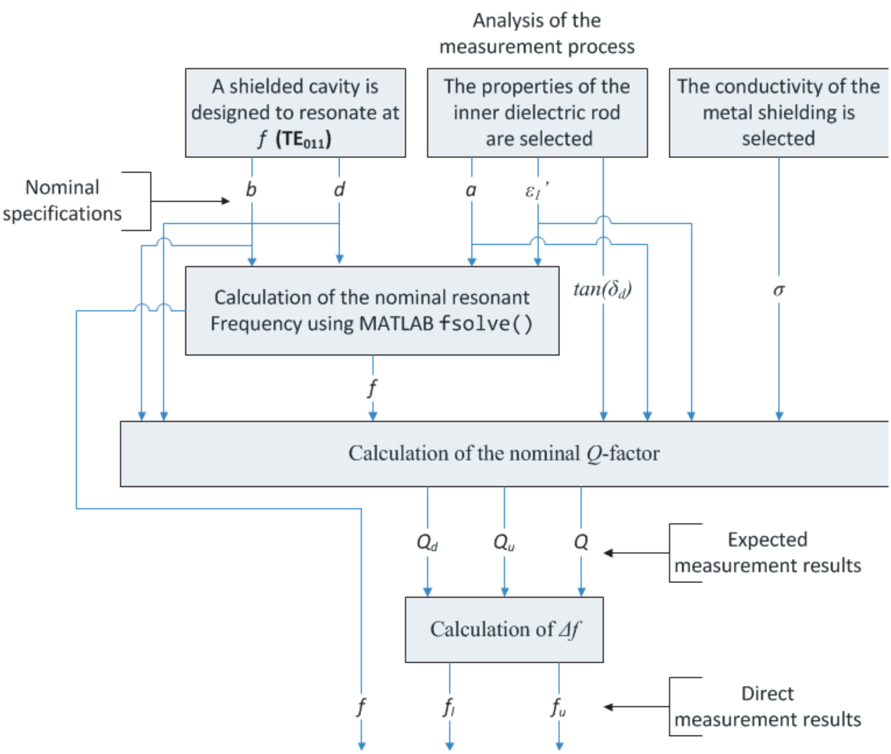

Figure 2. Analysis of the measurement process.

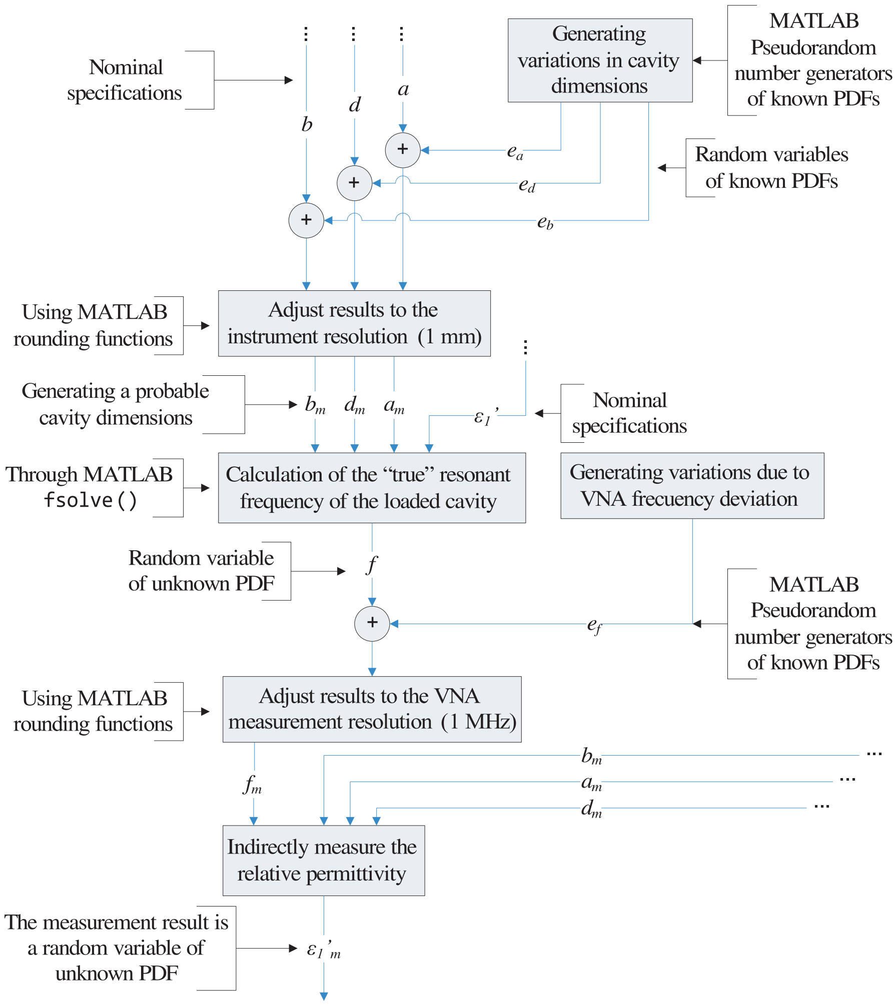

Figure 3. The model of measurement process of ε'_{1m}.

The cutoff wavenumber of both media, the unknown dielectric, k_{c1}, and the air k_{c0}, are related to the angular resonance frequency, $\omega = 2\pi f$, as follows,

$$k_{c1}^2 = \omega^2 \mu_1 \varepsilon_1 - \beta^2, \tag{4}$$

$$k_{c0}^2 = \omega^2 \mu_0 \varepsilon_0 - \beta^2. \tag{5}$$

In order to estimate the uncertainty in the complex permittivity measurements obtained by shielded dielectric resonator technique, we start from a set of nominal specifications such as the electromagnetic properties of a given dielectric for the sample under test, cavity dimensions (designed to resonate approximately at a desired frequency for the selected mode), and conductivity of the metal enclosure. From these values the nominal direct measurement results are calculated theoretically, as shown in Figure 2. The procedure assumes a real permittivity value (which is also valid for low loss complex permittivity materials).

The random errors associated to the uncertainty contributions of each contributing factor (Table 1) are added to the nominal specifications in order to calculate the new direct measurement results (f_m, f_{um} and f_{lm}) and consequently the indirect measurement results ε'_{1m} and $\tan(\delta_d)$ are random variables of unknown probability density function (PDF). The errors are generated using the pseudo-random number generators included in MATLAB$^{\mathrm{TM}}$, because they meet the requirements of the Monte Carlo Method with any specific probability distribution. Figures 2 and 3 show the relationships between the errors in the measurement process.

It is important to note that the variations in the resonant cavity depend on the variations in the dimensions of the cavity with respect to the nominal specifications. The measured resonance frequency is also affected by the measurement errors of the Vector Network Analyzer (VNA). Thus the experimental results, ε'_{1m} and $\tan(\delta_{dm})$, can only be evaluated after a great number of iterations.

Table 1. Sources of uncertainty in complex permittivity measurements by shielded dielectric resonator technique.

Factor	Error	Source of Uncertainty	Type of evaluation	Probability Distribution	Parameters
d	e_d	Accuracy	B	Uniform	MAE
		Resolution	B	Triangular	SD
		Repeatability	A	Normal	s_o
a	e_a	Accuracy	B	Uniform	MAE
		Resolution	B	Triangular	SD
		Repeatability	A	Normal	s_o
b	e_b	Accuracy	B	Uniform	MAE
		Resolution	B	Triangular	SD
		Repeatability	A	Normal	s_o
f	e_f	Calibration	B	Normal	U_{cal}
		Resolution	B	Uniform	SF

2.1. Sources of Uncertainty

The variables needed for the indirect measurement of the dielectric permittivity are mainly influenced by the following uncertainty contributions: a) the accuracy of the instruments used in the measurements; b) the resolution of the measuring instruments and c) the repeatability of the results. In each Monte Carlo iteration, the combination of all the mentioned uncertainty factors associated to the nominal specifications is treated as a measurement error. A summary of the sources of uncertainty in complex permittivity measurements by shielded dielectric resonator technique is shown in Table 1.

The errors associated with the dimensional factors, e_d, e_a and e_b, are given by the sum of the error related to the accuracy of the measurement instrument, e_α, the error related to the resolution of the measurement instrument, e_r, and other errors that affects the repeatability, e_o. The uncertainties contributions due to e_α and e_r are defined in terms of the maximum allowable error (MAE), and the scale division (SD), respectively. Hence, the probability density functions of e_α, $f(e_\alpha)$ and e_r, $f(e_r)$, are given in our previous work [1].

On the other hand, e_f is obtained as the sum of the error related to the accuracy of the calibration of the VNA, e_{cal}, and the error due to rounding off the value of the frequency measured in units of gigahertz to the third significant figure, e_{SF}. e_{cal} is modeled as an unbiased random variable of normal distribution with a standard deviation equal to σ_{cal}, where σ_{cal} (standard uncertainty) is obtained from the calibration certificate of the VNA as the expanded uncertainty reported, U_{cal}, divided by the coverage factor, k (usually $k = 2$ for a 95% of confidence level). In the same way, e_{SF} is modeled as an unbiased uniformly distributed random variable taking values within $-500\,\mathrm{kHz} \leq e_{\mathrm{SF}} \leq 500\,\mathrm{kHz}$. In order to reduce the influence of the repeatability of the frequency measurements, it is recommended to configure the VNA to perform the sample averaging of the readings automatically.

Finally, other factors that might contribute to the uncertainty in complex permittivity measurements, such as sample properties heterogeneities and eccentricity of the cavity walls, were not considered, since it is assumed that the sample under test is well prepared and that the cavity has been carefully constructed and validated.

3. VALID FREQUENCY RANGE FOR TE AND TM MODES IN PERMITTIVITY MEASUREMENTS

When a permittivity measurement is performed in a shielded dielectric resonator is important to know which resonant modes are excited in the cavity. For a specific dielectric permittivity and radius of the dielectric cylindrical rod, the resonant frequency is calculated as solution of the corresponding TE or TM equations set. It means that for a selected working mode of order n the resonant frequency decreases when the dielectric permittivity increases just to the point where the cutoff wave number k_{c1} and k_{c0} produce different frequencies values using Equations (4) and (5). This means that we are dealing with an attenuated mode due to the resonant frequency of the medium 2 (air) becomes a complex quantity. This implies that the existent mode inside the cavity corresponds to a higher order mode. This is the usable range of a particular resonant mode in a specific cavity.

In order to determine the usable frequency range as a function of the dielectric permittivity of the sample for a specific mode with a fixed dielectric rod radius in a cavity resonator, we first solve the Equations (1) and (3) to find the cutoff wave numbers k_{c1}, k_{c0} and then we calculate the resonant frequency using Equations (4) and (5). If the solution is valid, both frequency results — in air and in the dielectric medium — must agree. The permittivity value at which the frequency calculated in both medium are no longer equal is the upper bound that limits the maximum dielectric permittivity that could be measured for this resonant mode and for this sample radius using the specified cavity. To avoid this situation the sample radius could be reduced or a higher resonant mode could be excited.

In order to illustrate our procedure we consider an empty cavity designed for the TE_{021} mode with a resonant frequency of 3.2 GHz, whose radius is twice the length. A dielectric rod with the same height as the cavity is placed at its center, having a radius 3 cm. The relative dielectric permittivity value is assessed in the interval $[1, 18]$, and the resonant frequency in both medium is calculated using (4) and (5). The point where both resonant frequency becomes different ($\varepsilon_1 \approx 17$), illustrated in the Figure 4, marks the usable permittivity range for the TE_{021} mode.

If the permittivity value of the sample is expected to be within this range, then is possible to use this mode for the measurement of the real part of the dielectric permittivity.

4. METHODOLOGY FOR UNCERTAINTY MINIMIZATION

As shown in Figure 4, the greater variations in the resonant frequency in both extremities of the ε_1 vs f curve corresponds to small changes in the dielectric permittivity, in other words, the real part of the dielectric permittivity is less sensitive to the variations in the measured resonant frequency. This can be analyzed in terms of the first derivative of this data. Where the first derivative of this curve is higher, it implies a lower sensitivity of the calculated dielectric permittivity with respect to the uncertainty related in the measured resonant frequency. This effect is mainly caused by the fact that the measurement system is highly non-linear, resulting in variations of the resonant frequency that are not proportional to the changes in the real permittivity measurement. The first derivative of the ε_1 vs f curve is shown in Figure 5.

All dimensional uncertainty sources affect directly the resonant frequency of the empty cavity and this mean that they are the most important parameters to be considered for the reduction of the uncertainty measurement. Therefore, the resonant frequency is used as the most important variable affecting the determination of the real part of the dielectric permittivity.

As a numerical example, lets consider the same cavity resonator and the same radius for the sample under test ($a = 3\,\text{cm}$) for which the valid frequency range for TE and TM modes in permittivity measurements was calculated previously in Section 3. For the aforementioned configuration the measurement results confidence

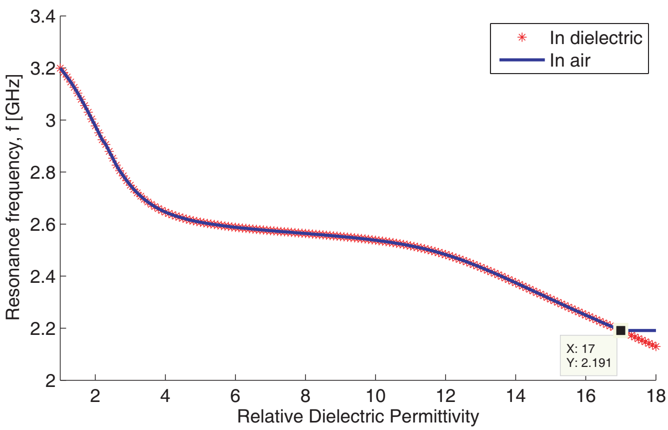

Figure 4. Frequency range for TE_{021} for $a = 3\,\text{cm}$.

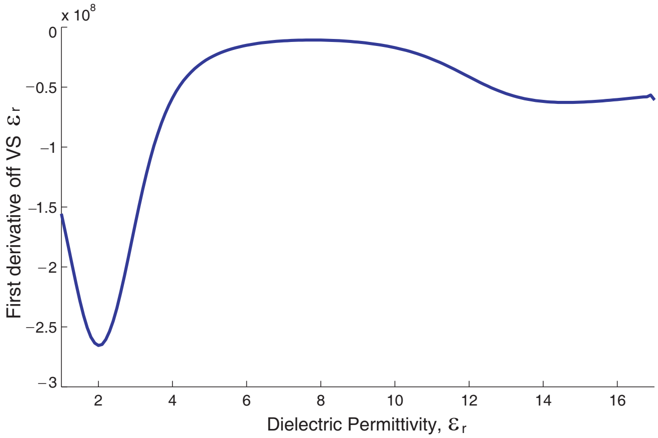

Figure 5. First derivative of resonant frequency with respect to the dielectric permittivity.

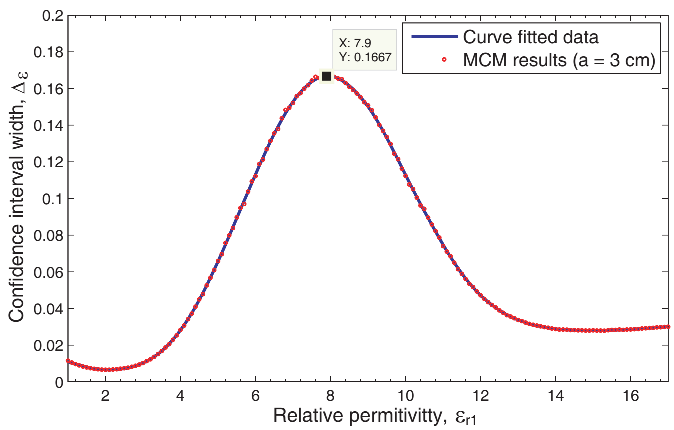

Figure 6. Confidence interval for a range of dielectric permittivity with Radius of 3 cm.

interval will be iteratively calculated within the valid permittivity range, using the MCM with a set of 10^4 runs for each value of the dielectric permittivity considered in the range, which for this particular case, had a resolution of 0.1. It is important to note that the step size in the dielectric permittivity evaluation and the number of Monte Carlo runs must be chosen to meet the computational capabilities. Figure 6 shows the 95% confidence interval width, calculated for every dielectric permittivity in the valid range. Figure 6 shows that there is a dielectric permittivity value ($\varepsilon_1 = 7.9$) for which the measurement uncertainty is maximized when using a dielectric rod radius of 3 cm. This fact allow us to interpret that there must be radius size for which a sample under test characterized by $\varepsilon_1 = 7.9$ can have its measurement uncertainty minimized. Thus, for any dielectric permittivity value there is an optimum radius where the sensitivity of the measurement system is reduced allowing us to measure the dielectric permittivity with a minimum uncertainty associated to our measurement capabilities.

Returning to Figure 5, the point with the smallest absolute value (the central region), represent the range where there is a larger uncertainty in the measurement. Once the expected dielectric permittivity is approximately known (performing a preliminary measurement), the MCM is once again used to estimate the confidence interval width for this expected dielectric permittivity changing the sample under test radius size with a sweep starting from the smallest possible radius for which the sample could be prepared (1 cm in our case) to the maximum allowed radius calculated accordingly the explanation found in Section 3. Then, it is possible to select the optimum radius for which the uncertainty is the minimized. Finally, the sample under test should be prepared with the optimum radius size and the measurement must be repeated. This process can be repeated continuously in order to reduce the uncertainty as much as it is possible or practical.

As indicated previously, the least accurate dielectric permittivity to be measured using TE_{021} in the cavity with a 3 cm of radius is $\varepsilon_{r1} = 7.9$. In order to illustrate the measurement uncertainty minimization process we will use this value of permittivity for the sample under test. Hence, we have calculated the width of the 95% confidence interval of the measurement result for a the range $1\,\text{cm} < a < 4.5\,\text{cm}$, with a step size of 0.5 mm. The results are shown on Figure 7, where we used a 10^4 MCM iterations for every value of radius. The results shown on Figure 7 indicate that the optimum radius has a value of 3.8 cm (minimum uncertainty) while a radius of 2.7 cm will correspond to maximum uncertainty.

The results shown on Figure 8 are similar to those shown on Figure 6 but now they correspond to a 3.8 cm fixed radius. These results verify that there is a minimum in the confidence interval for $\varepsilon'_{r1} = 7.9$.

We can summarize our technique as follows:

(i) Once that the desired working frequency and the electromagnetic mode are selected, the first measurement of the dielectric permittivity is done with a sample of any radius within the valid range. Thus a first value of the dielectric permittivity is obtained.

(ii) The next step corresponds to the selection the optimum radius obtained by calculating the confidence interval using MCM for a range of radius, considering the practical limitations related to the sample preparation. The value of the dielectric permittivity is taken as the expected value measured previously.

(iii) Finally, the dielectric permittivity measurement is repeated with a sample with the optimum radius. This measurement will provide a more accurate result.

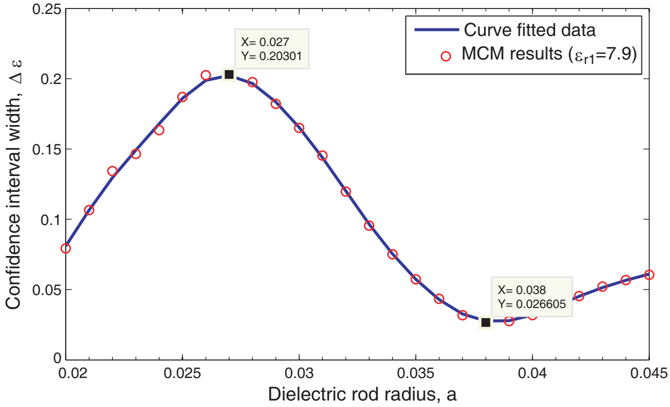

Figure 7. Confidence interval for a sample with a dielectric permittivity of 7.9 with a variable radius.

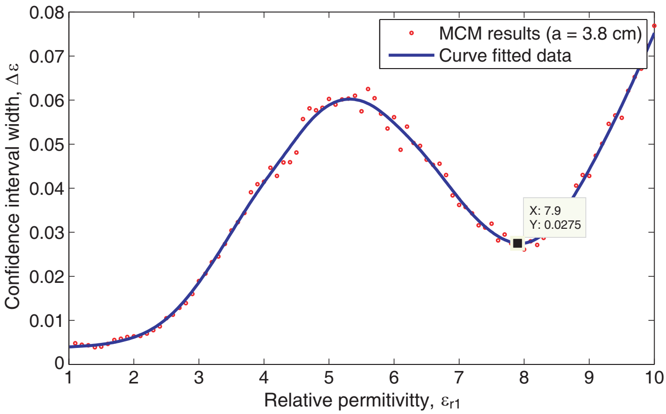

Figure 8. Confidence interval for a range of dielectric permittivity with Radius of 3.8 cm.

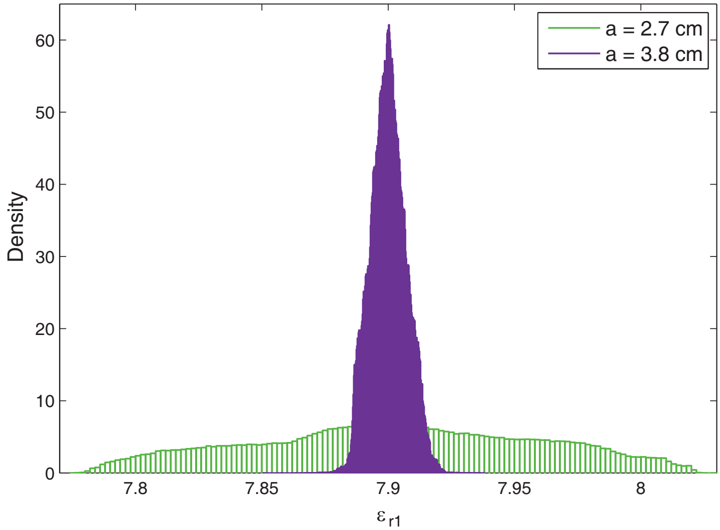

Figure 9. Coverage interval for a confidence level of 95%.

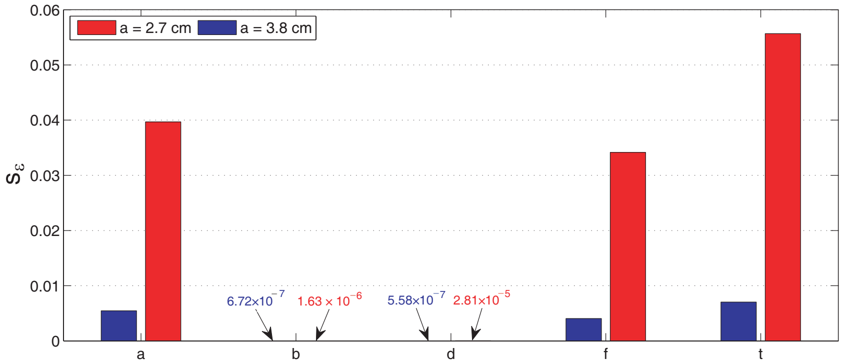

Figure 10. Sensitivity analysis.

5. RESULTS

Figure 9 shows the probability density function using the MCM with 10^6 iterations for $\varepsilon'_{r1} = 7.9$ and radius 2.7 cm and radius 3.8 cm, corresponding to the values of the worst case and the optimum radius, respectively. For the first case the coverage interval for a confidence level of 95% was $7.9^{+1.32\%}_{-1.28\%}$. In the second case the confidence interval width for the same confidence level was $7.9^{+0.18\%}_{-0.16\%}$. It means a reduction of the uncertainty in approximately 87% with respect to the worst case configuration.

This methodology does not implies a minimization in uncertainty in the measurement of the loss tangent.

The sensitivity analysis for each input uncertainty source is shown in Figure 10. We can observe how the system measurement is highly insensitive at the optimum radius to the uncertainty of the height and radius of the cavity, the radius of the sample under test and the measurement of the resonant frequency in comparison to the sample prepared for the case of the worst radius.

6. CONCLUSIONS

This article shows an efficient way to minimize the uncertainty in permittivity measurement using a shielded dielectric resonator for any resonant mode and any expected dielectric permittivity. This technique, based on the Monte Carlo Method, determines the optimum radius where the system exhibits an insensitivity to the dimension uncertainty and the frequency measurement.

The method is validated through MCM in the optimum radius and was compared to the worst radius case using 10^6 iterations. The example analyzed showed a reduction of 87% in the uncertainty between the optimum and the wort case, using the characteristics of our measurement system and its particular sources of uncertainty. The technique presented can be used for any resonant frequency and any electromagnetic mode inside a shielded dielectric resonator.

The technique discussed does not predict the effect on the measurement of the complex part of the permittivity (or equivalently of the loss tangent). Further studies are required in this respect and in the case of high loss materials where the real and the imaginary parts of the permittivity are comparable.

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