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SIMPLE FORMULA AND ITS EXACT ANALYTIC SOLUTION OF MUTUAL IMPEDANCE FOR NONPLANAR-SKEW DIPOLES

J. H. Han*, **W. Y. Song, K. S. Oh, and N. H. Myung**

Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), 291 Daehak-ro, Yuseong-gu, Daejeon 305-701, Republic of Korea

Abstract—The analysis of an antenna mutual coupling is a significant issue for designing the wireless communication system especially includesan array mutual coupling problem. The accurate analysis of the mutual coupling between antennas is needed. Accordingly, several methods for the mutual impedance calculation of dipoles have been studied in cases of coplanar-skew and nonplanar-skew. This paper proposes an exact and simple method for analyzing the mutual impedance between two arbitrarily located and slanted dipoles using the modified induced EMF method; their expressions and the exact analytic solution. The proposed formula and their closed-form solutions are verified by numerical solution using HFSS and give good agreement.

1. INTRODUCTION

Antenna mutual coupling analysis is an important and significant issue for the mobile communication system, radio frequency identification (RFID) system, near field communication (NFC) system and even the array antenna. For example, the mutual coupling may affect the performance of localization algorithms if the mutual coupling is not properly considered [1–5]. However, it is hard to predict such mutual effects because the presence of another element, which could be an antenna, can produce quite different radiated fields, current distribution and input impedance at the end. For the effective design of an antenna system, the input impedance has to be considered together with the mutual effects between antennas. That is called the driving

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* Corresponding author: Jung Hoon Han (junghoon_han@kaist.ac.kr).

point impedance, which consists of self-impedance, mutual impedance and the ratio of their current distributions. Based on this physical view, the exact calculation of mutual impedance is important for efficient performances of the antenna system [6].

There are several applications using the mutual coupling effect of dipoles and relative antennas. Some studies are focused on the impedance computation by various modeling methods. The computed mutual impedance can be used to reduce the radar cross section (RCS) or synthesize the optimal radiation pattern of the conformal array [7–14]. Also, it is useful for an interaction between the dipole particles, for example, analyzing the array impedance [15–22]. And, an analyzing the response of arbitrary configuration of two dipoles in reverberation chamber is also useful [23–25].

Several studies have been reported on the mutual impedance between parallel dipoles in echelon configurations. King [26] proposed exact expressions, developed for mutual impedance between two staggered parallel center-fed, infinitely thin antennas of unequal lengths. From this derivation, other advanced analysis techniques were introduced [27–32]. The mutual impedance is calculated by multiplying the radiated electric fields from the transmitting dipole and the current distribution on the receiving dipole. This method is called the induced electromotive force (EMF) method and is wellmatched with results from the method of moment (MoM), but is basically limited to straight, parallel, and echelon cases [6].

In the case of coplanar-skew configurations, several studieshave been described [33–37]. Representatively, Richmond [36] introduced the induced EMF formulation of mutual impedance between coplanar-skew dipoles. However, this approach is complicated because the integral path for calculating the mutual impedance lies along the r-directions from the origin point that is the intersection point of two coplanar-skew dipoles. The integration then requires a different axis, which isthe r-direction via radiated fields from the transmitting dipole, and transformations of variables are also needed.

Furthermore, advanced nonplanar-skew cases have been studied [38–41]. Representatively, Richmond [38] also introduced an expression for the mutual impedance of nonplanar-skew dipoles. The proposed configuration for analysis is similar to the coplanar-skew case However, the origin point for the r-direction, which is an integral path, is at $(x, y, z) = (0, d, 0)$. Then, the relative electric field direction for the integration has to be properly changed. Therefore, the geometrical structure is also complicated and the proposed formula needs transformations of variables as well.

This paper proposes an effective analysis method of mutual impedance between two arbitrarily located and slanted dipoles. This paper characterizes an exact, simple and intuitive analysis using the effective length vector (ELV) concept; their mutual impedance expression and the exact analytic solution. The proposed formula and their closed-form expressions, which is the exact analytic solution, are verified by numerical solution using HFSS and show well-matched results. Several configurations are examined and compared, including cases of varying distance, height slant angle, and even for nonplanar cases, utilizing the proposed method and numerical results by HFSS. Section 2 presents details of the proposed analysis method; Section 3 provides the expressions of the closedform followed by several examples in Section 4; and Section 5 gives conclusions.

2. MUTUAL IMPEDANCE ANALYSIS

An expression of the mutual impedance for two parallel dipoles in echelon was already introduced [26]. For new expressions of coplanar or nonplanar skew configuration, the proposed modified induced EMF method introduces the concept of the ELV and its application and formulations.

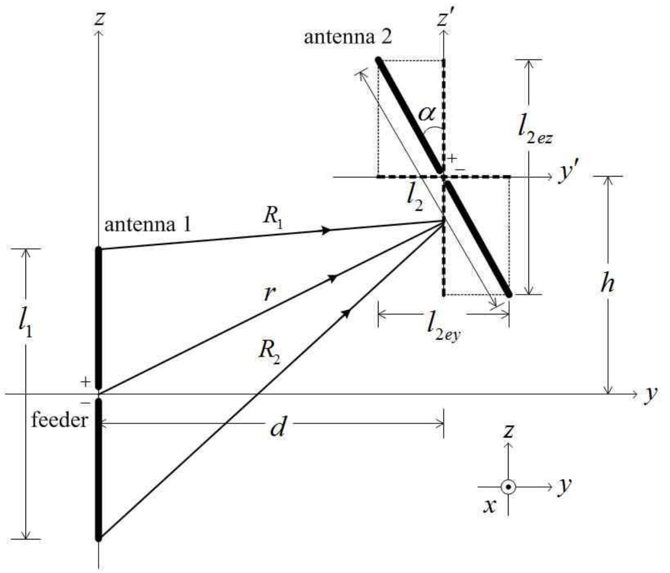


Figure 1. Geometry of two arbitrarily located and slanted dipoles in coplanar.

2.1. Modified Induced EMF Method Using Effective Length Vector

Figure 1 shows the geometry of two arbitrarily located and slanted dipoles in coplanar. Mutual impedance is calculated by multiplying the radiated E-fields from antenna 1 and the current distribution on antenna 2 by the induced EMF method. The radiated E-fields from the transmitting dipole lying on the z-axis for the Cartesian coordinate exist only along the z- and y-axes. Antenna 2, which is a receiving dipole, is slanted by an arbitrary angle α on the same plane with antenna 1, which is the transmitting dipole. Antenna 2 can be considered to consist of two effective lengths by orthogonal projections, which are on the z'- and y'-axes with a basis of the feed point of antenna 2 as the center. Thus, mutual impedance can be calculated by integration along these effective lengths and their sum. At this time, the selection of an integral path is also important. The dipole antenna has two poles which are plus and minus. The induced potential developed at the terminal of the dipole is calculated by integrating from the minus end to the plus end of the dipole. Therefore, this integral path direction can be defined as a vector concept.

Figure 2 shows the integral paths of effective length for slant angles at each quadrant. In the case of Figure 2(a), the plus pole direction is for the second quadrant. Then, the integral paths of effective lengths are plus direction for the z'-axis and minus direction for the y'-axis effective length. In this way, each integral path would be properly changed according to the slant angle α. The defined effective lengths for z- and y-axes are given by

$$l_{2ez} = l_2 \cdot \cos(\alpha) \tag{1a}$$

$$l_{2ey} = -l_2 \cdot \sin(\alpha) \tag{1b}$$

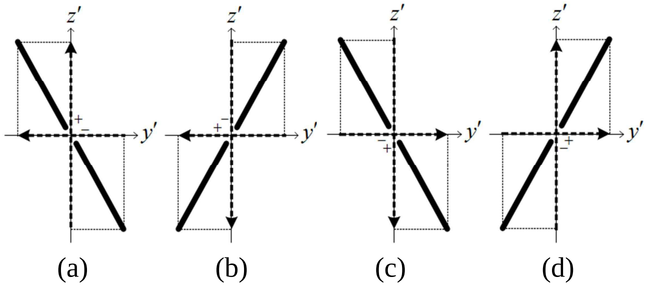


Figure 2. Integral paths of effective lengths for slant angle α at each quadrant: (a), (b), (c), (d) are for 1, 2, 3, 4 quadrant.

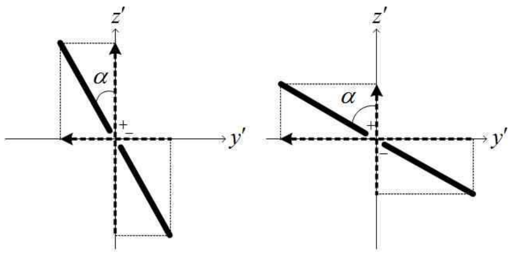


Figure 3. Changes of effective length in accordance with slant angle α.

where l_2 represents the length of antenna 2, α is the slant angle of antenna 2 on the yz-plane. Figure 3 indicates the changes of effective length in accordance with slant angle α for the same quadrant. From this ELV concept, we can define the integral path lengths and directions for any slanted angles.

2.2. Coplanar-skew Dipoles

According to the ELV concept, exact and efficient calculation of the mutual impedance is possible without changing the relative axes for integration and transformations of variables. Then, the mutual impedance can be expressed by the sum of the integrals of y- and z-axes, and is given according to the modified induced EMF method as

$$Z_{21}=\frac{-1}{I_{1i}I_{2ezi}}\int_{-\frac{l_{2ez}}{2}}^{\frac{l_{2ez}}{2}} E_z(z)\cdot I_{2ez}(z)\,dz+\frac{-1}{I_{1i}I_{2eyi}}\int_{-\frac{l_{2ey}}{2}}^{\frac{l_{2ey}}{2}} E_y(y)\cdot I_{2ey}(y)dy \quad (2)$$

where I_{1i}, I_{2ezi} and I_{2eyi} are the currents at each input terminal of the antennas. E_z and E_y are the radiated electric fields from antenna 1 to the receiving dipole, which is lying on the z-axis. The total radiated electric fields from antenna 1 are given by

$$E_x(x)=0 \quad (3a)$$

$$E_z(z)=-j\frac{\eta I_1}{4\pi}\left[\frac{e^{-jkR_1(z)}}{R_1(z)}+\frac{e^{-jkR_2(z)}}{R_2(z)}-2\cos\left(\frac{kl_1}{2}\right)\frac{e^{-jkr(z)}}{r(z)}\right] \quad (3b)$$

$$E_y(y)=j\frac{\eta I_1}{4\pi y}\left[\left(z-\frac{l_1}{2}\right)\frac{e^{-jkR_1(y)}}{R_1(y)}+\left(z+\frac{l_1}{2}\right)\frac{e^{-jkR_2(y)}}{R_2(y)}-2z\cos\left(\frac{kl_1}{2}\right)\frac{e^{-jkr(y)}}{r(y)}\right] \quad (3c)$$

where η is intrinsic impedance, k the wave number of medium, and l_1 the length of antenna 1. R_1, R_2 and r are lengths from the end of the positive, negative poles and center of dipole, respectively, to the observation point. Among these three electric fields, z- and y-oriented electric fields are concerned with mutual impedance and the x- oriented electric field is zero. I_{2ez} and I_{2ey} are the sinusoidal current distributions for each effective length by the modified induced EMF method and are defined by

$$I_{2ez}(z)=I_2\sin\left[k\left(-|z-h|+\frac{l_{2ez}}{2}\right)\right],\quad \text{for}\quad h-\frac{l_{2ez}}{2}\le z\le h+\frac{l_{2ez}}{2} \tag{4a}$$

$$I_{2ey}(y)=I_2\sin\left[k\left(-|y-d|+\frac{l_{2ey}}{2}\right)\right],\quad \text{for}\quad d-\frac{l_{2ey}}{2}\le y\le d+\frac{l_{2ey}}{2} \tag{4b}$$

where I_2 is the current maximum, h is the height and d is the distance between the center of feed points of antenna 1 and 2. Finally, the mutual impedance of the coplanar-skew dipoles can be written by substituting the relations (3), (4) into (2) and following the formula is given as

$$\begin{aligned}
Z_{21} &= Z_{21z}+Z_{21y}\\
&= \frac{-30}{\sin\left(\frac{kl_1}{2}\right)\sin\left(\frac{kl_{2ez}}{2}\right)}\int_{h-\frac{l_{2ez}}{2}}^{h+\frac{l_{2ez}}{2}}\sin\left[k\left(\frac{l_{2ez}}{2}-|z-h|\right)\right]\\
&\quad \left\{\frac{-je^{-jkR_{1z}(z)}}{R_{1z}(z)}+\frac{-je^{-jkR_{2z}(z)}}{R_{2z}(z)}+j2\cos\left(\frac{kl_1}{2}\right)\frac{e^{-jkr_z(z)}}{r_z(z)}\right\}dz\\
&\quad +\frac{-30}{\sin\left(\frac{kl_1}{2}\right)\sin\left(\frac{kl_{2ey}}{2}\right)}\int_{d-\frac{l_{2ey}}{2}}^{d+\frac{l_{2ey}}{2}}\sin\left[k\left(\frac{l_{2ey}}{2}-|y-d|\right)\right]\\
&\quad \left\{\left(h-\frac{l_1}{2}\right)\frac{je^{-jkR_{1y}(z)}}{R_{1y}(z)}+\left(h+\frac{l_1}{2}\right)\frac{je^{-jkR_{2y}(z)}}{R_{2y}(z)}\right.\\
&\quad \left.-j2h\cos\left(\frac{kl_1}{2}\right)\frac{e^{-jkr_y(z)}}{r_y(z)}\right\}\frac{dy}{y}
\end{aligned} \tag{5}$$

The geometry with respect to z- and y-axes of Figure 1 reveals that

$$\begin{aligned}
r_z(z) &= \sqrt{d^2+z^2}, & r_y(y) &= \sqrt{h^2+y^2}\\
R_{1z}(z) &= \sqrt{d^2+(z-l_1/2)^2}, & R_{1y}(y) &= \sqrt{(h-l_1/2)^2+y^2}\\
R_{2z}(z) &= \sqrt{d^2+(z+l_1/2)^2}, & R_{2y}(y) &= \sqrt{(h+l_1/2)^2+y^2}
\end{aligned} \tag{6}$$

Therefore, Equation (5) includes the characteristics for arbitrarily located and slanted coplanar dipoles using two effective length vectors and their sum of integrations.

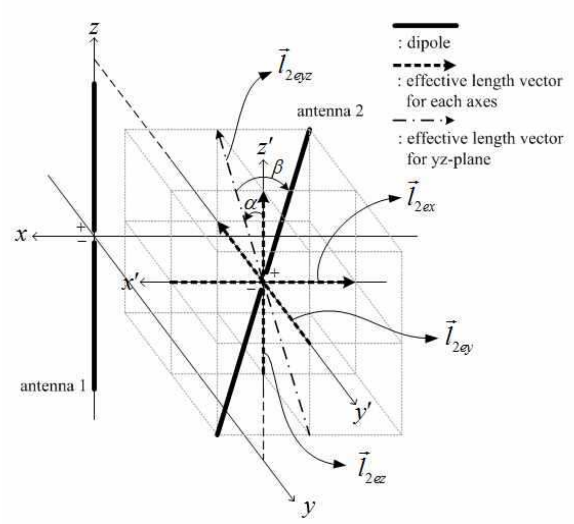


Figure 4. Geometry of two arbitrarily located and slanted dipoles in nonplanar.Antenna 2 consists of three effective length vectors.

2.3. Nonplanar-skew Dipoles

Figure 4 describes the geometry of two arbitrarily located and slanted nonplanar dipoles. The nonplanar slant angle is defined as β which is the angle between the ELV for $y'z'$-plane and antenna 2. Then, the effective lengths of each x'-, y'- and z'-axis are equal to

$$l_{2ex} = -l_2 \cdot \sin(\beta) \tag{7a}$$

$$l_{2ey} = -l_2 \cdot \sin(\alpha) \cdot \cos(\beta) \tag{7b}$$

$$l_{2ez} = l_2 \cdot \cos(\alpha) \cdot \cos(\beta) \tag{7c}$$

Antenna 2 is defined on the local coordinate system and consists of three ELVs for x'-, y'- and z'-axes. The expression can be written as

$$\vec{l}_2 = \vec{l}_{2ex} + \vec{l}_{2ey} + \vec{l}_{2ez} \tag{8}$$

This is the vector sum of components of each axis.

Nevertheless, the component for x'-axis does not need to be considered since a result of the integration along the x-direction will be zero. Figure 5 shows the cancellation of electric field of the effective length in the x-direction. The ELV for the x'-axis consists of x- and y-components of electric fields from antenna 1 at each point of the effective length. Then, each x-direction electric field of each point is zero from (3a). And, the y-direction electric fields of each point of the

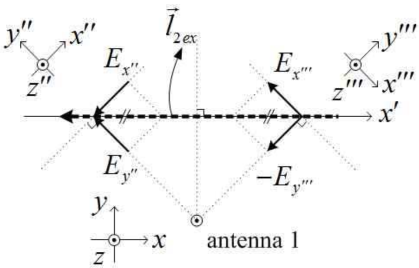


Figure 5. Cancellation of electric fields of the effective length to x-direction.

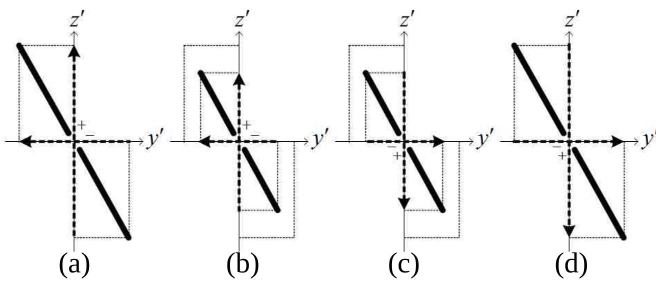


Figure 6. Changes of effective length and integral path in accordance with slant angle β and $\alpha = 30°$: (a), (b), (c), (d) are for $\beta = 0°$, $45°$, $135°$, $180°$ angles and their effective lengths are l_2, $l_2/\sqrt{2}$, $l_2/\sqrt{2}$, l_2, respectively.

effective length are canceled when we consider the effective length is symmetric and divided into two parts from the center of the vector length. The vector direction is then only the opposite. Thus the x- and y-directed fields cancel from antenna 1 along the effective length in the x'-direction. The explanation can be written as

$$E_{x''} = E_{x'''} = 0 \tag{9a}$$

$$-E_{y''} + E_{y'''} = 0 \tag{9b}$$

Therefore, in case of two nonplanar slanted dipoles by the angle β, the mutual impedance can be defined as (5) which has to be considered by changing the effective lengths for the z' and y'-axes, except for the x'-direction component.

When the angle β is varied with fixed angle α, the changes of effective length and integral path are as indicated in Figure 6. In accordance with slant angle β, the effective length for integration is reduced or increased by varying the angle, and the integral paths are also changed.

3. EXACT ANALYTIC SOLUTION OF THE PROPOSED FORMULA

Now, the mutual impedance for arbitrarily located and slanted dipoles is related to just two integrals for z- and y-axes. Each integral path is concerned with effective lengths and vector directions. Equation (5) can be evaluated by mechanical integration. However, to make arithmetical computations be possible the integration is done mathematically, so that a convenient form is available for calculations. To obtain the closed-form expression of the mutual impedance Equation (5), the following subchapters include the processes of solving the integral $Z_{21} = Z_{21z} + Z_{21y}$.

3.1. Closed-form Expression for Integration Z_{21z}

King solved the mutual impedance of two unequal length parallel dipoles in echelon. For the proposed mutual impedance Z_{21z} for z-direction, the effective antenna length for z'-axis is changed by the slant angle α of antenna 2. Thus, the method from King can be properly employed for the proposed z-direction mutual impedance Z_{21z}.

$$\begin{aligned}
Z_{21z} = \frac{-30}{\sin\left(\frac{kl_1}{2}\right)\sin\left(\frac{kl_{2ey}}{2}\right)} \Bigg[-j \Bigg\{ &\int_{h-\frac{l_{2ez}}{2}}^{h} \sin\left[k\left(z-h+\frac{l_{2ez}}{2}\right)\right] \frac{e^{-jkR_{1z}}}{R_{1z}} dz \\
&+ \int_{h}^{h+\frac{l_{2ez}}{2}} \sin\left[k\left(-z+h+\frac{l_{2ez}}{2}\right)\right] \frac{e^{-jkR_{1z}}}{R_{1z}} dz \Bigg\} \\
&-j \left\{ \int_{h-\frac{l_{2ez}}{2}}^{h} \sin\left[k\left(z-h+\frac{l_{2ez}}{2}\right)\right] \frac{e^{-jkR_{2z}}}{R_{2z}} dz \right. \\
&\left. + \int_{h}^{h+\frac{l_{2ez}}{2}} \sin\left[k\left(-z+h+\frac{l_{2ez}}{2}\right)\right] \frac{e^{-jkR_{2z}}}{R_{2z}} dz \right\} \\
&+j2\cos\left(\frac{kl_1}{2}\right) \left\{ \int_{h-\frac{l_{2ez}}{2}}^{h} \sin\left[k\left(z-h+\frac{l_{2ez}}{2}\right)\right] \frac{e^{-jkr_z}}{r_z} dz \right. \\
&\left. + \int_{h}^{h+\frac{l_{2ez}}{2}} \sin\left[k\left(-z+h+\frac{l_{2ez}}{2}\right)\right] \frac{e^{-jkr_z}}{r_z} dz \right\} \Bigg]
\end{aligned} \tag{10}$$

The formula can be evaluated to the closed form expression [26] and successively reduced to simply,

$$Z_{21z}=\frac{-30}{\sin\left(\frac{kl_1}{2}\right)\sin\left(\frac{kl_{2ez}}{2}\right)}\cdot\left\langle\frac{1}{2}\sum_{t=-1}^{1}\sum_{n=-1}^{1}\sum_{s=0}^{1}\left[\left\{1+2\cos\left(\frac{kl_1}{2}\right)\right\}\delta_t-1\right]\right.$$
$$\left[\begin{array}{l}\cos(kq_{n,t})\left\{\begin{array}{l}ci\left(k\sqrt{d^2+q_{n,t}^2}+(-1)^s kq_{n,t}\right)\\-ci\left(k\sqrt{d^2+q_{0,t}^2}+(-1)^s kq_{0,t}\right)\end{array}\right\}\\+(-1)^s\sin(kq_{n,t})\left\{\begin{array}{l}si\left(k\sqrt{d^2+q_{n,t}^2}+(-1)^s kq_{n,t}\right)\\-si\left(k\sqrt{d^2+q_{0,t}^2}+(-1)^s kq_{0,t}\right)\end{array}\right\}\end{array}\right]$$
$$-j\frac{1}{2}\sum_{t=-1}^{1}\sum_{n=-1}^{1}\sum_{s=0}^{1}\left[\left\{1+2\cos\left(\frac{kl_1}{2}\right)\right\}\delta_t-1\right]$$
$$\left.\left[\begin{array}{l}\cos(kq_{n,t})\left\{\begin{array}{l}si\left(k\sqrt{d^2+q_{n,t}^2}+(-1)^s kq_{n,t}\right)\\-si\left(k\sqrt{d^2+q_{0,t}^2}+(-1)^s kq_{0,t}\right)\end{array}\right\}\\-(-1)^s\sin(kq_{n,t})\left\{\begin{array}{l}ci\left(k\sqrt{d^2+q_{n,t}^2}+(-1)^s kq_{n,t}\right)\\-ci\left(k\sqrt{d^2+q_{0,t}^2}+(-1)^s kq_{0,t}\right)\end{array}\right\}\end{array}\right]\right\rangle \quad (11)$$

where δ_t is the Kronecker's delta function and

$$q_{n,t}=h+n\frac{l_{2ez}}{2}+t\frac{l_1}{2} \quad (12)$$

3.2. Closed-form Expression for Integration Z_{21y}

For the proposed mutual impedance Z_{21y} for the y-direction, the integration process is similar to the mutual impedance Z_{21z} for the z-direction. But, the y-direction radiated field from antenna 1, E_y of (3c), has the variable y in the denominator. Thus, the integration for the Z_{21y} also includes the variable y in their denominator. The Z_{21y} can be developed to

$$Z_{21y}=\frac{-30}{\sin\left(\frac{kl_1}{2}\right)\sin\left(\frac{kl_{2ey}}{2}\right)}\left[j\left(h-\frac{l_1}{2}\right)\left\{\int_{d-\frac{l_{2ey}}{2}}^{d}\sin\left[k\left(y-d+\frac{l_{2ey}}{2}\right)\right]\frac{e^{-jkR_{1y}}}{R_{1y}}\frac{dy}{y}\right.\right.$$
$$\left.+\int_{d}^{d+\frac{l_{2ey}}{2}}\sin\left[k\left(-y+d+\frac{l_{2ey}}{2}\right)\right]\frac{e^{-jkR_{1y}}}{R_{1y}}\frac{dy}{y}\right\}$$

$$+j\left(h+\frac{l_1}{2}\right)\left\{\int_{d-\frac{l_{2ey}}{2}}^{d}\sin\left[k\left(y-d+\frac{l_{2ey}}{2}\right)\right]\frac{e^{-jkR_{2y}}}{R_{2y}}\frac{dy}{y}\right.$$
$$\left.+\int_{d}^{d+\frac{l_{2ey}}{2}}\sin\left[k\left(-y+d+\frac{l_{2ey}}{2}\right)\right]\frac{e^{-jkR_{2y}}}{R_{2y}}\frac{dy}{y}\right\}$$
$$-j2h\cos\left(\frac{kl_1}{2}\right)\left\{\int_{d-\frac{l_{2ey}}{2}}^{d}\sin\left[k\left(y-d+\frac{l_{2ey}}{2}\right)\right]\frac{e^{-jkr_y}}{r_y}\frac{dy}{y}\right.$$
$$\left.\left.+\int_{d}^{d+\frac{l_{2ey}}{2}}\sin\left[k\left(-y+d+\frac{l_{2ey}}{2}\right)\right]\frac{e^{-jkr_y}}{r_y}\frac{dy}{y}\right\}\right] \tag{13}$$

For a comfortable evaluation of the equation, the relative current distributions of the integration path is divided into two parts which are the lower and upper sides of the dipole from the feed point of antenna 2.

For the first cosine term of Equation (13), the $Z'_{21y,\cos}$,

$$Z'_{21y,\cos}=\int_{d-\frac{l_{2ey}}{2}}^{d}\sin\left[k\left(y-d+\frac{l_{2ey}}{2}\right)\right]\cos\left(k\sqrt{h_y^2+y^2}\right)\frac{dy}{y\sqrt{h_y^2+y^2}}$$
$$=\frac{1}{2}\int_{d-\frac{l_{2ey}}{2}}^{d}\left\{\sin\left[k\left(\sqrt{h_y^2+y^2}+y\right)\right]\cos\left[k\left(d-\frac{l_{2ey}}{2}\right)\right]\right.$$
$$\left.-\cos\left[k\left(\sqrt{h_y^2+y^2}+y\right)\right]\sin\left[k\left(d-\frac{l_{2ey}}{2}\right)\right]\right\}\frac{dy}{y\sqrt{h_y^2+y^2}}$$
$$+\frac{1}{2}\int_{d-\frac{l_{2ey}}{2}}^{d}\left\{-\sin\left[k\left(\sqrt{h_y^2+y^2}-y\right)\right]\cos\left[k\left(d-\frac{l_{2ey}}{2}\right)\right]\right.$$
$$\left.-\cos\left[k\left(\sqrt{h_y^2+y^2}-y\right)\right]\sin\left[k\left(d-\frac{l_{2ey}}{2}\right)\right]\right\}\frac{dy}{y\sqrt{h_y^2+y^2}} \tag{14}$$

which is the cosine integral term and can be derived by the Euler's formula except for the constant. The $Z_{21y,\cos}$ is developed by the sum and difference identities of the trigonometric functions. For convenience, let $h_y = h - l_1/2$. Then, by changing in the variables of

$$u=k\left(\sqrt{h_y^2+y^2}-y\right) \tag{15a}$$

$$v=k\left(\sqrt{h_y^2+y^2}+y\right) \tag{15b}$$

And from this relation, the variable y with u is derived as

$$y = \frac{k}{2u}\left(h_y^2 - \frac{u^2}{k^2}\right) = \frac{1}{2ku}\left(k^2 h_y^2 - u^2\right) \tag{16}$$

Similarly for the variable v, Equation (14) simply reduces to

$$\begin{aligned} &Z'_{21y,\cos} \\ &= \frac{1}{2}\cos\left[k\left(d - \frac{l_{2ey}}{2}\right)\right]\int_{v_1}^{v_2}\frac{2k\sin v}{v^2 - k^2h_y^2}dv - \frac{1}{2}\sin\left[k\left(d - \frac{l_{2ey}}{2}\right)\right]\int_{v_1}^{v_2}\frac{2k\cos v}{v^2 - k^2h_y^2}dv \\ &\quad - \frac{1}{2}\cos\left[k\left(d - \frac{l_{2ey}}{2}\right)\right]\int_{u_1}^{u_2}\frac{2k\sin u}{u^2 - k^2h_y^2}du - \frac{1}{2}\sin\left[k\left(d - \frac{l_{2ey}}{2}\right)\right]\int_{u_1}^{u_2}\frac{2k\cos u}{u^2 - k^2h_y^2}du \end{aligned} \tag{17}$$

The integration to be solved is shortly reduced. Then, the reduced integration can be expressed by the partial fraction decomposition. That is given as

$$\int_{u_1}^{u_2}\frac{2k\sin u}{u^2 - k^2h_y^2}du = \frac{1}{h_y}\int_{u_1}^{u_2}\left(\frac{\sin u}{u - kh_y} - \frac{\sin u}{u + kh_y}\right)du \tag{18}$$

The equation is more simply divided into two parts and expressed with the sine and cosine integral functions using equation

$$\begin{aligned} \int\frac{\sin wx}{a + bx}dx &= \frac{1}{b}\left\{\cos\left(\frac{wa}{b}\right)\mathrm{si}\left[\frac{w}{b}(a+bx)\right] - \sin\left(\frac{wa}{b}\right)\mathrm{ci}\left[\frac{w}{b}(a+bx)\right]\right\} \\ \int\frac{\cos wx}{a + bx}dx &= \frac{1}{b}\left\{\cos\left(\frac{wa}{b}\right)\mathrm{ci}\left[\frac{w}{b}(a+bx)\right] + \sin\left(\frac{wa}{b}\right)\mathrm{si}\left[\frac{w}{b}(a+bx)\right]\right\} \end{aligned} \tag{19}$$

where $w = b = 1$, $x = u$ and $a = \pm kh_y$ [42]. In the manner described, Equation (13) can be derived as the closed-form expression with the sine and cosine integral functions and successively reduced to simply,

$$\begin{aligned} Z_{21y} = &\frac{-30}{\sin\left(\frac{kl_1}{2}\right)\sin\left(\frac{kl_{2ey}}{2}\right)}\cdot\left\langle\left\{-1 + \left[1 + 2\cos\left(\frac{kl_1}{2}\right)\right]\delta_t\right\}\times\frac{1}{2}\sum_{t=-1}^{1}\sum_{m=-1}^{1}\sum_{u=0}^{1}\sum_{s=0}^{1}\right. \\ &\left\{\begin{array}{l} (-1)^u(-1)^s\cos\{kp_{m,0} + (-1)^u(-1)^s kq_{0,t}\} \\ \left[\begin{array}{l} ci\left(k\sqrt{q_{0,t}^2 + p_{m,0}^2} + (-1)^u kp_{m,0} + (-1)^s kq_{0,t}\right) \\ -ci\left(k\sqrt{q_{0,t}^2 + p_{0,0}^2} + (-1)^u kp_{0,0} + (-1)^s kq_{0,t}\right) \end{array}\right] \\ +(-1)^s\sin\{kp_{m,0} + (-1)^u(-1)^s kq_{0,t}\} \\ \left[\begin{array}{l} si\left(k\sqrt{q_{0,t}^2 + p_{m,0}^2} + (-1)^u kp_{m,0} + (-1)^s kq_{0,t}\right) \\ -si\left(k\sqrt{q_{0,t}^2 + p_{0,0}^2} + (-1)^u kp_{0,0} + (-1)^s kq_{0,t}\right) \end{array}\right] \end{array}\right\} \end{aligned}$$

$$+\left\{1-\left[1+2\cos\left(\frac{kl_1}{2}\right)\right]\delta_t\right\}\times j\frac{1}{2}\sum_{t=-1}^{1}\sum_{m=-1}^{1}\sum_{u=0}^{1}\sum_{s=0}^{1}$$

$$\left\langle\left\{\begin{array}{l}(-1)^u(-1)^s\cos\left\{kp_{m,0}+(-1)^u(-1)^s kq_{0,t}\right\}\\ \left[\begin{array}{l} si\left(k\sqrt{q_{0,t}^2+p_{m,0}^2}+(-1)^u kp_{m,0}+(-1)^s kq_{0,t}\right)\\ -si\left(k\sqrt{q_{0,t}^2+p_{0,0}^2}+(-1)^u kp_{0,0}+(-1)^s kq_{0,t}\right)\end{array}\right]\\ -(-1)^s\sin\left\{kp_{m,0}+(-1)^u(-1)^s kq_{0,t}\right\}\\ \left[\begin{array}{l} ci\left(k\sqrt{q_{0,t}^2+p_{m,0}^2}+(-1)^u kp_{m,0}+(-1)^s kq_{0,t}\right)\\ -ci\left(k\sqrt{q_{0,t}^2+p_{0,0}^2}+(-1)^u kp_{0,0}+(-1)^s kq_{0,t}\right)\end{array}\right]\end{array}\right\}\right\rangle \quad (20)$$

where

$$p_{m,t}=d+m\frac{l_{2ey}}{2}+t\frac{l_1}{2} \quad (21)$$

Equations (11) and (20) are exactly and simply expressed formulas with the sine and cosine integral functions. Finally, the Equation (5) is derived as the closed form expressions, which is the exact analytic solution, by the sum of Equations (11) and (20).

4. VERIFICATION AND ANALYSIS

Section 2 presents the mutual impedance formulas for the two arbitrarily located and slanted dipoles. For the verification of the proposed formula, several configurations of the cases are performed. The dimensions of dipoles used for the HFSS simulation are shown in Table 1.

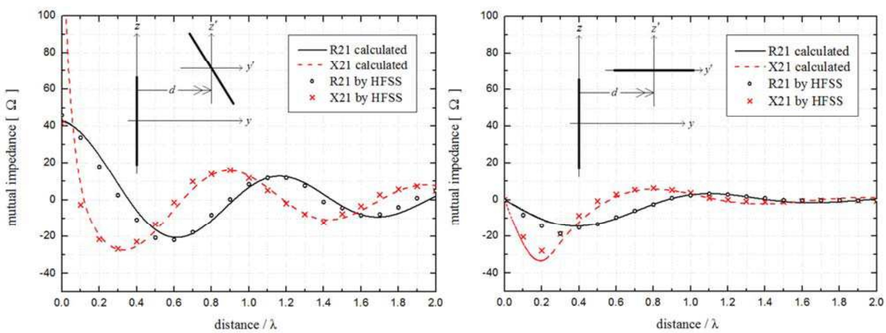
The designed antennas 1, 2 are assumed to be identical and 50 Ω matched half-wavelength thin dipoles. The cases are for the varying distance, angle and height for the coplanar, and even the nonplanar configurations, to ensure the reliability of the proposed formulas.

Table 1. Dimensions of dipoles for the HFSS simulation.

parameter	radius	port length	50-ohm matched half-length of dipole
dimension	$0.5\times10^{-3}\lambda$	$0.2\times10^{-2}\lambda$	0.227λ

4.1. Coplanar-skew: Varying Distance

The comparisons of mutual impedances calculated by the proposed method, and simulated by HFSS, are indicated in Figure 7. We assume that the distance d is varying from 0 to 2λ with $h = 0.3\lambda$ and fixed angle α. The proposed method is well matched with the HFSS simulation results. In particular, when the angle $\alpha = 90°$ in Figure 7(b), the result shows good agreement. The calculated result only depends on the effects of integration along the y-axis direction. The contribution of y-axis is reliable according to the proposed method for calculating the mutual impedance of dipoles.



(a) (b)

Figure 7. Mutual impedances of the calculated by the proposed method and the simulated by the HFSS. For the cases of varying distance $d = 0 - 2\lambda$ and $h = 0.3\lambda$ and (a) $\alpha = 30°$, (b) $\alpha = 90°$.

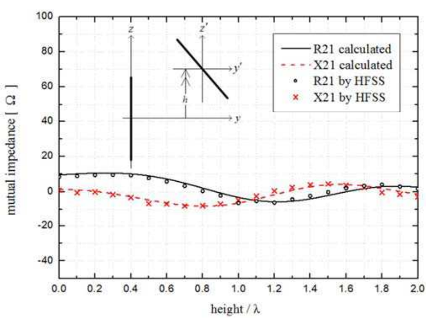


Figure 8. Mutual impedances of the calculated by the proposed method and the simulated by the HFSS. For the case of varying height (h) and $d = 1.2\lambda$, $\alpha = 45°$.

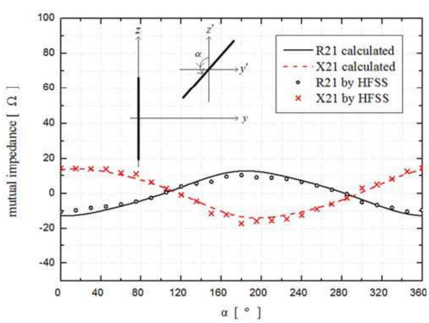


Figure 9. Mutual impedances of the calculated by the proposed method and the simulated by the HFSS. For the case of varying angle α and $h = 0.3\lambda$, $d = 0.8\lambda$

4.2. Coplanar-skew: Varying Height

In the case of varying height from 0 to 2λ between the center points of two dipoles, Figure 8 shows the comparison of mutual impedances calculated by the proposed method and simulated by HFSS, with $d = 1.2\lambda$, $\alpha = 45°$ and the results are almost the same.

4.3. Coplanar-skew: Varying Angle

Figure 9 shows mutual impedances for the case of varying angle α from 0° to 360° on the yz-plane with $h = 0.3\lambda$, $d = 0.8\lambda$. The effective length and their directions of the integration path to be calculated are changed according to the angle α variation. The results are also well matched.

4.4. Nonplanar-skew

Figure 10 shows the comparisons of mutual impedances calculated by the proposed method and simulated by HFSS in the cases of $h = 0.3\lambda$, $\alpha = 45°$, $d = 1.2\lambda$, varying angle β and $h = 0.3\lambda$, $\alpha = 45°$, $\beta = 45°$, and varying distance d with respect to Figures 10(a), (b), respectively. For the case of varying β according to the dimensions of Figure 10(a), the result shows good agreements. At the $\beta = 90°$ and 270°, particularly, the real and imaginary values are zeros because the two dipoles are located perpendicularly to each other. Thus, the radiated polarized fields are orthogonal and the mutual impedance becomes zero. Figure 10(b) shows the mutual impedance for the varying

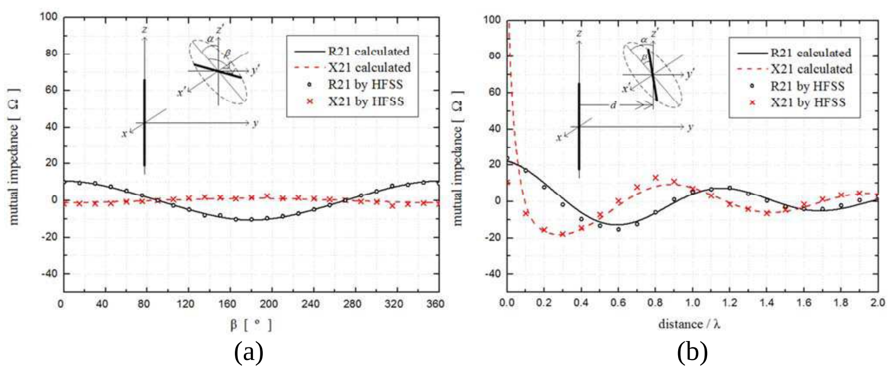


Figure 10. Mutual impedances of the calculated by the proposed method and the simulated by the HFSS. For the cases of $h = 0.3\lambda$, $\alpha = 45°$ and (a) varying angle β, $d = 1.2\lambda$ and (b) varying distance (d), $\beta = 45°$.

distance d with the fixed β for the nonplanar configuration. The results indicate the proposed method and concept for the nonplanar case shows good agreement with the numerical solution.

4.5. Closed-form Solution

Figure 11 depicts the mutual impedance for the integral form and the closed form when $h = 0.3\lambda$, $\alpha = 45°$, $\beta = 0°$ and $d = 0$–2λ. The closed form solution is well matched to the integral form solution (5) which is solved by the numerical integration. However, the closed form solution is the exact analytic solution of the proposed formula. The specific values of the certain distances are shown in Table 2.

There are some differences from the results in Table 2. It is clear that the closed form solution is the exact solution of the proposed method. The proposed integral form can be easily used to calculate the mutual impedance. But, the closed form can also be used for needs of the exact solution. Therefore, the developed formulas prove trustworthy and credible and even useful.

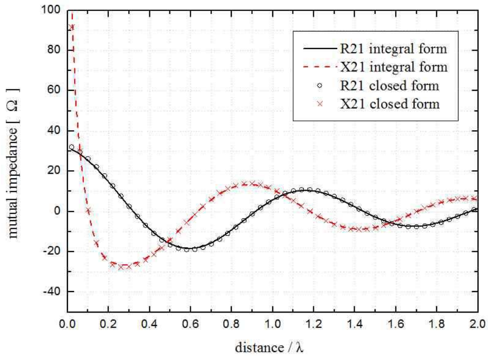


Figure 11. Mutual impedances of the integral form and the closed form by the proposed method when $h = 0.3\lambda$, $\alpha = 45°$, $\beta = 0°$ and $d = 0$–2λ.

Table 2. Comparisons of the closed-form and integral form solution.

distance		R_{21}	X_{21}
0.42λ	integral	-10.4280380401191	-21.3703524834806
	closed	-10.8004417091625	-21.3704155486934
1.22λ	integral	9.8781082328936	-2.26338823892638
	closed	10.2342456680168	-2.35063493055845

5. CONCLUSION

In this work, the effective analysis method of mutual impedance between two arbitrarily located and slanted dipoles has been developed. The proposed modified induced EMF method uses the concept of the effective length vector (ELV), their integral expressions and the closed-form solution even the simply reduced final form. The proposed method characterizes an exact, simple and intuitive analysis. In addition, the proposed method is demonstrated to be reliable by various verifications. The proposed method thus provided a reliable and useful solution for analysis of the mutual coupling between dipoles.

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