Progress In Electromagnetics Research M, Vol. 26, 251–265, 2012

EFFICIENT EVALUATION OF THE LONGITUDINAL COUPLING IMPEDANCE OF A PLANE STRIP

D. Assante[1, *] and L. Verolino[2]

[1]Facoltà di Ingegneria, Università Telematica Internazionale Uninettuno, Corso Vittorio Emanuele II 39, Roma 00186, Italy

[2]Dipartimento di Ingegneria Elettrica, Facoltà di Ingegneria, Università degli Studi di Napoli Federico II, Via Claudio 21, Napoli 80125, Italy

Abstract—We discuss the electromagnetic interaction between a traveling charge particle and a perfectly conducting strip of a negligible thickness. The particle travels at a constant velocity along a straight line parallel to the axis of symmetry of the strip. The efficiency of the proposed solution is proved by evaluating the longitudinal coupling impedance in a wide range of parameters.

1. INTRODUCTION

The current stored in an high-energy accelerator is limited by the particle instabilities, arising from the interaction of the particles among them in the same bunch or with the accelerator structures [1]. The main parameter to discuss the electromagnetic coupling between the particle and surrounding structure is the longitudinal coupling impedance [2, 3], widely studied in literature for different kinds of structures [4]. This important design parameter is directly related to the energy that the charge loses during his motion due to the interaction with the scattered fields produced by the surrounding structures. Equivalently, it is proportional to the energy that has to be fed to the traveling charge in order to keep its speed constant and contrast the slowing effect of the surrounding structures.

Modal analysis has been mostly adopted, and diffraction models have been used for high-frequency calculations [5, 6]. Integral formulations are not so common for this problem, although they can

Received 13 September 2012, Accepted 30 October 2012, Scheduled 31 October 2012
* Corresponding author: Dario Assante (d.assante@uninettunouniversity.net).

be really efficient for the analysis of some classes of structures, for instance in presence of edges [7, 8].

Aim of this paper is to evaluate the electromagnetic coupling between a traveling charge q and an infinite strip, as shown in Fig. 1. The proposed method is quite general and can be adopted for a wide class of scattering and diffraction problems [9–12]. It is shown how to efficiently represent and compute the induced density current on the strip, as well as the longitudinal coupling impedance.

The classical definition of longitudinal coupling impedance is usually adopted to take into account the effect of transversal discontinuities of the structure. In problems like the present one, where the structure is invariant along the traveling direction, it is well known that the classical definition leads to an infinite value. In order to deal with a finite quantity, it is usual in literature to introduce a longitudinal coupling impedance *per unit length* [3, 13], defined as

$$\hat{Z}_{||}(k,\beta) = -\frac{1}{q}\frac{1}{L}\int\limits_{-L/2}^{L/2} E_x(x, y=0, z=h, k)\, e^{jkx/\beta} dx\,, \qquad (1)$$

where $E_x(x, y, z, k)$ is the x-component of the electric field in the frequency domain, k is the wavenumber, L an unit length, and the charge is moving at constant velocity $v = \beta c$.

The problem is formulated in the particle frame at first, where an electrostatic model can be adopted. The formulation leads to an integral equation that is efficiently solved by means of an expansion in terms of Neumann series. Then, the Lorentz transformations allow to obtain the fields in the strip frame and to evaluate the longitudinal coupling impedance.

A large amount of computations will be performed in the strip and/or in the charge reference frames. The unprimed notation is used

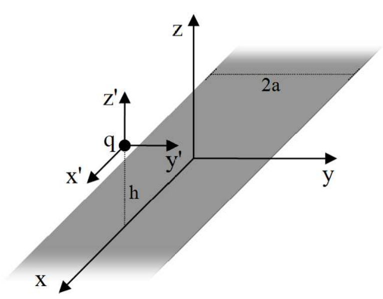


Figure 1. Geometry of the problem.

to express quantities in the strip reference frame, whereas the primed notation is used in the charge reference.

The proposed method of solution is efficient and accurate, since the longitudinal coupling impedance is reconstructed by means of few expansion terms. In addition, the proposed approach allows to automatically obtain an analytical low-frequency solution of the problem, that correctly fits the exact solution for a certain range of parameters.

2. FORMULATION OF THE PROBLEM

Let us consider a charge particle q traveling at constant velocity $v = \beta c$ along a straight line, parallel to the axis of symmetry of a strip, as shown in Fig. 1. The strip $\mathbb{S} = \{x, |y| \leq a, z = 0\}$ is wide $2a$ and is infinite along the x axis: its thickness is negligible. The induced current density has to be evaluated to solve the electromagnetic problem.

The simplest way to face the problem is to formulate and solve it in the particle reference frame, as already done in [14]. Due to the constant velocity of the motion and to the symmetry along the x axis, a steady state problem has to be considered in the particle frame, where the induced charge density has to be found. The Lorentz transformations will give the electromagnetic field in the strip frame.

In the particle frame, the electrostatic potential due to the fixed charge is:

$$V'_q = \frac{q}{4\pi\varepsilon_0\sqrt{x'^2 + y'^2 + (z' - h)^2}}\,, \tag{2}$$

whereas the electrostatic potential sustained by the induced charge density $\sigma'(x', y')$ on the strip can be written as

$$V' = \frac{1}{4\pi\varepsilon_0} \int\limits_{\mathbb{S}} \frac{\sigma'(x_0, y_0)\, dx_0 dy_0}{\sqrt{(x' - x_0)^2 + (y' - y_0)^2 + z'^2}}. \tag{3}$$

The boundary condition to be imposed on the strip is the vanishing of the tangential components E'_x and E'_y on the strip. This is equivalent to impose that the total electrostatic potential is constant on the strip. Since the constant can be arbitrary chosen, it is posed as zero, so the boundary condition is definitely

$$V'\left(x', y', z' = 0\right) + V'_q\left(x', y', z' = 0\right) = 0, \tag{4}$$

for every $\{x', y'\} \in \mathbb{S}$. Considering (2) and (3), this means that

$$\int\limits_{\mathbb{S}} \frac{\sigma'(x_0, y_0)\, dx_0 dy_0}{\sqrt{(x' - x_0)^2 + (y' - y_0)^2}} = -\frac{q}{\sqrt{x'^2 + y'^2 + h^2}}\,. \tag{5}$$

It is worth noting to observe that there is complete induction on the strip: the previous (5) can be rewritten as

$$\int\limits_{\mathbb{S}} \frac{\sigma'(x_0, y_0)\sqrt{x'^2 + y'^2 + h^2}}{\sqrt{(x' - x_0)^2 + (y' - y_0)^2}}\, dx_0 dy_0 = -q, \tag{6}$$

and evaluating the limit of (6) for x' going to $+\infty$, it is found that

$$\int\limits_{\mathbb{S}} \sigma'(x_0, y_0) dx_0 dy_0 = -q, \tag{7}$$

namely the total induced charge on the strip is equal to the charge q.

In order to solve the integral Equation (5), it can be useful to introduce a spatial Fourier transform of the charge density along the symmetry axis, defined as

$$\tilde{\sigma}'(u, y) = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} \sigma'(x, y) e^{jux} dx. \tag{8}$$

By using the relevant integral [15]

$$\frac{\pi}{\sqrt{D^2 + Z^2}} = \int\limits_{-\infty}^{+\infty} K_0\left(Du\right) e^{-juZ} du\,, \tag{9}$$

and applying the inverse Fourier transform to both members, it is possible to rewrite (5) as

$$\int\limits_{-a}^{a} \tilde{\sigma}'(u, y_0) K_0\left(u|y' - y_0|\right) dy_0 = -\frac{q}{2\pi} K_0\left(u\sqrt{y'^2 + h^2}\right). \tag{10}$$

Equation (10) has to be verified for $|y'| \leq a$, whereas the induced charge density has to vanishes outside the strip.

A representation in terms of Neumann series in the transformed domain can be used to efficiently solve such a kind of problem [7, 16]:

$$\tilde{\sigma}'(u, y') = -\frac{q}{2\pi a} \sum_{n=0}^{\infty} \tilde{\sigma}_n(u) \frac{T_{2n}\left(y'/a\right)}{\sqrt{1 - (y'/a)^2}}. \tag{11}$$

where $T_{2n}(\cdot)$ is the Chebyshev polynomial of order $2n$ [17]. Note that coefficients $\tilde{\sigma}_n(u)$ are dimensionless with the considered normalisation.

Even order polynomials are considered only, since the charge is centered with respect to the strip in the y' direction, so the charge density has an even behaviour with respect to y'. A more general representation could be adopted [18], including odd polynomials too, but it could be easily proven that the odd expansion coefficients would be zero in this problem.

This kind of representation satisfies the right behaviour at the edges of the strip [19]. In addition, as relevant advantage, this representation automatically vanishes outside the strip [20].

Then, substituting (11) in (10), it is possible to obtain

$$\sum_{n=0}^{+\infty} \tilde{\sigma}_n(u) \int_{-a}^{a} \frac{T_{2n}\left(y_0/a\right)}{\sqrt{1-\left(y_0/a\right)^2}} K_0\left(u|y'-y_0|\right) dy_0 = aK_0\left(u\sqrt{y'^2+h^2}\right). \tag{12}$$

This equation is verified for every $|y'| \leq a$. In order to impose this condition, (12) can be projected on the same basis functions adopted to represent the unknown (Galerkin scheme). In this way, (12) turns in the linear system

$$\underline{\underline{A}}\,\underline{\tilde{\sigma}} = \underline{b}, \tag{13}$$

where $\underline{\tilde{\sigma}}$ is the vector of unknowns $\tilde{\sigma}_n$, $\underline{\underline{A}}$ is a matrix whose coefficients are defined as

$$A_{nm} = \int_0^{\pi}\int_0^{\pi} K_0\left(au\left|\cos\varphi - \cos\varphi_0\right|\right)\cos\left(2n\varphi_0\right)\cos\left(2m\varphi\right) d\varphi_0 d\varphi \tag{14}$$

and $\underline{b}$ is the vector of know term whose coefficients are

$$b_m = \int_0^{\pi} K_0\left[au\sqrt{\cos^2\varphi + \left(h/a\right)^2}\right]\cos\left(2m\varphi\right) d\varphi, \tag{15}$$

with the positions $y' = a\cos\varphi$ and $y_0 = a\cos\varphi_0$.

For every u, the solution of this linear system allows to compute the coefficients $\tilde{\sigma}_n(u)$ and so the representation of the induced charge density in the transformed domain.

3. LONGITUDINAL COUPLING IMPEDANCE

Once the coefficient $\tilde{\sigma}_n(u)$ are computed, the induced density charge (11) is known in the transformed domain and in the charge reference frame. In this section the expression of the longitudinal coupling impedance is shown as function of this quantity.

The per unit length longitudinal coupling impedance is defined as (1), being function of the x-component of the electric field. This quantity is the sum of the electric field produced by the traveling charge and by the one produced by the induced current density on the strip. Anyway, since it is well known that the traveling charge electric field doesn't produce any contribution to the longitudinal coupling impedance, only the electric field produced by the induced density current is considered hereafter. In the charge reference frame, according to (3) its expression is

$$e'_x(x',y',z')=\frac{1}{4\pi\varepsilon_0}\int_{\mathbb{S}}\frac{\sigma'(x_0,y_0)\,(x'-x_0)\;dx_0dy_0}{\left[(x'-x_0)^2+(y'-y_0)^2+z'^2\right]^{3/2}}. \tag{16}$$

Since the electric field is required in the strip frame in order to evaluate the coupling impedance, the Lorentz transformations have to be applied [21]. In this case they appear as

$$\begin{aligned} e'_x &= e_x, \\ \sigma' &= \sigma\gamma, \\ x' &= \gamma\,(x-vt)\,, \quad y=y', \quad z=z', \end{aligned} \tag{17}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor.

So, after applying the Lorentz transformations, (16) becomes

$$e_x(x,y,z,t) = \frac{\gamma}{4\pi\varepsilon_0}\int_{\mathbb{S}}\frac{\sigma(x_0,y_0)\left[\gamma\,(x-vt)-x_0\right]}{\left\{\left[\gamma\,(x-vt)-x_0\right]^2+(y-y_0)^2+z^2\right\}^{3/2}}dx_0dy_0. \tag{18}$$

By taking a derivative of (9), it is found that

$$\frac{\pi\,Z}{\left(D^2+Z^2\right)^{3/2}} = j\int_{-\infty}^{+\infty} uK_0\,(Du)\,e^{-juZ}du. \tag{19}$$

Using this integral in (18) and then applying a spatial Fourier transform according to (8), it is found that

$$\begin{aligned} e_x(x,y,z,t) \;=\; & \frac{j\gamma}{2\pi\varepsilon_0}\int_{-a}^{+a}\int_{-\infty}^{+\infty}\tilde{\sigma}(u,y_0)ue^{ju\gamma vt} \\ & K_0\left[u\sqrt{(y-y_0)^2+z^2}\right]e^{-ju\gamma x}dy_0du. \end{aligned} \tag{20}$$

Since the electric field in the frequency domain is required for the evaluation of the longitudinal coupling impedance, by performing the time Fourier transform of (20), then it is easy to perform the integral with respect to u, obtaining with some manipulations

$$E_x(x,y,z,k)=\frac{jk\zeta_0}{\beta^2}\int_{-a}^{+a}\tilde{\sigma}\,(\kappa,y_0)K_0\left[\kappa\sqrt{(y-y_0)^2+z^2}\right]e^{-jxk/\beta}dy_0, \tag{21}$$

being $\kappa = k/\beta\gamma$ and $\zeta_0 = \sqrt{\mu_0/\varepsilon_0}$ the characteristic impedance of free space.

By substituting (21) into (1), it is finally found that the per unit length longitudinal coupling impedance has the expression $\hat{Z}_{||}(k, \beta) = j\hat{X}_{||}(k, \beta)$, where

$$\hat{X}_{||}(k, \beta) = -\frac{k\zeta_0}{q\beta^2} \int_{-a}^{+a} \tilde{\sigma}(\kappa, y_0) K_0 \left(\kappa \sqrt{y_0^2 + h^2}\right) dy_0. \tag{22}$$

This expression comes from the consideration that, since the coefficients $\tilde{\sigma}_n$ are purely real (all terms in (13) are real), the longitudinal coupling impedance is purely imaginary. This result is expected since there are not diffraction losses.

Substituting the representation of the charge density (11) into (22), the explicit expression of the per unit length coupling reactance is found

$$\hat{X}_{||}(k, \beta) = -\frac{k\zeta_0}{2\pi a\beta^2} \sum_{n=0}^{+\infty} \tilde{\sigma}_n(\kappa) \int_{-a}^{+a} \frac{T_{2n}(y_0/a)}{\sqrt{1-(y_0/a)^2}} K_0\left(\kappa\sqrt{y_0^2+h^2}\right) dy_0. \tag{23}$$

According to (15), the per unit length longitudinal coupling impedance can be finally expressed in the very compact form

$$\hat{X}_{||}(k, \beta) = \frac{k\zeta_0}{2\pi\beta^2} \sum_{n=0}^{+\infty} \tilde{\sigma}_n(\kappa) b_n(\kappa). \tag{24}$$

This expression evidences that, thanks to the adopted method for the solution of the problem, once the coefficients b_n and $\tilde{\sigma}_n$ are evaluated for a given $u = \kappa$, the per unit length longitudinal coupling impedance is practically computed.

4. NUMERICAL TREATMENT

In order to calculate the longitudinal coupling impedance, an accurate and easy to compute evaluation of integrals (14) and (15) is required: the integrands exhibit some singularities that create numerical implementation problems. This means that a specific treatment is required.

Both integrals A_{nm} and b_m behave a logarithmic singularity when the argument of the modified Bessel function $K_0(\cdot)$ vanishes.

Since for low values of the argument

$$K_0(z) \cong -\log\frac{z}{2} - \gamma_0, \tag{25}$$

where $\gamma_0 \approx 0.57721$ is the Eulero-Mascheroni constant, it is not difficult to put (14) and (15) in the variational form

$$A_{nm} = A_{nm}^{acc} + A_{nm}^{0}, \tag{26}$$

$$b_m = b_m^{acc} + b_m^{0}, \tag{27}$$

where

$$A_{nm}^{acc} = \int_0^\pi \int_0^\pi \left[K_0 \left(au \left| \cos\varphi - \cos\varphi_0 \right| \right) + \log\left(\frac{au \left| \cos\varphi - \cos\varphi_0 \right|}{2} \right) + \gamma_0 \right] \cos\left(2n\varphi_0\right) \cos\left(2m\varphi\right) d\varphi_0 d\varphi, \tag{28}$$

$$A_{nm}^{0} = -\int_0^\pi \int_0^\pi \left[\log\left(\frac{au \left| \cos\varphi - \cos\varphi_0 \right|}{2} \right) + \gamma_0 \right] \cdot \cos\left(2n\varphi_0\right) \cos\left(2m\varphi\right) d\varphi_0 d\varphi, \tag{29}$$

$$b_m^{acc} = \int_0^\pi \left[K_0 \left(au\sqrt{\cos^2\varphi + (h/a)^2} \right) + \log\left(\frac{au\sqrt{\cos^2\varphi + (h/a)^2}}{2} \right) + \gamma_0 \right] \cos\left(2m\varphi\right) d\varphi, \tag{30}$$

$$b_m^{0} = -\int_0^\pi \left[\log\left(\frac{au\sqrt{\cos^2\varphi + (h/a)^2}}{2} \right) + \gamma_0 \right] \cos\left(2m\varphi\right) d\varphi. \tag{31}$$

In such a way, the accelerated coefficients don't exhibit any more the logarithmic singularities and they can be quickly evaluated, while the remaining parts A_{nm}^0 and b_m^0 have to be analytically computed. It is worth noting that the accelerated coefficients approach to zero when u vanishes. This means that A_{nm}^0 and b_0^m represent the approximations of (14) and (15) for small values of u.

By means of the relevant expansions (see Appendix)

$$\log\left|\sin(x/2)\right| = -\sum_{n=1}^{\infty} \frac{\cos nx}{n} - \log 2 \tag{32}$$

and

$$\log\left[2\left(\cosh t + \cos x\right)\right] = t - 2\sum_{n=1}^{\infty} \frac{(-1)^n e^{-nt}}{n} \cos nx \tag{33}$$

it is possible to conclude that

$$A^0_{nm} = \begin{cases} -\pi^2 \left[\log\left(\frac{au}{4}\right) + \gamma_0\right], & n = m = 0, \\ \pi^2/4n, & n = m \neq 0\,, \\ 0, & n \neq m\,, \end{cases} \tag{34}$$

and similarly

$$b^0_m = \begin{cases} -\pi\gamma_0 - \pi\log\left(aus/4\right), & m = 0\,, \\ \frac{\pi}{2m}(-1)^m s^{-2m}\,, & m \neq 0\,, \end{cases} \tag{35}$$

being $s = h/a + \sqrt{1 + (h/a)^2}$.

Note that the acceleration (30)–(31) is efficient for small values of $au < 1$ (i.e., at low frequencies), otherwise the logarithmic term becomes much greater than the modified Bessel function, (15) is more accurate instead.

A similar consideration can be done for the acceleration (28) supplying to the singularities for $\cos\varphi = \cos\varphi_0$ and for au going towards zero. Only the first one occurs for higher values of au, in this case the acceleration (28) has to be partially modified.

5. NUMERICAL RESULTS

In the previous sections, the advantages of the proposed method in the analytical formulation and formal solution of the problem have been discussed.

In this section, its efficiency is also shown by the numerical point of view, this is particularly connected to the fast convergence of the series of coefficients $\tilde{\sigma}_n$.

A Simpson rule with an adaptive spacing is adopted to compute the coefficients (28) and (30). The ratio h/a and the parameter $\beta\gamma$ have been considered in the range $0.1 - 10$. Expansion coefficients $\tilde{\sigma}_n$ and the normalised per unit length reactance $\hat{X}_{||}/\zeta_0$ are computed.

In Fig. 2(a), the absolute values of expansion coefficients are shown for different frequencies, in semilogarithmic scale. Even coefficients are positive, odd ones are negative. They quickly drops at higher frequencies too, a small number of expansion coefficients is then needed to achieve convergency. This confirms the efficiency of the method.

In Fig. 2(b), the expansion coefficients are represented for different distances of the charge from the particle. Of course as far the particle, as lower the interaction between the particle and the strip, as lower the number of coefficients required to achieve the convergence.

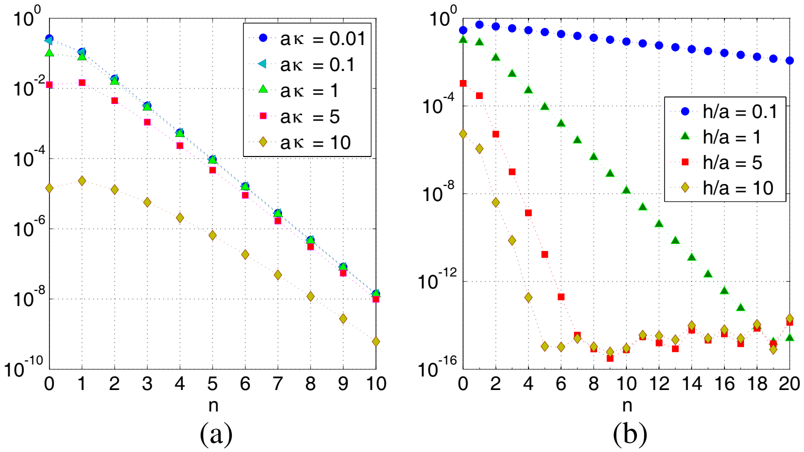


Figure 2. Absolute values of the expansion coefficients: (a) for different frequencies ($h/a = 1$, $\beta\gamma = 1$), (b) for different heights of the charge ($a\kappa = 1$, $\beta\gamma = 1$).

In the two following figures, the normalised per unit length coupling reactance is computed. A frequency sweep is performed for $a\kappa$ from 0.01 to 10 with 301 logarithmically spaced samples. The calculation has been performed on a Intel® Pentium®-4 CPU at 3.20 GHz clock speed and with 2 GB RAM. The computational time for a frequency sweep, depending on the values of h and $\beta\gamma$, vary from 16 to 18 minutes.

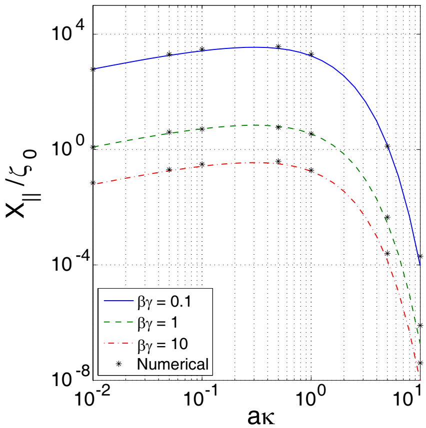
In Fig. 3(a), the reactance is shown as function of the frequency, for different values of $\beta\gamma$. The reactance magnitude decreases with the particle speed, being anyway the shape slightly influenced.

In Fig. 3(b), the reactance is shown as function of the frequency, for different values of h instead. Increasing the distance, the reactance drop is observed at smaller frequencies.

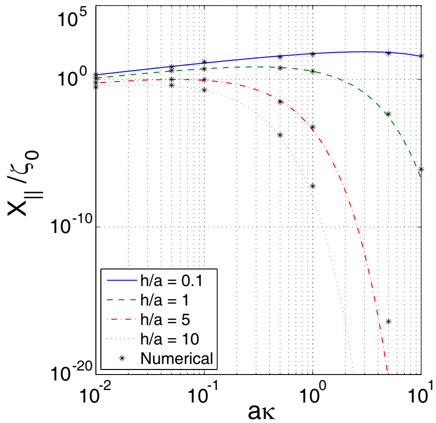
The results obtained with the proposes semi-analytical method have been compared with the ones obtained with a FEM solver, by means of the MATLAB PDE Toolbox. The comparison is shown in the Figures and a good agreement is found for the considered ranges of parameters.

6. SPECIAL CASES

In this section, some relevant cases are discussed, where approximate analytical results can be obtained. They can be used to quickly computed the parameters of interest in some conditions, as well as to improve the numerical convergence in the general case.



(a)



(b)

Figure 3. Normalised per unit length longitudinal coupling reactance: (a) for different frequencies and speeds ($h/a = 1$), (b) for different frequencies and heights ($\beta\gamma = 1$).

6.1. Particle Far from the Strip

When the particle is far from the strip, although there is complete induction on the strip, the induced current density has a smoother behaviour, due to the smaller interaction between the particle and the strip. A smaller number of expansion coefficients is required to achieve a good convergence of the method.

Anyway a simple analytical approximation can be found for coefficients b_m, in order to make some considerations on the solution.

It is trivial to observe that for $h/a \gg 1$, (15) can be well approximated as

$$b_m = \begin{cases} \pi K_0\left(hu\right), & m = 0\,, \\ 0\,, & m \neq 0\,, \end{cases} \tag{36}$$

This means that for high values of h, the magnitude of the coefficients reduces exponentially. This result is confirmed by Fig. 2(b), where the absolute value of the coefficients quickly drops for $h/a > 1$.

6.2. Particle Close to the Strip

As shown in Fig. 2(b), the number of coefficients required to achieve the convergence increases when the particle is closer to the strip. Studying the case $h/a \to 0$ can be useful to obtain a solution enhancing the convergence of the numerical method.

In this condition, coefficients (15) can be rewritten by means of the integral representation of $K_0(\cdot)$ as

$$b_m = 2 \int_{-\infty}^{+\infty} \int_{0}^{\pi/2} \cos\left(au \sinh t \cos\varphi\right) \cos\left(2m\varphi\right) d\varphi dt \tag{37}$$

.

By using the integral representation of $J_m(\cdot)$ and performing the outer integral [15], it is finally obtained

$$b_m = (-1)^m \pi K_m\left(au/2\right) I_m\left(au/2\right). \tag{38}$$

When the particle get closer to the strip and a more accurate evaluation of the coefficients b_m is required also for higher values of m, this result can be used to accelerate and improve their numerical evaluation.

6.3. Low Frequency Solution

As previously discussed, when k approaches to zero, the relevant part of (14) and (15) are (29) and (31) respectively. The matrix of the relevant system (13) becomes diagonal and therefore the expansion coefficients can be analytically evaluated:

$$\tilde{\sigma}_n = \frac{1}{\pi} \begin{cases} 1 + \dfrac{\log s}{\gamma_0 + \log\left(a\kappa/4\right)}, & n = 0\,, \\ 2(-1)^n s^{-2n}\,, & n \neq 0. \end{cases} \tag{39}$$

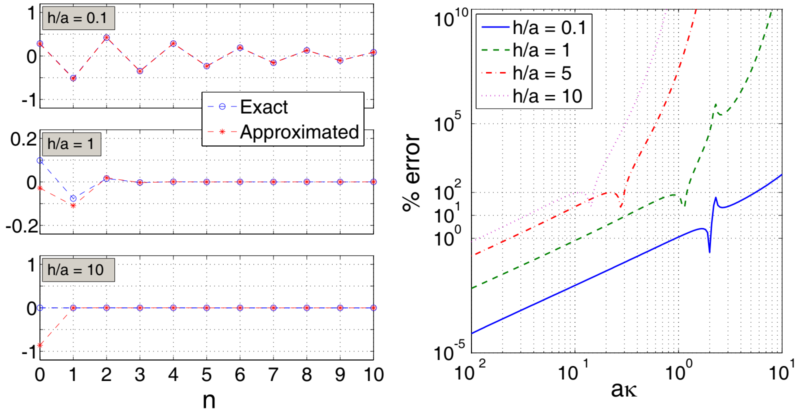
An easy limit is required to compute $\tilde{\sigma}_0$. Its expression can also be verified by substituting (11) into (7), after some manipulations. Note that this coefficient is much more sensitive to the frequency than the other ones.

Substituting this result in (24), then a low frequency expression of the longitudinal coupling reactance can be found

$$\hat{X}_{||}\left(k,\beta\right) = -\frac{k\zeta_0}{2\pi\beta^2}\left[\gamma_0 + \log\left(\frac{a\kappa}{4}\frac{s^4-1}{s^2}\right) + \frac{\log^2 s}{\gamma_0 + \log\left(a\kappa/4\right)}\right], \tag{40}$$

It is trivial that the longitudinal coupling impedance goes to zero when k goes to zero too.

In order to show the efficiency of this solution, the approximated expansion coefficients are computed and compared with respect to the exact ones, as shown in Fig. 4(a). At the considered frequency the coefficients are well approximated for small values of the ratio h/a. Increasing the distance, although the number of required coefficients is



(a) (b)

Figure 4. Comparison between exact and approximated solution: (a) Expansion coefficients ($a\kappa = 1$, $\beta\gamma = 1$). (b) Percentage error of the approximated p.u.l. coupling reactance with respect to the exact one ($\beta\gamma = 1$).

smaller, the error increases, especially for the first coefficient. This is expected, since the coefficients vanish for high values of h/a, as shown in (36), the solution (39) converges to a constant value instead.

The coefficient $\tilde{\sigma}_0$ is more sensitive than the other ones to the frequency an to the ratio h/a.

This result is confirmed in Fig. 4(b), where the absolute percentage error of the approximation (39) with respect to the exact solution is shown. Also in this case it is clear that, for similar frequencies, the error considerably increases with the particle distance.

7. CONCLUSIONS

A method for the evaluation of the longitudinal coupling impedance of a particle travelling parallel to a perfectly conducting strip has been presented. The method is accurate and effective, and easily can be generalised to the analysis of more complex structures.

APPENDIX A.

In this Appendix, the relationship (33) is proved.

For a generic complex number z, let us consider the geometrical series

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n. \tag{A1}$$

By integrating this expansion, it is found that

$$\log(1-z) = -\sum_{n=1}^{\infty} \frac{z^n}{n}. \tag{A2}$$

If $z = e^{-t+jx}$, it is trivial to observe that

$$|1-z|^2 = 2\, e^{-t} (\cosh t - \cos x)\,. \tag{A3}$$

Therefore the real part of (A2) gives

$$\log[2(\cosh t + \cos x)] = t - 2\sum_{n=1}^{\infty} \frac{(-1)^n\, e^{-nt}}{n} \cos nx\,, \tag{A4}$$

namely Equation (33). It is worth noting that the special case $t = 0$

$$\log[2(1+\cos x)] = -2\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos nx\,, \tag{A5}$$

after some trigonometric manipulations, gives the formula (32).

REFERENCES

1. Heifets, S. A. and S. A. Kheifets, "Coupling impedance in modern accelerators," *Reviews of Modern Physics*, Vol. 63, 631–673, 1993.
2. Kheifets, S. A. and B. W. Zotter, *Impedances and Wakes in High-Energy Particle Accelerators*, World Scientific, Singapore, 1998.
3. Palumbo, L., V. G. Vaccaro, and M. Zobov, "Wake fields and impedance," *Proceedings of CAS CERN Accelerator School CERN 95-06*, 331–390, 1995.
4. Chao, A. W. and M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific, Singapore, 1998.
5. Kuehn, E., "A mode-matching method for solving field problems in waveguide and resonator circuits," *Archiv fuer Elektronik und Uebertragungstechnik*, Vol. 27, 511–518, Dec. 1973.
6. Legenkiy, M. N. and A. Y. Butrym, "Method of mode matching in time domain," *Progress In Electromagnetics Research B*, Vol. 22, 257–283, 2010.
7. Assante, D., D. Davino, S. Falco, F. Schettino, and L. Verolino, "Coupling impedance of a charge travelling in a drift tube," *IEEE Trans. on Mag.*, Vol. 41, No. 5, 1924–1927, 2005.
8. Fujita, K. and H. Kawaguchi, "Time domain numerical simulation method based on EFIE and MFIE for axis-symmetric structure objects," *Progress In Electromagnetic Research Symposium*, 627–630, Pisa, Italy, 2004.

9. Censor, D., "Free-space relativistic low-frequency scattering by moving objects," *Progress In Electromagnetics Research*, Vol. 72, 195–214, 2007.
10. Censor, D., "Application-oriented relativistic electrodynamics," *Progress In Electromagnetics Research*, Vol. 29, 107–168, 2000.
11. Idemen, M. and A. Alkumru, "Influence of motion on the edge-diffraction," *Progress In Electromagnetics Research B*, Vol. 6, 153–168, 2008.
12. Censor, D., "Broadband spatiotemporal differential-operator representations for velocity depending scattering," *Progress In Electromagnetics Research*, Vol. 58, 51–70, 2006.
13. Mostacci, A., et al., "Wakefields due to surface waves in a beam pipe with a periodic rough surface," *Phys. Rev. ST Accel. Beams*, Vol. 5, Apr. 2002.
14. Miano, G., L. Verolino, and V. G. Vaccaro, "Time domain analysis of a charge particle travelling along the axis of a circular waveguide," *Il Nuovo Cimento*, Vol. 111B, 659–664, 1996.
15. Gradshteyn, I. S. and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th Edition, Academic Press, New York, 2007.
16. Eswaran, K., "On the solutions of a class of dual integral equations occurring in diffraction problems," *Proc. R. Soc. Lond.*, Vol. A429, 399–427, 1990.
17. Cesarano, C., "Identities and generating functions on Chebyshev polynomials," *Georgian Mathematical Journal*, Vol. 19, 427–440, 2012.
18. Assante, D., S. Falco, M. Lucido, G. Panariello, F. Schettino, and L. Verolino, "Shielding effect of a strip of finite thickness," *Electrical Engineering*, Vol. 89, No. 2, 79–87, 2006.
19. Sautbekov, S. S., "Diffraction of plane wave by strip with arbitrary orientation of wave vector," *Progress In Electromagnetics Research M*, Vol. 21, 117–131, 2011.
20. Assante, D., S. D'Agostino, and L. Verolino, "Shielding effect of a thin strip," *Electrical Engineering*, Vol. 91, Nos. 4–5, 177–185, 2009.
21. Handapangoda, C. C., M. Premaratne, and P. N. Pathirana, "Plane wave scattering by a spherical dielectric particle in motion: A relativistic extension of the Mie theory," *Progress In Electromagnetics Research*, Vol. 112, 349–379, 2011.