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EXCITATION OF AZIMUTHAL EIGEN MODES BY MODULATED ANNULAR ELECTRON BEAM

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Abstract—Excitation of extraordinarily polarized azimuthal eigen modes by modulated annular electron beam is shown to be characterized by the increase of instability growth rates compared with the case of non-modulated electron beam. Interaction between the modulated beam and azimuthal eigen modes happens in the range of electron cyclotron frequency in waveguides with metal walls, which are partially filled with cold magneto-active plasma. Nonlinear set of differential equations, which describs excitation of these azimuthal modes by an annular electron beam is derived and analyzed numerically. Different scenarios of the beam-plasma interaction depending on relation between azimuthal mode number of the exited waves and periodicity of azimuthal modulation of the beam density, degree and manner of the beams' modulation are studied numerically.

1. INTRODUCTION

Active research of plasma filled structures during the last years is motivated first of all by a request to elaborate electronic devices, which will be able to generate electromagnetic emission in higher frequency band, be continuously tunable (over a broad frequency range) and be of a relatively small-sized. Application of plasma in plasma filled metal waveguides is intended mainly for neutralization of a space charge of the transported beams and for creation favorable conditions for propagation of eigen modes there. A plasma filling strongly effects on dispersion properties of such waveguide structures. Various aspects of the problem of an interaction between electron beams and the eigenmodes of plasma filled waveguides are investigated

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in [1–3]. The main attention is paid there to studying the processes of waves' excitation in magneto-active plasma waveguides. However, one can't consider the theory of beam-plasma instabilities as a completely developed theory because process of interaction between charged particle beams and eigen waves of plasma waveguides essentially depends on a lot of factors, including dispersion properties of these waves, their polarization, spatial distribution of their fields, geometrical peculiarities and design features of the utilized waveguides.

Properties of waveguides with different designs and their eigen modes are actively studied during last thirty years in order to elaborate small-sized electromagnetic generators and amplifiers, which can operate in a wide frequency band and can be smoothly tuned. Taking into account advantages of plasma filled waveguides over the conventional vacuum tubes applied in vacuum electronics one can understand why the more and more attention is paid to elaboration of magneto-active plasma filled waveguides and/or to waveguides, which are made of special meta-materials [4, 5]. This allows one to develop radio-electronic devices, which operate at modes with interesting features like non-reciprocal waves or unidirectional waves. Among them, it should be indicated azimuthal eigen modes, which propagate in cylindrical plasma filled waveguide structures across an external steady axial magnetic field [6]. As it is shown in [7] azimuthal modes (AMs) can be excited by charged particle beams rotating above plasma interface across a strong external steady axial magnetic field.

Results of studying beam-plasma interaction are widely utilized in different branches of plasma physics, e.g., for studying plasma heating in fusion devices [8, 9], for studying generation of high power radiation in various plasma electronic devices [10–15]. Moreover, the relativistic electron beams are offered to be used even for plasma confinement. In the paper [16], authors present an example of calculating a steady state of compact toroidal beam-plasma system consisting of the immobile ion background and relativistic electron beam.

The possibility of AMs excitation by a relativistic electron beam rotating over plasma column was shown at the first time in the linear approximation in the paper [17]. The peak value of a growth rate of this instability and its dependence on plasma density, the azimuthal mode number, value of an external steady magnetic field and a thickness of the vacuum gap, which separates plasma column form metal wall of the waveguide have been studied analytically and numerically. Non-linear theory of the AM excitation by a non-modulated charged particle beam has been presented in [18]. Influence of different plasma waveguides' parameters have been investigated therein, including effect of an external radial electric field. But utilization of an annular charged particle beams for excitation of electromagnetic waves is a wide theme, since its investigation is far from completion. For instance, mechanisms for the generation and amplification of electromagnetic waves by a thin-walled annular beam of electrons rotating in a radial electric field in free space are studied theoretically in [19]. It is shown that electromagnetic waves can be generated and amplified under the Cherenkov resonance conditions.

To increase efficiency of a beam-plasma interaction, one can apply a preliminary spatial modulation of charged particle beams [1, 2, 4]. One of the main branches of a modulated beam application is construction of powerful super high frequency generators, see, e.g., [20]. The evolution of a velocity distribution function of a modulated beam during its motion in dense plasma is considered in [21]. There an influence of background plasma inhomogeneity on the modulated beam evolution is studied numerically and comparison of the results obtained by simulation with laboratory experiments data is carried out as well.

Since excitation of AMs by uniform annular beam was studied in [18] then in order to develop theory of AMs excitation the present paper is devoted to non-linear theory of azimuthal eigen modes' excitation by the annular electron beam, which density is preliminary modulated. The paper is organized as follows. Formulation of the problem is presented in Section 2. Section 3 is devoted to discussion of the obtained results. Conclusions are summarized in the Section 4.

2. BASIC EQUATIONS

Let's consider cylindrical metal waveguide of radius R_2 inside which a column of cold magneto-active plasma of radius R_1 is located co-axially. It's supposed that an external constant magnetic field $\vec{B}_0$ is oriented along the waveguides' axis of symmetry $\vec{z}$. Fields of extraordinary polarized AMs is assumed to depend on azimuthal angle φ and time t as follows: $\sim f(r)\exp(im\varphi - i\omega t)$, where natural number m is azimuthal mode number, ω is eigen frequency of the AM, dependence upon radial coordinate r can be found from solution of Maxwell equations. Along z direction, the waveguide is assumed to be uniform, so that the studied electromagnetic perturbations are independent upon z coordinate. An annular electron beam rotates in the vacuum layer $R_2 > r > R_1$ between the plasma column and a metal wall of the waveguide.

The set of differential equations, which describes a nonlinear stage of the excitation of these modes by an electron beam, can be obtained from the hydrodynamic equations for the plasma, Maxwell's equations for AMs fields, and momentum equation for the beam electrons. The beam is described by model of macro-particles. Since the beam and plasma densities are such that inequality $n_b \ll n_p$ is valid then one can neglect both the effect of the beam on the AMs dispersion properties and the effect of the self-field of the beam on the AMs electromagnetic field. We restricted our consideration by the case of weak beam instability, where the ratio $n_b/n_p \ll 1$ is a small parameter of the problem. Therefore our theory describes excitation of the AM, increasing of the wave envelop amplitude up to maximum value, and saturation of the instability due to the capturing of the beams' particles into potential wells of the wave. Thus such strong non-linear effects like an oscillation of dense electron bunches in potential wells of the excited waves are out of scope of the paper.

Tangential component of the AM magnetic field is described by inhomogeneous Bessel equation [22], which right-hand part $F_b(r)$ is determined by the electron beam that moves above the plasma column. In the case, when flows of charged particles are absent in the region $R_2 > r > R_1$ (it means $j_r = j_\varphi = 0$ and hence $F_b = 0$) expressions for the AMs coincide with that are obtained in [6] for studying dispersion properties of these modes in the case of a dense plasma (Langmuir frequency is larger than electron cyclotron frequency). Thus for tangential components of the AMs fields one can derive the following expressions in the region occupied by the electron beam:

$$\begin{aligned} H_z^{(B)}(r) \;=\; & C_1 J_m(\varsigma) + C_2 N_m(\varsigma) - \frac{\pi}{2} J_m(\varsigma) \int\limits_{\varsigma_1}^{\varsigma} x N_m(x) F_b(x) dx \\ & + \frac{\pi}{2} N_m(\varsigma) \int\limits_{\varsigma_1}^{\varsigma} x J_m(x) F_b(x) dx, \end{aligned} \tag{1}$$

$$\begin{aligned} E_\varphi^{(B)}(r) \;=\; & -iC_1 J'_m(\varsigma) - iC_2 N'_m(\varsigma) + \frac{i\pi}{2} J'_m(\varsigma) \int\limits_{\varsigma_1}^{\varsigma} x J_m(x) F_b(x) dx \\ & - \frac{i\pi}{2} N'_m(\varsigma) \int\limits_{\varsigma_1}^{\varsigma} x J_m(x) F_b(x) dx + \frac{4\pi}{i\omega} j_\varphi, \end{aligned} \tag{2}$$

here we omit dependence of the fields upon azimuthal angle φ and time t, which is mentioned at the beginning of the Section, in order to simplify these expressions. They can be considered as Fourier-coefficients for tangential components of magnetic and electric fields in the space of time-azimuthal angle, thus they are functions of radius, azimuthal mode number m, and eigen frequency ω. The other notations are as follows: $F_b(x) = -\frac{4\pi}{\omega x}[\frac{\partial}{\partial x}(x j_\varphi) - imj_r]$, $\varsigma = kr$,

$\varsigma_1 = kR_1$, $\delta(y - y_0)$ is delta function. Applying model of macro-particles, the expressions for electric current density for the beam are written here as follows: $j_r = -|e| \sum_{j=1}^{N} \delta(r - r_j)\delta(\varphi - \varphi_j)\frac{\partial r}{\partial t}$, $j_\varphi = -|e| \sum_{j=1}^{N} r\delta(r - r_j)\delta(\varphi - \varphi_j)\frac{\partial \varphi}{\partial t}$, in expressions for electric current densities the summation index "j" indicates the number of the beam particle, N is total quantity of the macro-particles, which models this electron beam. More detailed derivation of the Equations (1) and (2) is presented in [18].

In the plasma region ($r \leq R_1$) spatial distribution of the AMs axial magnetic field $H_z^{(P)}(r)$ is described by the modified Bessel function $I_m(r)$ [22] and the radial and azimuthal electric fields can be find out using its solution [18]. Absence of MacDonald function in expression for $H_z^{(P)}(r)$ is explained by necessity to satisfy the boundary condition on restriction of the value of AMs fields on the axis of the waveguide. So let's write down expression for AMs fields in the plasma region:

$$H_z^{(P)}(r, \varphi, t) = A_0 \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} I_m(\xi) \exp(im\varphi - i\omega t) d\omega, \tag{3}$$

$$\begin{aligned} E_\varphi^{(P)}(r, \varphi, t) &= \left(\frac{im\varepsilon_2}{\varepsilon_1 \xi \psi} + \frac{i}{\psi}\frac{\partial}{\partial \xi}\right) H_z^{(P)}, \\ E_r^{(P)}(r, \varphi, t) &= \left(\frac{m}{\xi \psi} + \frac{\varepsilon_2}{\psi \varepsilon_1}\frac{\partial}{\partial \xi}\right) H_z^{(P)}, \end{aligned} \tag{4}$$

here A_0 is constant of integration, and argument of the Bessel function is as follows: $\xi = \varsigma\psi$, $\psi = \sqrt{(\mu^2 - 1)\varepsilon_1}$, $\mu = \varepsilon_2 \varepsilon_1^{-1}$, ε_1 and ε_2 are components of plasma dielectric permittivity in the fluid approximation [23]. Let's restrict our consideration by the case of collisionless plasma, then explicit expressions for the indicated components of the dielectric permittivity tensor can be written as follows:

$$\varepsilon_1 = 1 - \sum_\alpha \frac{\Omega_\alpha^2}{\omega^2 - \omega_\alpha^2}, \quad \varepsilon_2 = \sum_\alpha \frac{\omega_\alpha \Omega_\alpha^2}{(\omega^2 - \omega_\alpha^2)\, \omega}, \tag{5}$$

here Ω_α and ω_α are plasma and cyclotron frequencies of the plasma, respectively (subscript α applied here indicates type of the plasma particles, for ions $\alpha = i$, and for electrons $\alpha = e$).

Using standard procedures [5, 18] of determination of a mean value and separation of the amplitude and phase of the excited waves' envelope, one can derive equations for amplitude and phase of the envelope of AM under the condition of resonant excitation of the eigen mode. Their explicit forms are as follows:

$$\begin{aligned}\frac{dE}{dt} &= -\frac{\alpha_b D_p}{NzPL}\sum_{j=1}^{N}\Big[\frac{m}{w}\frac{dR_j}{dt}L_1(\varsigma_j)\cdot\sin(m\varphi_j+\Theta-\omega t)\\ &\qquad +R_j^2\frac{d\varphi_j}{dt}L_2(\varsigma_j)\cdot\cos(m\varphi_j+\Theta-\omega t)\Big], \end{aligned} \tag{6}$$

$$\begin{aligned}\frac{d\Theta}{dt} &= \frac{\alpha_b D_p}{ENzPL}\sum_{j=1}^{N}\Big[R_j^2\frac{d\varphi_j}{dt}L_2(\varsigma_j)\cdot\sin(m\varphi_j+\Theta-\omega t)\\ &\qquad -\frac{m}{w}\frac{dR_j}{dt}L_1(\varsigma_j)\cdot\cos(m\varphi_j+\Theta-\omega t)\Big], \end{aligned} \tag{7}$$

here $E = E_\varphi^{(P)}/B_o$ is dimensionless amplitude of the envelope, Θ is its phase, $\alpha_b = n_b/n_p$ is dimensionless density of the beam, $L = J_m(\varsigma_1)N'_m(\varsigma_2) - J'_m(\varsigma_2)N_m(\varsigma_1)$,

$$\begin{aligned}L_1(\varsigma_j) &= J_m(\varsigma_j)\,N'_m(\varsigma_2) - J'_m(\varsigma_2)\,N_m(\varsigma_j)\,,\\ L_2(\varsigma_j) &= J'_m(\varsigma_j)\,N'_m(\varsigma_2) - J'_m(\varsigma_2)\,N'_m(\varsigma_j)\,,\\ D_p &= \frac{I'_m(\psi\varsigma_1)}{\psi I_m(\psi\varsigma_1)} + \frac{m\varepsilon_2}{\varepsilon_1\psi^2\varsigma_1},\\ D &= D_p + \frac{J'_m(\varsigma_1)\,N'_m(\varsigma_2) - J'_m(\varsigma_2)\,N'_m(\varsigma_1)}{J_m(\varsigma_1)\,N'_m(\varsigma_2) - N_m(\varsigma_1)\,J'_m(\varsigma_2)}, \quad P = \frac{dD}{dw},\end{aligned}$$

$\varsigma_2 = kR_2$, $w = \omega/\Omega_e$, $D(w, m, \varsigma_{1,2}) = 0$ is dispersion equation of the AMs propagating in the considered waveguide (see [6]) under the condition of $\alpha_b = 0$ (that means absence of the beam), $R_j = r_j\Omega_e c^{-1}$ is dimensionless radial co-ordinate of the j-th particle of the beam.

It is suitable for formulate momentum equation, which describes the beams' electrons motion, using terms of their impulses $\vec{p} = \gamma m_e\vec{V}$ (γ is relativistic factor), because it allows one to take into the account a weak relativism of the beam:

$$\frac{d\vec{p}}{dt} = e\vec{E}^{(B)} + \frac{e}{c}\left[\vec{V}\times\left(\vec{H}^{(B)} + \vec{B}_0\right)\right]. \tag{8}$$

Analyzing Equation (8) that describes motion of the beams' particles, one can see that it is a non-linear equation because fields of the AM in the region occupied by the beam depend upon the current density of the beams' electrons j_r and j_φ (see Equations (1) and (2)). Radial v and azimuthal u dimensionless impulses of the beam particles can be determined using derivatives of the radial co-ordinate R and angular co-ordinate φ over time, respectively. Let's write down them in the following form:

$$v = p_r/(cm_e), \quad u = p_\varphi/(cm_e). \tag{9}$$

Expressions for radial and azimuthal impulses of the beam particles can be written using derivatives of radial and angular particles' coordinates over time, respectively:

$$dR_j/dt = |\omega_e|\, v_j/(z\gamma_j), \quad d\varphi_j/dt = |\omega_e|\, u_j/(z\gamma_j R_j). \tag{10}$$

By the way of substituting expressions (1) and (2), which are obtained for the AM fields in the region $R_1 < r < R_2$ into the Equation (8), one can drive the following equations for derivatives of radial v_j and azimuthal u_j components of the impulse for j-th particle of the beam over time (meaning of the index j possess the value from 1 until N):

$$\frac{dv_j}{|\omega_e|\, dt} = \frac{ER_1}{\delta}\left(\frac{m\delta}{R_1} - w\frac{u_j}{\gamma_j}\right)\left(\frac{1}{\varsigma_1 D_p} - R_{1j}\right)\sin\left(m\varphi_j + \theta - \omega t\right) - \frac{u_j}{\gamma_j}\left(1 - \frac{u_j}{zR_j}\right), \tag{11}$$

$$\frac{du_j}{|\omega_e|\, dt} = \frac{EwR_1}{\delta\varsigma_1}\left[R_{1j} - 1 + \frac{R_{1j}}{D_p}\left(\frac{m^2}{\varsigma_1} - \varsigma_1\right)\right]\cos\left(m\varphi_j + \theta - \omega t\right) + \frac{v_j}{\gamma_j}\left(1 - \frac{u_j}{zR_j}\right) + \frac{EwR_1 v_j}{\delta\gamma_j}\left(\frac{1}{\varsigma_1 D_p} - R_{1j}\right)\sin(m\varphi_j + \theta - \omega t), \tag{12}$$

here $R_{1j} = r_j R_1^{-1} - 1$, $\delta = c/\Omega_e$ is skin-depth, $z = |\omega_e|/\Omega_e$. According to results of [6], AM can propagate in the range of electron cyclotron frequency in such waveguides if inequality $z < 1$ is satisfied, so we apply here this condition.

To derive a complete set of differential equations, which describe beam-plasma interaction one can apply the following boundary conditions. Tangential electric field of the AMs is equal to zero on the surface of a metal wall of the waveguide:

$$E_\varphi^{(B)}\left(R_2\right) = 0. \tag{13}$$

There is no azimuthal electric current on the surfaces of a metal wall of the waveguide:

$$j_\varphi\left(R_2\right) = 0. \tag{14}$$

Axial magnetic field of the AMs is continuous on the plasma column interface $r = R_1$:

$$H_z^{(B)}\left(R_1 + 0\right) = H_z^{(P)}(R_1 - 0). \tag{15}$$

And the last condition is concerned with azimuthal electric field of the AMs; its form is similar to the form of the previous condition (15):

$$E_\varphi^{(B)}\left(R_1 + 0\right) = E_\varphi^{(P)}\left(R_1 - 0\right). \tag{16}$$

But really it is a non-linear condition, because expression of tangential electric field of the AMs in the region occupied by the electron beam $E_{\varphi}^{(B)}(r)$ is determined by electric current density (see Equation (2)), unlike the expressions for an AM fields in the region $r \leq R_1$.

Solution of the set of equations for amplitude (6) and phase (7) of the AMs envelope, motion equations for the beams' electrons (10)–(12) has been found by the Runge-Kutta method of the fourth order, which is widely used for the research of such problems [1, 2, 14, 15]. Quantity of the beams' macro — particles was assumed to be here $N = 2000$. To describe the interaction between charged particle beam and a metal wall of the waveguide, the model of mirror reflection of the particles from the wall has been applied (the particles do not disappear after such an interaction with the wall) [1, 18].

Results of the numerical analysis of the resonant plasma-beam instability of the AMs are presented in Figures 1–6. Condition of the resonant instability means that eigen frequency ω of the AM is equal to N_0-th harmonic of electron cyclotron frequency ($\omega = N_0|\omega_e|/\gamma$). Taking into the account results of linear theory of interaction between annular electron beams and AMs [7, 17] azimuthal wave number and radius of the waveguide have been chosen in such way that the following equation was satisfied: $|m|\delta \approx 0.4R_1$. This chose is determined by the fact that just in this case value of the growth rates of resonant plasma-beam instability was found to be maximal.

3. DISCUSSION OF THE OBTAINED RESULTS

Making numerical analysis of this resonant plasma-beam interaction the following values of the plasma filled waveguides' parameters and initial values of the beams' parameters have been applied. The following meanings of initial values have been chosen: for the amplitude of the AM envelope $E(\tau = 0) = 10^{-3}$, for the envelopes' phase $\Theta(\tau = 0) = 0$, for the radial impulses of the beams' particles $v_j(\tau = 0) = 0$ and for the angular impulses of the particles $u_j(\tau = 0) \approx zR_j(\tau = 0)$. This approximate equation for azimuthal impulse of the beams' particles was valid with accuracy 2%. It means that at the first time of the beam-plasma interaction, particles of the electron beam move with azimuthal velocity, which value satisfies the condition $V_{\varphi} \approx |\omega_e|(R_1 + R_2)/(2\gamma)$. Thus initially the particles move in the gap between the plasma column and wall of the waveguide without collisions with both the column and the wall. Dimensionless density of the beam was $\alpha_b = 10^{-2}$, the AM azimuthal mode number was chosen as $m = 3$. Initial location of the beams' particles along azimuthal angle φ was described by sine-law, along radial co-ordinate the particles were distributed randomly within middle third part of the gap between the plasma column and metal wall of the waveguide. Geometrical parameters of the waveguide were as follows: $\Delta = R_2/R_1 - 1 = 0.3$, external magnetic field and density of the plasma were chosen by such manner that the following relation between electron cyclotron frequency and plasma frequency was valid: $z = 0.099$. The specific meaning of the small parameter $z < 1$ has been chosen arbitrary, but condition of its smallness was obtained in [6], while the AM eigen frequency can be related to the range of electron cyclotron resonance. The present research has been performed for the cases when azimuthal angular circumference 2π of the beam was equal to two, three and four periods ($n = 2$, 3 and 4) of the spatial modulation of the beam, respectively. Location of the macro-particles in the both co-ordinate space and phase space are indicated in the corresponding figures as dots. Hence one can visually estimate: where concentration of the beams' particles is larger and where its value is smaller.

In Figure 1, one can see that electron beam, whose density has been preliminary spatially modulated, interacts with the AM more effectively than in the case of spatially uniform electron beam. Numeral 1 marks the curve obtained in the case of application the beam with initial spatial uniform distribution. Numeral 2 marks the curve calculated in the case of continuous beam with weakly modulated density. And the last curve marked by numeral 3 presents result obtained in the case of strongly modulated beam, which initially was composed of specially separated bunches. In the lower half-space of Figure 1, dependence of the AMs phases Θ on time is presented. One

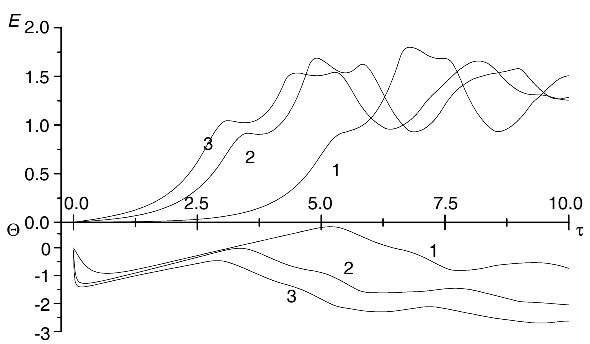

Figure 1. Variation of the amplitude of AM envelope (in upper half-space) and its phase (in lower half-space) with a time in the cases: 1 — spatially uniform beam; 2 — weakly modulated beam; 3 — stronly modulated beam; azimuthal mode number $m = 3$, $\Delta = 0.1$.

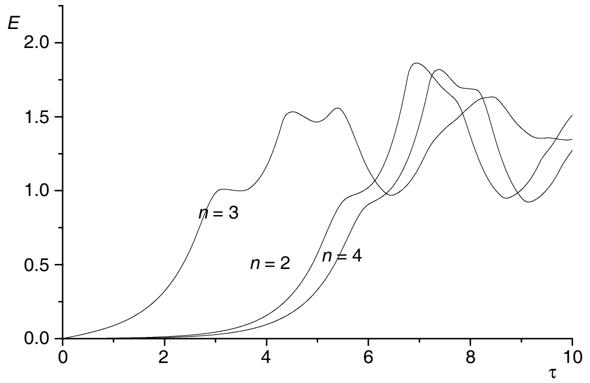

Figure 2. The same as in Figure 1, but for the cases of different meanings of the period of initial spatial modulation of the beam: $n = 2$, 3 and 4.

can see that changing of the envelope phase Θ for all presented cases correlates with changing of the corresponding AM envelop amplitudes with time.

Influence of the value of spatial modulation period of the beam electrons on development of the resonant plasma-beam instability of the AM has also been investigated. Results of this investigation are represented in Figure 2. One can see that if period n of the spatial modulation of the annular electron beam at the initial time of beam-plasma interaction coincides with the azimuthal mode number then the resonant beam instability develops more quickly. If the period n of initial spatial modulation is not equal to azimuthal mode number of the excited AM then the resonant beam instability develops during the time interval, which is approximately equal to that interval, which was typical for the case of application of spatially uniform (non-modulated) electron beam. The calculations are performed until the time, when the AM envelop amplitude starts to oscillate nearby some value. This means that instability comes into the strongly non-linear stage, where dense small-sized bunches interact with the wave, which has a finite value amplitude.

Figure 3 illustrates dynamics of spatial distribution of the beam particles in the case when weakly modulated electron beam initially is characterized by the period of its modulation $n = 4$. One can see that development of the resonant beam instability leads, at first, to the formation of three groups of beam electrons and then capturing of them into potential wells of the AM with azimuthal mode number $m = 3$. Gradually the electrons, which initially were separated into

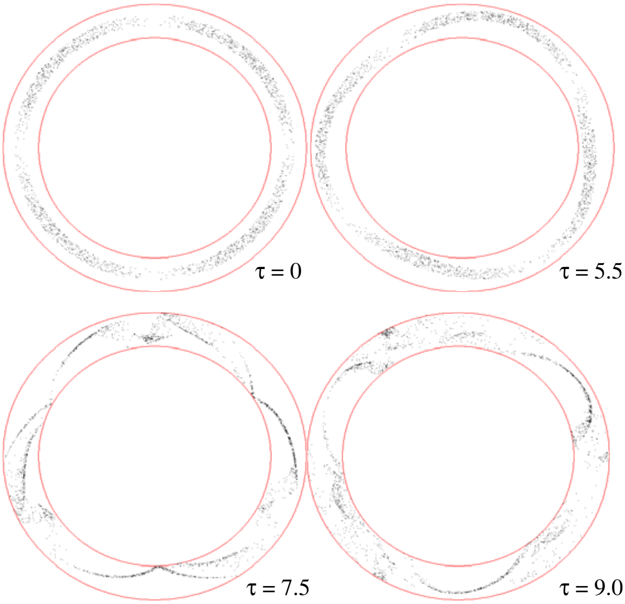

Figure 3. Variation of spatial distribution of the beams' particles with time. Parameters $m = 3$, $n = 4$, $\Delta = 0.3$. The following moments of dimensionless time were chosen $\tau = 0.0$, 5.5, 7.5 and 9.0.

four groups, start to roll down into the bottoms of three (because $m = 3$) potential wells of the AM and then three groups of the captured beams' particles start to create three dense bunches. To illustrate such development of the instability we have chosen the following moments of time: the first moment corresponds to initial value of time, the second moment corresponds to intermediate value of time interval (between the starting moment and the moment, when the amplitude of envelope reaches its first maximum), the third moment corresponds to the first maximum of the amplitude of the AM envelope and the fourth moment corresponds to its first minimum. In spite of relatively small value of the tangential component of AM field [6] as compared with magnetic component of its field, one can see that finally just three bunches have been formed, which correlates with the value of azimuthal mode number $m = 3$.

Phase portraits of the beams' particles (azimuthal impulses vs azimuthal angles) calculated during development of this instability are presented in Figure 4 for the same moments of time, which are illustrated in Figure 3. Here one can see creation of three groups of the captured beams' particles from initial four separated groups of the

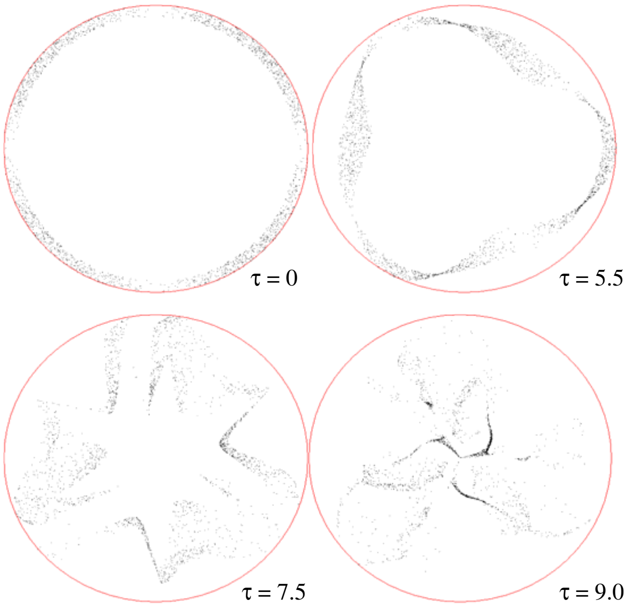

Figure 4. Phase portraits of the beams' particles: azimuthal impulse vs azimuthal angle obtained under the same conditions as it is done in Figure 3. External continuous circle represents the maximal meaning of the azimuthal impulse $u = zR_2$, at the center of the circle $u = 0$.

beam electrons as well. The presented phase portraits illustrate losses of initial energy of the electron beam. One can see that gradually azimuthal impulses of the electrons decrease (the dots, which represent the particles move to the center of the phase portrait) and values of the electrons impulses become such as located closer to the bottoms of potential wells of the AM. As well one can see that the most part of the beam electrons are gathered in relative narrow angular sectors, which quantity is equal to the value of azimuthal mode number $m = 3$.

To compare the case of $n = 4$ with the case of $n = 3$, one can look at the pictures presented in Figures 5 and 6. Distribution of the beams' particles in co-ordinate space and phase space are shown there, respectively. The co-ordinate distribution and phase portraits are presented there once again at the initial moment of time, intermediate moment of time, moment of the first maximum of the amplitude of the AM envelope and the moment of its first minimum. Comparison of the second pictures in Figures 3 and 5 allows one to conclude that reformation of the electron beam shape due to influence of the AM with azimuthal mode number $m = 3$ happens faster in the case $n = 3$

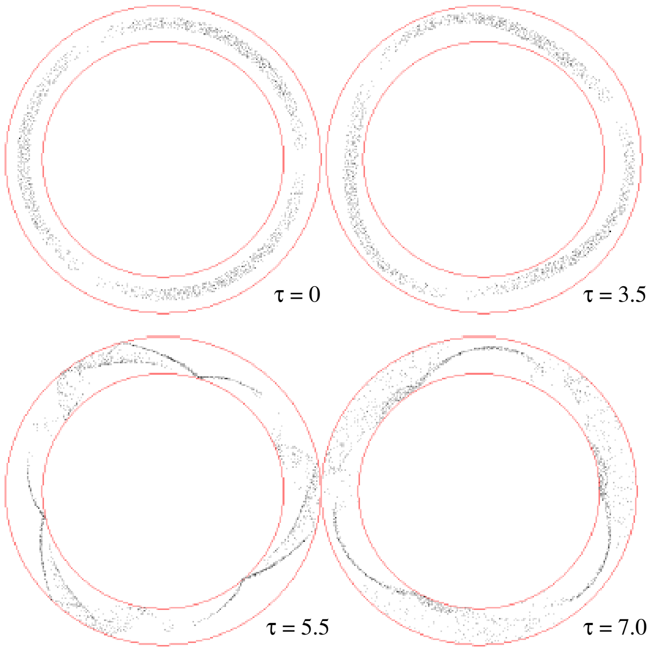

Figure 5. The same as in Figure 3, but $n = 3$, $\Delta = 0.3$.

as compared with the case $n = 4$. Process of gathering the beam electrons into three potential wells of the AM field is also happens faster in the case $n = 3$ rather than in the case $n = 4$ (see the third pictures in Figures 3 and 5). This means that the AM growth rate in the case $n = m = 3$ is larger than in the case $n = 4$. This conclusion is confirmed by results presented in Figure 2 (compare lines marked by $n = 3$ and $n = 4$).

The second conclusion concerned with distinguishing features of the AM instabilities in the cases $n = 4$ and $n = 3$ can be made after comparison the corresponding phase portraits. This peculiarity is also seen in the pictures with spatial distributions of the beams' electrons illustrated as dots, but it is less pronounced as compared with the phase spaces filled with the illustrating dots. Thus one can see that distribution of the beams' electrons in the AM field for the case $n = m = 3$ (Figure 6) is practically symmetrical. In other words, the dots in all three groups are located in the same manner. But in the case $n = 4$ (see last three pictures in Figure 4), location of the dots in these three groups is not symmetrical. This is connected with the fact that at initial stage of the resonant beam-plasma instability, the AM field has

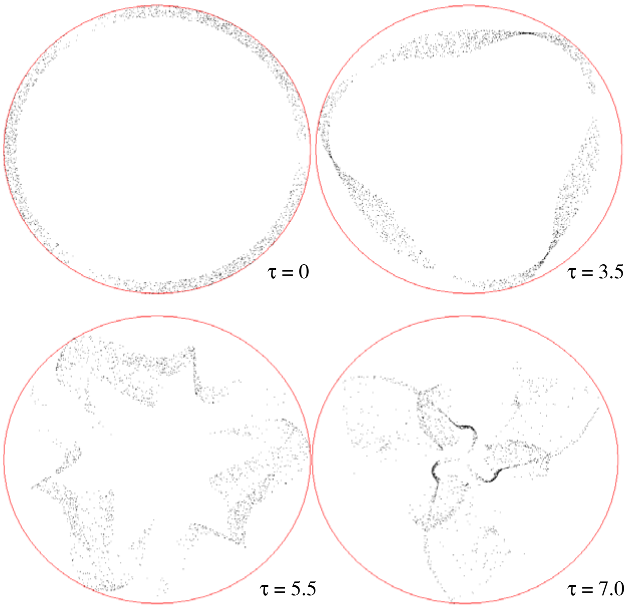

Figure 6. The same as in Figure 4, but $n = 3$, $\Delta = 0.3$.

no time and no ability to separate four initial groups of electrons into three identical groups. Thus in the case $n = 4$, one group of electrons consists of the lager quantity of the particles as compared with two other groups at the initial stage of the instability (it means until the beginning of the stage of saturation of the instability). Therefore, this dense group of electrons being in the phase synchronism with the excited AM can transfer more quantity of its kinetic energy into the wave energy as compared with the case $n = 3$. This is confirmed by results presented in Figure 2, namely maximum of the AM envelope amplitude in the case of $n = 3$ is less than that in the case of $n = 4$.

4. CONCLUSIONS

Resonant beam instability of azimuthal modes (AMs) has been studied numerically using one mode approximation. Nonlinear self-consistent set of differential equations has been derived and analyzed by Runge-Kutta method of the fourth order. Obtained results allow one to make the following conclusions:

1. Preliminary spatial modulation of the electron beam leads to increasing growth rates of the resonant beam instability, the time interval needed for the attainment of saturation stage of this instability

becomes shorter as compare with the case of uniform (non-modulated) electron beam. This is explained by absence of a necessity to perform separation of the beam electrons into such quantity of groups, which will be able to move under the condition of phase synchronism with the excited AM.

2. Correlation between the period n of initial spatial modulation of the beam electrons and value of the azimuthal mode number of the studied AM influences strongly on the development of its resonant beam instability. If the beam is initially composed of groups of electrons, whose quantity is not equal to azimuthal mode number then development of AM instability has approximately similar scenario as it was in the case of non-modulated electron beam, namely the AM first of all will separate the electrons into such quantity of groups that will be equal to the mode number.

3. Strengthening of a degree of a spatial modulation of the electron beam and decreasing of the beam thickness lead to increasing of the AMs instability growth rates. During the first time the instability is developed accordingly to prediction of the linear theory created for interaction between an annular electron beam and AMs, which can propagate in different frequency ranges [7, 17]. At the non-linear stage of the instability changing of the AM envelop amplitude is similar to the results obtained in [18] for the case of the non-modulated beam.

4. During development of the resonant beam instability the beams' electrons loss their energy for excitation of the azimuthal mode; the waves' field captures the electrons into its potential wells; these electrons roll down into the bottom of the wells and create there groups of the captured electrons with a small value of azimuthal impulse. Quantity of these groups is equal to azimuthal mode number of the excited AM. Saturation of AM instability is characterized by creation of high density bunches from the captured electrons, which are localized in a space with a narrow angular sizes.

The obtained results can be useful for investigations in the branches of plasma electronics and gas discharges in plasmas, where flute modes can be excited.

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