

USE OF JONSCHER MODEL FOR ESTIMATING THE THICKNESS OF A CONCRETE SLAB BY TECHNICAL GPR

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Abstract—The thickness measurement of concrete is one of the most important commercial applications of ground-penetrating radar (GPR) technique. This paper describes a procedure for estimating the thickness of concrete slab for different moisture contents (MCs) in frequency domain, as in Impulse-Response (IR) Method, over the radar frequency band (100 MHz–2 GHz). The method is based on predicting the reflected frequency spectrum through a concrete slab using Jonscher model. The procedure is explained and examples of results are presented.

1. INTRODUCTION

In civil engineering, pavement layer thickness is an important factor determining the quality and durability of pavements, since the deficiencies in thickness reduces the life of the pavement. It is then necessary to have an accurate and reliable method for measuring thickness. Cores that are extracted from pavements determine pavement thickness accurately, but they are time consuming. They damage the pavement and they represent a very limited sample of the actual pavement. Impact Echo (IE) is a nondestructive testing method that is used to determine concrete slab thickness. This test method is

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described in the ASTM C 1383 [1]. It's reliability is acceptable, but the test is punctual and cannot be used to cover large surface areas in short period of time [2]. The IR method, which will be discussed later in this paper present the same limitation as the IE method. The GPR technique is a thickness measuring method which is quick, non destructive and which can generate a representative population of pavement thickness data points. GPR is based on the propagation and reflection of high frequency (100 MHz–2 GHz) electromagnetic waves. The electromagnetic waves behaviour is characterised by the dielectric permittivity properties of the medium and conductivity. The modeling of these parameters can predict waves that can be captured by the GPR [3]. Also, the modeling of the expected GPR signal is a powerful tool to better understand the measured signals.

In this paper, we want to extend the basic knowledge about GPR when used as an assessment tool for concrete structures. This application in civil engineering illustrates the modeling of the radar wave reflected on a concrete slab for different MCs and its use for measuring the thickness of concrete slabs.

2. GPR

2.1. The Incident Frequency Spectrum of GPR

In practice, for real radar measurements, the initial pulse from GPR antenna is either not fully known or is very susceptible to the medium-antenna coupling conditions.

For this study and due to the fact that, the incident pulse is necessary information for any modelling, a GPR system, with an air-coupled antenna, has been chosen [4]. The system is composed of a control unit, a transceiver box, and a pair of air-coupled antennas. The antennas are TEM horns with a manufacturer reported centre frequency of 1 GHz. Since the air-coupled antennas were usually placed at a distance of 475 mm from the surface, polystyrene plastic pieces with a total thickness of 475 mm were placed on top of the slab surface. The antennas were then placed on top of the polystyrene plastic pieces as shown in Fig. 1.

The incident frequency spectrum is calculated from the data collected over a copper plate placed on the pavement surface. A fast Fourier transform (FFT) algorithm is used on the time-domain signal to achieve the input frequency spectrum F_i [4]. The incident frequency spectrum is shown in Fig. 2. Using the 3 dB criteria, a bandwidth of 0.5 GHz is found (from 0.5 to 1 GHz). The centre frequency of the nominal 1 GHz antenna has dropped to 666 MHz.

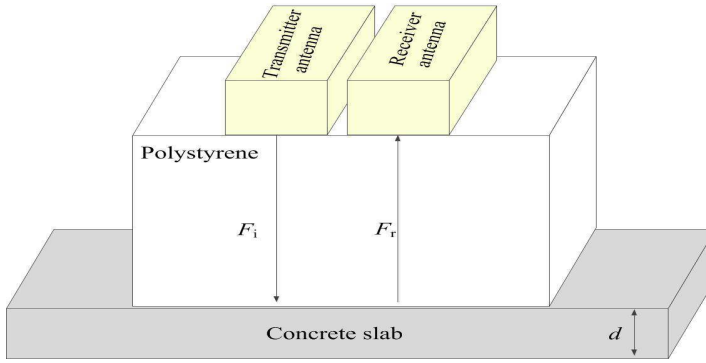


Figure 1. Air-coupled data collection from concrete slabs.

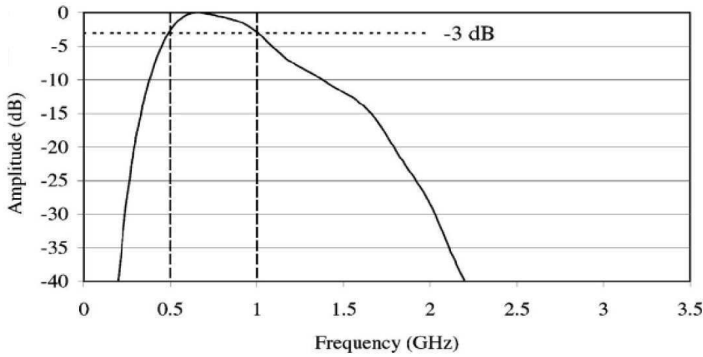


Figure 2. Incident frequency spectrum F_i [4].

2.2. Frequency Spectrum of Reflected Wave

The theory of propagation of electromagnetic waves is very complicated. The following presentation is simplified based on suitable assumptions in civil engineering applications. More detailed treatments are available in [5,6]. A model was developed by [3] to predict the radar signal over a concrete slab. The predicted frequency spectrum F_r was obtained by using an Equation (1) with only one unknown, ε_r .

$$F_r = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} - \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} \left(1 - \left(\frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} \right)^2 \right) \left(e^{-i \frac{\omega}{c} d \sqrt{\varepsilon_r}} \right)^2 F_i \quad (1)$$

where ε_r real part of the complex relative permittivity of concrete; ω angular frequency; d (m) concrete slabs thickness; and c speed of light.

3. MODELING OF THE EFFECTIVE DIELECTRIC CONSTANT OF THE CONCRETE

The dielectric permittivity of concrete is affected by moisture content and change in the frequency of the electromagnetic field applied. In [7], the three-parameter Jonscher model was applied to show the dispersive aspect of the concrete. The validation of this model was carried out through tests on mortar and concrete and this for different moisture contents, heterogeneities and porosities of concrete. Compared with other models in [3], the Jonscher model is very effective and is the most appropriate to represent the electric properties of concrete. Using the Jonscher model the effective permittivity $\varepsilon_e(\omega)$ of concrete slab was modeled with a simple formulation law (2):

$$\varepsilon_e(\omega) = \varepsilon_0 \chi_r \left(\frac{\omega}{\omega_r} \right)^{n-1} \left[1 - i \cot \left(\frac{n\pi}{2} \right) \right] + \varepsilon_\infty \quad (2)$$

where $\varepsilon_0 = 8.854\,187\,8176 \times 10^{-12} \text{ Fm}^{-1}$ is the free-space permittivity. The $\varepsilon_\infty [\text{Fm}^{-1}]$ is the limiting high-frequency value of the real part of the effective permittivity. ω_r is a reference frequency, arbitrarily chosen. n is an empirical parameter without dimension that characterizes the change in amplitude of permittivity as a function of frequency, this parameter varies between 0 for materials with high dielectric loss and 1 for materials without dielectric losses; and χ_r is the real part of the susceptibility to the frequency reference.

The three parameters n , χ_r and ε_∞ in the Jonscher model of concrete were obtained by simple expressions without using a special algorithm for calculating [7]. In this section, these parameters are calculated from the experimental results published in [8] and [9] in the respective frequency ranges 10 MHz–1 GHz and 1 MHz–2 GHz, respectively. The input data (n , χ_r , ε_∞) relative to the modeling for different MCs are given in the Table 1.

4. IMPULSE-RESPONSE METHOD PRINCIPLE

A blow on the shaft head by a small sledgehammer equipped with a load cell generates a stress wave with a wide frequency content, which can vary from 0 to 1000 Hz for soft rubber-tipped hammers to 0 to 3000 Hz for metal-tipped hammers. The load cell measures the force input, and the vertical response of the shaft head is monitored by a geophone. The force and velocity time-base signals are recorded by a digital acquisition device, and then processed by the computer using the FFT algorithm to convert the data into the frequency domain. Velocity is then divided by force to provide the unit response, or

Table 1. Fitted parameters for concrete samples.

MC (%)	Reference	Parameters for Jonscher’s model [7]		
		n	χ_r	$\varepsilon_\infty/\varepsilon_0$
12	[8]	0.62	7.49	4.39
9.3	[9]	0.54	4.77	5.75
6.2	[9]	0.60	2.99	4.94
5.5	[8]	0.74	3.87	3.75
2.8	[8]	0.74	1.49	4.45
0.2	[8]	0.80	0.60	4.11

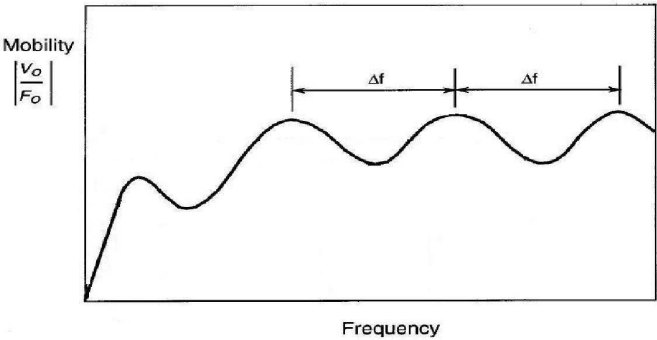


Figure 3. Theoretical mobility plot for impulse-response test of perfect pile in homogeneous soil.

transfer function, which is displayed as a graph of shaft mobility versus frequency [10, 11]. An example of a mobility plot for a pile shaft is given in Fig. 3.

When a perfect, laterally unrestrained (free) pile of length L resting on the surface of an elastic foundation is excited by a continuous sinusoidal axial force with peak value F_0 , the pile head achieves a maximum velocity V_0 . As the frequency of the applied force varies, the amplitude of the pile head velocity gets to the peak values at equally spaced frequencies (resonant frequencies). The frequency interval, Δf , between the peaks is given by (3).

$$\Delta f = \frac{C_b}{2L} \tag{3}$$

where C_b is the speed of stress-wave propagation along the pile axis. This method gives a very good results in terms the thickness measurement of concrete elements (such as a slab resting on the

ground). However, its ability to detect discontinuities in thin layers of pavement was not unanimously accepted. In fact, the test is relatively simple and requires access to only one side of the material investigated. However, the frequencies generated in practice hardly exceed 3 kHz, which leads to a low resolution. Moreover, the applications of this method to large structures are not economically viable because the tests are occasional and sometimes laborious.

5. ESTIMATED THICKNESS OF CONCRETE

Before exploring the measurement of the thickness of the concrete slab in the frequency domain, we study the effect of the change in MC and thickness of the slabs on the frequency spectrum of the reflected signal on a concrete slab.

5.1. Effect of the Change in MC and Thickness

The synthesis of air-coupled GPR waveforms for concrete slabs with different MCs was performed for $MC = 0.2\%$ and 12% . Fig. 4 shows these different waveforms for different thickness of the slab concrete (5 cm to 30 cm).

For small thicknesses of slabs (e.g., 5 cm at $MC = 0.2\%$), the amplitude of the reflected signal varies linearly on frequency intervals ($[0.3 \text{ } 0.7]$ GHz with a positive slope and $[0.7 \text{ } 1.4]$ GHz with a negative slope). For larger thicknesses of slabs, the reflected signal has a periodicity depending on the frequency. Minima and maxima correspond respectively to the destructive and constructive interference between the first reflection and secondary reflections. Minima and maxima of the amplitude of the reflected signal is very pronounced for concrete slabs with low dielectric loss as $MC = 0.2\%$ (the minimum tend to zero). However, these extremes are faintly visible for concrete slabs at high dielectric losses as $MC = 12\%$.

5.2. Measuring of the Thickness of a Slab of Concrete

The variation of permittivity with frequency in concrete implies that there will be some variation in the velocity of propagation with frequency. By cons, in most practical trial situations the relative permittivity will be unknown. The velocity of propagation must be measured in situ, estimated by means of direct measurement of the depth to a physical interface or target (i.e., by trial holing), or by calculation by means of multiple measurements. In our case, Jonscher model provides the variation of epsilon throughout the range of radar

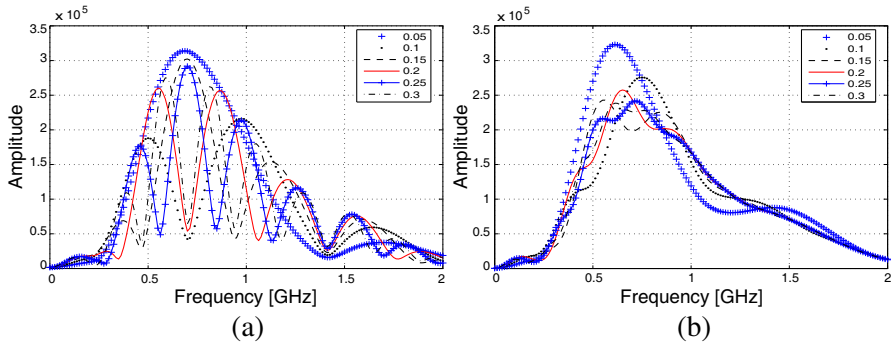


Figure 4. Frequency spectra of the reflected signal on a concrete slab for (a) $MC = 0.2\%$ and (b) $MC = 12\%$.

frequencies. However, for calculating the velocity of propagation of the electromagnetic wave in the concrete, an average complex value of permittivity provided accurate predictions of the air-coupled GPR system [4].

Using concrete slab with $MC = 0.2\%$, the average dielectric constant for concrete, over the frequency band of the radar system, would be 4.58, with a maximum of 4.95 and a minimum of 4.49. The wave velocity can be calculated from (4).

$$v = \frac{c}{\sqrt{\varepsilon_r}} \quad (4)$$

v is therefore equal to: 0.14 m/ns, which is a reasonable value for dry concrete. On the other hand, when the surveyed slab concrete is composed of relatively thick layer, the GPR reflected pulses would have more possibility to give the best results. This condition makes detecting the layer interface reflection easier than in the case of thin layers. A pavement layer could be considered as thin or thick depending on whether its thickness is smaller or larger than the GPR depth resolution Δd , which is given in the following Equation (5) [12]

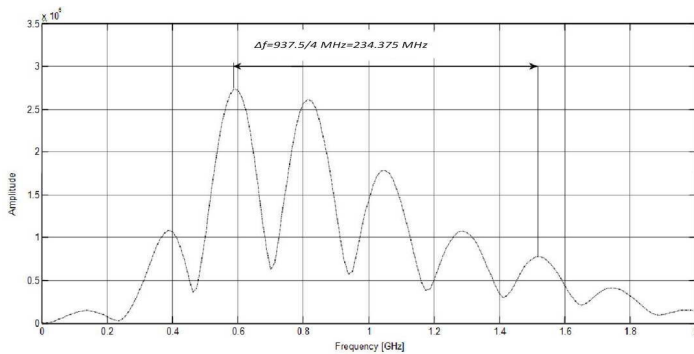
$$\Delta d = \frac{cT}{2\sqrt{\varepsilon_r'}} \quad (5)$$

where T is the incident pulse width and ε_r' the dielectric constant of the considered layer. In our case, the incident pulse width is approximately 1 ns.

In Table 2, the values for velocity of propagation and resolution obtained for different MCs are summarized.

Table 2. Velocity of propagation and resolution for concrete samples.

MC (%)	average dielectric constant	velocity (m/ns)	Resolution (m)
0.2	4.5838	0.1401227	0.07006134
5.5	6.5973	0.1167987	0.05839937
6.2	6.5805	0.1169477	0.05847387
12	9.0847	0.0995327	0.04976637

**Figure 5.** Amplitude plot for $d = 0.3$ m and 0.2%.

The thickness, d , of the concrete can be derived as in IR method (3), from the following Equation (6):

$$d = \frac{v}{2f} \quad (6)$$

where v is the wave velocity in concrete and f , is equal to the fundamental frequency of the reflected signal spectrum.

To illustrate how the method works, reflected signal spectra obtained from concrete is shown in Fig. 5. At frequency values corresponding to resonant frequencies of the wave in structure of concrete, amplitude values are maximums. The series of peaks corresponds to the fundamental; the difference between any two adjacent, Δf , is equal to the fundamental frequency.

$$\Delta f = 937.5/4 \text{ MHz} = 234.375 \text{ MHz} \quad (7)$$

The fundamental frequency of 234.375 MHz was calculated by determining the average frequency difference between four successive

Table 3. Estimated thickness for concrete samples with different MCs.

	MC = 0.2%	MC = 5.5%	MC = 6.2%	MC = 12%
Known thickness (m)	Estimated thickness (m)	Estimated thickness (m)	Estimated thickness (m)	Estimated thickness (m)
0.05	0.0700	0.0649	0.0668	0.0622
0.1	0.1231	0.0954	0.0995	0.1327
0.15	0.1716	0.1483	0.1508	0.1731
0.2	0.2050	0.2001	0.2034	0.1990
0.25	0.2548	0.2503	0.2487	0.3063
0.3	0.2989	0.3014	0.3119	0.3318

Table 4. RMSE between the real and estimated thickness for different MCs.

	MC = 0.2%	MC = 5.5%	MC = 6.2%	MC = 12%
RMSE	0.0155	0.0064	0.0085	0.0314

peaks. The thickness, d , of the concrete can be calculated using (6) and (7): $d = 0.2992$ m. The thickness of concrete was calculated to be 29.92 cm. The known thickness of concrete was 30 cm (as seen in Table 3).

Note the minima and maxima which correspond to destructive interferences (where $f = v/2d = 230, 460, 690$) and constructive interferences (where $f = v/4d = 115, 345, 575, 805$).

This method gives very good results for thin layers concrete slab [0.1 m–0.3 m], especially when MC is low [0.2%–6.2%]. But, this experiment shows that it is sensibly influenced by the high MC [12%] and very thin layer slab of concrete [0.05 m].

The root-mean squared error (RMSE) is then computed for all slabs, and the calculation of the RMSE between the real and estimated thickness are summarized in Table 4.

The maximum RMSE between the real and estimated thickness for different MCs obtained with the modeling procedure was 0.0314. For all slabs, the estimated thicknesses were very close to the real thicknesses.

6. CONCLUSION

The proposed method aims to show the interest of the GPR method to measure the thickness of a concrete slab based on processing signal frequency similar to that used in IR method. In particular, it was shown that when the concrete slab is composed of thick layer (relative to the incident GPR pulse width), this technique yields best thickness results. This was possible by using the Jonscher model to reproduce the dispersion of the dielectric permittivity of concrete, which was then used in the GPR frequency range to obtain the radar wave reflected from concrete slab. These results indicate that the frequency-domain approach, which is the less common approach used with GPR method, is a reliable means of estimating the slab layer thickness of concrete. In this study we considered concrete as a homogeneous material and did not take into account the size of heterogeneities composing the concrete. But this assumption may not be true for all types of concretes, particularly those where the aggregate size is larger than the electric wavelength. Moreover, this method could be extended to include more layers and could be used to model the reflected signal by defects in concrete. But it must be validated for such cases. A comparison with the most effective techniques available commonly used for thickness measurements in civil engineering could be also very interesting for to show the effectiveness of this method.

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