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ELECTROMAGNETIC WAVES RADIATION INTO THE SPACE OVER A SPHERE BY A SLOT IN THE END-WALL OF A SEMI-INFINITE RECTANGULAR WAVEGUIDE

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Abstract—The problem of electromagnetic waves radiation into a space outside a perfectly conducting sphere through a narrow slot, cut in an end-wall of a semi-infinite rectangular waveguide, excited by a fundamental wave of H_{10} type is solved using a rigorous self-consistent formulation. The starting point for the analysis is the one-dimensional integral equation for the equivalent magnetic current in the slot, obtained by using the effective thickness of the slot. The asymptotic solution of the equation was found by the generalized method of induced magnetomotive forces (MMF). The physical adequacy of the constructed mathematical model to the real physical process is confirmed by experimental data. Influence of the sphere radius upon energy characteristics of the slot radiator was investigated numerically. It was shown that at any frequency of waveguide single-mode range, the radiation coefficient of a spherical antenna can be made close to one by choosing the slot length, the sphere radius and the waveguide height. Conditions for correct application of infinite screen approximation for spherical scatterers with sufficiently large radii are formulated.

1. INTRODUCTION

Non protruding slot antennas (NPSA) are widely used on mobile objects since slot antennas make no substantial contribution in object's weight and size, and do not deteriorate its aerodynamic properties [1]. NPSA are used in very wide range of applications extending from spacecrafts [2] to autonomous microdevices [3]. Surface of a mobile objects or its constructive part is usually modeled by a spherical surface

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with a radius is comparable to the antenna's operating wavelength. Therefore, research of resonant slot spherical antennas has continued during antenna the last several decades.

Characteristics of zonal and azimuthal narrow slots located on spherical scatterers have been studied in [4–13]. Axisymmetric excitation of slot antennas is assumed in most of the above references. We will not do any comparative analysis of problem formulation and solution methods, but only note that spherical antennas with impedance surfaces are studied in [11, 13] along with perfectly conducting spherical slot antennas. Plane wave diffraction by a hollow conductive shell with an annular slot or a circular aperture, have been studied in [14, 15]. Characteristics of spherical antennas with slotted rectangular radiators have been investigated in [6, 16–18]. External electrodynamic characteristics of slot antennas under the assumption of a given cosine magnetic current distribution along a half-wave radiator with a narrow slot has been examined in [6, 18]. Thus, far zone radiation patterns in the equatorial plane for spherical antennas were obtained in [6]. Intrinsic and mutual admittances of half-wave slot radiators, oriented along parallels on a surface of a conducting sphere, were studied in [18].

Characteristics of narrow rectangular slots, cut in an infinitely thin spherical shell, were analyzed in [16, 17] by a method of moments. A spherical antenna in which a slot couples free space outside of a spherical scatterer and an inner antenna region in a form of hollow spherical cavity was solved in the first paper [16], and an analogues configuration with a spherical cavity containing a conducting sphere of smaller radius concentrically nested in it [17]. In both cases a slot element was excited by a hypothetical point voltage generator. In the present paper we will consider a real design of a spherical antenna, where a power to the rectangular slot located on the sphere is fed through the inner section of a rectangular waveguide transmission line. This electrodynamic problem is of direct practical interest and presents a new approach to the analysis of spherical antennas.

In terms of waveguide electrodynamics, this problem can be reformulated as an excitation problem of a slotted waveguide radiator with a conducting spherical flange. As far as we know a solution to this problem using a rigorous electrodynamic formulation does not exist. Usually, flanges in slot antennas modes are considered as flat perfectly conducting screens [1, 19]. Therefore, one of the aims of this study is to determine the conditions for correct application of infinite screen approximation for spherical scatterers in models of slotted waveguide radiators.

The authors of this paper have proposed and justified [19] a numerical-analytical method, named as generalized method of MMF, intended for diffraction problem at slotted elements. The problem of electromagnetic fields excitation by a coupling hole between two arbitrary electrodynamic volumes was preliminarily solved in a rigorous self-consistent formulation. This problem was reduced to a two-dimensional integral equation for equivalent surface magnetic currents defined on the coupling aperture. A physically correct transition from the resulting integral equation to a one-dimensional equation for the current in the narrow slot was justified. A general method for solving one-dimensional equations for slotted waveguide structures with a flat conductive flange was presented. A development of this method for solving the problem of electromagnetic wave radiation by a slotted waveguide structure into the space above a perfectly conducting sphere will be presented.

2. PROBLEM FORMULATION AND INTEGRAL EQUATION SOLUTION

Let a fundamental wave H_{10} propagates in a hollow semi-infinite rectangular waveguide with perfectly conducting walls (index V^i) from $z = \infty$ (Figure 1). The waveguide cross-section is $\{a \times b\}$. A Cartesian coordinate system $\{x, y, z\}$, related to the waveguide, is shown in Figure 1(b). A narrow transverse slot is cut in the waveguide end-wall symmetrically relative to the waveguide's longitudinal axis ($x_0 = a/2$). The width of slot aperture S_i is d and its length is $2L_i$ ($[d/(2L_i)] \ll 1$, $[d/\lambda] \ll 1$, λ is free space wavelength). The slot radiates into the free space outside a perfectly conducting sphere whose radius is R (index V^e). A spherical coordinate system, associated with the spherical scatterer (flange) is shown in Figure 1(a). The geometric center of the slotted element in the Cartesian coordinate system is defined by coordinates $(a/2, y_0, 0)$, the center coordinates of external aperture S_e in the spherical coordinate system $\{r, \theta, \varphi\}$ are $(R, \pi/2, 0)$. The length of slot aperture, S_e measured along the arc is $2L_e$. The tunnelling slot cavity is an area (index V^v), bounded between apertures S_i and, S_e represents a complex shape resonator, whose boundaries could not be described in either coordinate system (Figure 1(c)). This peculiarity of the electrodynamic problem defines the principal difficulty for its analytical analysis. Moreover, the cavity volume may depend upon the sphere radius and waveguide cross-sectional dimensions, since these geometrical parameters determine its resonant properties.

The system of equations for the spherical antenna under consideration can be formulated by considering continuity conditions for tangential components of the magnetic fields on the inner and outer

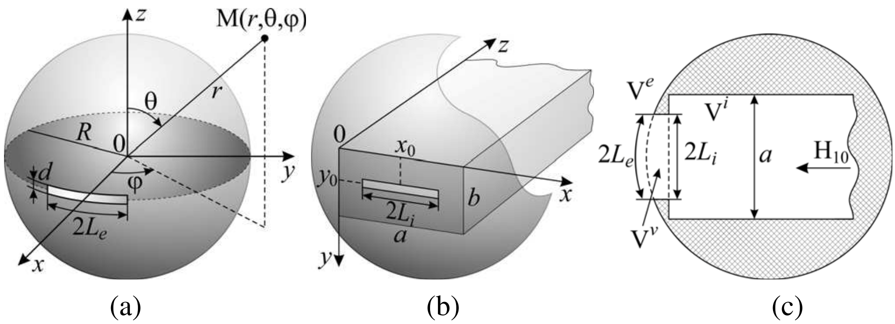

Figure 1. The local coordinate systems used for analysis of the spherical antenna.

slot apertures [19]

$$\begin{cases} \text{for } S_i: \quad \vec{H}^i_\tau\left(\vec{e}_{si}\right)+\vec{H}^i_{0\tau}=\vec{H}^v_\tau\left(\vec{e}_{si}\right)+\vec{H}^v_\tau\left(\vec{e}_{se}\right), \\ \text{for } S_e: \quad \vec{H}^v_\tau\left(\vec{e}_{si}\right)+\vec{H}^v_\tau\left(\vec{e}_{se}\right)=\vec{H}^e_\tau\left(\vec{e}_{se}\right), \end{cases} \tag{1}$$

where $\vec{e}_{si}$, $\vec{e}_{se}$ are the respective electric fields on the surfaces S_i and S_e, $\vec{H}^i_\tau(\vec{e}_{si})$, $\vec{H}^e_\tau(\vec{e}_{se})$, $\vec{H}^v_\tau(\vec{e}_{si})$, $\vec{H}^v_\tau(\vec{e}_{se})$ are tangential components of magnetic fields (with respect to the slot aperture), excited by fields $\vec{e}_{si}$, $\vec{e}_{se}$, in the corresponding electrodynamic volumes, and $\vec{H}^i_{0\tau}$ is the component of the extraneous magnetic field in the waveguide. The Equation (1) allows us, in principle, to use representations of magnetic fields in arbitrary local coordinate systems for each of the coupling volumes.

A rigorous mathematical substantiation, concerning reduction of simultaneous Equation (1) to a single equation

$$\vec{H}^i_\tau(\vec{e}_{si})+\vec{H}^i_{0\tau}=\vec{H}^e_\tau(\vec{e}_{se}), \tag{2}$$

which does not contain fields, defined in the slot cavity V^v can be found in [20]. This substantiation was performed for a coupling problem involving two arbitrary electrodynamic volumes through a narrow rectangular slot under the condition that the volume V^v is a rectangular parallelepiped with dimensions $2L_i \times d \times h$. In [20] it was also shown that under the condition $(hd/\lambda^2) \ll 1$ solutions for a slot, cut in the wall of finite thickness $h((h/\lambda) \ll 1)$, and for a slot cut in an infinitely thin wall are approximately equal, if the actual slot width d is replaced by d_e. The explicit formulas for d_e can be found in [19, 20]: $d_e = d(1 - \frac{h}{\pi d} \ln \frac{d}{h})$ at $\frac{h}{d} \ll 1$, $d_e = d(\frac{8}{\pi} e^{-(\frac{\pi h}{2d}+1)})$ at $\frac{h}{d} \geq 1$. This approach has been tested in the analysis of different slot problems, including that made by the authors of this paper [19, 21].

The concept of equivalent slot width was introduced in [20] for narrow rectangular slots, cut in flat regions of shielding surfaces. It was also concluded that, on the basis of physical principles, this concept can be valid for curved narrow slots, cut in an arbitrary smooth surface, if the radii of curvature of slot axial line and surface principal radii of curvature are much larger than d and h. The same arguments are relevant for the problem, considered here, when the slot cavity V^v is part of a rectangular parallelepiped in which one face is subjected to small perturbations which become infinitely small for large sphere radii. However, it should be understood that the exact formula for equivalent slot width d_e, mentioned in [19, 20] is approximate and its value may be corrected in the course of numerical calculations.

Thus, using a rigorous definition, the problem for the slotted spherical antenna will be solved using the Equation (2). To ensure mathematical correctness of the problem solution, we rewrite this equation in a system of generalized coordinates (ξ_1, ξ_2, ξ_3) and require that it will be satisfied in some imaginary cross-section S, virtually located in the vicinity of aperture S_e in the slot plane V^v. Without loss of generality we can assume that the unit vector of coordinate ξ_1, i.e., $\vec{\xi}_1^0$, coincides with the longitudinal axis of the cross-section, and the unit vector $\vec{\xi}_2^0$ is directed along its transverse axis. Multiplying both sides of Equation (2) by a prescribed scalar function $\psi(\xi_1)$, and integrating the result over the cross-section S, we get

$$\int_S \vec{H}_\tau^i(\vec{e}_s)\psi(\xi_1)\,ds + \int_S \vec{H}_{0\tau}^i\psi(\xi_1)\,ds = \int_S \vec{H}_\tau^e(\vec{e}_s)\psi(\xi_1)\,ds. \tag{3}$$

Since the slot cavity is assumed to be narrow, the field, constant in the direction of $\vec{\xi}_2^0$, in the cross section S can be written as $\vec{e}_s = I_0 f(\xi_1')\vec{\xi}_2^0$ [19]. Here (ξ_1', ξ_2', ξ_3') are the source coordinates, $f(\xi_1')$ is an unknown scalar function and I_0 is its complex amplitude.

Let the magnetic fields in coupled volumes V^e and V^i be presented in two different coordinate systems. To approximate a virtual cross-section S in each volume we consider two coordinate surfaces, S_e and S_i, respectively. Then, performing a formal summation operation we introduce presentations for perturbed surfaces $S_e = S + \Delta s_e$ and $S_i = S + \Delta s_i$, and rewrite Equation (3) as

$$\begin{aligned} &I_0 \int_{S+\Delta s_e} \vec{H}_\tau^e\left(f(\xi_1')\vec{\xi}_2^0\right)\psi(\xi_1)\,ds - I_0 \int_{S+\Delta s_i} \vec{H}_\tau^i\left(f(\xi_1')\vec{\xi}_2^0\right)\psi(\xi_1)\,ds \\ &= \int_{S+\Delta s_i} \vec{H}_{0\tau}^i\psi(\xi_1)\,ds, \end{aligned} \tag{4}$$

where $\Delta s_{e(i)}$ denotes symbols of small perturbations. Equation (4) is approximate and becomes exact only if $\Delta s_{e(i)} \to 0$. The physical meaning of the Equation (4) for small perturbations $\Delta s_{e(i)}$ can be easier to understand by comparing it with the classical problem of thin wire vibrator excitation, where electric current in a vibrator is assumed to be concentrated near its longitudinal axis, and the boundary conditions for the electromagnetic field must hold on its generating line [1, 22]. Similarly, in (4), the surface S can be regarded as a cross-section, where secondary magnetic current, equivalent to the field $\vec{e}_s$ is concentrated and boundary conditions for the fields are to be satisfied at S_i and S_e.

Since the Equation (4) is approximate, the correct relationship between the field amplitude $\vec{e}_{se}$ at the aperture S_e, which determines the total power, radiated by the slot, and the field amplitude $\vec{e}_{si}$ at aperture S_i, determining the slot matching with the waveguide will be inevitably violated. Therefore, the value of I_0 may be selected only so that either one or the other power be close to its true value. However, in this case, the balance between the input and the radiated power may be violated. By analogy with the method of induced electromotive force in theory of dipole antennas [22], we conclude that $\psi(\xi_1) = f(\xi_1)$. The fulfillment of this condition allows us to find the amplitude I_0, ensuring balance of powers, but both powers can be determined only with some error.

Application of local coordinate systems in (4) requires that the magnetic fields $\vec{H}^e_\tau(f(\xi_1)\vec{\xi}^0_2)$, $\vec{H}^i_\tau(f(\xi_1)\vec{\xi}^0_2)$ be presented in these coordinates too, and thereby disturb the surface S on which the secondary sources of these excitation fields are concentrated as it was assumed above. Then the magnetic fields will be determined by integral-differential operators where integration should be carried out over $S_e = S + \Delta s_e$ and $S_i = S + \Delta s_i$ instead of S as required in Equation (3). If $f(\xi_1)$ is a functional dependence of the exact solution of the Equation (2) at cross-section S, it will at the same time present an approximate solution of Equation (4) with a sufficiently small error. As is known from the general antenna theory, small errors in the determination of the current distribution does not lead to significant errors in the calculation of the integral values such as the excited electromagnetic fields. Therefore, if inequality $|\Delta s_{e(i)}| \ll \lambda^2$ holds, and equity of basis $f(\xi_1)$ and weight $\psi(\xi_1)$ functions is assumed, application of Equation (4) instead of (3) will not violate the correctness of the electrodynamic problem solution.

The assertions, listed above, may serve as a justification for the application of generalized method of induced MMF, proposed and tested for solving slotted-waveguide problems [19], in analysis of spherical antennas. As a starting point, we will use the Equation (4), and write it in the chosen local coordinate system (Figure 1) as

$$I_0 \int\limits_{-L_e/R}^{L_e/R} \int\limits_{\theta_0-\frac{d_e}{2R}}^{\theta_0+\frac{d_e}{2R}} \vec{H}_\tau^e \left(f\left(\varphi'\right)\vec{\theta}^0\right)\psi\left(\varphi\right)d\theta d\varphi$$

$$-I_0 \int\limits_{x_0-L_i}^{x_0+L_i} \int\limits_{y_0-d_e/2}^{y_0+d_e/2} \vec{H}_\tau^i \left(f\left(x'\right)\vec{y}^0\right)\psi\left(x\right)dydx = \int\limits_{x_0-L_i}^{x_0+L_i} \int\limits_{y_0-d_e/2}^{y_0+d_e/2} \vec{H}_{0\tau}^i \psi\left(x\right)dydx. \quad (5)$$

Here we use the parameter d_e instead of d and take into account that $\vec{e}_s = \vec{\theta}^0 \frac{I_0}{d_e}\delta(r'-R)f\left(\varphi'\right)$ in the spherical coordinate system and $\vec{e}_s = \vec{y}_0 \frac{I_0}{d_e}\delta(z')f\left(x'\right)$ in the rectangular coordinate system since the field $\vec{e}_s$ is constant in the transverse direction of the slot and the parameters $\theta_0 = \pi/2$ and $x_0 = a/2$ are fixed in the problem definition.

In accordance with generalized method of induced MMF we use, as the basic functions, the functional dependence, obtained in analytical solution of Equation (3) by the asymptotic averaging method for the key problem of electromagnetic wave radiation through a slot in the end wall of a semi-infinite rectangular waveguide into the half-space over a perfectly conducting plane [21]. Then, $f(\varphi) = \frac{1}{d_e}[\cos(kR\varphi)\cos\frac{\pi}{a}L - \cos kL\cos\frac{\pi R\varphi}{a}]$ in the spherical coordinate system and $f(x) = \frac{1}{d_e}[\cos k(x-\frac{a}{2})\cos\frac{\pi}{a}L - \cos kL\cos\frac{\pi}{a}(x-\frac{a}{2})]$ in the rectangular coordinate system. Here $k = \omega\sqrt{\varepsilon\mu} = 2\pi/\lambda$ is wave number, (ε, μ) are the permittivity and permeability of free space, ω is the angular frequency and time t dependence of electromagnetic fields was selected as $e^{i\omega t}$.

The complex current amplitude I_0 can be found from Equation (5) as

$$I_0 = F_0^m \big/ \left(Y^e + Y^i\right) \quad (6)$$

where the MMF is defined by the formula

$$F_0^m = \int\limits_{a/2-L_i}^{a/2+L_i} \int\limits_{y_0-d_e/2}^{y_0+d_e/2} H_{0x}^i f\left(x\right)dydx, \quad (7)$$

and slot admittances in the corresponding electrodynamic volumes by

$$
\begin{aligned}
Y^e &= \int\limits_{-L_e/R}^{L_e/R} \int\limits_{\frac{\pi}{2}-\frac{d_e}{2R}}^{\frac{\pi}{2}+\frac{d_e}{2R}} H_\varphi^e \left(f\left(\varphi'\right) \vec{\theta}^0 \right) f\left(\varphi\right) d\theta d\varphi, \\
Y^i &= -\int\limits_{a/2-L_i}^{a/2+L_i} \int\limits_{y_0-d_e/2}^{y_0+d_e/2} H_x^i \left(f\left(x'\right) \vec{y}^0 \right) f\left(x\right) dy dx.
\end{aligned} \tag{8}
$$

To find the explicit form of integrals in (7) and (8) we replace the electric field in the slot by the equivalent one-dimensional magnetic currents $\vec{J}^m_{se(si)} = -[\vec{n}, \vec{e}_{se(si)}]$ where $\vec{n}$ is the unit vector to the surface normal directed inside the area where the field is defined. Such an approach is often used to solve slot problems. It is well known that such substitution under conditional metallization of slot apertures allows us to use the Green's functions for volumes with integral boundary surfaces. Then we take into account the well-known formulas $\vec{H}^{e(i)}(\vec{r}) = (\text{graddiv} + k^2)\overrightarrow{\prod}^m_{e(i)}(\vec{r})$ and $\overrightarrow{\prod}^m_{e(i)}(\vec{r}) = \frac{1}{4\pi i\omega\mu} \int\limits_{S_{e(i)}} \hat{G}^{e(i)}(\vec{r}, \vec{r}') \vec{J}^m_{se(si)}(\vec{r}')dr'$, where $\overrightarrow{\prod}^m_{e(i)}(\vec{r})$ are magnetic Hertz vectors, $\vec{H}^{e(i)}(\vec{r})$ are magnetic field vectors, $\vec{J}^m_{se(si)}(\vec{r}')$ are magnetic currents, $\hat{G}^{e(i)}(\vec{r}, \vec{r}')$ are tensor Green's functions of magnetic type for respective volumes, and $\vec{r}$ and $\vec{r}'$ are position vectors of observation and source points, respectively.

We will find the slot admittance Y^e for the external slot by application of the Green's tensor components, constructed for space outside a perfectly conducting sphere [13]. At first we define the components of the magnetic Hertz vector

$$
\begin{aligned}
\Pi^m_{e\theta}(r,\theta,\varphi) =& \frac{4\pi R}{i\omega d_e} \sum_{n=0}^{\infty} \sum_{m=1}^{n} \frac{m Q_n(r)\, F S_m(\varphi)}{n(n+1)C_{nm}} \\
& \times \left[\frac{dP_n^m(\cos\theta)}{d\theta} F_n^m + \frac{P_n^m(\cos\theta)}{\sin\theta} \Phi_n^m \right], \\
\Pi^m_{e\varphi}(r,\theta,\varphi) =& \frac{2\pi R}{i\omega d_e} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{\varepsilon_m Q_n(r)\, F C_m(\varphi)}{n(n+1)C_{nm}} \\
& \times \left[m^2 \frac{P_n^m(\cos\theta)}{\sin\theta} F_n^m + \frac{dP_n^m(\cos\theta)}{d\theta} \Phi_n^m \right],
\end{aligned} \tag{9}
$$

where

$$Q_n(r) = \frac{h_n^{(2)}(kr)}{(n+1)h_n^{(2)}(kR) - kR\, h_{n+1}^{(2)}(kR)}, \quad F_n^m = \int\limits_{\frac{\pi}{2}-\frac{d_e}{2R}}^{\frac{\pi}{2}+\frac{d_e}{2R}} P_n^m\left(\cos\theta'\right) d\theta',$$

$$\Phi_n^m = \int\limits_{\frac{\pi}{2}-\frac{d_e}{2R}}^{\frac{\pi}{2}+\frac{d_e}{2R}} \frac{dP_n^m(\cos\theta')}{d\theta'} \sin\theta'\, d\theta',$$

$$FC_m(\varphi) = \int\limits_{-\frac{L}{R}}^{\frac{L}{R}} f\left(\varphi'\right) \cos\left(m(\varphi-\varphi')\right) d\varphi',$$

$$FS_m(\varphi) = \int\limits_{-\frac{L}{R}}^{\frac{L}{R}} f\left(\varphi'\right) \sin\left(m(\varphi-\varphi')\right) d\varphi', \quad C_{nm} = \frac{2\pi\ (n+m)\,!}{(2n+1)\ (n-m)!},$$

$\varepsilon_m = \left\{ \begin{array}{l} 1,\ m=0, \\ 2,\ m\neq 0, \end{array} \right.$ $P_n^m(\cos\theta)$ are associated Legendre functions of the first kind, $h_n^{(2)}(kr) = \sqrt{\frac{\pi}{2kr}} J_{n+1/2}(kr) - i\sqrt{\frac{\pi}{2kr}} N_{n+1/2}(kr) = \sqrt{\frac{\pi}{2kr}} H_{n+1/2}^{(2)}(kr)$ is the spherical Hankel function of the second kind, $J_{n+1/2}(kr)$ is Bessel and $N_{n+1/2}(kr)$ is Neumann functions of half-integer index.

Then we utilize the expression for $H_\varphi(r,\theta,\varphi)$ in the form

$$H_\varphi(r,\theta,\varphi) = \frac{2\pi R}{i\omega d_e} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{Q_n(r) FC_m(\varphi)}{n(n+1)C_{nm}} \left\{ \varepsilon_m k^2 \frac{dP_n^m(\cos\theta)}{d\theta} \Phi_n^m \right.$$
$$\left. -2m^2 \left[\frac{1}{r^2} n(n+1) - k^2 \right] \frac{P_n^m(\cos\theta)}{\sin\theta} F_n^m \right\}$$

and obtain the slot external admittance in the form

$$Y^e = \frac{8R}{i\omega d_e^2} \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \times \frac{1}{(n+1) - kRh_{n+1}^{(2)}(kR)/h_n^{(2)}(kR)} \times \left\{ k^2R^2 \tilde{C}_0^2 \left(A_n^0\right)^2 \right.$$
$$\left. -2\sum_{m=1}^{n} \tilde{C}_m^2 \left[m^2 \left(n(n+1) - k^2R^2\right) \left(B_n^m\right)^2 - k^2R^2 \left(A_n^m\right)^2 \right] \right\}. \tag{10}$$

Here

$$A_n^m = \int\limits_{\frac{\pi}{2}-\frac{d_e}{2R}}^{\frac{\pi}{2}+\frac{d_e}{2R}} \frac{d\bar{P}_n^m(\cos\theta)}{d\theta}\sin\theta\, d\theta$$

$$\approx \bar{P}_n^m\left[\cos\left(\frac{\pi}{2}+\frac{d_e}{2R}\right)\right] - \bar{P}_n^m\left[\cos\left(\frac{\pi}{2}-\frac{d_e}{2R}\right)\right],$$

$$B_n^m = \int\limits_{\frac{\pi}{2}-\frac{d_e}{2R}}^{\frac{\pi}{2}+\frac{d_e}{2R}} \bar{P}_n^m(\cos\theta)\, d\theta,$$

$$\tilde{C}_m = \frac{\cos(\pi L/a)}{m^2-(kR)^2}\left[m\sin\frac{mL}{R}\cos kL - kR\cos\frac{mL}{R}\sin kL\right]$$

$$-\frac{\cos kL}{m^2-(\pi R/a)^2}\left[m\sin\frac{mL}{R}\cos\frac{\pi L}{a} - \frac{\pi R}{a}\cos\frac{mL}{R}\sin\frac{\pi L}{a}\right] = \tilde{C}_m^I - \tilde{C}_m^{II}.$$

It is significant that $\lim\limits_{m\to kR}\tilde{C}_m^I = (\frac{L}{2R}+\frac{\sin(2kL)}{4kR})\cos\frac{\pi L}{a}$, $\lim\limits_{m\to\pi R/a}\tilde{C}_m^{II} = (\frac{L}{2R}+\frac{\sin(2\pi L/a)}{4\pi R/a})\cos kL$, and normalized associated Legendre functions are $\bar{P}_n^m(\cos\theta) = \sqrt{\pi/C_{nm}}P_n^m(\cos\theta)$.

Inside the semi-infinite waveguide we will use expression for the component of the Green's function $\hat{G}^i(\vec{r},\vec{r}')$ [19], perform the necessary conversion, and get the explicit expression for the internal slot admittance in the form

$$Y^i = -\frac{8\pi}{i\omega ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{k_z}(\cos k_y y_0)^2\left(\frac{\sin(k_y d_e/2)}{k_y d_e/2}\right)^2$$

$$\times\left[\Phi(kL)\cos\frac{\pi L}{a} - \Phi\left(\frac{\pi L}{a}\right)\cos kL\right]^2, \tag{11}$$

where

$$\Phi(kL) = \frac{2}{k^2-k_x^2}\left(k\sin kL\cos k_x L - k_x\cos kL\sin k_x L\right);$$

$$\Phi\left(\frac{\pi L}{a}\right) = \frac{\sin(2\pi L/a)+2\pi L/a}{(2\pi/a)};$$

$$k_x = \frac{m\pi}{a};\quad k_y = \frac{n\pi}{b};\quad k_z = \sqrt{k_x^2+k_y^2-k^2};$$

$$m = 1,\,3,\,5\ldots;\quad n = 0,\,1,\,2\ldots$$

Taking into account the field structure in a rectangular waveguide for the excitation mode H_{10}, the necessary field component may be determined as $H^i_{0x}(x) = -2H_0 i\gamma\frac{\pi}{a}\sin\frac{\pi x}{a}$. Here H_0 is the complex amplitude and $\gamma = \sqrt{k^2-(\pi/a)^2}$ is the propagation constant. Using formula (7), we can write in explicit form the expression for the MMF

$$F^m_0 = -2H_0 i\gamma\frac{\pi}{a}\left[2\frac{k\sin kL\cos\frac{\pi}{a}L-\left(\frac{\pi}{a}\right)\cos kL\sin\frac{\pi}{a}L}{k^2-(\pi/a)^2}\cos\frac{\pi}{a}L\right.$$
$$\left.-\frac{\sin\frac{2\pi}{a}L+\frac{2\pi}{a}L}{(2\pi/a)}\cos kL\right]. \tag{12}$$

Thus, the magnetic currents on the slot apertures in the local coordinates will be defined taking into account the selected above functional dependencies for the current distributions $f(\varphi)$ and $f(x)$

$$\vec{J}^m_{se(si)} = \begin{cases} \vec{J}^m_{si} = \frac{F^m_0}{d_e(Y^e+Y^i)}\left[\cos k\left(x-\frac{a}{2}\right)\cos\frac{\pi}{a}L\right. \\ \qquad \left.-\cos kL\cos\frac{\pi}{a}\left(x-\frac{a}{2}\right)\right]\vec{x}^0 \ \text{ at } \ S_i, \\ \vec{J}^m_{se} = \frac{F^m_0}{d_e(Y^e+Y^i)}\left[\cos(kR\varphi)\cos\frac{\pi}{a}L\right. \\ \qquad \left.-\cos kL\cos\frac{\pi R\varphi}{a}\right]\vec{\varphi}^0 \ \text{ at } \ S_e. \end{cases} \tag{13}$$

These expressions allow us to define such energy characteristics of spherical slot antenna, as the reflection coefficient in the waveguide

$$|S_{11}|^2 = \left|1-\frac{4\gamma F^2(kL)}{ab\omega\mu(Y^i+Y^e)}\right|^2, \ \text{ where } \ F(kL) = -F^m_0\Big/\left(2H_0 i\gamma\frac{\pi}{a}\right), \tag{14}$$

and the radiation efficiency

$$|S_\Sigma|^2 = \frac{P_\Sigma}{P_{10}} = \frac{|I_0|^2}{2}\mathrm{Re}Y^e. \tag{15}$$

Here P_Σ is the average power, radiated through the slot aperture, i.e., Umov-Pointing flux through the slot, P_{10} is the input power of waveguide wave H_{10}, $\mathrm{Re}Y^e$ is the real part of the external slot admittance (10).

The selection of the effective slot thickness h_e was made by using the energy balance equation $|S_{11}|^2+|S_\Sigma|^2 = 1$. Numerical calculations have shown that the optimum value of h_e, used to calculate the equivalent slot width d_e and ensuring the value of the power balance better than 0.993, roughly corresponds to the value $h_e \approx V^v/S_i$. Here V^v is the volume of the slot cavity, S_i is the area of the inner slot aperture. Of course, when the sphere radius is increased, the value of h_e tends to h and power balance becomes equal to unity.

3. NUMERICAL RESULTS

On the basis of our mathematical model for spherical slot antenna, we carried out the numerical analysis to find ranges of possible changes in energy characteristics, caused by varying the basic parameters of the problem. Analysis of expression (10) shows that for free space, outside a sphere ($(\varepsilon\mu)^{-1/2} = c[m/s]$ is the speed of light, $\sqrt{\mu/\varepsilon} = 120\pi$ [Ohm] and $k = \omega/c$ [1/m]), the external slot admittance Y^e is a function of dimensionless parameters (kR) and (d/R). These generalized parameters will be used for calculation and analysis of both slot external admittance and energy characteristics of spherical antennas. The parameter (kR) in problems of spherical scatterer analysis is known as diffraction radius.

To ensure correct modeling, we have tested the convergence of the infinite series in (10) for different values of parameters (kR) and (d/R), and determined the maximum value of the summation index to ensure adequate accuracy of Y^e calculations. Figure 2 shows the computational result for real $\mathrm{Re}(Y^e)$ and imaginary $\mathrm{Im}(Y^e)$ parts of the slot admittance, normalized, respectively, by the values of $\mathrm{Re}(Y^e_{scr})$ and $\mathrm{Im}(Y^e_{scr})$ for a slot in an infinite perfectly conducting screen, depending on number of series terms N, taken into account in (10), for the slot dimension $2L_i = 0.45\lambda$, ratio $d/2L_i = 0.05$ and diffraction radii, increasing in the interval $kR \in [1; 20]$. As expected, the convergence of the series significantly worsens with increasing kR values. If during calculations the results for real part of admittance $\mathrm{Re}(Y^e)$ are quickly stabilized, the $\mathrm{Im}(Y^e)$ should be calculated by the principle of relative intrinsic convergence of the series. That is, it is necessary to limit the number of series terms N by fixing the contribution of successive terms at a predefined level, for example, less than 1%.

The curves in Figure 2 are plotted only for values of N for which the specified accuracy is achieved. The plot shows that accuracy of up to 1% can be achieved if $N = 40$ for $kR = 1$, $N = 75$ for $kR = 2$ and so forth. The difference between external slot admittances Y^e, calculated by the formula (10) and by the formula for perfectly conducting screen [19], does not exceed the errors of their numerical determination. The calculation results have also revealed that variations in the slot length within $2L_i \in [0.4\lambda; 0.6\lambda]$ and width within $d/2L_i \in [0.025; 0.1]$ practically do not change the estimates for selection of N. The character of $\mathrm{Re}(Y^e)$ and $\mathrm{Im}(Y^e)$ behavior with increase in the number of series terms taken into account in (10) depends on the slot length.

The validity of the numerical simulation was confirmed by comparing the calculated and experimental data. A photograph of a prototype model which was made for experimental studies is shown in Figure 3. The model's geometrical parameters and waveguide wall thickness h for a standard rectangular waveguide are given in Figure 4 captions. The effective internal slot cavity thickness and effective slot width are $h_e = 0.1\,\text{mm}$ and $d_e = 1.414\,\text{mm}$, respectively.

The modulus of the reflection coefficient in the waveguide $|S_{11}|$ and radiation coefficient of the antenna $|S_\Sigma|^2$ are presented in Figure 4 for experimental and calculated data by circles and by solid lines, respectively. The plots show that the curves are in good agreement with each other. The maximum antenna efficiency, very close to one, is observed at $\lambda \approx 37.5\,\text{mm}$, for $2L_i \approx 0.48\lambda$ and $kR \approx 8.38$. This maximum is ensured by optimal matching between the spherical antenna and waveguide. The discrepancy between the wavelength of the calculated and experimental points of maximum radiation of the antenna can be explained by the difficulty of combining the slot longitudinal axis with the axis of the waveguide cross-section in the design of the prototype model. Notice that sufficiently large broadband operation is achieved for this antenna prototype. Thus, the ratio of antenna's operation band width at half-power level $\Delta\lambda \in [29.0; 42.5]\,\text{mm}$ to the working wavelength $\lambda \approx 37.5\,\text{mm}$ is 36%.

The observed effect of the resonant slot length reduction, determined by the maximum achieved power level, radiated by the antenna, as compared with $\lambda/2$, is associated with the influence of the spherical scatterer and the size of the slot cavity. As opposed to the case of an infinite screen, influence of the real slot width upon radiation properties for a spherical antenna can not be studied directly, if the waveguide cross-sectional dimensions are fixed. Indeed, sphere radius variation results in alteration of the internal slot cavity size. It

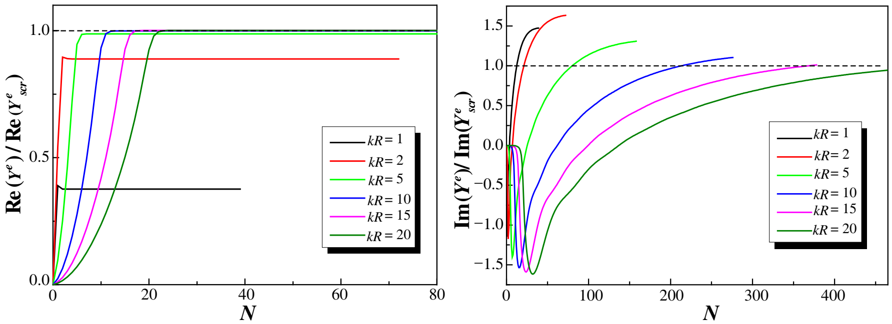

Figure 2. The normalized real and imaginary parts of the external admittance of slot radiator versus the numbers of series terms for various sphere diffraction radii ($2L_i = 0.45\lambda$, $d/2L_i = 0.05$).

Figure 3. The prototype model of the spherical antenna.

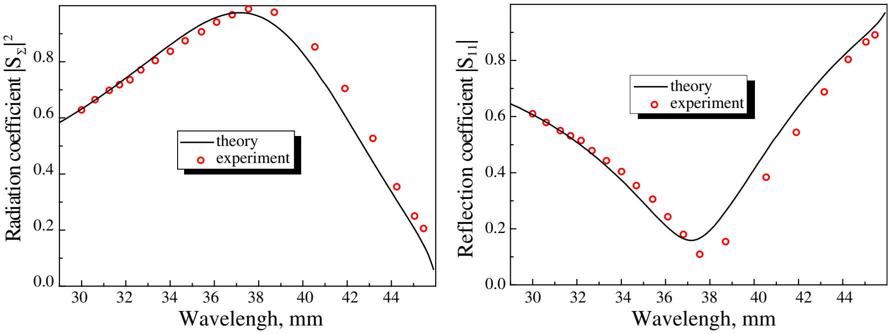

Figure 4. The energy characteristics of the antenna prototype with the parameters $R = 50\,\text{mm}$, $2L_i = 18\,\text{mm}$, $d = 1.5\,\text{mm}$, $a = 23\,\text{mm}$, $b = 10\,\text{mm}$ and $h = 1\,\text{mm}$.

is therefore of interest to study energy characteristics of a spherical antenna by varying the radius of the sphere in order to determine the antenna electrical parameters, ensuring maximal radiation at a given frequency from wavelength of waveguide single-mode regime.

Figure 5 shows the calculated energy characteristics of a spherical antenna for a waveguide with standard cross-section $\{23 \times 10\}\,\text{mm}^2$. Both the slot length and sphere radius were varied in the calculation. The ratio $d/2L_i = 0.05$ was chosen as in the above analysis of the external slot admittance. Numerical simulations have revealed the following properties. First of all, that by variation of slot length a maximal radiation level at any given frequency in the range of waveguide single-mode regime, except for a region close to the waveguide critical frequency, can be achieved. For small sphere radii, for example if $\pi R/(2L_e) = 3$, the slot resonant length is close to the value $2L_e \approx 0.5\lambda$, and the shortening effect begins to manifest itself if radius R is further increased. Maximum reduction of the slot resonant length is observed for the infinite screen and correspond to the slot length $2L_e \approx 0.48\lambda$. The bandwidth of the antenna radiation coefficient at the half-power level is maximal for the infinite screen and significantly reduces with decreasing sphere radius and the

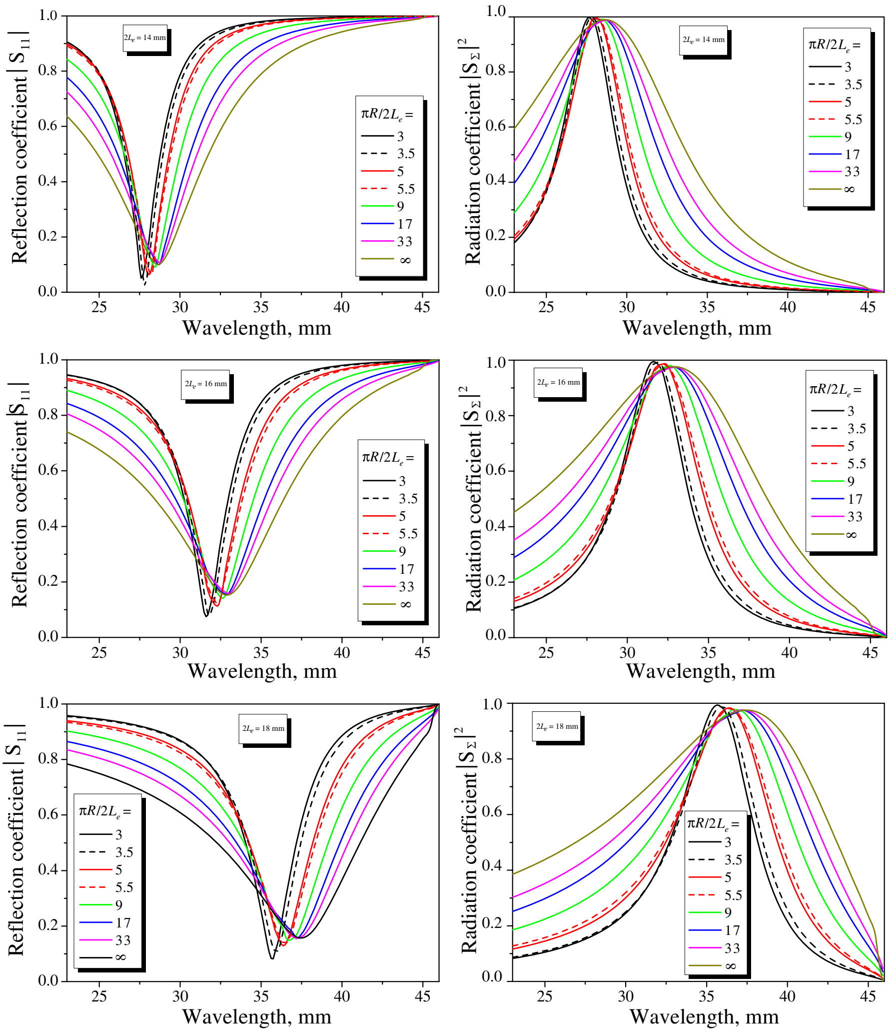

Figure 5. The energy characteristics of the spherical antenna with a waveguide of standard cross-section $\{23 \times 10\}\,\mathrm{mm}^2$.

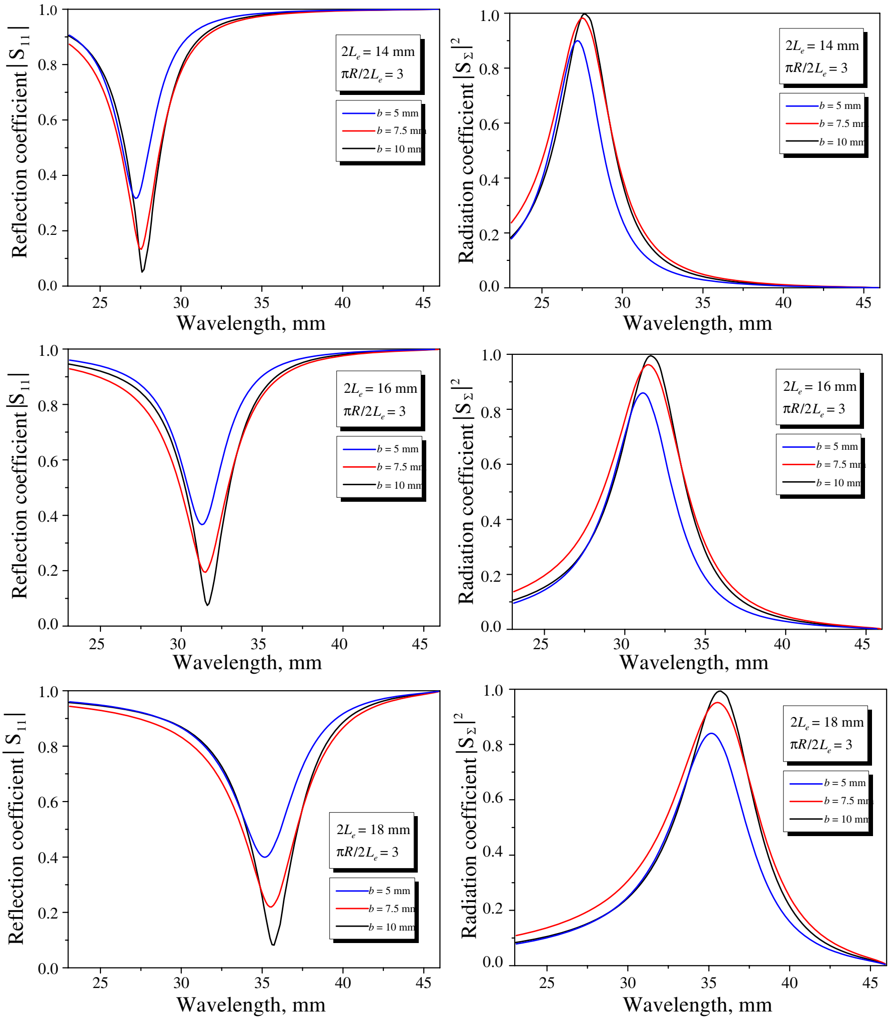

Figure 6. Energy characteristics of the spherical antenna with the low-profile waveguide $\{23 \times b\}\,\text{mm}^2$.

operating wavelength. In the vicinity of spherical antenna resonance the application of ideal screen approximation has become valid for $kR \geq 10$. This condition imposes much less constraints upon kR values as compared to that obtained above in the analysis of slot external admittance. However, far from resonance, the calculation of wavelength dependences for the energy characteristics of the spherical antenna cannot be done in the ideal screen approximation even for

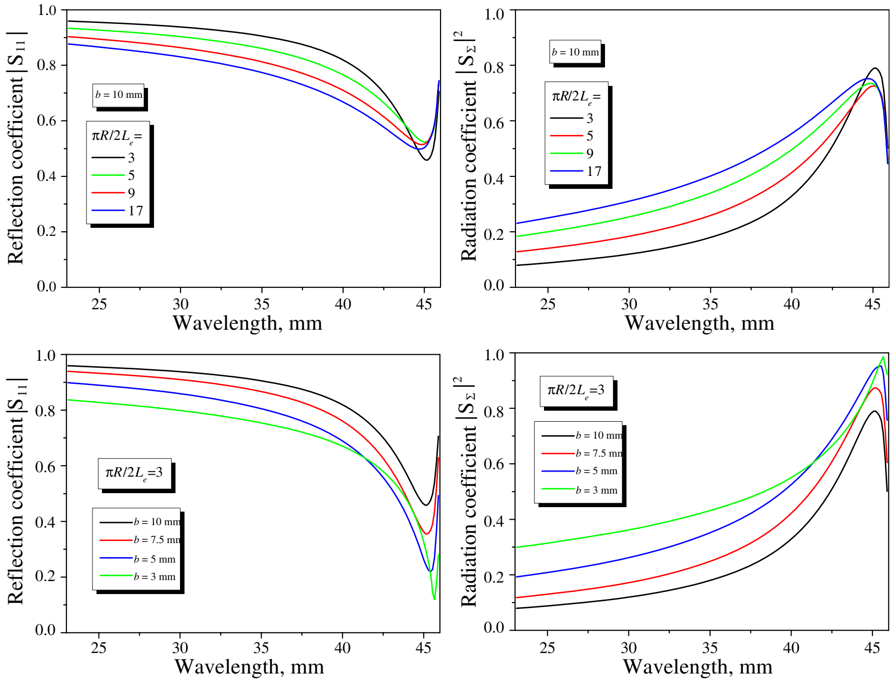

Figure 7. Energy characteristics of the spherical antenna with the low-profile waveguide $\{23 \times b\}\,\mathrm{mm}^2$ for $2L_e = a = 23\,\mathrm{mm}$.

sufficiently large diffraction radii, for example $kR \approx 33$. Additional calculations have shown that a satisfactory accuracy for practical modeling of spherical antennas with slot admittance using the infinite screen approximation may be achieved for $kR \geq 50$. In the frame of adopted constraints upon slot narrowness all above trends remain unchanged for other $d/2L_i$ if only shortening of slot resonant length is varied.

We have also calculated energy characteristics for a spherical antenna design with a low-profile waveguide. Figure 6 shows the calculated dependences of energy characteristics for a spherical antenna using a low-profile waveguide with heights $b = 5\,\mathrm{mm}$, $b = 7.5\,\mathrm{mm}$ and $b = 10\,\mathrm{mm}$. For these plots the diffraction radius is $\pi R/(2L_e) = 3$ and other parameters coincide with that in the previous series of calculations. The plots show that reduction of the rectangular waveguide height decreases slightly the resonant wavelength of the spherical antenna, and a slot length correction is required to ensure maximum radiation. The same wavelength dependences are valid for different sphere radii. However, all relations, inherent to spherical antennas with waveguides of standard cross-section, are applicable to antennas with low-profile waveguides.

Slot radiators, which have lengths equal or close to a rectangular waveguide width, i.e., if $2L_e \approx a$, are very interesting from the point of view of antenna manufacturability. The wavelength dependences of spherical antennas energy characteristics for $2L_e = a$ are shown in Figure 7. This antenna design is characterized by a fairly narrow operating band, shifted to the critical frequency of the rectangular waveguide. The plots show that the wavelength dependences are similar for all values of sphere radii and waveguide heights. Since the resonant length of spherical antenna is decreased if kR is increased and b is accordingly decreased, level of the maximum radiation could be somewhat increased by shortening of the slot.

4. CONCLUSION

The problem of electromagnetic wave radiation into the space above a perfectly conducting sphere through a narrow slot, cut in the end-wall of a semi-infinite rectangular waveguide was solved by the generalized method of induced MMF. The waveguide section was excited by a wave of H_{10} type. The concept of the equivalent slot width d_e, was used to eliminate the need for fields to be defined in the internal slot cavity. The magnetic current in the slot was determined by a single integral equation, rather than the traditional system of two equations for both apertures of the slot cavity. This paper presents a study of mathematical correctness of the approximate solution. The obvious advantage of the proposed approach is the application of local coordinate systems for coupling electrodynamic volumes. Thus the fields in the space outside the spherical scatterer and inside the waveguide section can be calculated using the appropriate Green's functions.

Physical validity of constructed mathematical models for a spherical antenna has been confirmed by experimental data, obtained for the prototype model. The influence of the sphere radius on energy parameters of the slot radiator was studied numerically. It was shown that by a proper selection of slot length the maximum level of radiation at any given frequency in the range of single-mode waveguide regime can be achieved. Naturally, such frequency tuning can be realized for a spherical antenna of any radius. It was revealed that calculation of energy characteristics for spherical antennas in the vicinity of resonance can be done using of the infinite screen approximation which becomes correct already for $kR \geq 10$. However, far from resonance, the characteristics of spherical antennas can be calculated with sufficient accuracy for practical purposes in the approximation of ideal screen only for $kR \geq 50$. A possibility of low-profile waveguide application in the design of spherical antenna is justified. Results, presented in this paper, can be directly used in the development and design of slotted spherical antennas.

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