Progress In Electromagnetics Research, Vol. 135, 245–260, 2013

AN UNCONDITIONAL STABLE 1D-FDTD METHOD FOR MODELING TRANSMISSION LINES BASED ON PRECISE SPLIT-STEP SCHEME

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Abstract—This paper presented a novel unconditional stable FDTD (US-FDTD) algorithm for solving the transient response of uniform or nonuniform multiconductor transmission line with arbitrary coupling status. Analytical proof of unconditional stability and detailed analysis of numerical dispersion are presented. The precise split-time-step scheme has been introduced to eliminate the restriction of the Courant-Friedrich-Levy (CFL) condition. Compared to the conventional US-FDTD methods, the proposed approach generally achieves lower phase velocity error for coarse temporal resolution. So larger time scales can be chosen for the transient simulation to achieve accurate results efficiently. Several examples of coupled uniform and nonuniform lines are presented to demonstrate the accuracy, stability, and efficiency of the proposed model.

1. INTRODUCTION

The increasing quest for higher signal speed and smaller feature sizes in electronic circuits have made transmission line system a dominant factor in determining circuit performance and reliability in very large scale integration designs [1]. To ensure the required quality of signal delivery, accurate and efficient transient simulations are required during the design process.

Many approaches have thus far been proposed for the multiconductor transmission line (MTL) transient analysis. The most general method is related to the quasi-TEM model. An important feature of the model is its formulation simplicity and its relative low computation

Received 30 October 2012, Accepted 17 December 2012, Scheduled 20 December 2012
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cost when compared to the full-wave numerical algorithms [2], such as time-domain integral equation (TDIE) [3, 4], the Crank-Nicolson finite-difference time-domain (CN-FDTD) approach [5], or finite element method [6]. The transmission line equations based on quasi-TEM model can be formulated in both frequency- and time-domains. As a result, the transient analysis is divided primarily into two parts: one is the frequency-domain transform method. The general approaches like the inverse Fast Fourier Transform [7], the numerical inverse Laplace transform [8], etc., can be used in most cases. However, these methods require special attention to avoid aliasing errors as described in [9]. As to the direct time-domain methods, it is an easiest way to handle transient problem of transmission lines. The related literature is vast [10–13]. The largest advantage of time-domain models is that it can achieve the broadband responses in frequency band with one single round of simulation.

The 1D finite-difference time-domain (FDTD) based on Yee's method [14], hereafter designated as "the traditional FDTD method" has been accepted extensively as one of the most popular and effective time-domain method for solving transient MTL problems. However, the discretization grid with an adequate temporal resolution must be chosen for the Courant-Friedrich-Levy (CFL) stability condition [15], which leads to expensive simulation cost. In an attempt to improve the FDTD computation efficiency, two main alternatives have been proposed: the first one is to incorporate an implicit difference scheme into the basic algorithm [16], and the second one is to use a semidiscrete model based on a time-step integration method [17, 18]. Both approaches can eliminate the CFL limitations. Nevertheless, for the former methods the numerical oscillations triggered by the rapid change in the voltage excitation will increase dramatically with larger time-step size. And the latter involves complicated matrix operation, requires huge computation memory.

Prompted by the above mentioned reasons, a novel FDTD method is developed in this paper which is combined with the precise split-time-step scheme for the transient analysis of MTL. And it is shown that the method is unconditionally stable. With this new approach, the numerical accuracy can be improved by increasing the number of sub-time-step. Therefore, it greatly enhances the computational efficiency due to the reduction of the overall time steps. Numerical results address that the presented method can increase the time step dramatically than that in the original unconditional stable FDTD.

2. DEVELOPMENT OF THE UNCONDITIONAL STABLE FDTD METHOD

This section presents the proposed methodology in detail. It begins with a brief review of the formulations for the nonuniform multiconductor transmission lines. The proposed unconditional stable technology is then described, the numerical stability and dispersion of the algorithm has been analyzed in the subsection.

2.1. Discretization of Transmission Line Equations

Considering a coupled lossy nonuniform multiconductor transmission line system, the nonuniform transmission line equations in time domain with per-unit-length parameter matrices $\mathbf{L}(x)$, $\mathbf{C}(x)$, $\mathbf{R}(x)$ and $\mathbf{G}(x)$ can conveniently be expressed as the matrix form [11],

$$\frac{d\mathbf{X}}{dt} = \mathbf{MX} \tag{1}$$

where the voltage-current vector $\mathbf{X}$ and the differential operator matrix $\mathbf{M}$ are given by

$$\mathbf{X} = \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} -\mathbf{C}^{-1} \cdot \mathbf{G} & -\mathbf{C}^{-1} \frac{\partial}{\partial x} \\ -\mathbf{L}^{-1} \frac{\partial}{\partial x} & -\mathbf{L}^{-1} \cdot \mathbf{R} \end{bmatrix} \tag{2}$$

By using ODEs theory and denoting the time step size as Δt, a recursive solution of (1) is obtained

$$\mathbf{X}^{n+1} = \exp(\mathbf{M}\Delta t)\mathbf{X}^n \tag{3}$$

where superscript n of $\mathbf{X}$ denotes its value at $n\Delta t$, and the exponential matrix $\exp(\mathbf{M}t)$ can be solved by using the precise split-time-step scheme [19], i.e.,

$$\exp(\mathbf{M}\Delta t) = [\exp(\mathbf{M}\Delta t/N)]^N \tag{4}$$

where N is the number of sub-time-step. It can be selected suitably to reduce the numerical dispersion error when the time step is large. Therefore, for the interval of sub-time-step $\Delta t/N$, the 2nd order Padé approximation [20] can be used to approximate the exponential operator $\exp(\mathbf{M}\Delta t/N)$), and combined with (3) yield,

$$\mathbf{X}^{n+1} = \left(\frac{\mathbf{U} + \frac{\mathbf{M}\Delta t}{2N}}{\mathbf{U} - \frac{\mathbf{M}\Delta t}{2N}} \right)^N \mathbf{X}^n \tag{5}$$

where $\mathbf{U}$ is the identity matrix. Noted that, when $N > 1$, the rational function in (5) is tended to the higher order Padé approximation. Namely, the precision of the numerical results could be improved by increasing the number of sub-time-step. Then a similar splitting technique [21] is applied for implementing (5) with the aim of reducing the computational complexity. Here (5) is divided into N sub-equations, from n to $n+1$, one time step is divided into N sub-steps accordingly. Considering $n+\frac{k-1}{N} \rightarrow n+\frac{k}{N}$, where $k = 1, 2, \ldots, N$, the sub-equation is given by

$$\left(\mathbf{U} - \frac{\mathbf{M}\Delta t}{2N}\right)\mathbf{X}^{n+\frac{k}{N}} = \left(\mathbf{U} + \frac{\mathbf{M}\Delta t}{2N}\right)\mathbf{X}^{n+\frac{k-1}{N}} \tag{6}$$

Assume that the line is divided into N_x segments with equal length Δx. Let $n + k/N$ denote $(n + k/N)\Delta t$ and m denote the points at $m\Delta x$, $(m + 0.5)\Delta x$ for superscripts V and I, respectively. The spatial derivatives of voltage and current points are approximated by the second-order central differences. Then, the update formulations for the k'th sub-step, after some manipulations, can be expressed as

$$\begin{aligned}
&-\frac{\Delta t^2}{\Delta x^2}\mathbf{A}_m^V \cdot \mathbf{A}_{m-1}^I \cdot \mathbf{V}_{m-1}^{n+\frac{k}{N}} - \frac{\Delta t^2}{\Delta x^2}\mathbf{A}_m^V \cdot \mathbf{A}_m^I \cdot \mathbf{V}_{m+1}^{n+\frac{k}{N}} \\
&+\left(\mathbf{U} + \left(\mathbf{A}_m^V \cdot \mathbf{A}_m^I + \mathbf{A}_m^V \cdot \mathbf{A}_{m-1}^I\right)\frac{\Delta t^2}{\Delta x^2}\right) \cdot \mathbf{V}_m^{n+\frac{k}{N}} \\
=& \frac{\Delta t^2}{\Delta x^2}\mathbf{B}_m^V \cdot \mathbf{A}_m^I \cdot \mathbf{V}_m^{n+\frac{k-1}{N}} - \frac{\Delta t}{\Delta x}\mathbf{A}_m^V \cdot \left(\mathbf{I}_m^{n+\frac{k-1}{N}} - \mathbf{I}_{m-1}^{n+\frac{k-1}{N}}\right) \\
&-\frac{\Delta t}{\Delta x}\mathbf{A}_m^V \cdot \left(\mathbf{B}_m^I \cdot \mathbf{A}_m^I \cdot \mathbf{I}_m^{n+\frac{k-1}{N}} + \mathbf{B}_{m-1}^I \cdot \mathbf{A}_{m-1}^I \cdot \mathbf{I}_{m-1}^{n+\frac{k-1}{N}}\right) \\
&+\frac{\Delta t^2}{\Delta x^2}\mathbf{A}_m^V \cdot \mathbf{A}_m^I\left(\mathbf{V}_{m+1}^{n+\frac{k-1}{N}} - \mathbf{V}_m^{n+\frac{k-1}{N}}\right) \\
&-\frac{\Delta t^2}{\Delta x^2}\mathbf{A}_m^V \cdot \mathbf{A}_{m-1}^I\left(\mathbf{V}_m^{n+\frac{k-1}{N}} - \mathbf{V}_{m-1}^{n+\frac{k-1}{N}}\right) \\
&\qquad m = 1, 2, \ldots, N_x - 1.
\end{aligned} \tag{7a}$$

$$\begin{aligned}
\mathbf{I}_m^{n+\frac{k}{N}} =& -\frac{\Delta t}{\Delta x}\mathbf{A}_m^I \cdot \left(\mathbf{V}_{m+1}^{n+\frac{k-1}{N}} - \mathbf{V}_m^{n+\frac{k-1}{N}}\right) \\
&+\mathbf{B}_m^I \cdot \mathbf{A}_m^I \cdot \mathbf{I}_m^{n+\frac{k-1}{N}} - \frac{\Delta t}{\Delta x}\mathbf{A}_m^I \cdot \left(\mathbf{V}_{m+1}^{n+\frac{k}{N}} - \mathbf{V}_m^{n+\frac{k}{N}}\right) \\
&\qquad m = 0, 1, \ldots, N_x - 1.
\end{aligned} \tag{7b}$$

where

$$\begin{aligned}
\mathbf{A}_m^V &= (2N\mathbf{C}_m + \mathbf{G}_m\Delta t)^{-1}, \quad \mathbf{B}_m^V = (2N\mathbf{C}_m - \mathbf{G}_m\Delta t) \\
\mathbf{A}_m^I &= (2N\mathbf{L}_m + \mathbf{R}_m\Delta t)^{-1}, \quad \mathbf{B}_m^I = (2N\mathbf{L}_m - \mathbf{R}_m\Delta t)
\end{aligned}$$

Next, considering that the line is terminated by linear loads $\mathbf{R}_s$, $\mathbf{R}_l$ and excited by a voltage source $\mathbf{V}_s$, the boundary condition can be derived as

$$\left(\frac{\Delta x}{2\Delta t}\left[\mathbf{A}_0^V\right]^{-1}+\mathbf{R}_s^{-1}+\frac{\Delta t}{\Delta x}\mathbf{A}_0^I\right)\cdot\mathbf{V}_0^{n+\frac{k}{N}}-\mathbf{R}_s^{-1}\cdot\mathbf{V}_s^{n+\frac{k}{N}}-\frac{\Delta t}{\Delta x}\mathbf{A}_0^I\cdot\mathbf{V}_1^{n+\frac{k}{N}}$$
$$=\mathbf{R}_s^{-1}\cdot\mathbf{V}_s^{n+\frac{k-1}{N}}+\left(\frac{\Delta x}{2\Delta t}\mathbf{B}_0^V-\mathbf{R}_s^{-1}-\frac{\Delta t}{\Delta x}\mathbf{A}_0^I\right)\cdot\mathbf{V}_0^{n+\frac{k-1}{N}}$$
$$+\frac{\Delta t}{\Delta x}\mathbf{A}_0^I\cdot\mathbf{V}_1^{n+\frac{k-1}{N}}-\left(\mathbf{U}+\mathbf{B}_0^I\cdot\mathbf{A}_0^I\right)\cdot\mathbf{I}_0^{n+\frac{k-1}{N}} \tag{8a}$$

$$\left(\frac{\Delta x}{2\Delta t}\left[\mathbf{A}_{N_x}^V\right]^{-1}+\mathbf{R}_l^{-1}+\frac{\Delta t}{\Delta x}\mathbf{A}_{N_x-1}^I\right)\cdot\mathbf{V}_{N_x}^{n+\frac{k}{N}}-\frac{\Delta t}{\Delta x}\mathbf{A}_{N_x-1}^I\cdot\mathbf{V}_{N_x-1}^{n+\frac{k}{N}}$$
$$=\left(\frac{\Delta x}{2\Delta t}\mathbf{B}_{N_x}^V-\mathbf{R}_l^{-1}-\frac{\Delta t}{\Delta x}\mathbf{A}_{N_x-1}^I\right)\cdot\mathbf{V}_{N_x}^{n+\frac{k-1}{N}}$$
$$+\frac{\Delta t}{\Delta x}\mathbf{A}_{N_x-1}^I\cdot\mathbf{V}_{N_x-1}^{n+\frac{k-1}{N}}+\left(\mathbf{U}+\mathbf{B}_{N_x-1}^I\cdot\mathbf{A}_{N_x-1}^I\right)\cdot\mathbf{I}_{N_x-1}^{n+\frac{k-1}{N}} \tag{8b}$$

Equations (7a) and (8) are linear systems with a block tridiagonal coefficient matrix, which can be solved efficiently with special numerical packages as like the Gauss-Seidel iterative method. The $\mathbf{I}_m^{n+\frac{k}{N}}$ in Equation (7b) can be solved directly since it only depends on the known values after $\mathbf{V}_m^{n+\frac{k}{N}}$ and $\mathbf{V}_{m+1}^{n+\frac{k}{N}}$ is updated. By repeating the updated processes, we can obtain the transient analysis values on all spatial-temporal discrete points.

The above processes include the inverse operation of the coefficient matrices, e.g., $\mathbf{A}_m^V$, $\mathbf{A}_m^I$, etc. Because these matrices do not change with time, the inverse operation only needs to be calculated once and then to be stored. Hence the simulation time is not significantly affected by it. On the other hand, the updated Equations (7a) and (8) involves matrix addition, subtraction and multiplication. All of these operations increase with the number of sub-time-step N linearly. Thus the computational load for each time step $n\Delta t$ is $O(N)$. It means that choose N reasonably will make the computation more effectively. We summarize our algorithm in Table 1.

2.2. Numerical Stability and Dispersion Analysis

The general way of analyzing the stability of the FDTD algorithm is to put a sinusoidal traveling wave into the algorithm and make sure that the propagation gain is no more than one for all frequencies. By using the Fourier method, which has been described and applied in many open literature such as [22, 23], we can analytically prove that the proposed method is unconditional stable.

Table 1. Precise split-step 1D-FDTD algorithm.

Compute the coefficient matrices: $\mathbf{A}_m^V$, $\mathbf{B}_m^V$, $\mathbf{A}_m^I$, $\mathbf{B}_m^I$
Input = $\mathbf{I}^n$, $\mathbf{V}^n$; Output = $\mathbf{I}^{n+1}$, $\mathbf{V}^{n+1}$
Begin Sub-Iteration k: ..for $k = 1 : N$; ....Equations 7(a) and (8) $\Rightarrow \mathbf{V}^{n+\frac{k}{N}}$ ....Equation 7(b) $\Rightarrow \mathbf{I}^{n+\frac{k}{N}}$ ..end End

For simplicity, assuming the transmission line to be lossless, the field components in spectral domain at the nth time step can be represented as

$$\phi_x^n = \phi_{x0} \exp\left(-jk_x m\Delta x\right) \tag{9}$$

where ϕ_x^n denotes the instantaneous value of V_x^n or I_x^n. Substitution of (9) into (5), the following equations can be generated

$$\mathbf{X}^{n+1} = \left(\frac{\mathbf{U} + \frac{\mathbf{M}'}{2N}\Delta t}{\mathbf{U} - \frac{\mathbf{M}'}{2N}\Delta t}\right)^N \mathbf{X}^n = \mathbf{\Lambda}\mathbf{X}^n \tag{10}$$

where $\mathbf{\Lambda}$ is the growth matrix and $\mathbf{M}'$ a coefficient matrix related to spatial frequencies k_x, spatial interval, Δx, the per-unit-length parameters as inductance L and capacitance C. According to the matrix theory, the matrix $\mathbf{M}'$ can be decomposed as

$$\mathbf{M}' = \mathbf{Y}\text{diag}(\lambda_i)\mathbf{Y}^{-1}, \quad i = 1, 2. \tag{11}$$

Here $\mathbf{Y}$ is the eigenvector and λ_i the eigenvalue of $\mathbf{M}'$, it is easy to get

$$\lambda_{1,2} = \pm j\frac{W_x}{\sqrt{LC}} \tag{12}$$

where $W_x = \frac{2\sin(k_x\Delta x/2)}{\Delta x}$. Substituting (12) into (11) and rearranging it, the growth matrix $\mathbf{\Lambda}$ can be rewritten as

$$\mathbf{\Lambda} = \mathbf{Y}\text{diag}(r_i)\mathbf{Y}^{-1}, \quad i = 1, 2. \tag{13}$$

the two eigenvalues r_i of the $\mathbf{\Lambda}$ can be found as

$$r_i = \left(\frac{1 + \frac{\Delta t\lambda_i}{2N}}{1 - \frac{\Delta t\lambda_i}{2N}}\right)^N, \quad i = 1, 2. \tag{14}$$

By substituting (12) into (14), the eigenvalue of $\mathbf{\Lambda}$ is

$$r_{1,2} = \left(\varsigma \pm j\sqrt{1-\varsigma^2}\right)^N \tag{15}$$

where $\varsigma = a/b$, the parameters a and b are found to be $a = 1 - \frac{\Delta t^2 W_x^2}{4N^2LC}$, and $b = 1 + \frac{\Delta t^2 W_x^2}{4N^2LC}$. It is obvious that the value of W_x corresponding to the second-order central finite-difference scheme is a real number. Subsequently, the eigenvalues associated with the proposed method can be represented as (15). Since $|r_{1,2}| = 1$, it can be concluded that the novel FDTD method is unconditionally stable.

For the numerical dispersion analysis [24], assuming that a plane-wave with an angular frequency ω propagates along the line axis x, we have

$$\mathbf{X}^n = \mathbf{X}\exp(j\omega n\Delta t) \tag{16}$$

Substituting (10) into (16), we arrive at

$$(\exp(j\omega\Delta t)\cdot\mathbf{U} - \mathbf{\Lambda})\,\mathbf{X} = 0 \tag{17}$$

From the determinant operator of the coefficient matrix in the (17), the dispersion relationship of the proposed method can be derived. So we have

$$\det\left(\exp(j\omega\Delta t)\cdot\mathbf{U} - \mathbf{\Lambda}\right) = 0 \tag{18}$$

From Equations (15) and (18), the numerical dispersion relation can be easily given as

$$\mathrm{tg}^2\left(\omega\frac{\Delta t}{N}\right) = \frac{1-\varsigma^2}{\varsigma^2} \tag{19}$$

It should be noted that the spatial frequencies $k_x = k$. k denotes the numerical wave number. For this analysis, the normalized phase velocity error is calculated as

$$error = \frac{|\omega/c_0 - k_{final}|}{k_{final}} \times 100\% \tag{20}$$

where k_{final} is the final iterative value of k from (19) and c_0 the speed of light in the free space. Figure 1 illustrates the normalized phase velocity error against the CFLN for different sub-time-step. The CFLN in the figure is the ratio between the time step taken and the CFL time step. For this analysis the mode velocity along the line axis is c_0, namely $1/\sqrt{LC} = c_0$, the operating frequency is chosen at 1 GHz and the cell size is calculated as

$$\Delta x = \frac{\lambda}{50} = 6\,\mathrm{mm}$$

Figure 1 shows that the dispersion (normalized phase velocity) error of the proposed method increases with increasing CFLN. The sub-time-step in Figure 1 is taken up to 4, just to see the dispersion trend. It can be found that the dispersion is reduced dramatically by introducing the precise split-time-step scheme. At CFLN $= 10$, for $N = 4$ the dispersion is approximately 130% better than the proposed algorithm for $N = 1$.

3. NUMERICAL EXPERIMENT

In this section, the novel 1D-FDTD-based transmission line solver with the proposed precise split-time-step scheme is applied to several numerical examples to verify the algorithm and demonstrate its performance and applications. As we know, the efficiency of the algorithm is affected by the performance of the computer, programming language, grid number and so on [25]. In order to compare effectively, we develop the algorithms in C-sharp language,

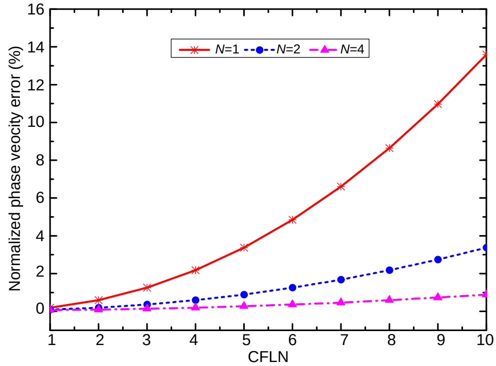

Figure 1. Normalized phase velocity error versus CFLN.

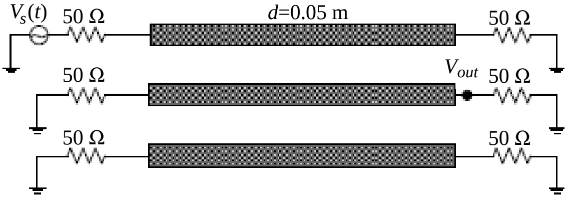

Figure 2. Uniform coupled three-conductor transmission line structure.

and perform on the LG PC with Intel Core(TM)2 (2.5 GHz) processors.

As the first example, we consider a coupled uniform transmission line system depicted in Figure 2. The per-unit-length parameter matrices of the MTL are given in [16]. The input signal voltage source is a 1 V rectangular pulse with rise and fall-times of 0.25 ns, a width of 1.5 ns. The maximum mode velocity of the line is 2.1408×10^8 m/s. For this simulation, the lines are divided into 40 segments along the length. Since the traditional FDTD algorithm uses the smallest time-step size, highly accurate solutions should be achieved. It can be regarded as a reference. A straightforward approach called "US-FDTD" [16], which is believed to be more efficient, is adopted here for comparison. It must be noted that the spatial equations in [16] (Equations (18) and (19)) can be written like (7) and (8) in a matrix form. The special numerical package has been used for both algorithms (the proposed method and "US-FDTD").

In Figure 3, the voltage waveforms across the load resistance versus time are shown. The time scales, time steps, CPU times, relative errors and memory requirements of the above simulations are given in Table 2. It can be seen from Figure 3 and Table 2 that the proposed method and US-FDTD can provide stable results for larger time scales and both of these algorithms require much less CPU time compared with the traditional FDTD. The memory requirement of the novel method is largest. This is because that the computational load of the proposed approach for each time step is $O(N)$. A large block tridiagonal coefficient matrix need to be saved. So the memory cost correspondingly increased. Besides, the CFLN adopted in the proposed scheme is larger than the existing US-FDTD, whereas the computation efficiency are not improved evidently. The reason is that both methods are recursive for Δt, the US-FDTD algorithm needs

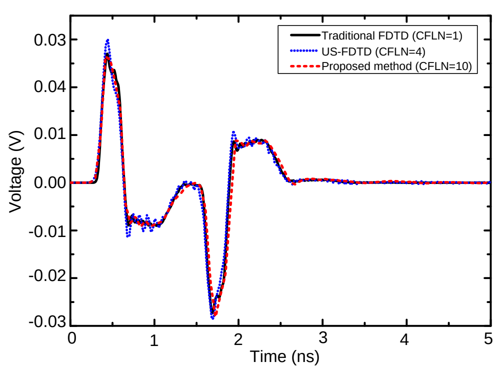

Figure 3. The output voltage V_{out} obtained by three FDTD methods with different CFLN.

215 loops for CFLN = 4. While the proposed algorithm needs 3×85 loops, where 3 is the number of sub-time-step. Therefore in the same conditions, CPU time of the proposed algorithm is almost the same as the US-FDTD method presented in [16].

However, there is a critical point should be considered in this problem. The proposed method and conventional US-FDTD are independent of the time scale; hence, the computation accuracy must be investigated. Table 2 shows the relative error of the maximum value of V_{out} taking the results calculated by traditional FDTD as the reference solution. Note that the relative error of the proposed algorithm is 0.756% as compared to 8.788% of the US-FDTD. This indicates that the proposed scheme can improve the computational accuracy significantly with the same solution efficiency.

The second example deals with a coupled lossless nonuniform three-line structure depicted in Figure 4. It is excited with a voltage source, generating trapezoidal 1-V pulses with 5 ns delay time, 0.5 ns rise and fall times and 10 ns duration. The line parameters are as follows [26]:

$$\mathbf{L}(x) = \begin{bmatrix} L(x) & L_m(x) & 0 \\ L_m(x) & L(x) & L_m(x) \\ 0 & L_m(x) & L(x) \end{bmatrix}$$

Table 2. Comparisons of results with three FDTD methods for uniform lines.

Algorithms	CFLN	Sub steps	Total time steps	Relative error	CPU time (s)	Memory cost (Mb)
Traditional FDTD	1	-	857	-	1.436	0.562
US-FDTD	4	-	215	8.788%	0.178	0.683
Proposed method	10	3	85	0.756%	0.183	0.811

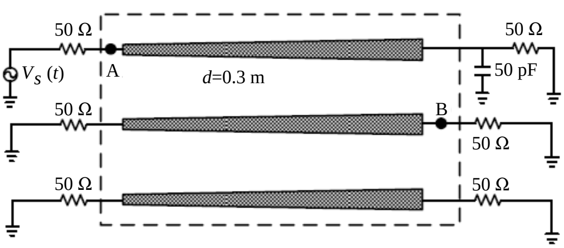

Figure 4. Nonuniform coupled three-conductor transmission line structure.

$$\mathbf{C}(x) = \begin{bmatrix} C(x) & C_m(x) & 0 \\ C_m(x) & C(x) & C_m(x) \\ 0 & C_m(x) & C(x) \end{bmatrix}$$

where

$$\begin{aligned} L(x) &= 387\left[1 + 0.1\left(1 + 0.6\sin\left(\pi x + 0.25\pi\right)\right)\right] \text{ nH/m} \\ L_m(x) &= 0.1\left(1 + 0.6\sin\left(\pi x + 0.25\pi\right)\right) L(x) \\ C(x) &= 104.13/\left[1 - 0.15\left(1 + 0.6\sin\left(\pi x + 0.25\pi\right)\right)\right] \text{pF/m} \\ C_m(x) &= -0.15\left(1 + 0.6\sin\left(\pi x + 0.25\pi\right)\right) C(x) \end{aligned}$$

In the simulation, the nonuniform line is divided into 30 segments of equal length. The maximum mode velocity of the line is 1.3801×10^8 m/s. Firstly, the transient analysis of MTL system is performed with CFLN $= 5$ and two choices of sub-time step ($N = 1$ and 3). Again, the results from traditional FDTD can be regarded as benchmarks to demonstrate the accuracy of other results. The voltage response at node A is shown in Figure 5. It is obviously that large discrepancies are occurred for $N = 1$. While $N = 3$, the results of this method is stable and in good agreement with the benchmarks. Consequently, the numerical dispersion characteristic of the novel FDTD can be improved greatly by implementing precise split-step scheme.

Similar to the first example, a comparison with traditional FDTD and US-FDTD is performed. The voltage waveforms of node B versus time is depicted as in Figure 6. The time scales, time steps, CPU times and memory cost are given in Table 3. The required memory size of the proposed method is 0.983 Mb which is larger than other methods. As stated before, the number of discretized points and the split steps increased the memory cost. Table 3 also indicated that with the time step enlarged, both US-FDTD and proposed method

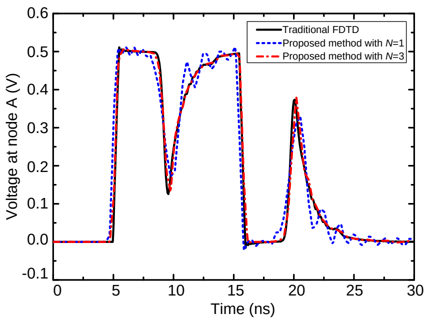

Figure 5. Transient response at node A of nonuniform interconnects.

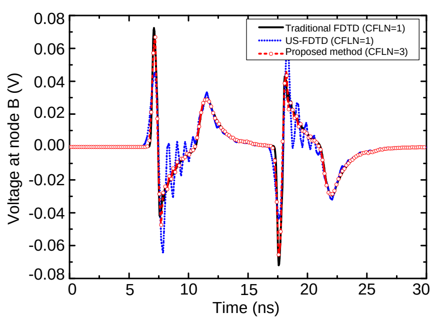

Figure 6. Transient response at node B of nonuniform interconnects.

can improve the computational efficiency compared to conventional FDTD. Nonetheless, from Figure 6, although the results of US-FDTD are stable and even more efficient, huge numerical oscillations can be observed, which are not tolerable for accurate circuit simulation. On the other hand, the novel method is able to capture the waveform propagation along the transmission line quite accurately. It means that with the same accuracy, much less temporal sampling rate can be used in the proposed method.

The third example deals with a three-coupled uniform transmission line network system. The transmission structure, as shown in Figure 7(a) and further referenced as "Block A", is used as a building block to construct more complex networks. Block A is terminated by

Table 3. Comparisons of results with three FDTD methods for nonuniform lines.

Algorithms	CFLN	Sub steps	Total time steps	CPU time (s)	Memory cost (Mb)
Traditional FDTD	1	-	450	1.285	0.324
US-FDTD	3	-	150	0.236	0.328
Proposed method	3	3	150	0.396	0.983

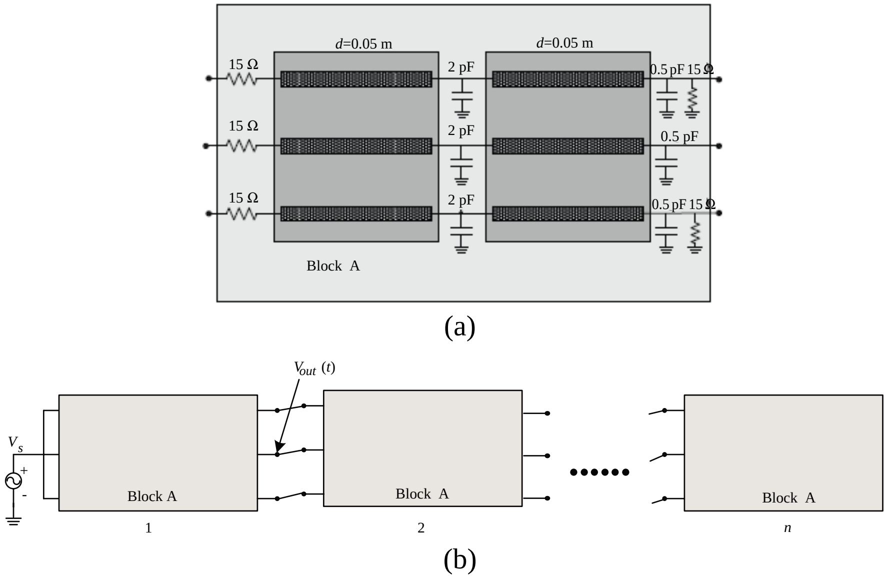

Figure 7. Multiconductor transmission line networks. (a) Block A used as the building block of the MTL networks in (b). (b) Three-conductor transmission line network constructed using Block As.

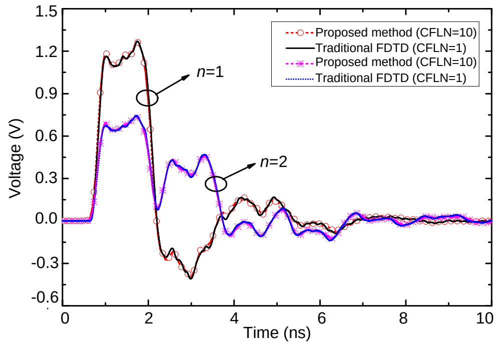

Figure 8. V_{out} of the transmission line network in Figure 7(b).

linear circuit elements. The cascaded MTL network in Figure 7(b) is constructed by connecting n Block As. The input voltage source and per-unit-length parameters of the lines are the same as that in the first example.

Two simulations are performed on the cascaded network. For both simulations, the lines are divided into 40 segments, and a comparison between the results of small time step (CFLN = 1) and those of large one (CFLN = 10) is depicted. In the first simulation, the number of blocks, n, is equal to 1. Figure 8 shows that V_{out} obtained using the proposed algorithm is in good agreement with those obtained by traditional FDTD method. However, the CPU time for the novel approach ($N = 2$, $Nt = 172$ time steps) is 0.276 s as compared to 5.675 s of the traditional FDTD method ($Nt = 1712$ time steps). In the second simulation, two Block As is chosen. The results are also depicted in Figure 8. It is obviously that the accuracy of the results is not changed with the increase of the time step. The CPU time are 13.298 and 0.687 s for traditional FDTD approach ($Nt = 1712$ time steps) and proposed approach ($N = 2$, $Nt = 172$ time steps), respectively. For this simulation, the novel unconditional stable FDTD based on precise split-step scheme is approximately 20 times faster than the conventional FDTD algorithm.

4. CONCLUSION

A precise split-step unconditional stable FDTD method for the MTL transient simulation has been developed in this paper. Its unconditional stability and numerical dispersion relationship have been demonstrated. By using the precise split-step scheme, larger time steps can be employed in the transient simulation with slight phase velocity error. Compared to the conventional unconditional stable approaches (e.g., [16]), it seems that the more the sub-time-step has been selected, the larger the computational resources occupied by the proposed method. However, the novel method has the lower dispersion error, a larger time step and a coarser mesh can be used. As a result, the total number of iterations required by the proposed scheme can be reduced. The numerical results fully illustrate that the novel method can significantly improve the simulation efficiency with high accuracy.

As a direct spatial-temporal discretization and simple model, it seems to be convenient to combine the proposed method with circuit simulator to solve the transmission line system with nolinear or passive terminations. However, the implicit difference schemes give some limitations for the complex circuits analysis. A future extension of the novel 1D-FDTD method would be to add terms to account for the MTL with arbitrary loads. On the other hand, the interconnect systems introduced in this paper are passive networks. Nevertheless, in some integrated circuits, with the operating frequency increasing to the millimeter wave range, the transmission line properties of some active devices, such as MESFET, should be considered [27]. In this situation, both the active and passive parts are included in the model. Can we still apply the novel techniques to such problems? This is a very important issue for future realistic applications.

ACKNOWLEDGMENT

This work was supported by the National Advanced Research Program of Science and Technology, China (Grant No. 9140A33020612KG01334).

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