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UNEQUAL WILKINSON POWER DIVIDER USING ASYMMETRIC MICROSTRIP PARALLEL COUPLED LINES

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Abstract—An effective method is introduced to overcome the narrow strip width in unequal Wilkinson power divider with high dividing ratio. In the proposed method, the separated microstrip lines of the Wilkinson power divider are replaced by a uniform asymmetrical microstrip coupled lines. It is shown that in the proposed method, the width of the narrow microstrip line is significantly wider in comparison to the narrow microstrip line in the conventional Wilkinson power divider with the same specification. To design the power divider, a suitable error function is defined and then minimized which led to the final dimensions of the power divider. A sample of the designed power divider with 4 : 1 dividing ratio is fabricated and tested, which indicate the effectiveness of the proposed method.

1. INTRODUCTION

The Wilkinson power divider is widely used in planar antennas as a feed structure and in microwave circuits due to its straightforward design, simple configuration and ease of construction [1–4]. It not only is able to provide almost flat power division and appropriate ports matching over its bandwidth, but also can effectively isolate the output ports. In many applications, there is a need to use microstrip Wilkinson power divider with unequal power division ratio. However, for the unequal Wilkinson power divider with high power dividing ratio, the characteristic impedance of one of the microstrip lines becomes high. This leads to the narrow strip width which is undesirable due to

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the manufacturing difficulty, extra insertion loss and power handling reduction.

Several methods have been proposed to overcome this shortcoming in literatures [5–16]. Achieving high characteristic impedance and avoiding narrow strip width of the microstrip line can be simultaneously reached using defected ground structure (DGS) [5–7]. However, this solution causes another difficulty, because the whole structure should be isolated from the other grounded conductors.

The double-sided parallel strip-lines have been proposed to realize high characteristic impedances and an unequal Wilkinson power divider has been fabricated by this technique [8]. However, implementations of these methods have some difficulties, because the proper transition between microstrip and double-sided parallel strip-line is needed.

The grooved substrate has been proposed for realizing the power divider with high dividing ratio [9]. However, fabrication of the grooved substrate is a little difficult in comparison to the conventional microstrip line.

In [10], an extra transmission line between the output ports and in series with the isolation resistor in Wilkinson power divider is added which leads to a new class of power divider with unequal power division. The lengths of the transmission lines are the only parameters that must be derived and the output ports can directly connect to the power divider without using the impedance transformers. However, the power division bandwidth is somewhat reduced.

Replacing the very low characteristic impedance transmission lines with the dual transmission lines leads to a new class of power dividers with unequal power division ratio [11]. However, such a solution increases the power divider dimensions slightly.

Application of the T-shaped microstrip lines has also been proposed to design the Wilkinson power divider with high power dividing ratio [12]. In this approach, the microstrip lines with very low characteristic impedances have been replaced by the T-shaped microstrip lines. The physical dimensions of each T-shaped microstrip line are selected such that they would be equivalent to that in the original Wilkinson power divider, but such a solution decreases the power divider bandwidth a bit.

In [13], the loaded transmission line has been proposed to design the Wilkinson power divider with high dividing ratio. All the microstrip lines in the conventional Wilkinson power divider have been replaced by the transmission lines which are loaded by the short or the open circuit stubs. Optimizing the various geometrical dimensions of the divider has led to easy implemented Wilkinson power divider with high power dividing ratio.

In Another approach two shunt stubs have been added to the conventional Wilkinson power divider and at the output nodes [14]. The amount of the power division ratio has been controlled by the stub lengths. However, the bandwidth of the proposed divider is small in comparison to the conventional Wilkinson power divider.

The application of the asymmetrical coupled lines for designing Wilkinson power divider was first introduced in [15]. The design method starts by decomposing the three port network into two networks and the even- and odd-mode analysis is applied to one network. The parameters of the other network are related to the first network from the impedance scaling factor. The equivalent even-mode network is used to derive the even-mode characteristic impedance of each section of the power divider in order to obtain the desired power division ratio and input VSWR. The odd-mode network including the coupling scheme is used to derive the isolation resistor of each section.

In [16] the asymmetrical microstrip coupled-line has been mainly proposed for circuit size reduction. The design procedure has been proposed based on the even- and odd- mode analysis. Application of this method leads to desired power division between output ports which are terminated by arbitrary impedances.

The application of asymmetrical uniform microstrip coupled lines is mainly proposed to overcome the shortcoming of the narrow strip width of the conventional Wilkinson power divider with high dividing ratio. The proposed method overcomes this problem by replacing the separated microstrip branches with a uniform asymmetrical microstrip coupled lines. It is shown that for 4 : 1 power division, the strip width of the designed Wilkinson power divider is significantly greater than the strip width of the conventional Wilkinson power divider with the same power division.

2. PROPAGATING MODE AMPLITUDES

Figure 1 shows the layout of the Wilkinson power divider implemented by using the asymmetrical microstrip parallel coupled lines including its various parameters. As the figure shows, the separated microstrip lines are replaced by a uniform asymmetrical microstrip coupled lines. In order to derive a design method, a simple model of the Wilkinson power divider is considered which is shown in Fig. 2. The resistor is not included in this model for simplicity. Assume a sinusoidal voltage source of 1 volt amplitude excites the input of the model as shown in Fig. 2. Then, according to the analysis provided in [17], the voltage along the asymmetrical microstrip transmission line can be expressed

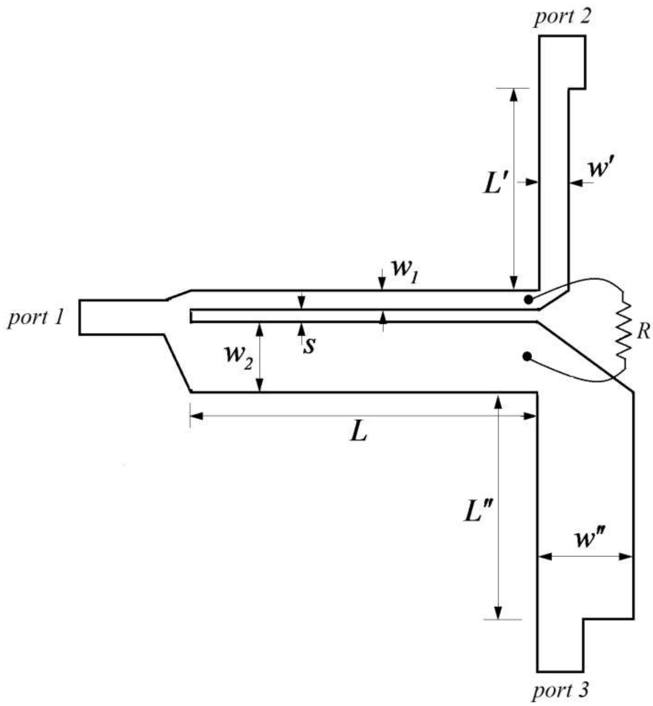

Figure 1. Layout of the Wilkinson power divider including its parameters.

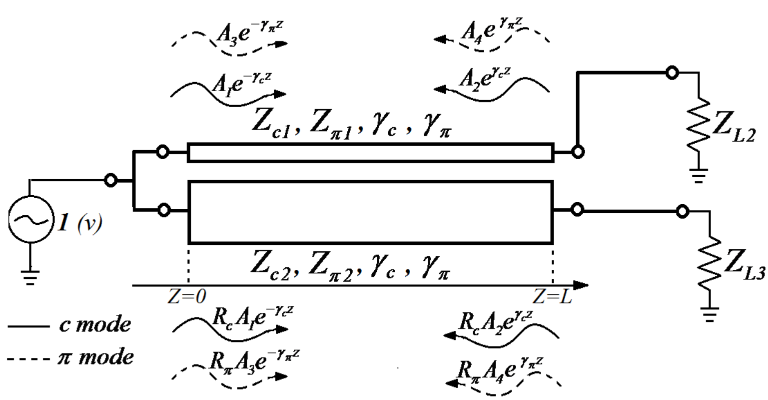

Figure 2. Simple model of the Wilkinson power divider.

as

$$\begin{cases} V_1(z) = A_1 e^{-\gamma_c z} + A_2 e^{\gamma_c z} + A_3 e^{-\gamma_\pi z} + A_4 e^{\gamma_\pi z} \\ V_2(z) = A_1 R_c e^{-\gamma_c z} + A_2 R_c e^{\gamma_c z} + A_3 R_\pi e^{-\gamma_\pi z} + A_4 R_\pi e^{\gamma_\pi z} \end{cases} \quad (1)$$

where A_1 to A_4 are the amplitudes of the forward and backward voltage waves of c and π modes along the transmission line, γ_c and γ_π are the propagation constants of the c and π modes, respectively, and R_c and R_π are the ratio of the voltages on the two lines of the $c$and π modes, respectively. As Fig. 2 shows, the asymmetric coupled-line is terminated by Z_{L2} and Z_{L3}, so the backward mode voltage amplitudes A_2 and A_4 can be expressed easily versus the forward mode voltage amplitudes A_1 and A_3. So, we will have

$$\begin{cases} V_1(z) = A_1 \left(e^{-\gamma_c z} + \Gamma^c_{L2} e^{\gamma_c z}\right) + A_3 \left(e^{-\gamma_\pi z} + \Gamma^\pi_{L2} e^{\gamma_\pi z}\right) \\ V_2(z) = A_1 R_c \left(e^{-\gamma_c z} + \Gamma^c_{L3} e^{\gamma_c z}\right) + A_3 R_\pi \left(e^{-\gamma_\pi z} + \Gamma^\pi_{L3} e^{\gamma_\pi z}\right) \end{cases} \quad (2)$$

where Γ^c_{L2} and Γ^π_{L2} are the reflection coefficients which can be defined by the following relation

$$\begin{cases} \Gamma^c_{L2} = \dfrac{Z_{L2} - Z_{c1}}{Z_{L2} + Z_{c1}} e^{-2\gamma_c z}, \quad \Gamma^\pi_{L2} = \dfrac{Z_{L2} - Z_{\pi 1}}{Z_{L2} + Z_{\pi 1}} e^{-2\gamma_\pi z} \\ \Gamma^c_{L3} = \dfrac{Z_{L3} - Z_{c2}}{Z_{L3} + Z_{c2}} e^{-2\gamma_c z}, \quad \Gamma^\pi_{L3} = \dfrac{Z_{L3} - Z_{\pi 2}}{Z_{L3} + Z_{\pi 2}} e^{-2\gamma_\pi z} \end{cases} \quad (3)$$

The forward voltage amplitudes A_1 and A_3 can be easily derived by setting $V_1(z=0) = V_2(z=0) = 1$ and solving a resultant system of equations as

$$\begin{cases} A_1 = \dfrac{R_\pi \left(1 + \Gamma^\pi_{L3}\right) - \left(1 + \Gamma^\pi_{L2}\right)}{\left[R_\pi \left(1 + \Gamma^c_{L2}\right)\left(1 + \Gamma^\pi_{L3}\right) - \left(1 + \Gamma^\pi_{L2}\right) R_c \left(1 + \Gamma^c_{L3}\right)\right]} \\ A_3 = \dfrac{R_c \left(1 + \Gamma^c_{L3}\right) - \left(1 + \Gamma^c_{L2}\right)}{\left[R_c \left(1 + \Gamma^c_{L3}\right)\left(1 + \Gamma^\pi_{L2}\right) - \left(1 + \Gamma^c_{L2}\right) R_\pi \left(1 + \Gamma^\pi_{L3}\right)\right]} \end{cases} \quad (4)$$

For better demonstration of how the proposed method can avoid narrow strip width, the forward voltage amplitudes A_1 and A_3 are derived for various strip widths and gap spacing of the a typical

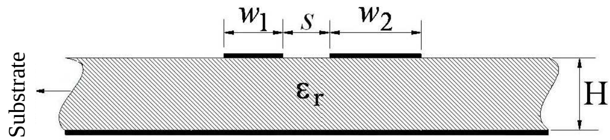

Figure 3. Cross section of the asymmetrical parallel coupled line.

Table 1. Various parameters of the asymmetrical coupled line including the normalized amplitudes of the forward propagating voltages along the line.

w_2 (mm)	s (mm)	Z_{c1} (Ω)	Z_{c2} (Ω)	$Z_{\pi1}$ (Ω)	$Z_{\pi2}$ (Ω)	β_c $\frac{\text{rad}}{\text{m}}$	β_π $\frac{\text{rad}}{\text{m}}$	R_c	R_π	A_1 (v)	A_3 (v)	A'_1 (v)	A'_3 (v)
$w_1 = 5$													
1	0.5	131	39.5	66	20	39.6	34.7	1.1	−0.26	0.77	0.16	0.76	0.09
1	0.25	140	39.9	58	16.5	39.5	34.3	1.14	−0.26	0.74	0.18	0.73	0.096
1.5	0.5	102	39.9	56	21.7	39.7	35	1.12	−0.34	0.79	0.27	0.77	0.16
$w_1 = 5.5$													
1	0.5	133	36.7	67	18.5	39.7	34.7	1.15	−0.24	0.76	0.17	0.74	0.09
1	0.25	142	37	58.4	15.3	39.7	34.3	1.11	−0.23	0.77	0.12	0.76	0.06
1.5	0.5	103	37	56	20	39.9	35	1.14	−0.31	0.79	0.27	0.77	0.15
$w_1 = 4.5$													
1	0.5	130	42.7	66	22	39.4	34.7	1.13	−0.29	0.74	0.2	0.72	0.11
1	0.25	138	43	57.2	18	39.3	34.2	1.1	−0.28	0.75	0.17	0.74	0.1
1.5	0.5	101	43	55	23.5	39.5	35	1.11	−0.38	0.79	0.28	0.76	0.17
$w_1 = 4.0$													
1	0.5	128	46.6	65	24	39.2	34.6	1.11	−0.32	0.73	0.21	0.72	0.13
1	0.25	136	47	56.4	19.6	39.2	34.2	1.08	−0.32	0.75	0.18	0.74	0.11
1.5	0.5	100	47	54.4	26	39.4	34.9	1.1	−0.43	0.78	0.29	0.76	0.18

asymmetrical coupled line with length of 40 mm. The cross section of the considered asymmetrical coupled line is shown in Fig. 3. The asymmetrical coupled line is on an FR-4 with $H = 1.57\,\text{mm}$ and relative dielectric constant of 4.4. The first load impedance (Z_{L2}) is selected equal to $100\,\Omega$ while the second load impedance (Z_{L3}) is selected equal to $25\,\Omega$. The inductance and capacitance matrices of the asymmetrical coupled line are obtained. Then, all parameters of the c and π modes are easily derived using inductance and capacitance matrices [18]. Finally, the amplitudes of the forward propagating modes are derived using Equation (4). The results are shown in Table 1.

As the table shows, the voltage amplitude of the c mode propagating along the nonsymmetrical coupled line is larger than the voltage amplitude of the π mode. Similarly, the amount of power supplied to the output ports are majority due to the c mode. So, the performance of the power divider is predominantly affected by the characteristics of the c mode, rather than the characteristics of the π mode.

As it was mentioned, the resistor was not considered in the above analysis which obviously adds some error. The resistor effect can be easily considered by placing magnetic and electric walls for the c and π modes, respectively [15] and decomposing resistor to four resistors for each mode and each line. The new resistors are now parallel to the load resistors. Here the resistor is selected to $100\,\Omega$. The analysis is then repeated and the new amplitudes for the forward and backward propagating modes are derived. They are added to the last columns of the Table 1, known as A'_1 and A'_3. Comparing the new amplitudes of the forward and backward propagating modes shows again that the amount of power delivered to the loads are predominantly due to the c mode.

Taking into account this fact, it is easy to explain how the proposed method can simultaneously provide power division with high dividing ratio and prevent narrow strip width. In this regards, the variation of the characteristic impedances of the c for various strip widths and gap spacing are shown in Fig. 4 to Fig. 6. An interesting fact indicated by the figures, the c mode characteristic impedance of the line with narrow strip width is impressively large in comparison to the isolated microstrip line with the same strip width. For example, for $w_1 = 0.5\,\text{mm}$, $w_2 = 5\,\text{mm}$ and $s = 0.2\,\text{mm}$ the c mode characteristic impedance of the narrow line is approximately equal to 200 ohm. While for an equivalent conventional microstrip line and for such impedance, the strip width is needed to be equal to just 0.046 mm.

3. ERROR FUNCTION CONSTRUCTION

According to the design method provided in [15], the couplings between asymmetrical coupled lines need to be first selected arbitrary. So, the designed Wilkinson power divider is not unique. In order to better control the strip width of the high impedance line, in the following design procedure both the strip width of the narrow microstrip line and the gap spacing are optimized. To achieve this goal, an error function is constructed as follows:

According to the Equation (2), the voltage across at nodes 2 and 3 can be represented as

$$\begin{cases} V_1(L) = A_1\left(e^{-\beta_c L} + \Gamma_{L2}^c e^{\beta_c L}\right) + A_3\left(e^{-\beta_\pi L} + \Gamma_{L2}^\pi e^{\beta_\pi L}\right) \\ V_2(L) = A_1 R_c\left(e^{-\beta_c L} + \Gamma_{L3}^c e^{\beta_c L}\right) + A_3 R_\pi\left(e^{-\beta_\pi L} + \Gamma_{L3}^\pi e^{\beta_\pi L}\right) \end{cases} \tag{5}$$

The input currents I_1 and I_2 are also found to be

$$\begin{cases} I_1(z=0) = \dfrac{A_1}{Z_{c1}}\left(1-\Gamma_{L2}^{c}\right) + \dfrac{A_3}{Z_{\pi 1}}\left(1-\Gamma_{L2}^{\pi}\right) \\ I_2(z=0) = \dfrac{A_1}{Z_{c2}} R_c \left(1-\Gamma_{L2}^{c}\right) + \dfrac{A_3}{Z_{\pi 1}} R_\pi \left(1-\Gamma_{L2}^{\pi}\right) \end{cases} \tag{6}$$

According to [4], for isolating the output ports, the resistor of the Wilkinson power divider have to be chosen equal to $1/\bar{Y}_{23}$. Where $\bar{Y}_{23}$ is the mutual admittance between the output ports. The mutual admittance $\bar{Y}_{23}$ can be derived as follows (see Appendix)

$$\begin{cases} \bar{Y}_{23} = -\dfrac{(C+D)(C+F)}{\Delta} + B \\ B = -\dfrac{Y_{c1}\coth\gamma_c L}{R_\pi\left(1-R_c/R_\pi\right)} - \dfrac{Y_{\pi 1}\coth\gamma_\pi L}{R_c\left(1-R_\pi/R_c\right)} \\ C = \dfrac{Y_{c1}}{(R_\pi - R_c)\sinh\gamma_c L} + \dfrac{Y_{\pi 1}}{(R_c - R_\pi)\sinh\gamma_\pi L} \\ D = -\dfrac{Y_{c1}}{(1-R_c/R_\pi)\sinh\gamma_c L} - \dfrac{Y_{\pi 1}}{(1-R_\pi/R_c)\sinh\gamma_\pi L} \\ F = -\dfrac{R_c Y_{c2}}{R_\pi\left(1-R_c/R_\pi\right)\sinh\gamma_c L} - \dfrac{R_\pi Y_{\pi 2}}{R_c\left(1-R_\pi/R_c\right)\sinh\gamma_\pi L} \end{cases} \tag{7}$$

The error function is constructed according to the following fact. Form Fig. 2, the desired power division is achieved if the voltage difference

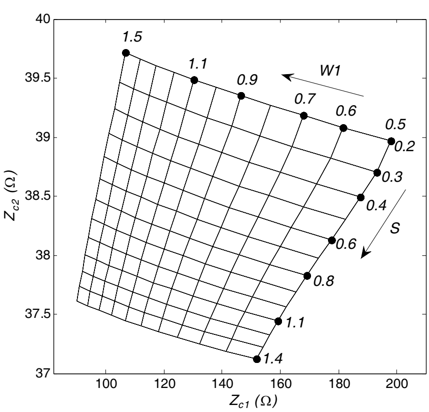

Figure 4. Variation of the c-mode characteristic impedances versus s (mm) and w_1 (mm) for $w_2 = 5$ mm.

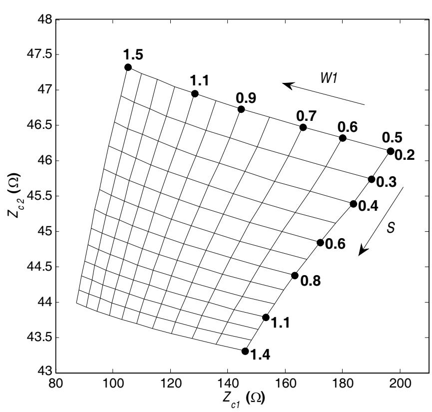

Figure 5. Variation of the c-mode characteristic impedances versus s (mm) and w_1 (mm) for $w_2 = 4$ mm.

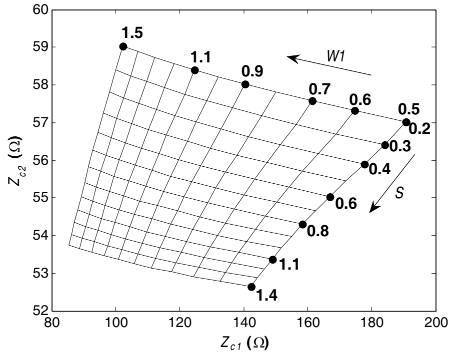

Figure 6. Variation of the c-mode characteristic impedances versus s (mm) and w_1 (mm) for $w_2 = 3$ mm.

across the resistor R becomes zero. The input port of the Wilkinson power divider shown in Fig. 1 is also matched if the input admittance of the model shown in Fig. 2 becomes equal to $1/Z_0$. Besides, the output ports are isolated if the imaginary part of $(\bar{Y}_{23})$ becomes zero and the resistor R is selected to be equal to $1/\bar{Y}_{23}$. So the following error function is defined

$$e = \left|\left[(I_1 + I_2) - \frac{1}{Z_0}\right]\right| + |(V_1(L) - V_2(L))| + \beta\,|\mathrm{Im}\,(Y_{23})| \qquad (8)$$

where the first term is related to the input matching. β is a weighting constant which is selected empirically equal to 100. Considering small values of β leads to low isolation between the output ports, while considering large values of it degrade the input matching and the power division ration of the power divider. The physical dimensions of the uniform asymmetrical coupled lines are derived by applying the optimization to the defined error function.

4. DESIGNING OF THE POWER DIVIDER AND RESULTS

In order to verify the proposed design method, a sample of the power divider with 4 : 1 power division is designed for the FR4 substrate with a thickness of 1.57 mm and approximate dielectric constant equal to 4.4. The minimization of the error function is done using the "Fmincon" function of MATLAB software. The c and π mode parameters of the asymmetric microstrip coupled lines are derived according to [17, 18] and they are used in the computer program. The terminated value of the error function is equal to 0.029.

The error function is minimized with respect to some dimensions of the power divider, namely w_1, s and L, while other dimensions are fixed. The results show that the selection of the initial dimensions is not critical. The results also show that the designed Wilkinson power divider is not unique. On the other hand, as the initial dimensions are changed the final dimensions are also changed. The initial and final dimensions of the designed Wilkinson power divider are shown in Table 2. The value of the isolation resistor is obtained by the program equal to $126\,\Omega$ ohm. However it is selected equal to $130\,\Omega$ instead of the required $126\,\Omega$ due to the unavailability of $126\,\Omega$ resistor.

Since the main goal of the paper is to solve the drawbacks of narrow strip width of the conventional unequal Wilkinson power divider. It is desirable to compare the dimensions of the designed power divider to that of the conventional unequal Wilkinson power divide with the same specification. In this regard, another microstrip Wilkinson power divider with the same unequal power-split ratio is designed. Fig. 7 shows the layout of the designed conventional Wilkinson power divider including its various parameters. The various dimensions of this divider are also shown in Table 3. A comparison between the results of Tables 2 and 3 shows that in the designed Wilkinson power divider both of the w_1 and w_2 are increased in comparison to the conventional Wilkinson power divider, while the other dimensions are almost unchanged. As the tables show, the width of the narrow line of the designed Wilkinson power divider is 0.59 mm while for the conventional Wilkinson power divider it is equal to just 0.15 mm. It is also indicated by the tables that the size of the entire circuit of the designed Wilkinson power divider is not considerably different form the size of the conventional Wilkinson power divider.

Table 2. Dimensions of the designed Wilkinson power divider on FR4 in millimeter.

<table>
<tr><th></th><th>Optimized Values</th><th>Fixed Values</th></tr>
<tr><td>Initial Values</td><td>$w_1 = 0.9$, $s = 0.3$,
$L = 40$</td><td rowspan="2">$w_2 = 5.0$, $w' = 1.62$,
$L' = 42$, $w' = 5.3$
$L'' = 40$</td></tr>
<tr><td>Final Values</td><td>$w_1 = 0.59$, $s = 0.5$,
$L = 42.8$</td></tr>
</table>

Table 3. Dimensions of the conventional Wilkinson power divider on FR4 in millimeter.

w_1	w_2	L_1	L_2	w'	L'	w''	L''
0.15	4.5	44.52	40.3	1.62	4	5.3	40

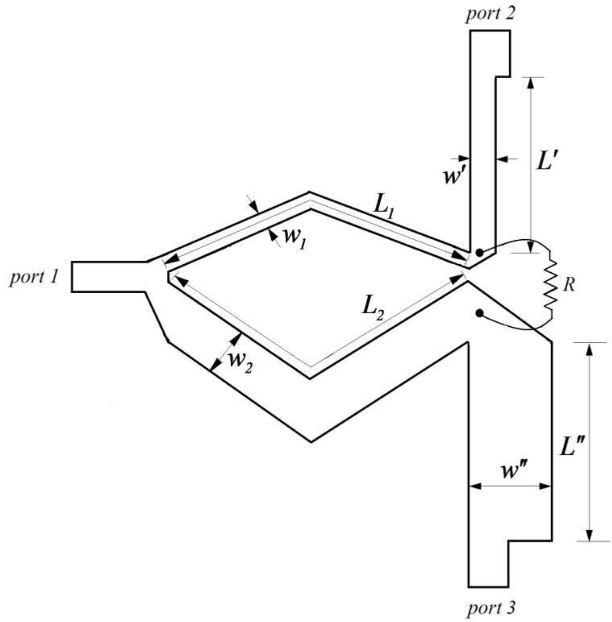

Figure 7. Layout of the conventional Wilkinson power divider.

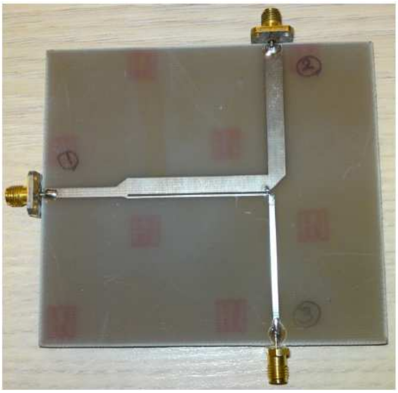

Figure 8. Photograph of the fabricated power divider.

The designed Wilkinson power divider has been fabricated over the FR4 board by employing the conventional photolithography and then tested. Fig. 8 shows the fabricated circuit. The simulation of the designed power divider has been carried out using HFSS. The

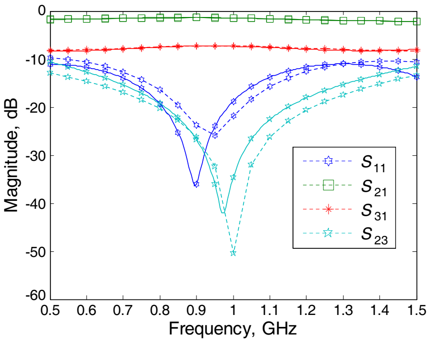

Figure 9. Measured (solid line) and simulated (dotted line) scattering parameters of the designed Wilkinson power divider.

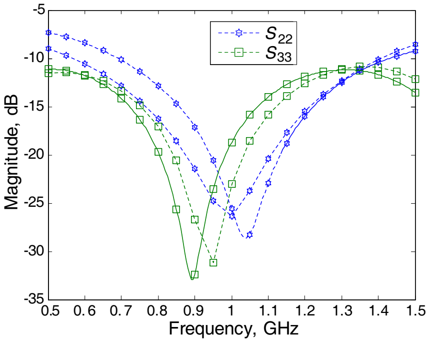

Figure 10. Measured (solid line) and simulated (dotted line) return losses at the output ports of the designed Wilkinson power divider.

simulation and the measurement results are shown in Fig. 9.

As the figure shows, the power division between output ports is approximately flat in entire bandwidth. The power division between port 2 and 3 at midband is equal to 1.4 dB and 7.3 dB, respectively. In comparison to the desired power division, there is an extra loss which is due to the high loss tangent of the FR4 substrate. Comparing the simulation and measurement return loss and isolation of the power divider also shows that they are in good agreement, except some frequency shift which may result from the fabrication tolerances.

Measured and simulated return losses at the output ports of the designed Wilkinson power divider are shown in Fig. 10. As the figure shows the return losses at the output ports are also acceptable in a large fraction of bandwidth.

5. CONCLUSION

The uniform asymmetrical microstrip coupled line was used to design the Wilkinson power divider with high dividing ratio. The results show that using the proposed method can eliminate the difficulty of narrow strip width of the conventional Wilkinson power divider with high dividing ratio. The simulation and measurement results of the designed power divider indicated the effectiveness of the proposed method and its design procedure.

APPENDIX A. DETERMINATION OF THE MUTUAL ADMITTANCE BETWEEN THE OUTPUT PORTS

In order to obtain the mutual admittance between the output ports namely $\bar{Y}_{23}$. One can considers the simple equivalent circuit of the proposed Wilkinson power divider, as shown in Fig. A1. This equivalent circuit is derived by terminating the input port of the proposed Wilkinson power divider to the match load and neglecting the load impedances. The admittance matrix of a uniform asymmetrical coupled-line can be expressed as

$$\begin{cases} Y_{11} = Y_{22} = \dfrac{Y_{c1}\coth\gamma_c l}{(1-R_c/R_\pi)} + \dfrac{Y_{\pi 1}\coth\gamma_\pi l}{(1-R_\pi/R_c)} = A \\ Y_{13}=Y_{32}=Y_{24}=Y_{42}=-\dfrac{Y_{c1}\coth\gamma_c l}{R_\pi(1-R_c/R_\pi)} - \dfrac{Y_{\pi 1}\coth\gamma_\pi l}{R_c(1-R_\pi/R_c)} = B \\ Y_{14}=Y_{41}=Y_{23}=Y_{32}=\dfrac{Y_{c1}}{(R_\pi-R_c)\sinh\gamma_c l}+\dfrac{Y_{\pi 1}}{(R_c-R_\pi)\sinh\gamma_\pi l}=C \\ Y_{12}=Y_{21}=-\dfrac{Y_{c1}}{(1-R_c/R_\pi)\sinh\gamma_c l}-\dfrac{Y_{\pi 1}}{(1-R_\pi/R_c)\sinh\gamma_\pi l}=D \\ Y_{33} = Y_{44} = -\dfrac{R_c Y_{c1}\coth\gamma_c l}{R_\pi(1-R_c/R_\pi)} - \dfrac{R_\pi Y_{\pi 1}\coth\gamma_\pi l}{R_c(1-R_\pi/R_c)} = E \\ Y_{34}=Y_{43}=\dfrac{R_c Y_{c2}}{R_\pi(1-R_c/R_\pi)\sinh\gamma_c l}+\dfrac{R_\pi Y_{\pi 2}}{R_c(1-R_\pi/R_c)\sinh\gamma_\pi l}=F \end{cases} \tag{A1}$$

The currents and voltages at the input ports of the asymmetrical coupled-line are related as

$$\begin{cases} I_1 = AV_1 + DV_2 + BV_3 + CV_4 \\ I_2 = DV_1 + AV_2 + CV_3 + BV_4 \\ I_3 = BV_1 + CV_2 + EV_3 + FV_4 \\ I_4 = CV_1 + BV_2 + FV_3 + EV_4 \end{cases} \tag{A2}$$

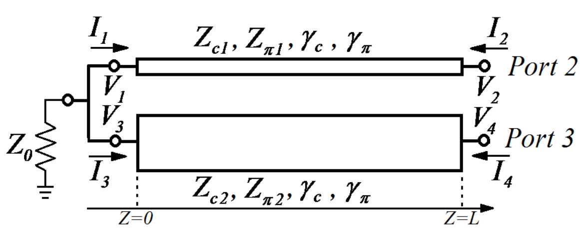

Figure A1. The equivalent circuit of the proposed Wilkinson power divider when the input port is terminated and outputs are excited.

The currents and voltages at the input port of the Fig. A1 are related as

$$\begin{cases} V_1 = V_3 = V \\ I_1 + I_3 = -VY_0 \end{cases} \tag{A3}$$

where Y_0 is equal to $1/Z_0$. Combining Equations (A1) and (A3) leads to

$$V = -\frac{[D+C]}{\Delta}V_2 - \frac{[C+F]}{\Delta}V_4 \tag{A4}$$

where:

$$\Delta = A + 2B + E + Y \tag{A5}$$

Substitution (A5) into (A3) through some manipulation yield:

$$\begin{cases} I_2 = V_2\left\{\left[-\frac{(D+C)(C+D)}{\Delta}\right]+A\right\}+V_4\left\{\left[-\frac{(F+C)(C+D)}{\Delta}\right]+B\right\} \\ I_2 = V_2\left\{\left[-\frac{(F+C)(C+D)}{\Delta}\right]+B\right\}+V_4\left\{\left[-\frac{(F+C)(C+D)}{\Delta}\right]+E\right\} \end{cases} \tag{A6}$$

Finally, from (A6), one can obtain the mutual admittance between output ports as

$$\bar{Y}_{23} = -\frac{(C+D)(C+F)}{\Delta} + B \tag{A7}$$

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