

AIRCRAFT-LIGHTNING ELECTRODYNAMICS USING THE TRANSMISSION LINE MODEL PART I: REVIEW OF THE TRANSMISSION LINE MODEL

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Abstract—This work presents a self-consistent and self-contained model to study and analyze aircraft-lightning electrodynamics. In this paper, we review the well developed and reported transmission line model of the cloud-to-ground (CG) lightning return stroke. Subsequently, the incorporation of a circuit model of the aircraft into the return stroke model is considered. The direct hit characteristics of aircraft body lightning currents for both CG and GC (ground-to-cloud) are important when designing protection, shielding and filtering systems for airborne electronic and electrical systems within the aircraft system. Moreover, the model will allow design of aircraft structure and geometry to minimize energy dissipation into the aircraft structure and systems. Basic electromagnetic theory is used to show the validity of considering the return stroke as a transverse magnetic wave along a transmission line. A distributed transmission line model for the aircraft and the return stroke channel of the lightning is used to simulate the return strokes of CG and GC flashes. The effects of the aircraft geometry with sharp edges are included in the computation of aircraft capacitance values, both distributed as well as lumped values.

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The paper compares electric currents, channel voltages, the rate of change of current and the frequency spectrum along the lightning channel of the return strokes for CG and GC flashes with aircraft attached to the channel.

1. INTRODUCTION

Lightning is a large electrical discharge process that has adverse impact on an aircraft and electronic devices within it. Every commercial airliner could be struck by lightning during landing or take-off under a thundercloud. Video frames obtained at Kamatzu Air Force Base, Japan and San Francisco, California show an airliner struck by lightning soon after take-off with no fatalities being reported [1]. A commercial aircraft can expect a minimum of one lightning strike between 1000 and 10000 hours of flight, where it is generally accepted that the lightning strikes at least once for every 3000 hours of flight or about once a year [2,3]. Several lightning strike events have been reported as well as structural damage to the aircraft, communication devices and passengers held within. The physical effects of the lightning on aircraft are generally minimal, although the consequences of the interaction in the form of direct and indirect effects on aircraft structure as well as electric power, communication and navigational systems can be costly or disastrous.

The common type of lightning strike is the cloud-to-ground (CG) flash which occurs when the negative electric charges in the thundercloud emanate and travel to ground. When the downward leader connects with ground, a return stroke is produced and a bright return stroked wave travels from ground to cloud (Fig. 1(a)). Another less common type of lightning strike is the ground-to-cloud (GC) flash which emanates from sky-scraping ground objects under the thundercloud such as mountains and tall towers. The return stroke of this type is from cloud to ground (Fig. 1(b)). While including the GC and CG flashes in the circuit model, this paper considers a third type of flash initiated by an aircraft flying under a thundercloud. The lightning return strokes radiate powerful electromagnetic fields which may cause damage to aircraft avionics, telecommunication systems and power systems [3].

The aircraft-lightning electrodynamics must take into account the following two facts: (a) the aircraft is an electric conductor carrying the lightning stroke wave [4–7], and (b) the lightning channel is an electric plasma phenomenon [8–12]. Section 2 explains the electromagnetic nature of the return strokes. Recent work on more detailed incorporation of the effects of grounding on the earth end of

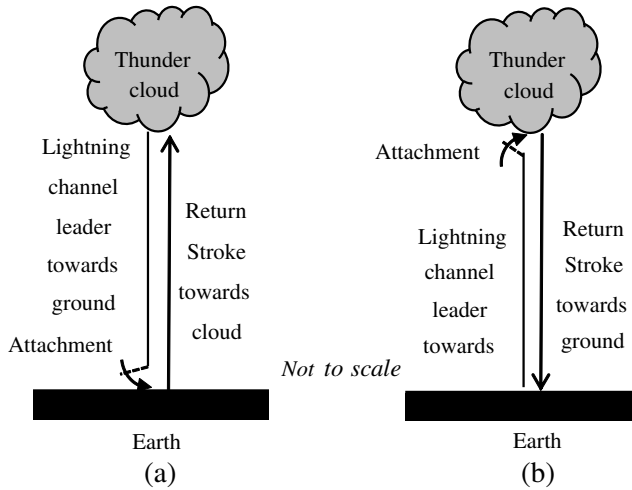


Figure 1. The representation of return strokes of (a) CG lightning, (b) GC lightning.

the return stroke [9] is not used in the work reported herein. The details of the transmission line elements used to model the return strokes [4–7] are given in Section 3. The transmission line model reported herein is based on one of the earliest reported works in book form [9, 12]. Section 4 describes the transmission line model of the return strokes and finally Section 5 reports the aircraft model incorporated into the transmission line model to find the current and voltage waveforms on the aircraft skin when it is struck by lightning.

2. ELECTROMAGNETIC NATURE OF THE RETURN STROKE

The transmission line model of guided electromagnetic waves in coaxial cables is well established [13]. In this section, we shall establish the validity of the transmission line model of a vertically standing electrically conducting channel with no outer conductor, as presented in [4]. We show that the wave number h (where $h = \beta - i\alpha$, $i^2 = -1$, and switching from engineering terminology to the subject literature, β is the attenuation constant and α is the phase constant) for a transverse magnetic (TM) wave along a single conductor transmission line models the lightning channel carrying a TM return stroke wave [4, 6]. From Maxwell's equations we obtain the following equation for the electric

field [13]:

$$\nabla^2 \underline{E} - \mu\sigma \frac{\partial \underline{E}}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 \underline{E}}{\partial t^2} = \nabla \rho / \varepsilon_0 \quad (1)$$

in standard notation. The charge relaxation time in a linear, homogeneous and isotropic conductor is given by $\tau = \varepsilon_0 / \sigma = 2 \times 10^{-5}$ seconds for $\sigma = 4242 \Omega^{-1} \text{m}^{-1}$ [4, 11]. So we may assume that the net free charge within the conductor rapidly vanishes and that any excess charge is located on the surface of the conductor. Hence we may drop the fourth term in Equation (1) and solve for E . For a wave propagating in the positive z direction:

$$\nabla^2 E_z - \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2} - \mu\sigma \frac{\partial E_z}{\partial t} = 0 \quad (2)$$

We set $E_z = E(R) \exp(i(-\omega t + hz))$, where R measures phase shift in the direction r and h is the vertical (z -directed) wave number. The electric field has a small component along the z axis, as well as in the r direction — both are traveling waves.

Defining the propagation constant k according to

$$k^2 = \varepsilon_0 \mu_0 \omega^2 - i\mu_0 \sigma \omega \quad (3)$$

$$R = r \sqrt{k^2 - h^2} \quad (4)$$

and permittivity, for $f \ll 1/2\pi(\sigma/\varepsilon_0) = 8 \times 10^{12} \text{ Hz}$ (with $\sigma = 4242 \Omega^{-1} \text{m}^{-1}$ for an ionized lightning channel [4])

$$\varepsilon_p = \varepsilon_0 + i\sigma/\omega \quad (5)$$

$$= i\sigma/\omega = \varepsilon_{pi} \quad \text{inside the lightning channel} \quad (6)$$

$$= \varepsilon_0 = \varepsilon_{pe} \quad \text{outside the lightning channel} \quad (7)$$

We shall consider the transverse magnetic wave, where only B_ϕ , E_z , and E_r have nonzero values. From Faraday's law we obtain,

$$i\omega B_\phi = ihE_r - \partial E_z / \partial r \quad (8)$$

From Ampere's law in Maxwell's equations, we get

$$-\mu_0 \varepsilon_p i\omega E_r = -ihB_\phi \quad (9)$$

From Equations (8) and (9), we get

$$B_\phi = [i\mu_0 \varepsilon_p \omega / (\omega^2 \mu_0 \varepsilon_p - h^2)] \partial E_z / \partial r \quad (10)$$

We have dropped the factor $\exp(i(-\omega t + hz))$ which is common to B_ϕ , E_z , and E_r . Note that $h = \beta - i\alpha$, $ih = \alpha + i\beta$. Once E_z is solved for, B_ϕ and E_r , may be determined from Equations (9) and (10). Now Equation (2) becomes the Bessel's equation

$$d^2 E_z / dR^2 + (1/R) dE_z / dR + E_z = 0 \quad (11)$$

For the axially symmetric solution of Equation (11), mode $n = 0$, and for E_z , to be finite at the axis of the conductor and everywhere else, the solution for $0 < r < a$ is

$$E_z = a_0 J_0(R) \quad (12)$$

where a_0 is a constant and J_0 the Bessel's function of zeroth order. Outside the conductor, remembering that open space surrounds the vertical lightning channel, for complex values of R only H_0^1 the Hankels' function of the first kind vanishes as r goes to infinity on the positive imaginary half plane of R . Hence for $a < r < \infty$, we get

$$E_z = b_0 H_0^1(R) \quad (13)$$

Substituting Equations (12) and (13) into Equation (10), we get,

$$B_\phi = ik\sqrt{[\mu_0\varepsilon_p/(k^2 - h^2)]} a_0 [dJ_0(R)/dR] \quad 0 < r \leq a \quad (14)$$

$$= ik\sqrt{[\mu_0\varepsilon_p/(k^2 - h^2)]} b_0 [dH_0^1(R)/dR] \quad a < r \leq \infty \quad (15)$$

The general permittivity has been retained to keep the expressions neat. Both B_ϕ and E_z must satisfy the continuity conditions at the boundary. When $r = a$, substituting $r = a$ into the two pairs of Equations (12) and (13), and (14) and (15), we obtain two equations; dividing one by the other and rearranging, we get,

$$\begin{aligned} & [\sqrt{(k_e^2 - h^2)}/k_e/\sqrt{(\mu_0\varepsilon_{pe})}] \times \{H_0^1(R_e)/[dH_0^1(R_e)/dR]\} \\ & = \sqrt{(k_i^2 - h^2)}/k_i/\sqrt{(\mu_0\varepsilon_{pi})} \times \{J_0(R_i)/[dJ_0(R_i)/dR]\} \end{aligned} \quad (16)$$

where subscripts i and e stand for internal to the conductor and external to the conductor, respectively. k_e (with $k_e^2 = \omega^2(\mu_0\varepsilon_0)$) is the k of Equation (3) external to the conductor and k_i (a complex number) is the k of Equation (3) inside the conductor. For small values of R_e , $H_0(R_e)$ can be approximately represented by

$$H_0(R_e) = (2i/\pi) \log_e(\eta R_e/2i); \quad dH_0^1(R_e)/dR = 2i/\pi R_e \quad (17)$$

where $\eta = 1.781$, the Euler-Mascheroni constant. Hence we may rewrite (16) as [4]

$$\begin{aligned} & -(2/\eta)^2 (\eta R_e/2i)^2 \ln_e[\eta R_e/2i]^2 \\ & = J_0(R_i)/[dJ_0(R_i)/dR] \times 2ak_e(\varepsilon_{pe}\varepsilon_{pi})^{1/2} \end{aligned} \quad (18)$$

The solution of Equation (18) is carried out on a digital computer. The lightning light intensity measured for the return stroke and the current peaks that are determined by the phase constant of the wave number of Equation (18) match as shown in Table 1 derived by us from plots in [4].

For large values of channel radius R_e , the transverse magnetic waves are illusory; the transverse electric values too are illusory and

Table 1. Time for current peak at different heights with return stroke light intensity.

Height from ground (m)	Time to peak of return stroke light intensity (μs)	Time to peak of transmission line transverse magnetic (TM) waves (μs)
400	5	4.9
800	10	9
1600	20	18
2000	28	25

will not be observed. The signal velocity is the velocity of the principal part of the disturbance. It is this main signal which would actuate a measuring device [14]. The axial electric field E_z , which travels at the signal velocity causes the free electrons to drift giving rise to impact excitation with a radiative life time of about 0.1 ns [11]. It is reasonable to expect the observed speed of the luminous process of the lightning return stroke to be associated with the signal velocity. The velocity photographically measured is about 80 m/ μs . In the case of normal dispersion, the signal velocity coincides with the group velocity [9]. The group velocities obtained from Equation (18) for a wave train centered around 5 kHz is about 90 m/ μs , a value in good agreement with observed return stroke velocities [14].

In the 3 km long transmission line model for a 3 km long lightning channel, we may take the concentration of field (or current) in space to indicate that the energy is localized in that region [6]. For resistances of 1, 2 and 5 Ω/m , the times at which the current peaks at different points on the line were compared with peak return stroke light intensity by Hoole and Hoole [4]. Both plots agree well as reported, showing a largely constant velocity of about 80 m/ μs along the channel [4].

3. THE TRANSMISSION LINE ELEMENTS

The return stroke of the lightning channel is considered as an electromagnetic wave [4] traveling over a lossy leader channel represented by a vertical, cylindrical conductor. The electrical parameters R (resistance), L (inductance) and C (capacitance) are calculated for the lightning channel. In order to determine the capacitance and inductance of the lightning electrode system, the simplified model described in Fig. 2 is used [6].

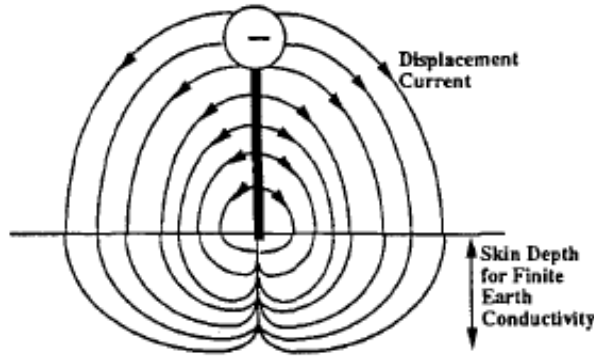


Figure 2. The assumed lightning electrode system [5, 6].

Capacitance values are calculated considering both the cloud and lightning channel separately from the skin effect of the earth which is neglected. Since the cloud capacitance will dominate at the cloud end, the capacitance along the channel will not have any significant effect at the cloud end when the cloud and channel are put together. Hence the increase in channel capacitance at the top is calculated without the presence of the lightning channel. A complete review is given by the references [5–7].

3.1. The Inductance

Figure 3 illustrates the current carrying wire adopted in the computation of inductance. The earth's effect is to extend the wire length. The current flows in the same direction as the image element, since lowering the negative electric charge will be seen as a movement of positive electric charge in the image. Hence the system appears as an isolated wire, for which the per unit length inductance, discarding the end effects, is given by [6]:

$$L = \frac{\mu_0}{2\pi} [0.25 + \log_e (D/a)] \text{ H/m} \quad (19)$$

where a is the radius of the conductor and D the far distance from the wire at which the field is considered zero. D should be kept large for transient fields since the radiation component of the transient magnetic field decays as with $1/D$ in contrast to $1/D^2$ for static magnetic fields. The fractional error is of the order of $1/\log_e(D/a)$ which is deemed to be insignificant, since $D \gg a$; for example, in a typical system with $D = 100 \text{ km}$ and $a = 1 \text{ cm}$. Be it noted that the internal

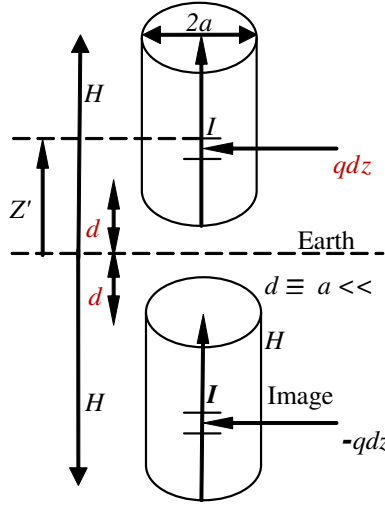


Figure 3. Definition of problem to determine L and C [5, 6].

inductance is included in Equation (19) to account for the uniform current distribution.

3.2. The Capacitance

To determine the capacitance, a charge simulation method is used in the computation of the charge to potential ratio q/V , where V is the potential of the wire with respect to earth. For the electrode system illustrated in Fig. 4 ignoring the presence of the image, the potential at point P , due to a line charge of $q \text{ cm}^{-1}$ placed at the center of the conductor of length l is given by

$$V_p = \frac{q}{4\pi\epsilon_0} \int_0^l \left(\frac{1}{\sqrt{(z' - z)^2 + a^2}} \right) dz \quad (20)$$

$$= \frac{q}{4\pi\epsilon_0} (\log_e z_1 - \log_e z_2) \quad (21)$$

$$\text{where } z_1 = z_A + \sqrt{z_A^2 + 1} \quad (22)$$

$$z_A = \frac{l - z'}{a} \quad (23)$$

$$z_2 = z_B + \sqrt{z_B^2 + 1} \quad (24)$$

$$z_B = -\frac{z'}{a} \quad (25)$$

As in Fig. 3, the image may be placed to substitute for the earth (which is assumed to be a perfect conductor), and its contribution added to the potential due to the real conductor. The earth's effect reduces the potential on the conductor, particularly at a point closer to the earth. The potential at three points are computed and an average is ironed out to calculate the capacitance more accurately for the lower segment along the channel closer to earth. An equivalent π -network [8] is constructed, the capacitance being determined from getting q/V_p from Equation (20). The lowest point at which the capacitance is calculated is at a point which is a m above the earth. Except for the capacitance values in the vicinity of the earth, the capacitance conforms to the relation of $\frac{1}{\sqrt{LC}} \approx c$, where c is the velocity of light. In the vicinity of the earth, at $a = 4$ mm, $C = 24.5$ pFm⁻¹, $L = 3.3$ μ Hm⁻¹, whereas at a height of 1 km above ground, when the $\frac{1}{\sqrt{LC}} \approx c$ relation applies, $C = 4.3$ pFm⁻¹, $L = 3.3$ μ Hm⁻¹.

In order to ensure that the charge structure used to determine the capacitance does not portray any significant discrepancy when plugged into the lightning channel, the charge distribution along the lightning channel and aircraft are examined in more detail in the following analysis. This is necessary to include the edge effects of lightning channel tip, as well as at sharp edges of the aircraft. As in Fig. 4, consider a drop in charge per unit length from the tip (q_2) to the cloud end of the channel (q_1) in a step like function which is linearized to give the charge at height z (zero at ground) to be:

$$q = q_2 + \frac{z}{l}(q_1 - q_2) \quad (26)$$

where l is the length of the channel. The potential at point P on the

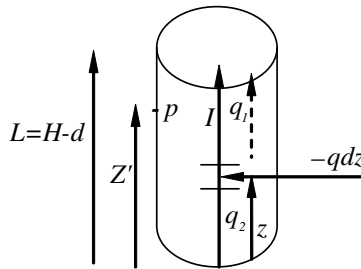


Figure 4. Application of the charge simulation method [5, 6].

conductor channel at height z' is given by

$$V_p = \frac{1}{4\pi\epsilon_0} \times \left[\frac{z'}{l} q_1 + \left(1 - \frac{z'}{l} \right) q_2 \right] (\log_e z_1 - \log_e z_2) + \frac{1}{4\pi\epsilon_0} (q_1 - q_2) \frac{a}{l} \left[\sqrt{z_A^2 + 1} - \sqrt{z_B^2 + 1} \right] \quad (27)$$

For a quick check on the validity of Equation (27), if the term containing $(q_1 - q_2)$ is negligibly small in Equation (27) then Equation (21) is obtained, and the ratio of q_1/q_2 will not significantly influence the final results.

Let:

$$T_1 = \frac{z'}{L} (\log_e z_1 - \log_e z_2) + I_1 \quad (28)$$

$$T_2 = \left(1 - \frac{z'}{L} \right) (\log_e z_1 - \log_e z_2) + I_2 \quad (29)$$

$$T_3 = \frac{a}{L} \left[\sqrt{z_A^2 + 1} - \sqrt{z_B^2 + 1} \right] + I_3 \quad (30)$$

where the earth has been replaced by the image of the source. I_1 , I_2 and I_3 indicate the contributions of the images carrying opposite polarity of charge. Using Equations (26) and (27), the capacitance q/V_p may be calculated [6].

The capacitance of the cloud is dependent on the radius of the sphere of charge attached to the leader. The radius is estimated to vary from 100–500 m [6]. In general, there are at least two electric charge centers containing a volume distribution of charges. One is the positive electric charge center at the top of the thundercloud. The other is the negative electric charge center in the lower part of the thundercloud, to which the lightning channel is attached. For a charge density of 10 Ckm^{-3} , the charge in a center is 5 C for a radius of 500 m, and the potential is about 80 MV. The cloud capacitance is calculated using the model of an isolated charged sphere from [20]

$$C_c = 4\pi\epsilon_0 r_1 \quad (31)$$

where r_1 is the radius of the cloud and ϵ_0 the permittivity constant. When the cloud is closer to the earth, the capacitance is pC_c , where $p = (1 - k + k^2)$, $k = R_1/2h$ and h is the height of the center of the sphere above the earth. For most cases $p = 1$. Note that the spherical region of the cloud charge is the region that is electrically active and connected to the lightning channel, and is sometimes visible to the naked eye.

3.3. The Resistance

The resistance of the column is given by $R = 1/\sigma\pi r^2 \Omega\text{m}^{-1}$. The radius of the return stroke channel is assumed to remain constant throughout the presence of the return stroke, where the constant radius is maintained by equal and counteracting magnetic and kinetic forces [9]. The conductivity of a highly ionized gas is given by $\sigma = j/E = eN\mu^-$ [10] where the electron mobility μ^- is given by $\mu^- = (e\lambda_e/mcT)$ [10]. The mean free path is $\lambda_e = 1/Nq$ and the root-mean-square velocity $C_T = (3\text{kT}/m)^{0.5}$. The collision cross-section for strongly scattered electrons is given by $q = (e^2/3\text{kT})^2/16\pi\epsilon_0^2$. Substituting these values into $\sigma = eN\mu^-$ gives $\sigma = 1.5 \times 10^{-5} \times T^{3/2} (\Omega\text{cm})^{-1}$ where a factor of 0.1 (from $1/1n\Lambda$, where Λ is the Debye length or critical impact parameter) is included for shielding effects [11]. The expression $\log_e V$ varies slowly with electron density and electron temperature [6]. Taking $r = 1\text{ cm}$ and $T = 20,000\text{ K}$ or $r = 4\text{ mm}$ and $T = 20,000\text{ K}$, channel resistances of approximately $1\Omega\text{m}^{-1}$ and $5\Omega\text{m}^{-1}$ respectively are obtained. For $T = 20,000\text{ K}$, the column conductivity $\sigma = 4242 (\Omega\text{m})^{-1}$ and for $T = 16,000\text{ K}$ [12], $\sigma = 3035 (\Omega\text{m})^{-1}$.

The earth resistance is included in the model, representing energy dissipation in the earth, accounting for the finite conductivity of the earth. In order to maintain a charge flow the finite earth conductivity maintains a small horizontal field, which has been ignored in previous studies in the calculation of capacitance values [5]. When lightning strikes the open ground, earthing is provided merely by loose attachment of the channel to the ground [15]. After a complete flash, the channel is found to have penetrated the earth to a depth of about 0.5 m. The channel radius is about 1 cm [13, 14]. Although the value of the earth resistance will slightly drop for subsequent strokes, its effects will not be significant over the few tens of micro-seconds. As for the flashes in an open terrain, the earth resistance is calculated for a spherical conductor, given by $R_E = 1/\sigma_E\pi a$, where a is the radius of the return stroke [15] and σ_E is the conductivity of the earth. As described in [15], a typical value of R_E is 200Ω , for earth conductivity σ_E of $0.01 (\Omega\text{m})^{-1}$. When lightning strikes an earthed conductor, R_E is taken to be $(1/2\pi\sigma_E l) \log_e(8l/1.36a)$, where l is the length of the conductor under the earth surface. Soil conductivity in hilly areas is an order less than that of flat terrains, because of different soil constituents, which influences R_E and terrain effects may also be examined using the transmission line model.

4. TRANSMISSION LINE MODEL OF LIGHTNING RETURN STROKES

A reliable distributed non-linear transmission line model with the elements of inductive (L), capacitance (C) and resistive (R) circuit parameters representing the lightning channel and accounting for the power dissipated and energy oscillations between L and C in the return strokes are presented here. The RLC model is considered consistent with the reported measurements [16, 17]. The model has already been verified for the CG lightning return stroke by Hoole and Hoole [7]. The transmission line characteristics of the lightning pulse are computed using the finite difference time domain method (FDTD) to solve the wave equation, with proper numerical computational steps ensuring the stability and the accuracy of this model [18]. A much ignored flash, that is the CG flash, is included in the transmission line computational technique reported herein. It is important to determine the GC lightning flash parameters (i.e., upward leader and downward return stroke) for safety testing of aircraft-lightning interaction. An RLC transmission line circuit model is adopted to represent the lightning channel of the first return stroke for both types of lightning strikes, i.e., CG and GC. The transmission line model uses the fact that the lightning channel may be likened to a long conductor and employs the typical RLC representation of a single conductor line [6]. Two independent RLC models are presented for both return stroke of CG and GC lightning strikes in Figs. 5(a) and 5(b), respectively.

The wave equations for current and the voltage V traveling on the distributed transmission line are given by Equations (32) and (33) [20]:

$$i(z, t) = -C \int_0^z \frac{\partial V}{\partial t} dz \quad (32)$$

and

$$\frac{\partial^2 V(z, t)}{\partial z^2} - RC \frac{\partial V(z, t)}{\partial t} - LC \frac{\partial^2 V(z, t)}{\partial t^2} = 0 \quad (33)$$

where V is the potential while R , L and C are the resistance per unit length, inductance per unit length and capacitance per unit length, respectively [6]. The actual value for these parameters would vary with the thunderstorm dependency of the geographical location.

The required parameters for R , L and C described in Section 3 are plugged into the transmission line lightning simulation parameters. The cloud voltage is taken to be -50 MV, assuming that the cloud's earth flash charge is stored in a spherical charge center at a presumed height of 1000 m above ground [6, 19]. The 1000 m long conducting

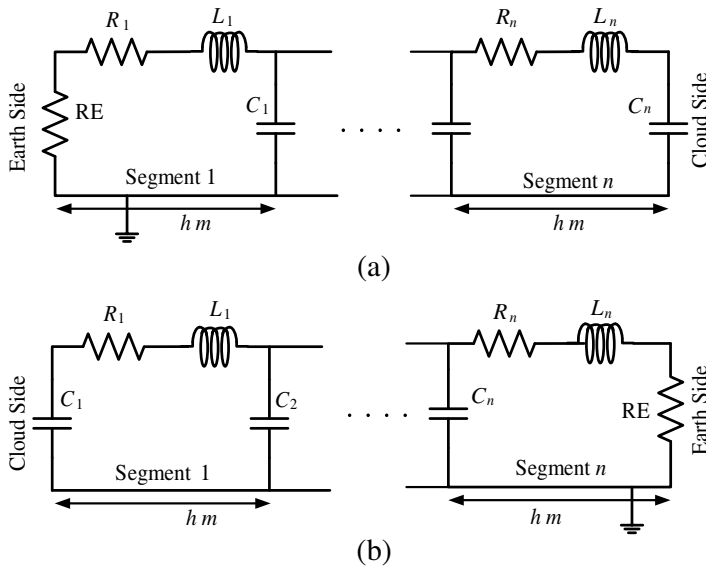


Figure 5. Transmission line models of return strokes. (a) CG lightning, (b) GC lightning.

channel is divided into 10 equivalent segments in the transmission line. Each segment is represented by a combination of the R , L and C elements in order to compute the current and voltage transients characteristic at different heights of the return stroke.

In the model described in Fig. 5(a), the GC return stroke of cloud to ground lightning strike initiates after the attachment to the ground. Thus, the left most end of the model starts with the earth resistance, which is taken to be $1500\ \Omega$, acting as the load resistance [7]. In an upward lightning strike, the switching is made at the cloud end and hence the CG return stroke travels from cloud to ground. Fig. 5(b) models this phenomenon, where the cloud side of the model commences with the cloud's capacitance as the source of energy and terminates with the earth's resistance at the earth end. The height of each RLC segment depends on the overall distance between the earth and cloud together with the number of segments adopted in the model. The profiles of current and voltage are observed to undergo rapid changes in the first few microseconds as will be observed in the next companion part of this two-part paper.

5. AIRCRAFT INTO TRANSMISSION LINE CIRCUIT MODEL

The distributed transmission line model (TLM) can be applied to represent the return stroke of a lightning channel with the elements of R , L and C incorporated with the aircraft segment [21]. The return stroke current flows along the surface of the aircraft connected to the cloud and earth through the lightning channels. The aircraft can therefore be a part of the natural lightning discharge process. All conductive parts of the aircraft skin become a part of the conductive part of the lightning current. However, the R , L and C values of the aircraft will be different to that of the lightning channel. The vulnerability of electronic devices to current and voltage surges generated by the lightning channel depends on the characteristics of the surges [22]. The lightning discharge path via the aircraft is effectively represented using the TLM. The calculated electrical elements R_a , L_a and C_a of the aircraft are lumped together and slotted in as one of the segments of the RLC transmission line model of the lightning return stroke (Fig. 6). The position of the lumped aircraft segment depends on the height of the aircraft from the ground. The transmission line model of combined aircraft and lightning channel is a distributed circuit model. In the next part of the paper, we consider a distributed TLM of the aircraft as well. A table showing the R , L and C values for the lightning channel and the distributed values of R , L and C for the F106B aircraft are given in Table 2, where the resistance value of the

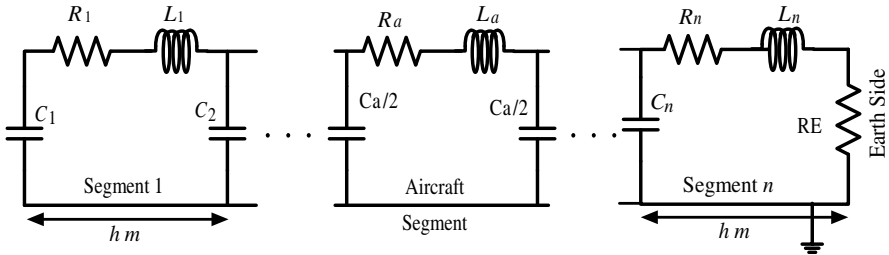


Figure 6. TLM with lumped aircraft.

Table 2. R , L and C values of the lightning channel and the aircraft.

	R (Ω/m)	L (H/m)	C (F/m)
Lightning channel [6]	1.00E-00	3.00E-06	4.60E-12
Aircraft [21]	2.00E-06	8.29E-07	1.34E-11

aircraft and the equations used to get the capacitance and inductance are from [21]. More exact calculations for a composite material radome need further considerations [23]. Moreover, the effects of non-vertical lightning channel [24] will also be considered in a future extension of the basic lightning-aircraft electrodynamics model presented here.

6. CONCLUSION

An RLC transmission line model for lightning return strokes of both cloud-to-ground and ground-to-cloud lightning strikes was developed to determine the normally inaccessible electric current, induced voltage, current rise rate and the frequency spectrum of current along the lightning return stroke. The R , L and C values of the aircraft are plugged into the transmission line model to compute the lightning current and voltage surges produced on an aircraft directly hit by lightning earth flash. The results obtained can be used for analysis of direct and indirect effects to the aircraft and to design the avionics to be used within the aircraft. Moreover, as the accepted evidence shows that the aircraft initiates lightning during take-off under a thundercloud, the model reported herein could be used to study the situation, where lightning interacts with the aircraft at different height as well as while moving between subsequent strikes, and to identify the effects on the surface of the aircraft and avionics within the aircraft.

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