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APPLICATION OF BILAYER ANISOTROPIC STRUCTURES FOR DESIGNING LOW-PASS FILTERS AND POLARIZERS

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Abstract—Arrangements of bilayer anisotropic structures are inspected in this paper that manifest wonderful behaviors. One of these arrangements can pass TM & TE modes spontaneously in low frequencies and reflect them in higher frequencies and function as a low-pass filter. Another arrangement can reflect TE mode and pass TM mode in a particular frequency band and vice versa and function as a polarizer. All the analyses are based on the calculations of the hybrid matrix of layers by means of a recursive algorithm. Also the effect of the μ & ε tensors on the specifications of the filters is discussed.

1. INTRODUCTION

For many years, the problem of multilayer structures had been the subject of interest because of its wide application in various areas. But recently, by the advent of novel metamaterials, the problem has been reconsidered. Primary works on this problem were based on 4×4 characteristic matrix of a single anisotropic material [1, 2]. Later works include expansion of the problem for multi layer anisotropic structures [3–5].

The characteristic matrix algorithm [1, 2] had a serious disadvantage and showed instability for thick layers compared to wavelength. To avoid this instability which was due to the numerical finite difference algorithm, the use of hybrid matrix of the structure has been suggested [6, 12].

Hybrid matrix is extracted using recursive methods. The advantage of the recursive method is that by increasing number of iterations, the thickness of the first sub-layer decreases exponentially, leading to a significant improvement in accuracy.

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By using the hybrid matrix, reflection and transmission matrices are extracted [12]. Furthermore, with realizing the negative permittivity and permeability, application of isotropic metamaterials was shown in reducing the electromagnetic scattering via coupling with conventional isotropic materials [7–11]. Ultimately, by using an appropriate arrangement of materials in layers, we can approach our desire to make low-pass & polarizer filters [13–15].

This paper is organized as follows:

First the process of extracting reflection and transmission matrices from the hybrid matrix is briefly illustrated [12]. In the second step, by using a MATLAB GUI that is based on the calculation of hybrid matrix, the reflection and transmission matrices are computed for particular μ & ε tensors of layers, and it is checked which arrangements of materials can be used for our purposes.

Then, the effect of factors such as μ & ε components length of layers and the angle of incidence on the type of filters and the operating bandwidth is discussed. Also, the procedure of designing a TM polarizer is explained.

2. FORMULATION

The geometry of the problem to be discussed is shown in Figure 1. Regions (I) and (III) are free space, and region (II) is composed of anisotropic layers. An incident plane wave traveling in the air encounters the boundary of a multilayered anisotropic planar structure, and after a series of reflections at discontinuities, part of its power is transmitted through the structure into air. The desired solution of the problem is the reflected wave from the boundary of regions (I) and (II), and transmitted wave into region (III). The method of extracting these matrices is based on hybrid matrix.

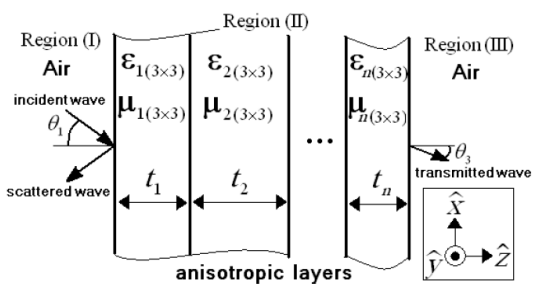


Figure 1. The geometry of a general multi layer structure.

Naming $\{E^i, H^i\}$, $\{E^r, H^r\}$ and $\{E^t, H^t\}$ respectively as phasers of incident fields, scattered fields in region (I) and transmitted fields in region (III), the reflection and transmission coefficient matrices can be defined as:

$$E^r = \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} = R \cdot E^i \tag{1}$$

$$E^t = \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} = T \cdot E^i \tag{2}$$

(R & T are 2×2 reflection and transmission matrices, respectively)

Equation (3) is the definition of the (4×4) hybrid matrix for fields perpendicular to z axis. The procedure of computing the hybrid matrix can be studied in [12].

($\bar{\bar{H}}_{ij}$ is a 2×2 hybrid matrix)

$$\begin{bmatrix} E^i + E^r \\ H^t \end{bmatrix} = \begin{bmatrix} \bar{\bar{H}}_{11} & \bar{\bar{H}}_{12} \\ \bar{\bar{H}}_{21} & \bar{\bar{H}}_{22} \end{bmatrix} \begin{bmatrix} H^i + H^r \\ E^t \end{bmatrix} \tag{3}$$

Also, the impedance relations between electric and magnetic fields in regions (I) and (III) are in this form:

$$E^i = Z_1 \cdot H^i, \quad E^r = -Z_1 \cdot H^r, \quad E^t = Z_3 \cdot H^t \tag{4}$$

in which:

$$Z_i = \begin{bmatrix} 0 & \eta_i \cos(\theta_i) \\ -\eta_i \sec(\theta_i) & 0 \end{bmatrix} \tag{5}$$

θ_1 and θ_3 are the angles of the wave vector with normal to boundaries of the regions. These two angles satisfy Snell's law ($\frac{\sin\theta_3}{sin\theta_1} = \sqrt{\mu_1\varepsilon_1/\mu_3\varepsilon_3}$). η is the wave impedance in each region.

By doing some matrix calculations, the reflection and transmission matrices can be computed using the hybrid matrix [12]

$$\begin{aligned} R &= \left[-Z_1 + H_{11} + H_{12}Z_3\left(I - H_{22}Z_3\right)^{-1} H_{21}\right] \\ &\quad \cdot \left[Z_1 + H_{11} + H_{12}Z_3\left(I - H_{22}Z_3\right)^{-1} H_{21}\right]^{-1} \end{aligned} \tag{6}$$

$$\begin{aligned} T &= \left[Z_3^{-1} - H_{22} + H_{21}\left(Z_1 + H_{11}\right)^{-1} H_{12}\right]^{-1} \\ &\quad \cdot \left[H_{21}Z_1^{-1}\left(I - (H_{11} - Z_1)\cdot(H_{11} + Z_1)^{-1}\right)\right] \end{aligned} \tag{7}$$

Extraction of the features of TE and TM polarizations needs more considerations. TE polarization consists of $\{E_y, H_x, H_z\}$ field components and TM polarization consists of $\{H_y, E_x, E_z\}$ components.

$$\text{TE}: \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \begin{bmatrix} r_{12} \\ r_{22} \end{bmatrix} E_y^i \qquad \text{TM}: \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix} E_x^i \tag{8}$$

The returned power of the TM polarization is therefore proportional to $(r_{11}^2 + r_{21}^2)$, and in case of TE polarization it is proportional to $(r_{12}^2 + r_{22}^2)$.

It is clear that if the electric and magnetic tensors are diagonal matrices, r_{12} and r_{21} are zero, and the returned power will be proportional to r_{11}^2 and r_{22}^2 for TM and TE modes, respectively.

The structure discussed in this paper consists of layers by diagonal anisotropic matrices. As a consequence, r_{12} and r_{21} are zero, and only r_{11} and r_{22} identify the characteristic of the filters.

3. RESULTS

The geometry considered here is a general bilayer structure perpendicular to z direction and illuminated by an electromagnetic plane wave. The layers are lossless in all the examples, and the outside region of structure is free space.

The results are extracted from a MATLAB GUI that is based on formulation discussed above. By using this GUI, different arrangements of materials are checked in order to find the best arrangement. In this paper, only two of these arrangements are compared to show which arrangement is better.

In general, by considering the following relations (9), no reflections occur [12].

$$\bar{\bar{\boldsymbol{\varepsilon}}}_{\mathbf{2}} = \left(-\frac{\mathbf{1}}{\mathbf{m}}\right) \times \bar{\bar{\boldsymbol{\varepsilon}}}_{\mathbf{1}}, \quad \bar{\bar{\boldsymbol{\mu}}}_{\mathbf{2}} = \left(-\frac{\mathbf{1}}{\mathbf{m}}\right) \times \bar{\bar{\boldsymbol{\mu}}}_{\mathbf{1}} \tag{9}$$

in (9), $\bar{\bar{\varepsilon}}_i$ & $\bar{\bar{\mu}}_i$ are the permittivity and permeability of the layer (i), and (m) is the ratio of the length of layers $(m = d_2/d_1)$.

However, even by considering this relation, there are some exceptions that experience reflections in higher frequencies. These exceptions are the base of our works [12].

In the following examples, different arrangements of materials are considered, and it is shown which arrangement is appropriate for our purposes.

In this paper, the amounts of μ & ε tensors are arbitrary, and only the relation between μ & ε tensors of two layers follow the above relation (9).

3.1. DPS-DNG Arrangement of Anisotropic Materials Using Tensors of $\boldsymbol{\mu}$ & $\boldsymbol{\varepsilon}$

As shown in Figure 2, approximately all the TE and TM modes passed through the medium.

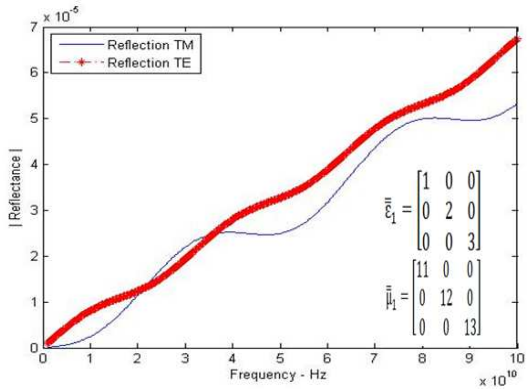


Figure 2. DPS-DNG arrangement of anisotropic materials.

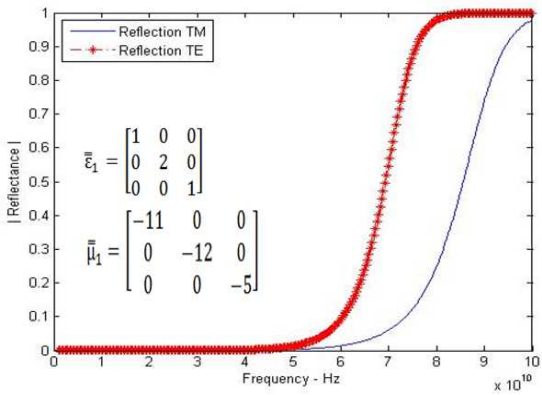


Figure 3. MNG-ENG arrangement of anisotropic materials.

So, this arrangement is not suitable for our purpose. The same result is extracted for DNG-DPS arrangement.

3.2. MNG-ENG Arrangement of Anisotropic Materials Using Tensors of μ & ε

As shown in Figure 3, low-pass filters and polarizers can be achieved by modifying the components of μ & ε tensors of this arrangement. It is obvious that in Figure 3, we can have a polarizer filter that pass TM mode in some frequencies and reflect TE mode in the same frequencies. The same result is extracted for ENG-MNG arrangement (Figure 4).

In the next section, some variables effective in the specifications of filters are considered, and it is discussed how these variables are related

to each other. In our investigations, only the MNG-ENG arrangement is considered.

3.2.1. Effect of Materials

In this status, the dimensions are definite; the angle of incidence is zero; only the tensors $\{\boldsymbol{\varepsilon_{11}}, \boldsymbol{\varepsilon_{22}}, \boldsymbol{\varepsilon_{33}}\}$ & $\{\boldsymbol{\mu_{11}}, \boldsymbol{\mu_{22}}, \boldsymbol{\mu_{33}}\}$ change. $\mathbf{d_1} = \mathbf{1\,mm}$, $\mathbf{d_2} = \mathbf{3\,mm}$, $\boldsymbol{\theta_i} = \mathbf{0}$.

(a) $\boldsymbol{\varepsilon_{33}}$ & $\boldsymbol{\mu_{33}}$

It can be resulted that the specifications of the filters do not relate to $\boldsymbol{\varepsilon_{33}}$ & $\boldsymbol{\mu_{33}}$. By changing the amounts of $\boldsymbol{\varepsilon_{33}}$ & $\boldsymbol{\mu_{33}}$, respectively, from 3 to 1 and from -13 to -5, no change happens for frequency response of the filter. These results are shown in Figure 5.

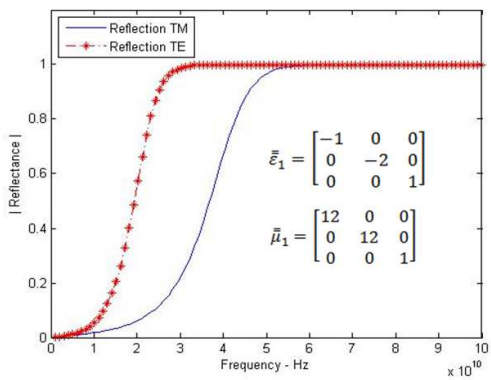
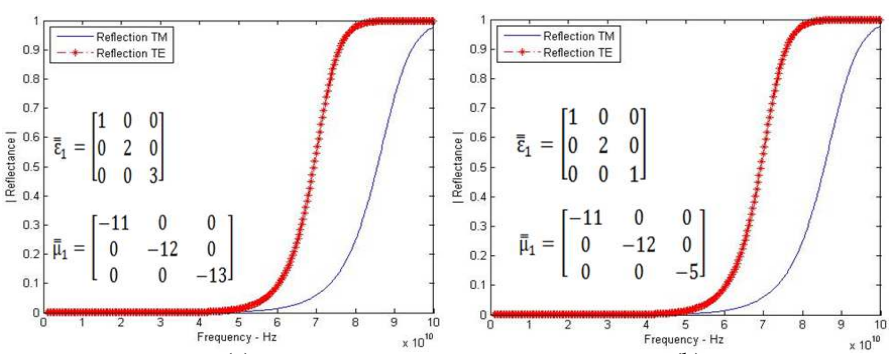


Figure 4. ENG-MNG arrangement of anisotropic materials.



(a) (b)

Figure 5. The effect of $\boldsymbol{\varepsilon_{33}}$ & $\boldsymbol{\mu_{33}}$ on TE and TM reflections ($m = 3$).

(b) $\{\boldsymbol{\varepsilon_{11}}, \boldsymbol{\varepsilon_{22}},\}$ & $\{\boldsymbol{\mu_{11}}, \boldsymbol{\mu_{22}},\}$

First, it should be mentioned that since the reflection pattern does not depend on ε_{33} & μ_{33}, these amounts are chosen $\boldsymbol{\varepsilon_{33}}=\boldsymbol{\mu_{33}}=\mathbf{1}$.

We consider the case that $\boldsymbol{\varepsilon_{11}}=\boldsymbol{\varepsilon_{22}}$ & $\boldsymbol{\mu_{11}}=\boldsymbol{\mu_{22}}$. As shown in Figure 6 by this assumption, we achieve a lowpass filter. Also we can handle the cutoff frequency for this low pass filter. By observing Table 1, it can be resulted by increasing the amounts of both $\boldsymbol{\varepsilon_{11}}=\boldsymbol{\varepsilon_{22}}$ & $|\boldsymbol{\mu_{11}}|=|\boldsymbol{\mu_{22}}|$, the cutoff frequency decrease and low pass filters with different cutoff frequency can be achieved.

As shown in Figure 6, in the case of $\boldsymbol{\varepsilon_{11}} = \boldsymbol{\varepsilon_{22}}$ & $\boldsymbol{\mu_{11}} = \boldsymbol{\mu_{22}}$ the TE & TM reflections have the same pattern. By knowing this, TE and TM reflections can be handled by $\{\boldsymbol{\varepsilon_{22}}, \boldsymbol{\mu_{11}}, \boldsymbol{\mu_{33}}\}$ and $\{\boldsymbol{\varepsilon_{11}}, \boldsymbol{\mu_{22}}, \boldsymbol{\epsilon_{33}}\}$, respectively. These patterns can be separated, and polarizers can be designed.

First, we investigate the effect of $\{\boldsymbol{\varepsilon_{22}}, \boldsymbol{\mu_{11}}\}$ on TE reflections, by increasing the amount of $\boldsymbol{\varepsilon_{22}}$ from 2 to 4 and decreasing $\boldsymbol{\mu_{11}}$ from -12 to -18. The results are shown in Figure 7. By comparing Figures 6

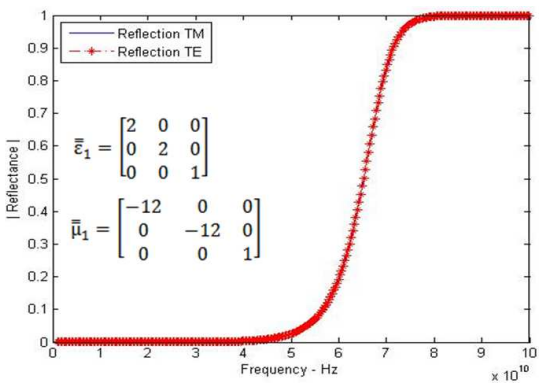


Figure 6. The reflection of a two layer structure ($\boldsymbol{\varepsilon_{11}} = \boldsymbol{\varepsilon_{22}}$ & $\boldsymbol{\mu_{11}} = \boldsymbol{\mu_{22}}$) ($m = 3$).

Table 1. The effect of μ & ε tensors on cutoff frequency ($m = 3$).

$\varepsilon_{11} = \varepsilon_{22}$	$\boldsymbol{\mu_{11}} = \boldsymbol{\mu_{22}}$	Cutoff frequency (GHz)
2	-12	65
2	-18	56
4	-12	43
4	-18	38

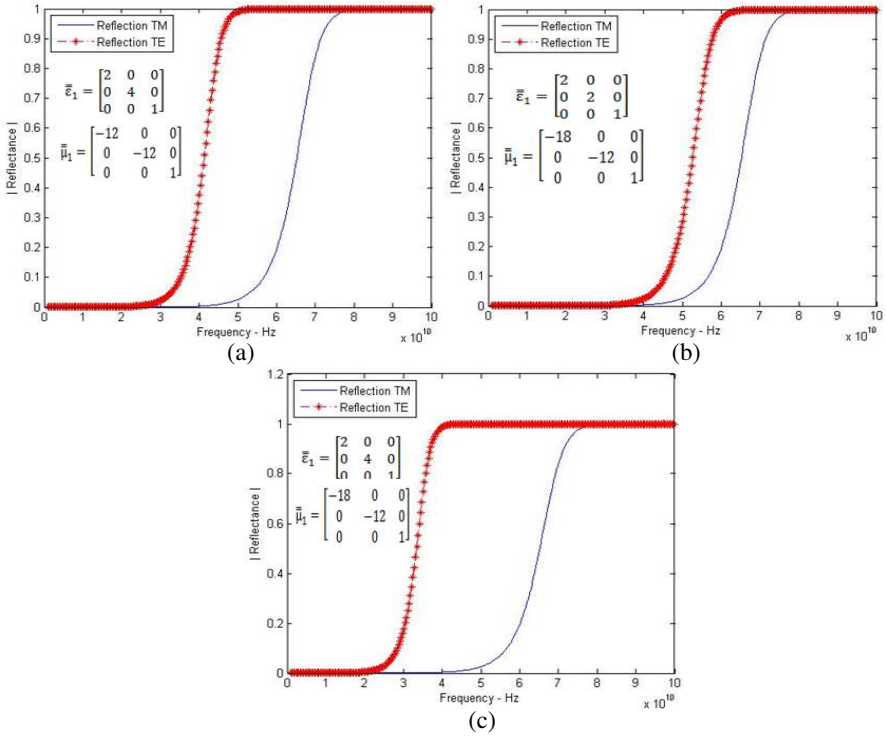


Figure 7. The effect of $\{\varepsilon_{22}, \mu_{11}\}$ on TE reflection ($m = 3$).

& 7, it can be derived that by increasing each of $\boldsymbol{\varepsilon_{22}}$ & $\boldsymbol{\mu_{11}}$, a TM polarizer can be designed. Also the fact that TM reflection does not depend on $\boldsymbol{\varepsilon_{22}}$ & $\boldsymbol{\mu_{11}}$ is observable by comparing Figures 6 & 7.

It can be concluded from the above results that whenever in MNG-ENG arrangement, these conditions are satisfied, i.e., $\boldsymbol{\varepsilon_{22}}>\boldsymbol{\varepsilon_{11}}$ and $|\boldsymbol{\mu_{11}}|>|\boldsymbol{\mu_{22}}|$, a TM polarizer is obtainable that it's operating bandwidth increase as the amounts of $\{\boldsymbol{\varepsilon_{22}}/\varepsilon_{11}, \mu_{11}/\boldsymbol{\mu_{22}}\}$ increase. Also it is noticeable that the effect of increasing $\boldsymbol{\varepsilon_{22}}/\boldsymbol{\varepsilon_{11}}$ is more than that of $\boldsymbol{\mu_{11}}/\boldsymbol{\mu_{22}}$, i.e.. If the amount of $\boldsymbol{\varepsilon_{22}}/\boldsymbol{\varepsilon_{11}}$ is doubled and the amount of $\boldsymbol{\mu_{11}}/\boldsymbol{\mu_{22}}$ is halved, the filter is still a TM polarizer. This effect can be extracted by comparing Figures 6 & 8.

The same procedures are performed for TM reflections. By increasing the amount of $\boldsymbol{\varepsilon_{11}}$ from 2 to 4 and decreasing $\boldsymbol{\mu_{22}}$ from -12 to -18. The results are shown in Figure 9. It can be derived that if the conditions of $\boldsymbol{\varepsilon_{22}}<\boldsymbol{\varepsilon_{11}}$ and $|\boldsymbol{\mu_{11}}|<|\boldsymbol{\mu_{22}}|$ are satisfied, a TE polarizer can be designed that its operating bandwidth increases as the amounts of $\{\boldsymbol{\varepsilon_{22}}/\boldsymbol{\varepsilon_{11}}, \boldsymbol{\mu_{11}}/\boldsymbol{\mu_{22}}\}$ decrease. Also it is shown in Figure 9

that TE reflection does not depend on $\boldsymbol{\varepsilon_{11}}$, $\boldsymbol{\mu_{22}}$.

Also it is noticeable that the effect of decreasing $\boldsymbol{\varepsilon_{22}/\varepsilon_{11}}$ is more than that of $\boldsymbol{\mu_{11}/\mu_{22}}$, i.e., if the amount of $\boldsymbol{\varepsilon_{22}/\varepsilon_{11}}$ is halved and the amount of $\boldsymbol{\mu_{11}/\mu_{22}}$ is doubled, the filter is still a TE polarizer. This effect can be extracted by comparing Figures 6 & 10.

3.2.2. Size of Layers

If the relative length of layers ($\mathbf{m} = \mathbf{d_2/d_1}$) is constant, by increasing the absolute lengths, the cutoff frequencies of both TE & TM reflections and the operating bandwidth of layers decrease. The results can be extracted by comparing Figures 7(a) & 9(a) & (11).

In the second step, the length of first layer $(\mathbf{d_1})$ does not change, and the relative length of layers $(\mathbf{m{=}d_2/d_1})$ increases. What happens

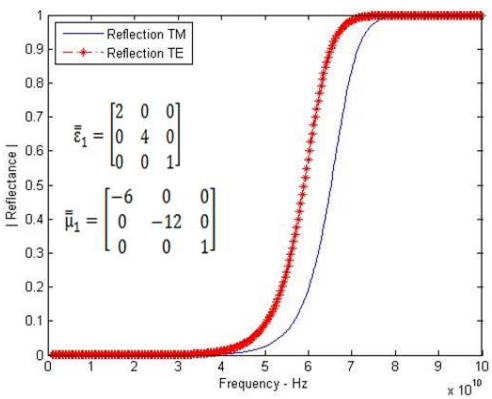


Figure 8. The effect of $\{\varepsilon_{22}/\varepsilon_{11}, \mu_{11}/\mu_{22}\}$ $(m = 3)$.

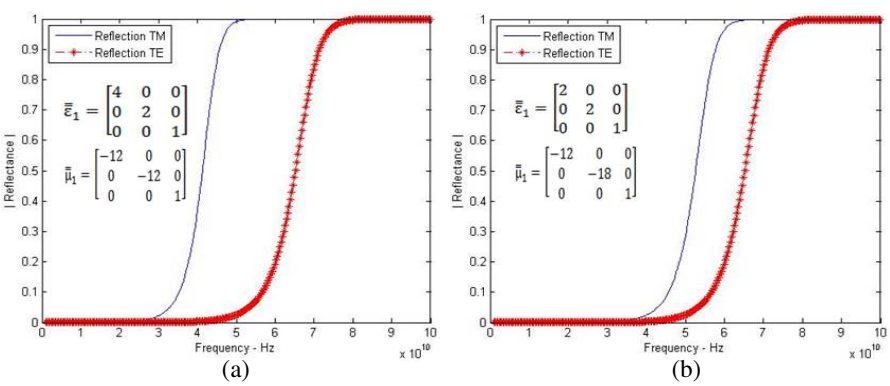


Figure 9. The effect of $\{\boldsymbol{\varepsilon_{11}}, \boldsymbol{\mu_{22}}\}$ on TM reflection $(m = 3)$.

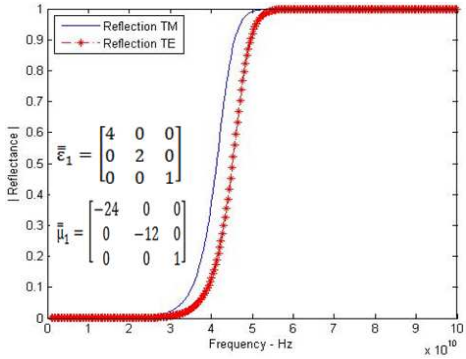


Figure 10. The effect of $\{\varepsilon_{22}/\varepsilon_{11},\ \mu_{11}/\mu_{22}\}$ $(m = 3)$.

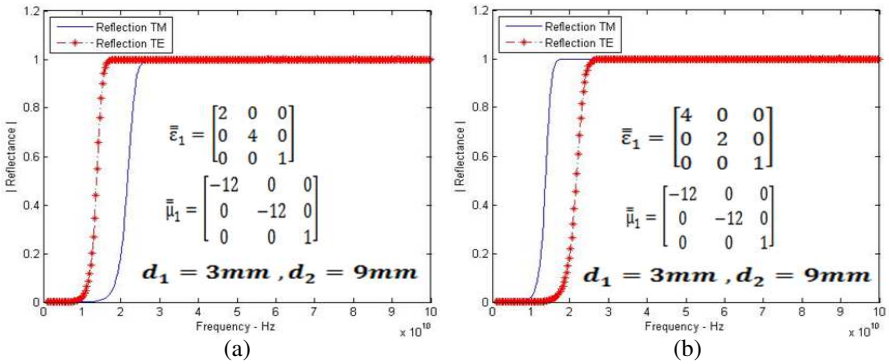


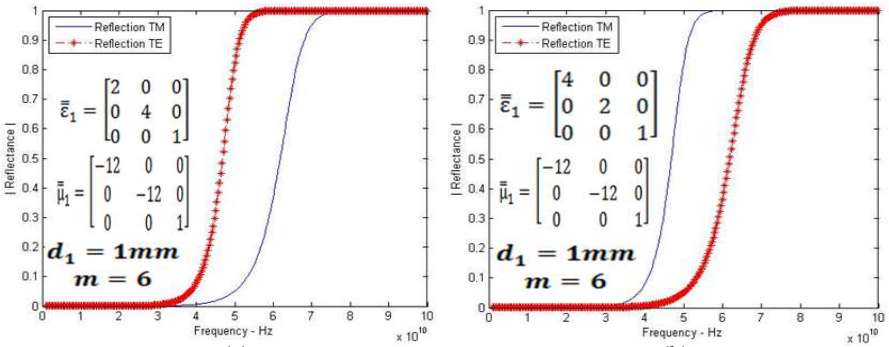
Figure 11. The effect of the absolute length of layers when (m) is constant.

is that the operating bandwidth of filter decreases. The results can be extracted by comparing Figures 7(a) & 9(a) & (12).

3.2.3. The Angle of Incidence

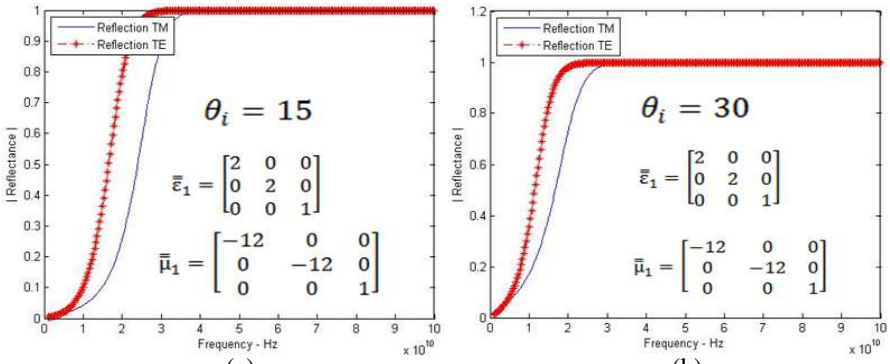
All the results that were extracted in the previous examples, were considered by zero angle of incidence, while TE & TM reflections depend on the angle of incidence. The results of increasing the angle of incidence from 0 to 15 and 30 are shown in Figure 13.

By comparing Figures 13(a) & 13(b), it is concluded that these filters behave differently in the response to different angles of incidence. Also, it is resulted that by increasing the angle of incidence, the cutoff frequencies of both TE and TM reflections and the operating bandwidth of these filers decrease. For solving the problem of dependence of filter frequency response to the angle of incidence,



(a) (b)

Figure 12. The effect of the relative length of layers ($m = d_2/d_1$) when (d_1) is constant.



(a) (b)

Figure 13. Effect of the incidence angle on reflection ($m = 3$).

a special case is proposed. By choosing $m = 1$, an interesting phenomenon happens. The angle of incidence does not affect the behavior of our filters (Figure 14).

4. DISCUSSION

Materials used in these cases show the same electromagnetic properties in all frequencies and therefore are not dispersive. In reality such materials cannot be found, and their presence violates the thermodynamic principles governing the universe [17]. Also the materials were assumed lossless in a wide range in frequency domain which is in contrast to the dependency of real and imaginary parts of ε and μ via Kramers-Kronig relations.

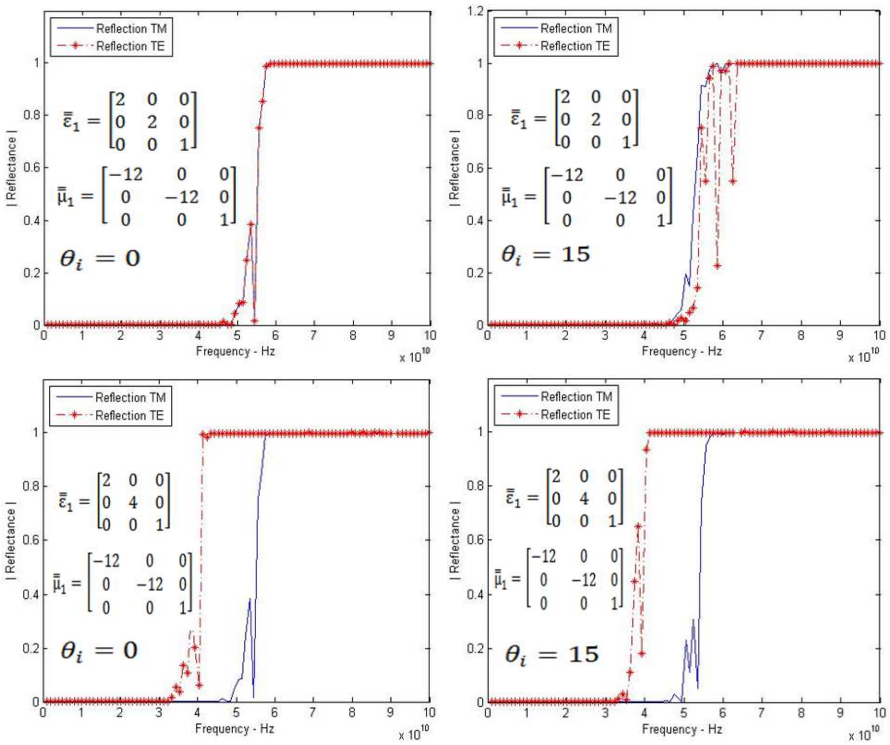


Figure 14. Effect of the incidence angle on reflection ($m = 1$).

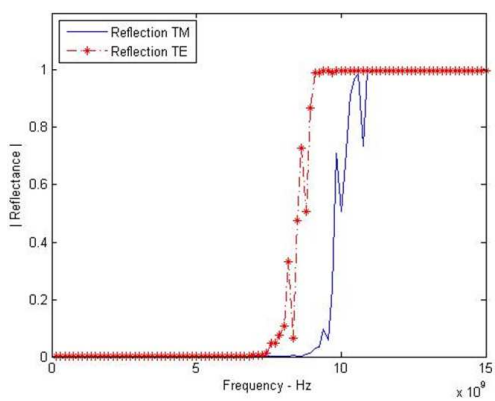


Figure 15. A TM polarizer with cutoff frequency of 10 GHz and bandwidth of 2 GHz ($m = 1$).

The procedure of designing a TM polarizer with cutoff frequency of 10 GHz and bandwidth of 2 GHz is explained. The first step is to design a low pass filter with cutoff frequency of 10 GHz. Also if the filter is supposed to have the same frequency response in response to the angle of incidence, this condition should be satisfied. ($m = 1$)

By considering $\mathbf{d_1} = \mathbf{d_2} = \mathbf{15\,mm}$ and $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_{22} = -\mathbf{8}$, the appropriate amount of $\boldsymbol{\varepsilon}_{11} = \boldsymbol{\varepsilon}_{22}$ for the required cutoff frequency is found by using the GUI. ($\boldsymbol{\varepsilon}_{11} = \boldsymbol{\varepsilon}_{22} = \mathbf{3.5}$). In the second step, four parameters are available for changing the lowpass filter to polarizer. In this case, it is adequate to increase the amount of $\boldsymbol{\varepsilon}_{22}$ from 3.5 to 5. It should be mentioned that in the first step, the amounts of $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_{22}$ or $\boldsymbol{\varepsilon}_{11} = \boldsymbol{\varepsilon}_{22}$ & $\mathbf{d_1} = \mathbf{d_2}$ are chosen arbitrarily, and according to these amounts, other parameters are extracted by the MATLAB GUI. (Figure 15).

5. CONCLUSION

By using the hybrid matrix and extracting the reflection and transmission matrices and using the conditions for zero reflection, a suitable arrangement of materials for designing bilayer anisotropic lowpass filters and polarizers was found. The effects of the permittivity and permeability of layers, size of layers and the angle of incidence were investigated. It is shown that the filters behave differently by the change in the angle of incidence. A special case ($m = 1$) was proposed that made the filter unchangeable to any angle of incidence.

By considering ($m = 1$) for designing bilayer anisotropic lowpass filters and polarizers and also the size of layers, 4 other parameters are available. There is a trade-off between these parameters. By using the GUI and also optimization, the optimum amounts of these parameters for achieving the specifications of the filter are determined.

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