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RECONSTRUCTING CONSTITUTIVE PARAMETERS OF INHOMOGENEOUS PLANAR LAYERED CHIRAL MEDIA BASED ON THE OPTIMIZATION APPROACH

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Abstract—This paper presents a frequency domain technique for reconstructing the constitutive parameters of inhomogeneous planar layered chiral media based on an optimization approach. The measured co- and cross-reflection and transmission coefficients are used to extract profiles of electromagnetic parameters of the inhomogeneous chiral media. To identify the functions of constitutive parameters of the chiral media, Fourier series expansions and Genetic Algorithm (GA) are utilized. Since the optimization problem is highly non-linear, enhanced GA in which a fuzzy system is used for improving the speed and accuracy of GA. The performance and feasibility of the proposed reconstruction method is proven using two typical examples.

1. INTRODUCTION

Unlike ordinary materials, which are described by electric permittivity and magnetic permeability, chiral media include a magneto electric coupling yielding to interesting properties of the electromagnetic fields. An object is called chiral if it cannot be superimposed on its mirror image by translations and rotations. Interaction of electromagnetic fields with chiral media has been the subject of many studies over the past decade and has led to the introduction of its wide application in different microwave devices [1–12] such as twist polarizer, polarization transformer, microwave radar absorbers, negative refraction, etc. Assuming a time harmonic field with $e^{-j\omega t}$, the constitutive relations

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of isotropic and homogeneous chiral media are as follows [1]:

$$\begin{bmatrix} \bar{D} \\ \bar{B} \end{bmatrix} = \begin{bmatrix} \varepsilon_0 \varepsilon_r & +j\kappa/c \\ -j\kappa/c & \mu_0 \mu_r \end{bmatrix} \begin{bmatrix} \bar{E} \\ \bar{H} \end{bmatrix}, \tag{1}$$

where $\bar{E}$, $\bar{B}$, $\bar{D}$, and $\bar{H}$ are electromagnetic fields, and ε_r and μ_r are relative permittivity and permeability of the material, while κ is the chirality parameter which describes electromagnetic coupling and c is speed of light in vacuum. Although the solving of the wave equation in homogeneous chiral media is straightforward and it can be easily seen that the right and left circularly polarized waves are the eigenpolarization, the study of wave propagation in inhomogeneous chiral media with some applications in aperture and lens antenna [2, 13] is a daunting task.

Waves and fields in inhomogeneous media is a recognized subject of major importance in electromagnetics research. Exact solution of the wave equation in such media is known for only a few particular profiles; thus, scattering from inhomogeneous media has been intensively investigated and several approaches have been presented [14–20]. Recently, several methods have been introduced for the analysis of scattering from inhomogeneous planar layered chiral media [21, 22].

This paper presents a method for reconstructing constitutive parameters of a chiral slab from the knowledge of the transmission and reflection coefficient. Generally, application of common procedures of reconstructing the media's parameters to composite chiral media is not simply possible [23–27]. A rather complicated approximation technique was presented in [28] to simultaneously evaluate the permeability, permittivity and chirality parameter of an inhomogeneous chiral slab. Here, the proposed method consists of an enhanced GA method.

2. FORWARD PROBLEM ANALYSIS

In this section, the frequency domain analysis of the inhomogeneous planar layered chiral media is concisely reviewed by the notation of propagators and wave splitting technique [22, 29]. Consider a plane wave that impinges with an angle θ_0 from free space on the first interface between the left half space and the slab with thickness d having the electromagnetic parameters of $\varepsilon(z) = \varepsilon_0 \varepsilon_r(z)$, $\mu(z) = \mu_0 \mu_r(z)$, and $\kappa(z)$, as shown in Fig. 1.

A linearly polarized wave propagating in a chiral medium undergoes a rotation of its polarization which shows that the chiral media are optically active media. Thus, TE and TM electromagnetic waves scattered by a chiral medium are coupled. In the forward

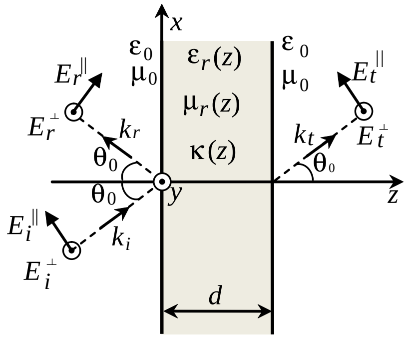

Figure 1. An inhomogeneous chiral slab with constitutive parameters $\varepsilon_r(z)$, $\mu_r(z)$ and $\kappa(z)$.

problem, the reflection and transmission coefficients of inhomogeneous chiral slab are calculated by the notation of propagators technique [22]. By substituting the constitutive relations in the Maxwell's equations, the following system of differential equations is obtained in terms of the tangential components of the fields, i.e., $\bar{E}_{xy}$ and $\bar{H}_{xy}$:

$$\frac{d}{dz}\begin{bmatrix}\bar{E}_{xy}(z)\\ \eta_0\bar{\bar{J}}\cdot\bar{H}_{xy}(z)\end{bmatrix} = jk_0\bar{\bar{M}}(z)\cdot\begin{bmatrix}\bar{E}_{xy}(z)\\ \eta_0\bar{\bar{J}}\cdot\bar{H}_{xy}(z)\end{bmatrix} \tag{2}$$

where k_0 is the wave number in the free-space, η_0 the free-space intrinsic impedance, $\bar{\bar{J}}$ the 2-D rotation dyadic, and $\bar{\bar{M}}$ the fundamental dyadic of the chiral medium, the elements of which can be found in [29]. At the boundary $z = d$, the following can be written:

$$\begin{bmatrix}\bar{E}_{xy}(d)\\ \eta_0\bar{\bar{J}}\cdot\bar{H}_{xy}(d)\end{bmatrix} = e^{jk_0d\bar{\bar{M}}}\begin{bmatrix}\bar{E}_{xy}(z=0)\\ \eta_0\bar{\bar{J}}\cdot\bar{H}_{xy}(z=0)\end{bmatrix} = \bar{\bar{P}}\begin{bmatrix}\bar{E}_{xy}(z=0)\\ \eta_0\bar{\bar{J}}\cdot\bar{H}_{xy}(z=0)\end{bmatrix} \tag{3}$$

where $\bar{\bar{P}}$ is the propagator dyadic. The transverse reflection and transmission dyadics $\bar{\bar{r}}_{xy}$ and $\bar{\bar{t}}_{xy}$, are defined by

$$\begin{cases}\bar{E}^r_{xy}(0) = \bar{\bar{r}}_{xy}\cdot\bar{E}^i_{xy}(0)\\ \bar{E}^t_{xy}(d) = \bar{\bar{t}}_{xy}\cdot\bar{E}^i_{xy}(0)\end{cases} \tag{4}$$

where i, r, and t supscripts indicate incident, reflected, and transmitted waves, respectively. The reflection and transmission dyads for the tangential electric field are determined by:

$$\begin{cases}\bar{\bar{r}}_{xy} = -\bar{\bar{T}}_{22}^{-1}\cdot\bar{\bar{T}}_{21}\\ \bar{\bar{t}}_{xy} = \bar{\bar{T}}_{11} + \bar{\bar{T}}_{12}\cdot\bar{\bar{r}}_{xy}\end{cases} \tag{5}$$

in which

$$\begin{bmatrix} \bar{\bar{T}}_{11} & \bar{\bar{T}}_{12} \\ \bar{\bar{T}}_{21} & \bar{\bar{T}}_{22} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \bar{\bar{I}}_t & -\bar{\bar{O}} \\ \bar{\bar{I}}_t & \bar{\bar{O}} \end{bmatrix} \cdot \begin{bmatrix} \bar{\bar{P}}_{11} & \bar{\bar{P}}_{12} \\ \bar{\bar{P}}_{21} & \bar{\bar{P}}_{22} \end{bmatrix} \cdot \begin{bmatrix} \bar{\bar{I}}_t & \bar{\bar{I}}_t \\ -\bar{\bar{O}}^{-1} & \bar{\bar{O}}^{-1} \end{bmatrix} \tag{6}$$

where $\bar{\bar{I}}_t = \hat{a}_x\hat{a}_x + \hat{a}_y\hat{a}_y$ is two-dimensional identity dyadic, $\bar{\bar{O}}^{-1} = k_0/k_z[\bar{\bar{I}}_t + l/k_0^2\bar{k}_t \times (\bar{k}_t \times \bar{\bar{I}}_t)]$, and $\bar{k}_t$ and k_z are tangential and normal components of the incident and reflected wave vectors, respectively.

3. OPTIMIZATION APPROACH TO THE INVERSE PROBLEM

The methods of the electromagnetic inversion problems can be generally divided to direct based inversion methods and model based inversion methods. In the based inversion methods, the constitutive parameters of the medium are obtained from direct calculations applied to the scattered data; while in the model based inversion ones, the data calculated from the forward problem should be matched with the measurements or simulation data. Here, an electromagnetic inversion method is proposed for the reconstruction of an inhomogeneous chiral slab based on enhanced GA method.

In reconstructing the inhomogeneous slab, it is recommended to use any suitable expansion function for the continuous profile models. Expansion functions such as the linear, cosine and Legendre forms can be used. However, it is recommended to use the model which can perform the profile reconstruction with minimum number of terms. Through the inversion process, it is found that the Fourier series expansion model has better performance for the many of profiles rather than other expansion functions. In the inverse problem, the constitutive parameters of inhomogeneous chiral layer are optimized so that the calculated co- and cross-reflection and transmission coefficients match the measured ones. Here, to identify the functions of relative permittivity, relative permeability, and chirality parameter of inhomogeneous chiral layer, their functions are considered as the following Fourier series expansions:

$$\varepsilon_r(z) = \sum_{n=0}^{N} X_n \cos\left(\frac{n\pi z}{d}\right) \tag{7}$$

$$\mu_r(z) = \sum_{n=0}^{N} Y_n \cos\left(\frac{n\pi z}{d}\right) \tag{8}$$

$$\kappa(z) = \sum_{n=0}^{N} Z_n \cos\left(\frac{n\pi z}{d}\right) \tag{9}$$

where X_n, Y_n, and Z_n are coefficients of Fourier expansion of electromagnetic parameters. Notice that the sinusoidal Fourier series expansions could also be used. By finding the coefficients of Fourier expansions, the inhomogeneous chiral layer will be exactly identified. The objective function is defined as follows here:

$$F = \sum_{TE,TM} \sum_{\theta=\theta_s}^{\theta e} \left[\left| \tilde{R}_{co}(\theta) - R_{co}^m(\theta) \right| + \left| \tilde{T}_{co}(\theta) - T_{co}^m(\theta) \right| + \left| \tilde{R}_{cr}(\theta) - R_{cr}^m(\theta) \right| + \left| \tilde{T}_{cr}(\theta) - T_{cr}^m(\theta) \right| \right] \tag{10}$$

where $\tilde{R}(\theta)$ and $\tilde{T}(\theta)$ are the calculated reflection and transmission coefficients, and $R^m(\theta)$ and $T^m(\theta)$ are the measured ones. Also, the indices "*co*" and "*cr*" represent the co- and cross-polarized coefficients, and θ_s and θ_e are the limits of incident angle. The optimization method seeks at minimizing the objective function, which is obtained when the measured and calculated transmission and reflection coefficients are identical and indicates that the reconstructed constitutive parameters are identical to the original ones.

In the retrieval of inhomogeneous chiral layer, the enhanced Genetic Algorithm (GA) is employed [30]. In GA, the optimization problem is given a chromosome structure. Next, an initial random population is generated. Then, members of the population with higher fitness are selected. The fitness of members is calculated by an evaluation function. A member with a better fitness has more selection chance; therefore, weaker members with worse fitness are gradually replaced with stronger ones. The selected members mate two by two randomly and the next population is generated. This procedure is repeated until the stop condition is reached. The stop condition should be a fixed iteration or obtaining a member with desirable fitness.

Since the optimization problem Eq. (10) is highly non-linear, enhanced GA with a fuzzy system for improving the speed and accuracy of GA was applied. In this method, when the algorithm is about to converge the mutation coefficient is increased and when the algorithm fails to converge, the coefficient of crossover is increased by a fuzzy system. So, the algorithm is forced to coverage at the global optimum quickly.

Surely, the first and perhaps critical step in such an optimization problem is to find an initial estimate solution. Serving as the initial input value, this estimate solution reduces the computation memory and time. Here, to find such an initial estimate solution, we assume that the measured or simulated reflection and transmission coefficients of the inhomogeneous chiral layer are pertained to a hypothetical homogeneous chiral slab. Although, this assumption is inaccurate, we can simply find an initial solution using GA, so that the measured and evaluated transmission and reflection coefficients are close to each other as possible. In addition, a multi-search GA is used, so that the GA is run to explore the whole search space and obtain primary results. Next, the search space is limited to find only the first N-th unknown coefficients, while the other unknown coefficients are assumed to be the primary results. This process is done for the other electromagnetic parameters. The whole method is repeated twice or more until the best solution is found.

4. NUMERICAL EXAMPLES AND RESULTS

In this section, two examples are provided to illustrate the applicability of the proposed method for the reconstruction of the profiles of electromagnetic parameters of inhomogeneous chiral layers.

4.1. Example 1 (Inhomogeneous Chiral Slab in Free Space)

In the first example, consider complex profiles for the relative permittivity, relative permeability, and chirality parameter as

$$\varepsilon_r(z) = \begin{cases} 3 & 0 \le z \le 0.5d \\ 1 & 0.5\,d \le z \le d \end{cases} \tag{11}$$

$$\mu_r(z) = \frac{2\,e^{(4z/d)}}{1+\,e^{(4z/d)}} \tag{12}$$

$$\kappa(z) = \frac{4\,e^{(8z/d)}}{1+\,2e^{(4z/d)}+e^{(8z/d)}} \tag{13}$$

which are similar to numerical scattering examples given in [28, 31]. The thickness of the slab and the excitation frequency, are selected 0.03 m and 1 GHz, respectively. The amplitudes of co- and cross-reflection and transmission coefficients versus the angle of incidence obtained from the notation of propagators method are illustrated in Fig. 2.

We assume that the reflection and transmission coefficients are pertained to a hypothetical homogeneous chiral slab, and find a initial estimate solution as $\varepsilon_r = 2$, $\mu_r = 1.6$, and $\kappa = 2.86$. Then, assuming $N = 14$ in (7)–(9), the proposed reconstruction technique is applied to the obtained reflection and transmission coefficients. The unknown coefficients of the truncated Fourier series are evaluated, as written in Table 1. In Fig. 3, the final retrieved unknown functions are compared with the true ones. The comparison between profiles generated by the proposed technique and the corresponding exact profiles illustrated the excellent behavior of the technique.

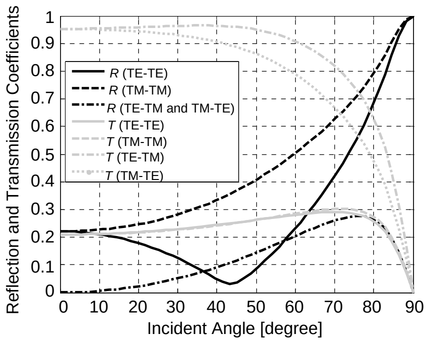

Figure 2. Amplitudes of co- and crossreflection and transmission coefficients as a function of incident angle θ_0 for the inhomogeneous chiral slab in free space.

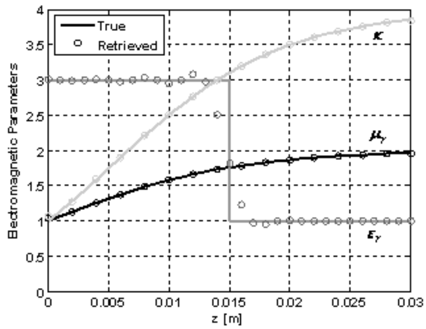

Figure 3. Comparison of the retrieved and the true relative permittivity, relative permeability, and chirality parameter of the inhomogeneous chiral slab in free space.

Table 1. Optimization results for the truncated Fourier series expansions of relative permittivity, relative permeability, and chirality parameter of inhomogeneous chiral slab.

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_n	1.984	1.268	0.031	−0.411	−0.029	0.230	0.026	−0.148	−0.023	0.098	0.021	−0.061	−0.019	0.029	0.016
Y_n	1.663	−0.374	−0.117	−0.053	−0.025	−0.017	−0.010	−0.008	−0.005	−0.004	−0.003	−0.002	−0.001	−0.001	−0.000
Z_n	2.843	−1.192	−0.296	−0.121	−0.049	−0.038	−0.019	−0.018	−0.010	−0.010	−0.006	−0.006	−0.003	−0.002	−0.001

Table 2. Optimization results for the truncated Fourier series expansions of relative permittivity, relative permeability, and chirality parameter of PEC backed inhomogeneous chiral slab.

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_n	2.149	1.449	0.594	0.394	0.238	0.183	0.125	0.100	0.071	0.058	0.042	0.033	0.022	0.015	0.007
Y_n	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Z_n	1.023	0.636	0.002	−0.381	0.315	−0.089	0.001	−0.041	0.059	−0.022	−0.000	−0.012	0.019	−0.008	−0.000

4.2. Example 2 (PEC Backed Inhomogeneous Chiral Slab)

As the second example, a PEC backed inhomogeneous chiral slab with thickness of 0.03 m is considered. Expressions of the relative permittivity, relative permeability, and chirality parameter used for this example are

$$\varepsilon_r(z) = \frac{6}{1 + 5\,(z/d)} \tag{14}$$

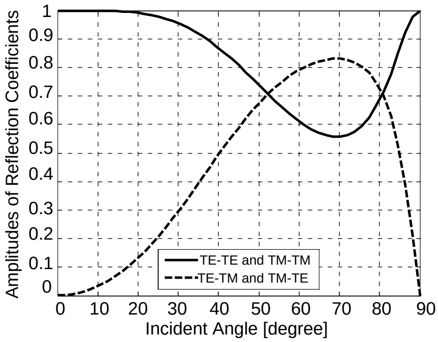

Figure 4. Amplitudes of co- and crossreflection coefficients as a function of incident angle θ_0 for PEC backed inhomogeneous chiral slab.

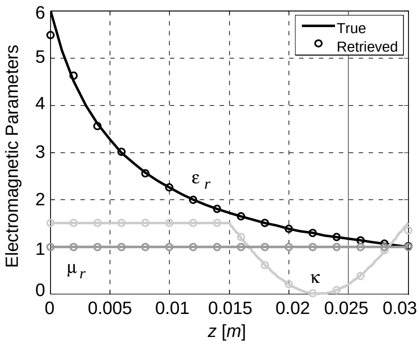

Figure 5. Comparison of the retrieved and the true relative permittivity, relative permeability, and chirality parameter of a PEC backed inhomogeneous chiral slab.

$$\mu_r(z) = 1 \quad (15)$$

$$\kappa(z) = \begin{cases} 1.5 & 0 \leq z \leq 0.5\,d \\ 1.5 + 1.5\sin\left(\frac{2\pi z}{d}\right) & 0.5\,d \leq z \leq d \end{cases} . \quad (16)$$

The amplitudes of co- and cross-reflection coefficients versus the angle of incidence are measured, as shown in Fig. 4. Assuming the reflection and transmission coefficients are pertained to a hypothetical homogeneous chiral slab, we find an initial estimate solution as $\varepsilon_r = 1.75$, $\mu_r = 1$, and $\kappa = 1.03$. With defining the objective function as

$$F = \sum_{TE,TM} \sum_{\theta=0}^{\pi/2} \left[\left| \tilde{R}_{co}(\theta) - R_{co}^{m}(\theta) \right| + \left| \tilde{R}_{cross}(\theta) - R_{cross}^{m}(\theta) \right| \right] \quad (17)$$

and assuming $N = 14$, the proposed reconstruction technique is applied to the reflection coefficients. The coefficients of the truncated Fourier series of electromagnetic parameters are evaluated, as written in Table 2. The final retrieved unknown functions are compared with the true ones in Fig. 5. The profiles generated by the proposed technique and the corresponding exact values are in very good agreement.

5. CONCLUSIONS

An optimization approach is used to retrieve the constitutive parameters of an inhomogeneous chiral slab from the knowledge of the co- and cross-reflection and transmission coefficients. At first, the constitutive parameters of inhomogeneous chiral layer are expanded in a truncated Fourier series. Next, the optimum values of the coefficients of the series are obtained through enhanced GA. The comparison between profiles obtained by the proposed technique and the corresponding exact profiles of chiral layers indicate that the constitutive parameters are retrieved successfully, and the method is feasible and robust. In future, the proposed retrieval method is expected to be used for optimally designing inhomogeneous chiral layers as microwave absorbers.

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