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SCINTILLATION INDEX OF A GAUSSIAN SCHELL-MODEL BEAM ON SLANT ATMOSPHERIC TURBULENCE

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Abstract—Based on the altitude-dependent model of the ITU-R slant atmospheric turbulence structure constant model, we present scintillation index calculations for a Gaussian Schell-model (GSM) beam under all irradiance fluctuation conditions. The longitudinal and radial components of the scintillation index are treated separately. Our results correctly reduce to the result of the horizontal path with atmospheric structure constant fixed, and simplify to a fully coherent beam with source coherence parameter ζ_s representing unit. The numerical conclusions indicate that within specific source and parameter ranges, the GSM beam is capable of offering less scintillation than the full coherent Gaussian beam. Before the maximum value of the scintillation, the scintillation index of the GSM beam will decrease with increased altitude. However, the off axis radial scintillation index will vanish when the Rytov variance is infinity.

1. INTRODUCTION

On propagation through the turbulent atmosphere, an optical wave will experience random irradiance fluctuations, called scintillation, due to perturbations from refractive index fluctuations. Scintillation can lead to power losses at the receiver, resulting in lower signal-to-noise ratios and eventually to fading of the received signal below a prescribed threshold [1]. So, optical scintillation is considered one of the most crucial atmospheric effects and has attracted much attention in practical applications, particularly in laser tracking and ranging systems [2–4] and imaging systems [5–7].

The topic of optical scintillation has been extensively studied for many years. In the early 1960s, Tatarskii and Cherenkov adopted

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Rytov approximation method [8–10] and obtained the scintillation index of unbounded plane wave and spherical wave, but their scintillation results were limited to weak fluctuations. The limitation of the weak fluctuation theory was clearly demonstrated, and saturation phenomenon of the scintillation was discovered by the experimental data of Gracheva and Gurvich [11] in 1965. A number of theoretical and experimental studies devoted to irradiance fluctuation under strong turbulence regimes then followed. With a theory of saturation developed by Gochelashvili and Shishov [12], several qualitative models of scintillation were subsequently developed and modified by others [13, 14]. In 1999, Hopen and Andrews [15] intensity studied fluctuation variance of Gaussian beam propagation in moderate-to-strong fluctuation. Andrews et al. [1, 16] made a comprehensive summary of these studies about plane wave, spherical wave, and Gaussian beam. Wu and Wei [17] discussed the scintillation index of a Gaussian beam considering inner-scale and outer-scale on slant path. A few papers reported the scintillation of a partially coherent beam. Many researchers concerned with the spread and average intensity of partially coherent beam [18, 19].

Studies above focused on the horizontal path; however structure constant C_n^2 in many cases may vary along the propagation path as it does in vertical and slant paths. Beran and Whitman [20] calculated the scintillation index of a plane wave using the two-scale theory and assuming a Kolmogorov structure function with an altitude-dependent structure constant. Baker [21] developed a low-order turbulence (LOT) solution to describe Gaussian beam weak scintillation on ground-to-space propagation. Charnotskii [22] presented a more rigorous theory of beam scintillation on ground-to-space paths using path integrals and analytic techniques, where turbulence is concentrated in a relatively thin layer near the transmitter.

Especially after the introduction of free-space optical atmospheric communication links, the average BER can be significantly reduced by the use of a partially coherent source beam. Many researchers started to study the propagation characteristics of different types of sources in turbulence [23, 24]. Baykal and Eyyubogl [25] developed the scintillation index formula for a flat-topped Gaussian beam source on axis in turbulence. They found that using a flat-topped Gaussian beam in a FSO link will bring advantages in scintillations compared with Gaussian-beam incidence. Then Eyyuboglu and Baykal [26, 27] presented scintillation calculations in weak atmospheric turbulence for partially coherent general beams based on the extended Huygens-Fresnel principle and the scintillation index of cosh-Gaussian beams in a turbulent atmosphere using Rytov method.

In this paper, we use the techniques in [28], the altitude-dependent model of the ITU-R turbulence structure constant model, to develop a scintillation index model for a Gaussian Schell-model beam on the slant path that is applicable under weak-to-strong fluctuations. The result considers the focusing regime and also can be reduced to the result of the horizontal path when atmospheric structure constant is a fixed value. Finally, the numerical results are compared between the fully coherent beam and the partially coherent beam.

2. SCINTILLATION INDEX ON SLANT PATH

2.1. Weak Scintillation Index

The beam is off the optical axis, and the scintillation index is expressed as a sum of radial and longitudinal components in the form at a point.

$$\sigma_I^2(r,L) = \sigma_{I,r}^2(r,L) + \sigma_I^2(0,L), \tag{1}$$

here the first component is radial, and the second is the longitudinal component.

The radial component of the scintillation index in weak fluctuation is given by [16]

$$\sigma_{I,r}^2(r,L) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa) \exp(-\Lambda L \kappa^2 \xi^2/k) \times [I_0(2\Lambda r \kappa \xi) - 1] d\kappa d\xi. \tag{2}$$

And the longitudinal scintillation index is defined under the Rytov approximation by [6]

$$\sigma_l^2(0,L) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \Phi_n(\kappa)\kappa \exp\left(-\frac{\Lambda L \xi^2 \kappa^2}{k}\right) \left\{1 - \cos\left[\frac{L\kappa^2}{k}\left(1-\bar{\Theta}\xi\right)\xi\right]\right\} d\kappa d\xi. \tag{3}$$

here, $I_0(x)$ is a modified Bessel function, $\Phi_n(\kappa)$ the spatial power spectrum of refractive index fluctuations, and the Gaussian Schell-model source illumination beam in receiver plane without atmospheric effects is characterized by Λ, Θ beam parameters defined by

$$\Lambda = \frac{\Lambda_0 \varsigma_s}{\Theta_0^2 + \Lambda_0^2 \varsigma_s} = \frac{2L}{kW^2}, \quad \Theta = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2 \varsigma_s}, \quad \bar{\Theta} = 1 - \Theta. \tag{4}$$

where W represents the beam radius at the receiver. In arriving at Equation (4), here we have replaced the receiver beam parameters for a fully coherent beam in free space with the equivalent free-space receiver parameters for a partially coherent beam [28], where ς_s is the source coherence parameter, which is the degree of the partial coherence of the source laser beam at the transmitter. For $\varsigma_s = 1$, the effective beam parameters in Equation (4) reduce to parameters of the fully coherent Gaussian beam, i.e., $\Lambda_1 = \frac{\Lambda_0}{\Theta_0^2+\Lambda_0^2}$, $\Theta_1 = \frac{\Theta_0}{\Theta_0^2+\Lambda_0^2}$. The parameter Θ describes amplitude change in the wave caused by refraction, and Λ describes amplitude change caused by diffraction.

2.2. Radial Component

In the presence of optical turbulence, additional diffraction and refraction cause further broadening of the beam spot size and further focusing and defocusing [29, 31]. We will follow the work of Ricklin et al. [32] and replace the beam parameter Λ with the effective beam parameter Λ_e in the radial component, based on Equation (2). It has been shown that the use of effective beam parameters permits us to formally extend weak fluctuation expressions for the spatial coherence radius into the strong fluctuation regime [29, 30]. In the case where the inner scale of turbulence is smaller than the spatial coherence length of the optical wave, we base our calculations on the conventional Kolmogorov spectrum $\Phi_n(\kappa) = 0.033C_n^2(\xi H)\kappa^{-11/3}$, Equation (2) yields

$$\sigma_{I,r}^2(r,L) = 0.264\pi^2 k^2 L \int_0^1 C_n^2(\xi H) d\xi \times \int_0^\infty [I_0(2\Lambda_e r\xi\kappa) - 1] \exp\left(-\frac{\Lambda_e L\xi^2\kappa^2}{k}\right) \kappa^{-8/3} d\kappa, \quad (5)$$

Let $a = 2\Lambda_e r\xi$, $b = \Lambda_e L\xi^2/k$, the evaluation of Equation (5) leads to

$$\sigma_{I,r}^2(r,L) = 0.264\pi^2 k^2 L \int_0^1 C_n^2(\xi H) d\xi \int_0^\infty I_0(a\kappa) \exp\left(-b\kappa^2\right) \kappa^{-8/3} d\kappa - 0.264\pi^2 k^2 L \int_0^1 C_n^2(\xi H) d\xi \int_0^\infty \exp\left(-b\kappa^2\right) \kappa^{-8/3} d\kappa, \quad (6)$$

By use of the following formula

$$\int_0^\infty I_0(a\kappa) \exp\left(-b\kappa^2\right) \kappa^{-8/3} d\kappa = \frac{1}{2}\Gamma\left(-5/6\right) b^{5/6} {}_1F_1\left(-5/6; 1; \frac{a^2}{4b}\right), \quad (7)$$

where ${}_1F_1(a,\, b;\, c;\, x)$ is a hyper geometric function.

$$\int_0^\infty \exp\left(-b\kappa^2\right) \kappa^{-8/3} d\kappa = \frac{1}{2}\Gamma(-5/6) b^{5/6}. \quad (8)$$

In terms of Equation (7), Equation (8), the radial component Equation (6) leads to

$$\sigma_{I,r}^2(r,L) = 7.08\sigma_0^2\Lambda_e^{5/6}\left[1 - {}_1F_1\left(-5/6;1;2r^2/W_e^2\right)\right]\int_0^1 \frac{C_n^2(\xi H)}{C_{n0}^2}\xi^{5/3}d\xi$$
$$\cong 11.8\sigma_0^2\Lambda_e^{5/6}\int_0^1 \frac{C_n^2(\xi H)}{C_{n0}^2}\xi^{5/3}d\xi\frac{r^2}{W_e^2}, \quad r < W_e. \tag{9}$$

where $\sigma_0^2{=}1.23C_{n0}^2k^{7/6}L^{11/6}$ is the Rytov variance.

The effective wave radius $W_e(L)$ for the GSM beam through atmospheric turbulence and the global coherence parameter ζ are expressed by

$$W_e(L) = W_0\left(\Theta_0^2 + \zeta\Lambda_0^2\right)^{1/2}, \quad \zeta = \zeta_S + \frac{2W_0^2}{\rho_T^2}, \tag{10}$$

$$\rho_T = \left[1.46k^2L\int_0^1 C_n^2(\xi L)(1-\xi)^{5/3}d\xi\right]^{-3/5}. \tag{11}$$

$$\Lambda_e = \frac{2L}{kW_e^2}. \tag{12}$$

Physically speaking, the global coherence parameter is a measure of the global degree of coherence of light across each transverse plane along the propagation. It is used to define a related dimensionless quantity, the source coherence parameter ζ_S as the degree of partial coherence of the source beam at the transmitter. For $\zeta_S = 1$, Equation (10) represents fully coherent beam; ζ_S increasing, source beam becomes less coherent. Here ρ_T is the coherence length of a spherical wave propagating on the slant turbulent atmosphere, and $C_n^2(\xi L)$ is the model for the refractive-index structure constant in the atmosphere. This effective wave radius for the partially coherent GSM beam through atmospheric turbulences agrees with that by Korotkova [33], where $W_e = W_1(1 + 4q_c\Lambda_1 + 1.63\sigma_0^2\Lambda_1^{5/6})^{1/2}$. It is noted that Ref. [33] introduces a thin random phase screen to model the diffuser in front of the laser transmitter and characterize the phase screen by a power spectrum function rather than by a correlation function as in the Gaussian Schell-model. It should be noted that W_1 is Gaussian beam spot radius in receiver plane not partially coherent beam because the effects of partially coherent beam are accounted by q_c. We note that the scintillation index for the partially coherent Gaussian beam presented in Equation (9) correctly reduces to the well-known scintillation index for the Gaussian beam on horizontal path when ς_s is taken as unity, and atmospheric structure constant is a fixed value $C_n^2(\xi H) = C_{n0}^2$ in Equation (9). Under weak fluctuations, the result is simplified to

$$\sigma_{I,r}^2(r,L) \cong 4.42\sigma_0^2\Lambda_1^{5/6}\frac{r^2}{W_1^2}, \quad r < W_1. \quad \sigma_0^2 \ll 1. \tag{13}$$

where W_1 is the free-space spot size radius for Gaussian beam at receiver plane. Here Equation (13) agrees with that in [1].

2.3. Longitudinal Component of Scintillation Index

In weak fluctuation regimes where inner-scale effects are negligible, we base our calculations on the Kolmogorov spectrum. By use of this spectrum model and let $a = \Lambda L\xi^2/k, b = L\xi(1-\bar{\Theta}_\xi)/k$, Equation (6) along the optical axis ($r = 0$) becomes

$$\sigma_l^2(0,L) = 0.264\pi^2k^2L\int_0^1 C_n^2(\xi H) \int_0^\infty \kappa^{-8/3}\exp(-a\kappa^2)\left[1-\cos\left(b\kappa^2\right)\right]d\kappa d\xi, \tag{14}$$

There is a small error $a = \Lambda L\xi/k$ in Equation (3.55) in [33]. Using Equations (7), (8), the evaluation of Equation (14) leads to

$$\sigma_l^2(0,L) = 0.882\pi^2k^2L\int_0^1 C_n^2(\xi H) \left\{(a^2+b^2)^{5/12}\cos\left[\frac{5}{6}\tan^{-1}\left(\frac{b}{a}\right)\right] - a^{5/6}\right\}d\xi, \tag{15}$$

Substituting a, b into Equation (15), we have

$$\sigma_l^2(0,L) = 1.77\sigma_0^2\int_0^1 \frac{C_n^2(\xi H)}{C_{n0}^2} \times \left\{\left[\Lambda^2\xi^4 + (1-\bar{\Theta}\xi)^2\xi^2\right]^{5/12} \cos\left[\frac{5}{6}\tan^{-1}\left(\frac{1-\bar{\Theta}\xi}{\Lambda\xi}\right)\right] - \Lambda^{5/6}\xi^{5/3}\right\}d\xi, \tag{16}$$

Equation (16) is restricted to weak fluctuations. Under strong fluctuation conditions, we use the fluctuation theory developed by Andrews et al. [16], which yields

$$\sigma_{I,strong}^2(0,L) = \exp\left[\frac{0.49\sigma_l^2}{\left(1+0.56\sigma_l^{12/5}\right)^{7/6}} + \frac{0.51\sigma_l^2}{\left(1+0.69\sigma_l^{12/5}\right)^{5/6}}\right] - 1. \tag{17}$$

where σ_l^2 is defined by Equation (16). We combine the radial component Equation (9) with the longitudinal component Equation (17) to obtain

$$\sigma_I^2(r,L) = 11.8\sigma_0^2\Lambda_e^{5/6}\int_0^1 \frac{C_n^2(\xi H)}{C_{n0}^2}\xi^{5/3}d\xi\frac{r^2}{W_e^2} + \exp\left[\frac{0.49\sigma_l^2}{\left(1+0.56\sigma_l^{12/5}\right)^{7/6}}+\frac{0.51\sigma_l^2}{\left(1+0.69\sigma_l^{12/5}\right)^{5/6}}\right]-1 \quad (18)$$

Equation (18) represents our approximation to the scintillation index at any transverse position in the beam at the receiver plane of a general lowest-order Gaussian Schell-model beam wave under all irradiance fluctuation conditions.

3. ATMOSPHERIC TURBULENCE STRUCTURE CONSTANT MODEL

From above derivation, we can see that scintillation index depends largely on height distribution of turbulence atmospheric structure constant. There, we employ ITU-R atmospheric turbulence structure constant model [34] expressed by

$$C_n^2(h) = 8.148\times10^{-56}v_{RMS}^2h^{10}e^{-h/1000} + 2.7\times10^{-16}e^{-h/1500} + C_{n0}^2e^{-h/100}, \quad h=\xi H. \quad (19)$$

where $v_{RMS} = 21\,(\mathrm{m/s})$ is the rms wind speed in meters per second (m/s). h is in meters (m), C_{n0}^2 the structure parameter at the ground in $\mathrm{m}^{-2/3}$, (its typical value is $1.7\times10^{-14}\mathrm{m}^{-2/3}$).

We plot the logarithm of the ITU-R model Equation (19) in Figure 1 as a function of altitude for three values of the parameter C_{n0}^2 and three values of high-altitude wind speed. C_{n0}^2 affects the profile model only up to roughly 1 km, and high-altitude wind is the element most around altitudes of 10 km. It shows that the value of atmospheric structure constant C_n^2 is the biggest near ground. From 1 km to 4 km, the atmospheric structure constant does not depend on sub aerial atmospheric structure constant and wind velocity due to atmospheric absorption. The ITU-R atmospheric structure constant mode is closer to the actual turbulence model than negative exponent index model.

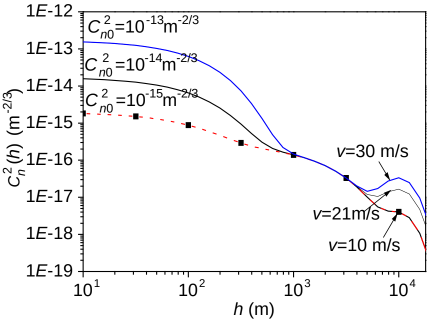

Figure 1. An altitude-dependent model of the ITU-R turbulence structure constant vs. altitude h. $v_{RMS} = 21\,\text{m/s}$, $30\,\text{m/s}$, $10\,\text{m/s}$. $C_{n0}^2 = 10^{-13}\,\text{m}^{-2/3}$, $10^{-14}\,\text{m}^{-2/3}$, $10^{-15}\,\text{m}^{-2/3}$.

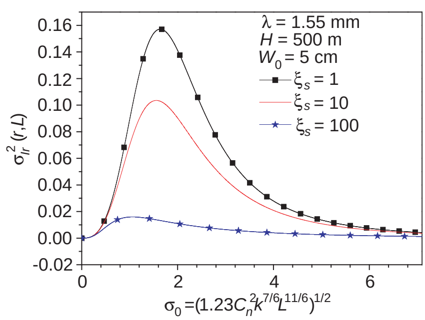

Figure 2. Radial component of the scintillation index versus the square root of the Rytov variance σ_0, $\lambda = 1.55\,\mu\text{m}$, $H = 400\,\text{m}$, $W_0 = 5\,\text{cm}$, $\zeta_s = 1, 10, 100$, $r = 0.05\,\text{m}$.

4. NUMERICAL ANALYSIS CONCLUSION

In this section, we have analyzed the characteristics of the scintillation index with the variations of propagation length and root of the Rytov variance. Our results are shown in Figures 2–6. We have chosen the wavelength of operation to be $\lambda = 1.55\,\mu\text{m}$, since this wavelength is the most widely used in the current FSO links. In computation, since the atmospheric structure constant is variable with height on slant propagation path, the computing formulas have to reserve $C_n^2(h)$ path integral calculus form; H is the height between transmitter and receiver. Then we deal with a Gaussian Schell-model collimated beam scintillation index.

Figure 2 shows the radial scintillation index as a function of the square root of the Rytov variance σ_0 for different values of the source coherence parameter ζ_s representing beams from the coherent to the partially coherent. The illustrated GSM beam is assumed collimated beam ($\Theta_0 = 1$), and the propagation distance is allowed to vary. At first, in weak fluctuation the radial scintillation index increases quickly with propagation distance, then it eliminates quickly once the beam has passed the focusing regime. In all irradiance fluctuation conditions, as the source beam becomes less coherent, the radial scintillation index decreases as expected. When σ_0 is very large, radial component of the scintillation index will vanish, independent of source coherence parameter.

In Figure 3, we compare behavior of the coherent beam with that

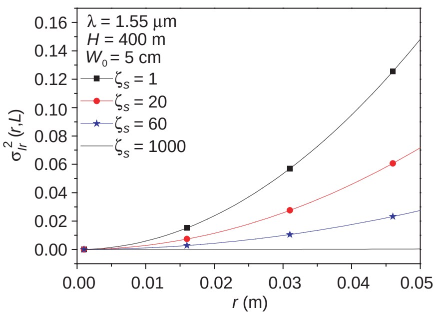

Figure 3. Radial component of the scintillation index versus radial distance r. $H = 400\,\mathrm{m}$, $\lambda = 1.55\,\mu\mathrm{m}$, $W_0 = 5\,\mathrm{cm}$, $\zeta_s = 1$, 20, 60, 1000.

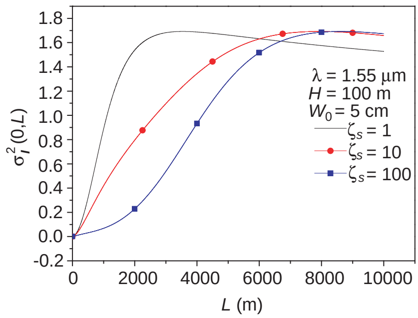

Figure 4. Longitudinal component of the scintillation index versus propagation distance L. $\lambda = 1.55\,\mu\mathrm{m}$, $H = 100\,\mathrm{m}$, $W_0 = 5\,\mathrm{cm}$, $\zeta_s = 1$, 10, 100.

of the partially coherent beam and plot the radial component of the scintillation index Equation (9) versus r for different values of the source coherence parameter ς_s. With increasing off-axis parameter r, the radial component of the scintillation index will increase, but the radial scintillation index for the incoherent source ($\varsigma_s = 1000$) is zero independent of the off-axis parameter r. The Gaussian beam scintillation index is bigger than the partially coherent beam. This agrees with the result that the partially coherent beam is less sensitive to the effects of turbulence than fully coherence one.

In Figure 4, we show Equation (17) as a function of the propagation distance. As the source beam becomes less coherent, the scintillation index decreases at a given distance. With the increase of distance, longitudinal scintillations index increases and tends to saturation. For short ranges (up to 6–8 km), the partially coherent and incoherent beams assume lower levels of scintillation index than a fully coherent beam; therefore the advantage of the partially coherent beam can be expected in weak or focusing atmospheric regimes.

The effect of receiver altitude H on the longitudinal scintillation index is illustrated in Figure 5. Before focusing atmospheric regimes (location of peak scintillation), the higher the receiver is, the smaller the scintillation index of the partially coherent GSM beam is. In Figure 6, we show the total scintillation Equation (15) as a function of σ_0 with several values of height. We set $C_{n0}^2 = 10^{-13}\,\mathrm{m}^{-2/3}$ and allow the propagation distance to vary. The result, as in Figure 5, shows that the total scintillation index is largely dependent on the longitudinal

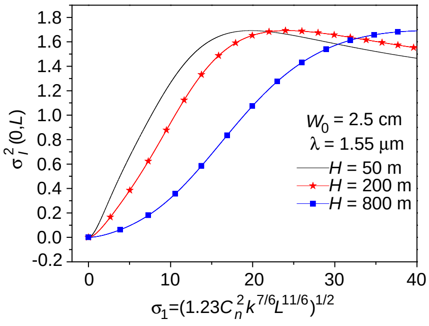

Figure 5. Longitudinal component of the scintillation index GSM beam versus σ_0 and various H. $W_0 = 2.5\,\text{cm}$, $H = 50\,\text{m}$, $200\,\text{m}$, $800\,\text{m}$, $\zeta_s = 10$, $\lambda = 1.55\,\mu\text{m}$.

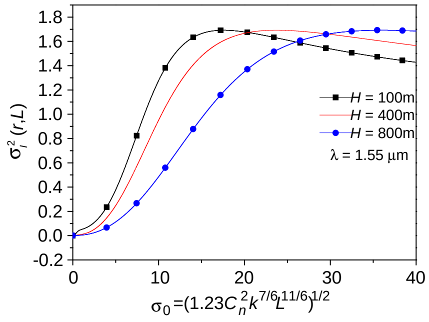

Figure 6. Scintillation index of GSM wave in the atmosphere versus σ_0 and various H. $\lambda = 1.55\,\mu\text{m}$, $H = 100\,\text{m}$, $400\,\text{m}$, $800\,\text{m}$, $\zeta_s = 10$, $W_0 = 2.5\,\text{cm}$.

scintillation index. When the atmospheric structure constant is a fixed value, our result is consistent with the result of horizontal path [30].

5. SUMMARY

With the motivation of understanding whether employing a GSM beam may bring some advantages in reducing the scintillation in long atmospheric optics telecommunications links, we evaluate the scintillation index of a GSM incidence on the slant path for all atmospheric turbulence conditions. With the change of altitude and source coherence parameters, the longitudinal and radial components of scintillation index are discussed, respectively. In general, the larger the source coherence parameter ς_s and the higher receiver H are, the smaller the scintillation index is. The scintillation index of the GSM is smaller on the slant path than in the horizontal path in optical turbulence. However, at relatively large σ_0, the scintillation advantage of the slant path tends to disappear. For incoherent beam, off axis radial scintillation index is zero. Combining the results in current work with previous result, we can assert that the GSM beam on slant path reduces scintillation index more than the fully coherent Gaussian beam on horizontal path. This scintillation index model of the GSM beam on slant path for laser beam propagation in atmosphere turbulence is significant for the visible and infrared imaging, laser tracking, controlling and guiding, and LIDAR systems.

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