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# TRAVELLING-WAVE MODELLING OF UNIFORM MULTI-CONDUCTOR TRANSMISSION LINE NETWORKS — PART I: ANALYTICAL DERIVATION

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**Abstract**—In Part I of this work, analysis of uniform multi-conductor transmission line networks is performed on travelling-wave basis, via "quasi-TEM" approach. Narrowband interpretation of the modal theory in the time domain and quantification of the multiple reflections effect are both included. Theoretical demonstration and analytical formulation are provided, along with guidelines towards computational implementation. Any network formed of lossy, diagonalisable uniform multi-conductor transmission lines of either distinct or degenerate eigenvalues is covered. This work applies especially in the field of Power-Line Communications, as High-Frequency transmission over the power electric network is dominated by multipath propagation.

## 1. INTRODUCTION

Provision of communication and/or smart grid services over the Low Voltage (LV) and Medium Voltage (MV) Distribution Network (DN) by use of the High Frequency (HF) band and also other effects of possible interest such as high-frequency transients resulting, e.g., from lightning or from inductive load switching, depend essentially on the response of unmatched transmission-line (TL) networks. The impedance mismatches found along the power electric network provoke intense multipath fading in the area of MHz [1–9]. Due to structural peculiarities, certain types of triple-pole (phase-neutral-ground) cables follow the two-conductor uniform TL model [10–16], whereas most tricels, as well as any multi-phase (two-phase,

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three-phase) cable, require a multiconductor transmission line (MTL) model [17–27]. Therefore, a computationally realisable travelling-wave analysis of MTL networks enabling in particular simulation of Power-Line Communication (PLC) channels in the Time Domain (TD) would be of considerable contribution. The principal transmission schemes are Orthogonal Frequency-Division Multiplexing (OFDM) and Code-Division Multiple Access (CDMA) [28–38], so that response to both narrowband and wideband signalling would be desired.

Numerical transition from the Frequency Domain (FD) [39–44], Finite Difference Time Domain (FDTD) analysis [45–52], and the Method of Characteristics (MoC) [53–55] are all common ways of obtaining the TD response of a MTL network. Other established techniques involve discretisation [56, 57], FD solution followed by TD transition [58–61], or polynomial/ rational function FD approximation of MTLs followed by numerical transition to the TD [62–72]. In all such methods, the MTL equations are numerically approximated — in the TD, or FD, or both — so that (i) the TD solution is only a computational result providing no connection with the channel (network) topology, and (ii) it is not possible to isolate either the several paths or the dominant path groups, or to trace back and identify the path routes.

The TD interpretation of modal decomposition introduced in this paper eliminates the above weaknesses (i) and (ii) inherent to all aforementioned methods. Propagation of travelling-wave components along uninterrupted uniform TL segments and reflection effects due to impedance mismatches are the elemental phenomena, which have already been partially addressed in previous articles, e.g., [73–81]. However, no generalised approach has up to now been presented that would reveal the exact correspondence with the standard FD analysis and provide a sound basis for time-response simulation, and here is where novelty of this work lies in, i.e., that the MTL network response and the performance parameters are determined not as the result of any numerical FD-to-TD transformation or/and numerical approximation of the MTL equations, but through analytical elaboration of the multipath propagation process that is the actual (physical) response mechanism, allowing thus (a) interlinking between the actual propagating components or/and the dominant groups of such components and the network topology, as well as (b) tracing back along the route of every single such component. At this point, it should be noted that the travelling-wave approach introduced herein is general and considers all reflections induced by all medium discontinuities (i.e., terminations, serial connections of different lines, line junctions).

The present travelling-wave analysis is *directly applicable in the* *case of narrowband signalling* (e.g., OFDM), i.e., when the MTL parameters, and therefore the network response as well, vary along the band of transmission only marginally and can be considered constant, so that upon any potential narrowband (either single- or multi-carrier) transmission scheme it is unrestrictedly valid. Moreover, although not in the scope of this work, treatment of wideband signalling, i.e., if variation of MTL parameters and network' response is considerable and cannot be neglected, is in addition feasible by use of recipes already established to approximate wideband channels via proper expansion of narrowband techniques. This aspect is further discussed in Subsection 3.1 of Part II, entitled "Bandwidth — Variation with Frequency".

In Section 2, the MTL reference model is presented. The modal theory in the FD is briefly referred in Section 3. In Section 4, the time-domain response is derived commencing from the single-mode travelling wave demonstration and then extending to the general multi-mode response through linear superposition. Verification through coincidence with the FD analysis in terms of (i) identical form of general solution and (ii) equivalent terminal conditions is demonstrated in Section 5. In Section 6, guidelines for computational implementation are suggested, and a concluding summation closes the paper in Section 7.

## 2. THE MTL REFERENCE MODEL

Referring to the $(\nu + 1)$-conductor MTL configuration of Fig. 1, the symmetric per-unit-length resistance, inductance, capacitance, and

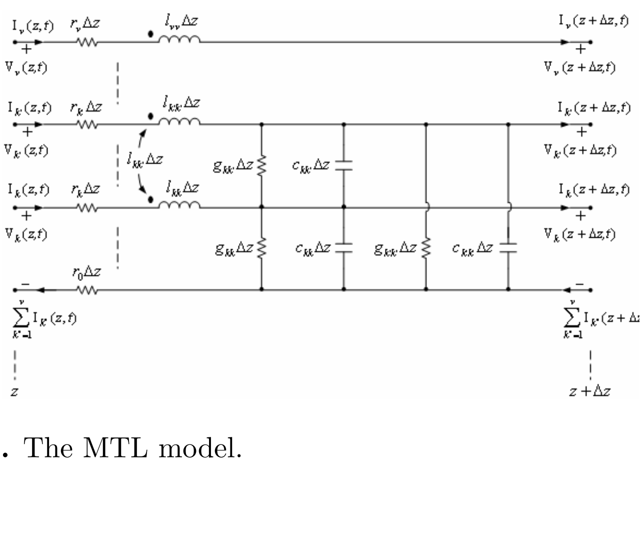


**Figure 1.** The MTL model.

conductance ($\nu \times \nu$) matrices are respectively:

$$[\mathbf{R}]_{kk'} \hat{=} \left\{ \begin{matrix} r_k + r_0, k = k' \\ r_0, k \neq k' \end{matrix} \right\} = [\mathbf{R}]_{k'k}\,, \quad k, k' = 1, \ldots, \nu, \tag{1a}$$

$$[\mathbf{L}]_{kk'} \hat{=} l_{kk'} = [\mathbf{L}]_{k'k}\,, \quad k, k' = 1, \ldots, \nu, \tag{1b}$$

$$[\mathbf{C}]_{kk'} \hat{=} \left\{ \begin{matrix} \sum\limits_{k''=1}^{\nu} c_{kk''}, k = k' \\ -c_{kk'}, k \neq k' \end{matrix} \right\} = [\mathbf{C}]_{k'k}\,, \quad k, k' = 1, \ldots, \nu, \tag{1c}$$

$$[\mathbf{G}]_{kk'} \hat{=} \left\{ \begin{matrix} \sum\limits_{k''=1}^{\nu} g_{kk''}, k = k' \\ -g_{kk'}, k \neq k' \end{matrix} \right\} = [\mathbf{G}]_{k'k}\,, \quad k, k' = 1, \ldots, \nu. \tag{1d}$$

At every point $z_0$ of structural discontinuity — termination, serial connection of different lines, or line junction — the line voltage and current vectors

$$\left. \begin{matrix} \mathbf{V}(z,t) \hat{=} [\; V_1(z,t) \quad \ldots \quad V_\nu(z,t) \;]^T \\ \mathbf{I}(z,t) \hat{=} [\; I_1(z,t) \quad \ldots \quad I_\nu(z,t) \;]^T \end{matrix} \right\} \tag{2a}$$

are subject to the voltage and current continuity conditions

$$\mathbf{V}(z_0^-, t) = \mathbf{V}\left(z_0^+, t\right), \quad \mathbf{I}(z_0^-, t) = \mathbf{I}\left(z_0^+, t\right) \tag{2b}$$

where superscripts ($-$) and ($+$) denote respectively longitudinal positioning infinitesimally before and after $z_0$. In the case of line junction, as shown in Fig. 2, the voltage vector $\mathbf{V}(z_0^+, t)$ is common to all *departure lines*, whereas the resultant current vector $\mathbf{I}(z_0^+, t)$ equals the sum of the current vectors of all departure lines.

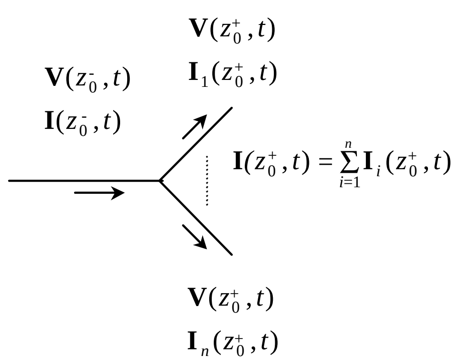


**Figure 2.** Junction of $(n + 1)$ transmission lines.

## 3. REFERENCE THEORY IN THE FREQUENCY DOMAIN

### 3.1. Frequency-domain Fundamentals

Pursuant to the FD approach, harmonic line voltages $V_k(z,t)$ and currents $I_k(z,t)$, $k = 1,\ldots,\nu$, are assumed, and respective *phasor* vectors are:

$$\hat{\mathbf{V}}(z)\hat{=}\left[\hat{V}_1(z)\ \ldots\ \hat{V}_\nu(z)\right]^T,\quad \hat{\mathbf{I}}(z)\hat{=}\left[\hat{I}_1(z)\ldots\hat{I}_\nu(z)\right]^T \tag{3}$$

The per-unit-length impedance and admittance matrices are then formed as:

$$\hat{\mathbf{Z}}\hat{=}\mathbf{R}+j2\pi f\mathbf{L},\quad \hat{\mathbf{Y}}\hat{=}\mathbf{G}+j2\pi f\mathbf{C} \tag{4a}$$

and the Input Admittance Matrix (IAM) $\hat{\mathbf{Y}}_{in}(z)$ at any longitudinal position $z$ is defined by:

$$\hat{\mathbf{I}}(z)=\hat{\mathbf{Y}}_{in}(z)\hat{\mathbf{V}}(z) \tag{4b}$$

The continuity conditions (2b) are applied to the phasors as:

$$\hat{\mathbf{V}}\left(z^-\right)=\hat{\mathbf{V}}\left(z^+\right),\quad \hat{\mathbf{I}}\left(z^-\right)=\hat{\mathbf{I}}\left(z^+\right) \tag{4c}$$

In MTL telecommunication networks, excitation is normally applied in the form of a finite-output-impedance voltage source, and no short-circuit termination is expected. Therefore, lumped transverse termination is herein represented by the coupling of a voltage source

$$V_{G_{kk'}}(t)=\mathrm{Re}\left\{\hat{V}_{G_{kk'}}e^{j2\pi ft}\right\},\quad \hat{V}_{G_{kk'}}\in\mathbf{C} \tag{5a}$$

with output impedance $\hat{Z}_{G_{kk'}}\in\mathbf{C}$ and output admittance

$$\hat{Y}_{kk'}\hat{=}\hat{Z}_{G_{kk'}}^{-1} \tag{5b}$$

between every $k$-th and $k'$-th conductor pair (reference conductor: $k,k'=0$), and the line termination interaction is expressed through the termination admittance matrix:

$$\left[\hat{\mathbf{Y}}_{0,L}\right]_{kk'}\hat{=}\left\{\begin{array}{c}\sum\limits_{\substack{k'=0\\k'\neq k}}^{\nu}\hat{Y}_{kk'},\,k=k'\\ -\hat{Y}_{kk'},\,k\neq k'\end{array}\right\}=\left[\hat{\mathbf{Y}}_{0,L}\right]_{k'k}^T,\quad k,k'=1,\ldots,\nu \tag{5c}$$

where subscripts "0" and "$L$" denote line's origin and end, respectively. If no source is coupled between the conductors $k$ and $k'$, $\hat{V}_{G_{kk'}}=0$ is set. An open circuit between the $k$-th and $k'$-th conductor gives $\hat{Y}_{kk'}\equiv 0$. No short circuit is expected, as already explained. From the continuity conditions (4c), the excitation/termination conditions at line's origin ($z = 0$) and end ($z = l$) become:

$$\begin{aligned}\hat{I}_k(0) &= \hat{Y}_{k0}\left[\hat{V}_{G_{k0}} - \hat{V}_k(0)\right] - \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left[\hat{V}_k(0) - \hat{V}_{G_{kk'}} - \hat{V}_{k'}(0)\right]\hat{Y}_{kk'},\\ \hat{I}_k(l) &= \hat{Y}_{k0}\left[\hat{V}_k(l) - \hat{V}_{G_{k0}}\right] + \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left[\hat{V}_k(l) - \hat{V}_{G_{kk'}} - \hat{V}_{k'}(l)\right]\hat{Y}_{kk'}\end{aligned} \tag{5d}$$

## 3.2. Ordinary Modes of Propagation

### *3.2.1. Decomposition*

Performing the modal decomposition (see [82] or equivalently [83]), the similarity transformation:

$$\begin{aligned}\hat{\mathbf{V}}(z) &= \hat{\mathbf{T}}_V\hat{\mathbf{V}}_m(z), \quad \hat{\mathbf{V}}_m(z) \hat{=} \left[\begin{array}{ccc}\hat{V}_{m1}(z) & \ldots & \hat{V}_{m\nu}(z)\end{array}\right],\\ \hat{\mathbf{I}}(z) &= \hat{\mathbf{T}}_I\hat{\mathbf{I}}_m(z), \quad \hat{\mathbf{I}}_m(z) \hat{=} \left[\begin{array}{ccc}\hat{I}_{m1}(z) & \ldots & \hat{I}_{m\nu}(z)\end{array}\right]\end{aligned} \tag{6a}$$

is applied and the diagonal matrix:

$$\hat{\gamma}^2 \hat{=} \mathrm{diag}\left[\hat{\gamma}_1^2, \ldots, \hat{\gamma}_\nu^2\right] = \hat{\mathbf{T}}_V^{-1}\hat{\mathbf{Z}}\hat{\mathbf{Y}}\hat{\mathbf{T}}_V = \hat{\mathbf{T}}_{\mathrm{I}}^{-1}\hat{\mathbf{Y}}\hat{\mathbf{Z}}\hat{\mathbf{T}}_{\mathrm{I}} \tag{6b}$$

of the common eigenvalues $\hat{\gamma}_\xi^2$, $\xi = 1, \ldots, \nu$, of the $(\hat{\mathbf{Z}}\hat{\mathbf{Y}})$ and $(\hat{\mathbf{Y}}\hat{\mathbf{Z}})$ matrices is formed, with the voltage and current transformation matrices chosen to satisfy:

$$\hat{\mathbf{T}}_I = \left(\hat{\mathbf{T}}_V^{-1}\right)^T \tag{6c}$$

providing

$$\begin{aligned}(d/dz)\,\hat{\mathbf{V}}_m(z) &= -\hat{\mathbf{z}}\hat{\mathbf{I}}_m(z), \quad \hat{\mathbf{z}} \hat{=} \hat{\mathbf{T}}_V^{-1}\hat{\mathbf{Z}}\hat{\mathbf{T}}_I = \mathrm{diag}\left[\hat{z}_1, ..., \hat{z}_\nu\right],\\ (d/dz)\,\hat{\mathbf{I}}_m(z) &= -\hat{\mathbf{y}}\hat{\mathbf{V}}_m(z), \quad \hat{\mathbf{y}} \hat{=} \hat{\mathbf{T}}_I^{-1}\hat{\mathbf{Y}}\hat{\mathbf{T}}_V = \mathrm{diag}\left[\hat{y}_1, ..., \hat{y}_\nu\right]\end{aligned} \tag{6d}$$

### *3.2.2. Modal Solution*

The $\xi$-th "ordinary mode of propagation", or simply "mode", consists of the modal voltage and current quantities:

$$\begin{aligned}V_{m\xi}(z,t) &\hat{=} \mathrm{Re}\left\{\hat{V}_{m\xi}(z)e^{j2\pi ft}\right\},\\ I_{m\xi}(z,t) &\hat{=} \mathrm{Re}\left\{\hat{I}_{m\xi}(z)e^{j2\pi ft}\right\}, \quad \xi = 1, \ldots, \nu\end{aligned} \tag{6e}$$

and the propagation constant

$$\hat{\gamma}_\xi = \hat{a}_\xi + j\hat{\beta}_\xi \in \mathbf{C} \tag{6f}$$

with $\hat{a}_\xi \in \Re$ the attenuation constant and $\hat{\beta}_\xi \in \Re$ the phase constant of the mode.

The general FD solution in modal terms is

$$\hat{\mathbf{V}}_m(z) = \underbrace{e^{-\hat{\gamma}z}\hat{\mathbf{V}}_m^+}_{\hat{\mathbf{V}}_m^+(z)} + \underbrace{e^{\hat{\gamma}z}\hat{\mathbf{V}}_m^-}_{\hat{\mathbf{V}}_m^-(z)}, \quad \hat{\mathbf{V}}_m^\pm = \begin{bmatrix} \hat{V}_{m1}^\pm & \dots & \hat{V}_{m\nu}^\pm \end{bmatrix} \tag{7a}$$

$$\hat{\mathbf{I}}_m(z) = \underbrace{e^{-\hat{\gamma}z}\hat{\mathbf{I}}_m^+}_{\hat{\mathbf{I}}_m^+(z)} - \underbrace{e^{\hat{\gamma}z}\hat{\mathbf{I}}_m^-}_{\hat{\mathbf{I}}_m^-(z)}, \quad \hat{\mathbf{I}}_m^\pm = \begin{bmatrix} \hat{I}_{m1}^\pm & \dots & \hat{I}_{m\nu}^\pm \end{bmatrix} \tag{7b}$$

$$e^{\pm\hat{\gamma}z} \hat{=} \mathrm{diag}\begin{bmatrix} e^{\pm\hat{\gamma}_1 z} & \dots & e^{\pm\hat{\gamma}_\nu z} \end{bmatrix} \tag{7c}$$

with $\hat{\mathbf{V}}_m^+(z)$, $\hat{\mathbf{I}}_m^+(z)$ the phasor vectors of the forward or positively evolving stationary modal voltage and current wave components respectively, and $\hat{\mathbf{V}}_m^-(z)$, $\hat{\mathbf{I}}_m^-(z)$ the phasor vectors of the backward or negatively evolving ones. The modal admittance matrix defined by:

$$\hat{\mathbf{I}}_m^\pm = \hat{\mathbf{Y}}_m\hat{\mathbf{V}}_m^\pm, \quad \hat{I}_{m\xi}^\pm = \hat{Y}_{m\xi}\hat{V}_{m\xi}^\pm, \quad \xi = 1, \dots, \nu \tag{8a}$$

is diagonal as:

$$\hat{\mathbf{Y}}_m = \hat{\mathbf{z}}^{-1}\hat{\gamma} = \mathrm{diag}\begin{bmatrix} \hat{Y}_{m1} & \dots & \hat{Y}_{m\nu} \end{bmatrix}, \quad \hat{Y}_{m\xi} \hat{=} \frac{\hat{\gamma}_\xi}{\hat{z}_\xi}, \quad \xi = 1, \dots, \nu \tag{8b}$$

with $\hat{Y}_{m\xi}$ the characteristic admittance of the $\xi$-th mode.

### *3.2.3. Line Quantities and Parameters*

In terms of the stationary modal components, the forward or positively evolving and backward or negatively evolving phasor vectors of the actual line quantities become

$$\begin{aligned} \hat{\mathbf{V}}(z) &= \hat{\mathbf{V}}^+(z) + \hat{\mathbf{V}}^-(z), \quad \hat{\mathbf{V}}^\pm(z) \hat{=} \hat{\mathbf{T}}_V e^{\mp\hat{\gamma}z}\hat{\mathbf{V}}_m^\pm, \\ \hat{\mathbf{I}}(z) &= \hat{\mathbf{I}}^+(z) - \hat{\mathbf{I}}^-(z), \quad \hat{\mathbf{I}}^\pm(z) \hat{=} \hat{\mathbf{T}}_I e^{\mp\hat{\gamma}z}\hat{\mathbf{I}}_m^\pm \end{aligned} \tag{9}$$

The Characteristic Admittance Matrix (CAM) $\hat{\mathbf{Y}}_c$ of the line is defined by:

$$\hat{\mathbf{I}}^\pm(z) = \hat{\mathbf{Y}}_c\hat{\mathbf{V}}^\pm(z), \quad \hat{\mathbf{Y}}_c = \hat{\mathbf{T}}_I\hat{\mathbf{Y}}_m\hat{\mathbf{T}}_V^{-1} \tag{10}$$

and the voltage reflection coefficient matrix or simply Reflection Coefficient Matrix (RCM) $\rho(z)$ is defined at any longitudinal position $z$ via

$$\hat{\mathbf{V}}^-(z) = \hat{\boldsymbol{\rho}}(z)\hat{\mathbf{V}}^+(z), \quad \hat{\boldsymbol{\rho}}(z) = \left[\hat{\mathbf{Y}}_c + \hat{\mathbf{Y}}_{in}(z)\right]^{-1}\left[\hat{\mathbf{Y}}_c - \hat{\mathbf{Y}}_{in}(z)\right] \tag{11}$$

The FD solution for the line voltages and currents is *uniquely determined via (6a) combining (i) the general modal solution (7), (ii) interdependence between modal voltages and currents imposed by (6d), and (iii) the applicable continuity (4c) or excitation/termination (5d) conditions.*

### *3.2.4. Matching*

The terminal IAM matrix of a TL of length $l$ terminated at the end $(z = l)$ at admittance matrix $\hat{\mathbf{Y}}_L$ is by definition:

$$\hat{\mathbf{Y}}_{in}(l) = \hat{\mathbf{Y}}_L \tag{12}$$

If the line is "matched", i.e., terminated at the end at its characteristic admittance matrix, the backward evolving line and modal electric quantities are cancelled, as:

$$\hat{\mathbf{Y}}_L = \hat{\mathbf{Y}}_c \Leftrightarrow \hat{\mathbf{Y}}_{in}(z) = \hat{\mathbf{Y}}_c \Leftrightarrow \left\{ \begin{matrix} \hat{\mathbf{V}}^-(z) = \hat{\mathbf{I}}^-(z) = \mathbf{0}_\nu, z \in [0, l] \\ \hat{\mathbf{V}}_m^- = \hat{\mathbf{I}}_m^- = \mathbf{0}_\nu \\ \hat{\boldsymbol{\rho}}(z) = \mathbf{0}_\nu \end{matrix} \right\} \tag{13}$$

### *3.2.5. Feasibility of Modal Decomposition*

The decoupling is frequency-independent if the transformation matrices result by default constant, as:

$$\hat{\mathbf{T}}_V = \mathbf{T}_V \forall f, \quad \hat{\mathbf{T}}_I = \mathbf{T}_I \forall f \tag{14}$$

which is feasible over the following TL structures [84]:

1) Perfect conductors inside homogeneous dielectric.
2) Perfect conductors inside transversely non-homogeneous dielectric.
3) Cyclic symmetric configurations, defined by cyclic symmetric per-unit-length admittance and impedance matrices as:

$$\left. \begin{matrix} [\mathbf{M}]_{kk'} = M_{|k-k'|+1} \\ M_{k\pm\nu} = M_k \\ M_{\nu+2-k} = M_k \end{matrix} \right\}, \qquad \left. \begin{matrix} \nu + k \to k \\ \nu + k' \to k' \end{matrix} \right\}, \quad \mathbf{M} = \hat{\mathbf{Y}}, \hat{\mathbf{Z}} \tag{15}$$

In the case of good conductors inside homogeneous dielectric, frequency-independent decoupling can also be feasible. However, there exist MTL structures (fortunately rare) of good conductors inside homogeneous dielectric where modal decomposition fails, i.e., non-singular transformation matrices cannot be found. This matter is concisely enlightened in [83]. Certain non-diagonalisable situations are described in [85], whereas in [83], the general non-diagonalisable

MTL case is presented leading to the generalised modal theory, where the similarity transformations $\hat{\mathbf{T}}_V^{-1}\hat{\mathbf{Z}}\hat{\mathbf{Y}}\hat{\mathbf{T}}_V$ and $\hat{\mathbf{T}}_\mathrm{I}^{-1}\hat{\mathbf{Y}}\hat{\mathbf{Z}}\hat{\mathbf{T}}_\mathrm{I}$ are not diagonal, but quasi-diagonal block matrices in the so-called Jordan form. The present travelling-wave TD analysis applies only if modal decomposition is feasible in the diagonal form of (6b).

## 4. TIME-DOMAIN RESPONSE DERIVATION

### 4.1. Single-mode Propagation

#### *4.1.1. Reference Scheme in the Frequency Domain*

4.1.1.1 Matched-line Configuration

For a $(\nu + 1)$-conductor TL of length $l$ excited at the origin $(z = 0)$ by a voltage source $V_{G_{kk'}}(t)$ of non-zero output impedance $\hat{Z}_{G_{kk'}} = (\hat{Y}_{kk'})^{-1}$ between every $k$-th and $k'$-th conductor pair, as in (5a) and (5b), and matched at the end according to (13), it is obtained:

$$\hat{\mathbf{V}}(z) = \hat{\mathbf{V}}^{+}(z), \quad \hat{\mathbf{I}}(z) = \hat{\mathbf{I}}^{+}(z) \tag{16}$$

and from (10) and (16), the terminal conditions (5d) become:

$$\begin{aligned}
&\hat{I}_k^{+}(0)=\hat{Y}_{k0}\left[\hat{V}_{G_{k0}}-\hat{V}_k^{+}(0)\right]-\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left[\hat{V}_k^{+}(0)-\hat{V}_{G_{kk'}}-\hat{V}_{k'}^{+}(0)\right]\hat{Y}_{kk'},\\
&\hat{I}_k^{+}(0)=\sum_{k'=1}^{\nu}\left[\hat{\mathbf{Y}}_c\right]_{kk'}\hat{V}_{k'}^{+}(0)
\end{aligned} \tag{17}$$

so that the forward-evolving standing wave components of the line electric quantities at the origin $(z = 0)$ get:

$$\begin{aligned}
&\hat{\mathbf{V}}^{+}(0)=\hat{\mathbf{T}}_V\hat{\mathbf{V}}_m^{+}\Leftrightarrow\hat{V}_k^{+}(0)=\sum_{\xi=1}^{\nu}\left[\hat{\mathbf{T}}_V\right]_{k\xi}\hat{V}_{m\xi}^{+},\\
&\hat{\mathbf{I}}_k^{+}(0)=\hat{\mathbf{T}}_\mathrm{I}\hat{\mathbf{I}}_m^{+}\Leftrightarrow\hat{\mathrm{I}}_k^{+}(0)=\sum_{\xi=1}^{\nu}\left[\hat{\mathbf{T}}_\mathrm{I}\right]_{k\xi}\hat{\mathbf{I}}_{m\xi}^{+},\quad k=1,\ldots,\nu
\end{aligned} \tag{18}$$

4.1.1.2 Single-mode Excitation

Since the eigenvectors:

$$\begin{aligned}
&\hat{\mathbf{t}}_{V\xi}\hat{=}\left[\ \left[\hat{\mathbf{T}}_V\right]_{1\xi}\ \ \cdots\ \ \left[\hat{\mathbf{T}}_V\right]_{\nu\xi}\right]^T,\\
&\hat{\mathbf{t}}_{I\xi}\hat{=}\left[\left[\hat{\mathbf{T}}_I\right]_{1\xi}\ \ \cdots\ \ \left[\hat{\mathbf{T}}_I\right]_{\nu\xi}\right]^T,\quad \xi=1,\ldots,\nu
\end{aligned} \tag{19}$$

of matrices $(\hat{\mathbf{Z}}\hat{\mathbf{Y}})$ and $(\hat{\mathbf{Y}}\hat{\mathbf{Z}})$ respectively are linearly independent (as the line is assumed diagonalisable), it is feasible to excite a single arbitrary $\xi$-th mode as:

$$\hat{V}^{+}_{m\xi'} = 0 \forall \xi' \in \{1, \ldots, \nu \backslash \xi\} \Leftrightarrow \hat{\mathbf{V}}^{+}_{m} = \begin{bmatrix} 0 & \ldots & 0 & \hat{V}^{+}_{m\xi} & 0 & \ldots & 0 \end{bmatrix} \tag{20a}$$

by selecting:

$$\hat{\mathbf{V}}^{+}(0) = \hat{\mathbf{V}}^{+}_{\xi}(0) \hat{=} \hat{\mathbf{t}}_{V\xi} \hat{V}^{+}_{m\xi} \tag{20b}$$

as (i) a total of $(\nu - 1)$ conditions:

$$\hat{\mathbf{t}}_{V\xi} = \left[\hat{\mathbf{T}}_{V}\right]_{1\xi} \left[1 \ \left(\left[\hat{\mathbf{T}}_{V}\right]_{2\xi} \Big/ \left[\hat{\mathbf{T}}_{V}\right]_{1\xi}\right) \ \ldots \ \left(\left[\hat{\mathbf{T}}_{V}\right]_{\nu\xi} \Big/ \left[\hat{\mathbf{T}}_{V}\right]_{1\xi}\right)\right]^{T} \tag{20c}$$

are applied to every $\xi' \in \{1, \ldots, \nu\}$, and (ii) up to $\nu!$ independent voltage sources may be coupled at the origin between the several conductor pairs, up to $\nu$ of which between the excitation conductors and the reference.

Since only the $\xi$-th mode is excited, from (16), (7), (9) and (8), the phasors of the line electric quantities become:

$$\begin{gathered}\hat{\mathbf{V}}(z) = \hat{\mathbf{V}}^{+}_{\xi}(z) \hat{=} \hat{\mathbf{t}}_{V\xi} \hat{V}^{+}_{m\xi}(z), \quad \hat{V}_{k}(z) = \hat{V}^{+}_{k\xi}(z) \hat{=} \left[\hat{\mathbf{T}}_{V}\right]_{k\xi} \hat{V}^{+}_{m\xi}(z), \\ \hat{V}^{+}_{m\xi}(z) \hat{=} \hat{V}^{+}_{m\xi} e^{-\hat{\gamma}_{\xi} z}, \quad k = 1, \ldots, \nu\end{gathered} \tag{21a}$$

$$\begin{gathered}\hat{\mathbf{I}}(z) = \hat{\mathbf{I}}^{+}_{\xi}(z) \hat{=} \hat{\mathbf{t}}_{I\xi} \hat{I}^{+}_{m\xi}(z), \quad \hat{I}(z) = \hat{I}^{+}_{k\xi}(z) \hat{=} \left[\hat{\mathbf{T}}_{I}\right]_{k\xi} \hat{I}^{+}_{m\xi}(z), \\ \hat{I}^{+}_{m\xi}(z) \hat{=} \hat{I}^{+}_{m\xi} e^{-\hat{\gamma}_{\xi} z}, \quad k = 1, \ldots, \nu\end{gathered} \tag{21b}$$

4.1.1.3 Single-mode Response

Supposing excitation to take place at the origin $(z = 0)$, the modal transfer function (TF) $\hat{H}_{\xi}(z)$ of every $\xi$-th mode is herein defined at any longitudinal position $z$ as:

$$\hat{H}_{\xi}(z) \hat{=} e^{-\hat{\gamma}_{\xi} z} = \frac{\hat{V}^{+}_{m\xi}(z)}{\hat{V}^{+}_{m\xi}(0)} = \frac{\hat{I}^{+}_{m\xi}(z)}{\hat{I}^{+}_{m\xi}(0)}, \quad \xi = 1, \ldots, \nu \tag{22a}$$

so that single $\xi$-th mode excitation gives:

$$\frac{\hat{V}^{+}_{k\xi}(z)}{\hat{V}^{+}_{k\xi}(0)} = \frac{\hat{I}^{+}_{k\xi}(z)}{\hat{I}^{+}_{k\xi}(0)} = \hat{H}_{\xi}(z), \quad k = 1, \ldots, \nu \tag{22b}$$

### *4.1.2. Time-domain Transition*

#### 4.1.2.1 Line Excitation and Matching

A $(\nu+1)$-conductor TL of length $l$ is considered, excited at the origin by a voltage source:

$$\begin{aligned} &V_{G_{kk'}}(t) \hat{=} \mathrm{Re}\left\{\widehat{V}_{G_{kk'}}(t)\right\}, \\ &\widehat{V}_{G_{kk'}}(t) = \tilde{V}_{G_{kk'}}(t)e^{j2\pi f_c t} \rightleftarrows F\left[\tilde{V}_{G_{kk'}}(t)\right]_{f-f_c} \end{aligned} \tag{23a}$$

of non-zero output impedance $\hat{Z}_{G_{kk'}} = (\hat{Y}_{kk'})^{-1}$ between every $k$-th and $k'$-th conductor pair, with a narrowband complex envelope $\tilde{V}_{G_{kk'}}(t)$ of bandwidth $W_{G_{kk'}}$ as:

$$F\left[\tilde{V}_{G_{kk'}}(t)\right] = 0, \quad |f| > \left(W_{G_{kk'}}/2\right) \tag{23b}$$

so that:

$$\begin{aligned} &\hat{\mathbf{Y}}_{0,L}, \hat{\mathbf{T}}_{V,I}, \hat{\mathbf{Y}}_{in}(z), \hat{\mathbf{Y}}_{c,m}, \quad \hat{\gamma} = \text{constant}, \\ &f \in \left(f_c - \frac{W_{G_{kk'}}}{2}, f_c + \frac{W_{G_{kk'}}}{2}\right) \end{aligned} \tag{24}$$

where the "∩" sign indicates a *pre-envelope* quantity, and the "$F$" operator denotes the Fourier transform.

Due to the narrowband spectral invariability of (24), the line voltages become:

$$V_k(0,t) = \mathrm{Re}\left[\tilde{V}_k(0,t)e^{j2\pi f_c t}\right] \tag{25a}$$

and the termination conditions at the origin become:

$$\left.\begin{aligned} &I_k(0,t) = \mathrm{Re}\left[\widehat{I}_k(0,t)\right], \quad \widehat{I}_k(0,t) = \tilde{I}_k(0,t)e^{j2\pi f_c t} \rightleftarrows F\left[\tilde{I}_k(0,t)\right]_{f-f_c} \\ &\tilde{I}_k(0,t) = \hat{Y}_{k0}\left[\tilde{V}_{G_{k0}}(t) - \tilde{V}_k(0,t)\right] \\ &\qquad - \sum_{\substack{k'=1 \\ k'\neq k}}^{\nu}\left[\tilde{V}_k(0,t) - \tilde{V}_{G_{kk'}}(t) - \tilde{V}_{k'}(0,t)\right]\hat{Y}_{kk'} \end{aligned}\right\} \tag{25b}$$

Supposing that (i) the line is matched, and (ii) no excitation is applied at line's end, from (25), (24), and (17), the line voltages and currents interrelate at the origin exactly as in the case of harmonic matched-line excitation, i.e.,

$$\tilde{I}_k(0,t) = \sum_{k'=1}^{\nu}\left[\hat{\mathbf{Y}}_c\right]_{kk'}\tilde{V}_{k'}(0,t) \tag{26}$$

4.1.2.2 Single-mode Excitation

Imposing over the whole complex-envelope set $\tilde{V}_{G_{kk'}}(t)$ of the excitation voltages (23) identical conditions as between respective phasors of the excitation voltages (5a)–(5b) for the satisfaction of (20b), it is obtained from (24):

$$\begin{aligned} V_k(0,t) &= V_{k\xi}(0,t) \hat{=} \mathrm{Re}\left\{\tilde{V}_{k\xi}(0,t)e^{j2\pi f_c t}\right\}, \\ \tilde{V}_{k\xi}(0,t) &\hat{=} \left[\hat{\mathbf{T}}_V\right]_{k\xi} \tilde{V}^{+}_{m\xi}(t), \quad k=1,\ldots,\nu \end{aligned} \tag{27a}$$

$$\begin{aligned} I_k(0,t) &= I_{k\xi}(0,t) \hat{=} \mathrm{Re}\left\{\tilde{I}_{k\xi}(0,t)e^{j2\pi f_c t}\right\}, \\ \tilde{I}_{k\xi}(0,t) &\hat{=} \left[\hat{\mathbf{T}}_I\right]_{k\xi} \tilde{I}^{+}_{m\xi}(t), \quad k=1,\ldots,\nu \end{aligned} \tag{27b}$$

$$\tilde{I}^{+}_{m\xi}(t) = \hat{Y}_{m\xi}\tilde{V}^{+}_{m\xi}(t) \tag{27c}$$

A single, $\xi$-th mode is thus exclusively excited at the origin, i.e., *the relation between line voltages and currents is identical as if only harmonic evolution of the $\xi$-th mode was present.*

4.1.2.3 Demonstration of Modal Propagation

Being from (24) the line and modal characteristic parameters constant along the bandwidth $W_{G_{kk'}}$ of every excitation voltage $V_{G_{kk'}}(t)$ defined in (23), upon exclusive $\xi$-th mode excitation at the origin ($z=0$) as in (27), each one of the excited line voltages $V_k(0,t)$, $k=1,\ldots,\nu$, defined in (27) is along the line subject to the $\xi$-th mode TF of (22) for $f=f_c$, as

$$\begin{aligned} V_k(z,t) &= V_{k\xi}(z,t) \hat{=} \mathrm{Re}\left\{\tilde{V}_{k\xi}(z,t)e^{j2\pi f_c t}\right\}, \\ \tilde{V}_{k\xi}(z,t) &= e^{-\hat{\gamma}_\xi z}\tilde{V}_{k\xi}\left(0, t-\frac{z}{2\pi f_c/\hat{\beta}_\xi}\right), \quad k=1,\ldots,\nu \end{aligned} \tag{28a}$$

Similarly, the line currents become

$$\begin{aligned} I_k(z,t) &= I_{k\xi}(z,t) \hat{=} \mathrm{Re}\left\{\tilde{I}_{k\xi}(z,t)e^{j2\pi f_c t}\right\}, \\ \tilde{I}_{k\xi}(z,t) &= e^{-\hat{\gamma}_\xi z}\tilde{I}_{k\xi}\left(0, t-\frac{z}{2\pi f_c/\hat{\beta}_\xi}\right), \quad k=1,\ldots,\nu \end{aligned} \tag{28b}$$

Therefore, *the narrowband excitation (27) applied at the origin propagates along the line forming voltage and current travelling waves, which cover distance $z$ within time $(\frac{z}{2\pi f_c/\hat{\beta}_\xi})$, attenuated by the factor $e^{-\hat{a}_\xi z}$, and having the complex envelope shifted by $(-\hat{\beta}_\xi z)$.*

### *4.1.3. Travelling-wave Quantification*

#### 4.1.3.1 Modal Parameters

The *propagation velocity* $v_\xi$ of the $\xi$-th mode is defined as the ratio of the angular velocity or cyclic frequency $\omega$ to the phase constant of the mode, as

$$v_\xi \hat{=} \omega \Big/ \hat{\beta}_\xi, \quad \omega \hat{=} 2\pi f_c \tag{29}$$

and represents along the line the travelling velocity of the electric quantities excited pursuant to (27).

The *wavelength* $\lambda_\xi$ of the $\xi$-th mode is defined as the ratio of the modal propagation velocity to the excitation frequency

$$\lambda_\xi \hat{=} v_\xi / f_c \tag{30}$$

and represents the travel distance of the electric quantities excited pursuant to (27), along which the angle of the $e^{j2\pi f_c t}$ harmonic term is shifted by $(2\pi)$.

The *propagation delay* $\tau_\xi(\Delta l)$ of the $\xi$-th mode along length $\Delta l$ is defined as the ratio of the covered length to the propagation velocity

$$\tau_\xi(\Delta l) \hat{=} \Delta l / v_\xi \tag{31}$$

and represents the time during which the electric quantities excited pursuant to (27) cover (arrive at) distance $\Delta l$ towards the travelling direction.

#### 4.1.3.2 Travelling Quantities — Characteristic Matrices

All the travelling components defined in (28) are *positively or forward travelling paths* as:

$$\begin{aligned} V_{k\xi}^{+}(z,t) &\hat{=} \mathrm{Re}\left\{ e^{-\hat{\gamma}_\xi z} \left[\hat{\mathbf{T}}_V\right]_{k\xi} \tilde{V}_{m\xi}^{+} \left[t - \tau_\xi(z)\right] e^{j2\pi f_c t} \right\}, \\ I_{k\xi}^{+}(z,t) &\hat{=} \mathrm{Re}\left\{ e^{-\hat{\gamma}_\xi z} \left[\hat{\mathbf{T}}_I\right]_{k\xi} \tilde{I}_{m\xi}^{+} \left[t - \tau_\xi(z)\right] e^{j2\pi f_c t} \right\}, \quad k=1,\ldots,\nu \end{aligned} \tag{32a}$$

and the corresponding *vectorial paths* are:

$$\begin{aligned} \mathbf{V}_\xi^{+}(z,t) &\hat{=} \left[\begin{array}{ccc} V_{1\xi}^{+}(z,t) & \ldots & V_{\nu\xi}^{+}(z,t) \end{array}\right]^T, \\ \mathbf{I}_\xi^{+}(z,t) &\hat{=} \left[\begin{array}{ccc} I_{1\xi}^{+}(z,t) & \ldots & I_{\nu\xi}^{+}(z,t) \end{array}\right]^T \end{aligned} \tag{32b}$$

Defining the *time-variant travelling phasors* of the line voltage and current paths as:

$$\left.\begin{aligned} \tilde{V}_{k\xi}^{+}(z,t) &\hat{=} e^{-\hat{\gamma}_\xi z} \left[\hat{\mathbf{T}}_V\right]_{k\xi} \tilde{V}_{m\xi}^{+} \left[t - \tau_\xi(z)\right] = e^{-\hat{\gamma}_\xi z} \tilde{V}_{k\xi}^{+} \left[0, t - \tau_\xi(z)\right] \\ \tilde{I}_{k\xi}^{+}(z,t) &\hat{=} e^{-\hat{\gamma}_\xi z} \left[\hat{\mathbf{T}}_I\right]_{k\xi} \tilde{I}_{m\xi}^{+} \left[t - \tau_\xi(z)\right] = e^{-\hat{\gamma}_\xi z} \tilde{I}_{k\xi}^{+} \left[0, t - \tau_\xi(z)\right] \end{aligned}\right\}, \quad k=1,\ldots,\nu \tag{33a}$$

and the corresponding *vectorial phasors* as:

$$\begin{aligned}\tilde{\mathbf{V}}_{\xi}^{+}(z,t) &\hat{=} \left[\tilde{V}_{1\xi}^{+}(z,t) \ \ldots \ \tilde{V}_{\nu\xi}^{+}(z,t)\right]^{T}, \\ \tilde{\mathrm{I}}_{\xi}^{+}(z,t) &\hat{=} \left[\ \tilde{\mathrm{I}}_{1\xi}^{+}(z,t) \quad \ldots \quad \tilde{\mathrm{I}}_{\nu\xi}^{+}(z,t)\ \right]^{T}\end{aligned} \tag{33b}$$

which travel along the line as:

$$\begin{aligned}\tilde{\mathbf{V}}_{\xi}^{+}(z,t) &= e^{-\hat{\gamma}_{\xi}z}\,\tilde{\mathbf{V}}_{\xi}^{+}\left[0,t-\tau_{\xi}(z)\right] = e^{-\hat{\gamma}_{\xi}z}\hat{\mathbf{t}}_{V\xi}\tilde{V}_{m\xi}^{+}\left[t-\tau_{\xi}(z)\right], \\ \tilde{\mathrm{I}}_{\xi}^{+}(z,t) &= e^{-\hat{\gamma}_{\xi}z}\tilde{\mathrm{I}}_{\xi}^{+}\left[0,t-\tau_{\xi}(z)\right] = e^{-\hat{\gamma}_{\xi}z}\hat{\mathbf{t}}_{I\xi}\tilde{I}_{m\xi}^{+}\left[t-\tau_{\xi}(z)\right]\end{aligned} \tag{33c}$$

it is obtained:

$$\mathbf{V}_{\xi}^{+}(z,t) = \mathrm{Re}\left\{\tilde{\mathbf{V}}_{\xi}^{+}(z,t)e^{j2\pi f_c t}\right\}, \quad \mathbf{I}_{\xi}^{+}(z,t) = \mathrm{Re}\left\{\tilde{\mathrm{I}}_{\xi}^{+}(z,t)e^{j2\pi f_c t}\right\} \tag{34}$$

The Travelling Admittance Matrix (TAM) $\tilde{\mathbf{Y}}_c$ is defined by:

$$\tilde{\mathbf{I}}_{\xi}^{+}(z,t) = \tilde{\mathbf{Y}}_c\tilde{\mathbf{V}}_{\xi}^{+}(z,t) \tag{35a}$$

and from (10), (27c), and (33c), it equals the CAM as:

$$\tilde{\mathbf{Y}}_c = \hat{\mathbf{Y}}_c \tag{35b}$$

Analogously to the FD, the voltage and current *modal travelling components*, or equivalently *modal paths* are defined as

$$\begin{aligned}V_{m\xi}^{+}(z,t) &\hat{=} \mathrm{Re}\left\{\tilde{V}_{m\xi}^{+}(z,t)e^{j2\pi f_c t}\right\}, \\ \tilde{V}_{m\xi}^{+}(z,t) &\hat{=} e^{-\hat{\gamma}_{\xi}z}\tilde{V}_{m\xi}^{+}\left[t-\tau_{\xi}(z)\right]\end{aligned} \tag{36a}$$

and

$$I_{m\xi}^{+}(z,t) \hat{=} \mathrm{Re}\left\{\tilde{I}_{m\xi}^{+}(z,t)e^{j2\pi f_c t}\right\}, \quad \tilde{I}_{m\xi}^{+}(z) \hat{=} e^{-\hat{\gamma}_{\xi}z}\tilde{I}_{m\xi}^{+}\left[t-\tau_{\xi}(z)\right] \tag{36b}$$

respectively, where $\tilde{V}_{m\xi}^{+}(z,t)$, $\tilde{I}_{m\xi}^{+}(z,t)$ are the phasors, and $\tilde{V}_{m\xi}^{+}(t)$, $\tilde{I}_{m\xi}^{+}(t)$ the *modal travelling excitation phasors.*

At this point, it should be noted that the concept of a *time-variant phasor* quantity carries in fact the meaning of a *complex-envelope*, which is obvious from the above definitions. However, for the sake of (i) term consistency, and (ii) term analogy with the FD approach, the term "phasor" is herein adopted to denote both the FD phasor parameter, and the aforementioned TD components.

## 4.2. Aggregate Response

### *4.2.1. Superposition of Modal Excitations*

#### 4.2.1.1 Superposition of Modal Quantities Excited at Origin

If the single-mode excitation conditions set in (27) for the complex envelopes $\tilde{V}_{G_{kk'}}(t)$ of the excitation voltages applied according to (23) are not satisfied, the whole set of line's modes are in the general case excited, as

$$V_{k0}^{+}(0,t) = \sum_{\xi=1}^{\nu} V_{k\xi 0}^{+}(0,t), \quad V_{k\xi\,0}^{+}(0,t) = \mathrm{Re}\left\{\tilde{V}_{k\xi 0}^{+}(0,t)e^{j2\pi f_c t}\right\},$$
$$\tilde{V}_{k\xi 0}^{+}(0,t) = \left[\hat{\mathbf{T}}_V\right]_{k\xi} \tilde{V}_{m\xi 0}^{+}(t), \quad \left\{\begin{matrix} k = 1,\ldots,\nu \\ \xi = 1,\ldots,\nu \end{matrix}\right\} \tag{37a}$$

$$I_{k0}^{+}(0,t) = \sum_{\xi=1}^{\nu} I_{k\xi 0}^{+}(0,t), \quad I_{k\xi 0}^{+}(0,t) = \mathrm{Re}\left\{\tilde{I}_{k\xi 0}^{+}(0,t)e^{j2\pi f_c t}\right\},$$
$$\tilde{I}_{k\xi 0}^{+}(0,t) = \left[\hat{\mathbf{T}}_I\right]_{k\xi} \tilde{I}_{m\xi 0}^{+}(t), \quad \left\{\begin{matrix} k = 1,\ldots,\nu \\ \xi = 1,\ldots,\nu \end{matrix}\right\} \tag{37b}$$

$$\tilde{I}_{m\xi}^{+}(t) = \hat{Y}_{m\xi}\tilde{V}_{m\xi}^{+}(t) \tag{37c}$$

where subscript "0" denotes the initial line voltages and currents excited at the origin — $V_{k0}^{+}(0,t)$ and $I_{k0}^{+}(0,t)$ respectively, $k = 1,\ldots,\nu$ — as well as the consequent positively travelling paths — as such "travelling" concept is defined in Sub-section (4.1.2.3) hereof — arisen as soon as the excitation voltages of (23) are applied. These positively travelling paths are the modal voltages $V_{k\xi 0}^{+}(0,t)$ and currents $I_{k\xi 0}^{+}(0,t)$, $k = 1,\ldots,\nu$, $\xi = 1,\ldots,\nu$, which constitute the components of the voltage $V_{k0}^{+}(0,t)$ and the current $I_{k0}^{+}(0,t)$ excited respectively at the line's origin between every $k$-th conductor and the reference (conductor "0"), and which start right upon excitation to travel (propagate) along the line covering distance $z$ within time $(\frac{z}{2\pi f_c/\hat{\beta}_\xi})$, attenuated by the factor $e^{-\hat{a}_\xi z}$, and having their complex envelope shifted by $(-\hat{\beta}_\xi z)$.

The *time-variant* voltage and current *travelling excitation phasors* are respectively defined as

$$\tilde{V}_{k0}^{+}(0,t) \hat{=} \sum_{\xi=1}^{\nu} \tilde{V}_{k\xi 0}^{+}(0,t), \quad \tilde{I}_{k0}^{+}(0,t) \hat{=} \sum_{\xi=1}^{\nu} \tilde{I}_{k\xi\,0}^{+}(0,t) \tag{38a}$$

and the *vectorial travelling excitation phasors* as

$$\begin{aligned}\tilde{\mathbf{V}}_0^+(0,t) &\hat{=} \left[\tilde{V}_{10}^+(0,t)\ldots\tilde{V}_{\nu 0}^+(0,t)\right]^T = \sum_{\xi=1}^{\nu}\tilde{\mathbf{V}}_{\xi 0}^+(0,t) \hat{=} \hat{\mathbf{T}}_V\tilde{\mathbf{V}}_{m0}^+(t),\\ \tilde{\mathbf{I}}_0^+(0,t) &\hat{=} \left[\tilde{I}_{10}^+(0,t)\ldots\tilde{I}_{\nu 0}^+(0,t)\right]^T = \sum_{\xi=1}^{\nu}\tilde{\mathbf{I}}_{\xi 0}^+(0,t) \hat{=} \hat{\mathbf{T}}_I\tilde{\mathbf{I}}_{m0}^+(t)\end{aligned} \tag{38b}$$

where

$$\begin{aligned}\tilde{\mathbf{V}}_{m0}^+(t) &\hat{=} \left[\ \tilde{V}_{m10}^+(t) \quad \ldots \quad \tilde{V}_{m\nu 0}^+(t)\ \right]^T,\\ \tilde{\mathbf{I}}_{m0}^+ &\hat{=} \left[\ \tilde{I}_{m10}^+(t) \quad \ldots \quad \tilde{I}_{m\nu 0}^+(t)\ \right]^T\end{aligned} \tag{38c}$$

the *modal vectorial travelling excitation phasors.*

4.2.1.2 Excitation Terminal Conditions at Origin

From (25b), (26), and (37), excitation at line's origin satisfies the following terminal conditions:

$$\begin{aligned}\tilde{I}_{k0}^+(0,t) &= \hat{Y}_{k0}\left[\hat{V}_{G_{k0}}(t) - \tilde{V}_{k0}^+(0,t)\right]\\ &\quad - \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left[\tilde{V}_{k0}^+(0,t) - \hat{V}_{G_{k\kappa}}(t) - \tilde{V}_{k'0}^+(0,t)\right]\hat{Y}_{kk'}\end{aligned} \tag{39a}$$

$$\tilde{I}_{k0}^+(0,t) = \sum_{k'=1}^{\nu}\left[\hat{\mathbf{Y}}_c\right]_{kk'}\tilde{V}_{k'0}^+(0,t) \tag{39b}$$

no matter whether the end is matched or not, since *no excitation effect appears and no related reaction arises at the end before arrival of the initial paths generated at the origin.*

4.2.1.3 Travelling Quantities — Superposition

By virtue of (i) linear independency between the eigenvectors defined in (19), and (ii) linearity of the TL under consideration, all the several excited paths travel independently along the line, as:

$$\begin{aligned}\mathbf{V}_{\xi 0}^+(z,t) &= \mathrm{Re}\left\{\tilde{\mathbf{V}}_{\xi 0}^+(z,t)e^{j2\pi f_c t}\right\},\\ \tilde{\mathbf{V}}_{\xi 0}^+(z,t) &= e^{-\hat{\gamma}_\xi z}\tilde{\mathbf{V}}_{\xi 0}^+\left[0,t-\tau_\xi(z)\right], \quad \xi = 1,\ldots,\nu\end{aligned} \tag{40a}$$

$$\begin{aligned}\mathbf{I}_{\xi 0}^+(z,t) &= \mathrm{Re}\left\{\tilde{\mathbf{I}}_{\xi 0}^+(z,t)e^{j2\pi f_c t}\right\},\\ \tilde{\mathbf{I}}_{\xi 0}^+(z,t) &= e^{-\hat{\gamma}_\xi z}\tilde{\mathbf{I}}_{\xi 0}^+\left[0,t-\tau_\xi(z)\right], \quad \xi = 1,\ldots,\nu\end{aligned} \tag{40b}$$

$$\tilde{\mathbf{I}}_{\xi 0}^{+}(z,t) = \tilde{\mathbf{Y}}_c \tilde{\mathbf{V}}_{\xi 0}^{+}(z,t), \quad \xi = 1,\ldots,\nu \tag{40c}$$

each one with the propagation velocity $v_\xi$ and the propagation constant $\hat{\gamma}_\xi$ of its respective $\xi$-th mode.

### *4.2.2. Medium Discontinuity — Terminal Conditions*

#### 4.2.2.1 Line-multitude Continuity

##### 4.2.2.1.1 Formulation

Incidence of a vectorial path (incident path) — modal by definition, since, in the general case, each mode travels at its own propagation velocity — against a longitudinal structural-discontinuity point $z_0$ — where by definition TAM discontinuity is as well observed — causes paths travelling away from the discontinuity to be raised, and in particular (i) the "transmitted" paths, propagating in line with the direction of incidence, and (ii) the "reflected" ones, opposite to the incident path. Let $\tilde{\mathbf{V}}_{i\xi}(t)$ and $\tilde{\mathbf{I}}_{i\xi}(t)$ be the $\xi$-th — mode (with no loss of generality) voltage and current respectively *incident* vectorial phasors, $\tilde{\mathbf{V}}_t(t)$ and $\tilde{\mathbf{I}}_t(t)$ the travelling-excitation vectorial phasors of the *transmitted* paths at the point of discontinuity, and $\tilde{\mathbf{V}}_r(t)$ and $\tilde{\mathbf{I}}_r(t)$ the travelling-excitation vectorial phasors of the *reflected* ones.

Due to (i) the *infinitesimal* longitudinal dimension of the medium-discontinuity point, and (ii) the consequent *instantaneous* generation of transmitted and reflected paths, the terminal conditions (2b) become:

$$\tilde{\mathbf{V}}_{i\xi}(t) + \tilde{\mathbf{V}}_r(t) = \tilde{\mathbf{V}}_t(t), \quad \tilde{\mathbf{I}}_{i\xi}(t') - \tilde{\mathbf{I}}_r(t) = \tilde{\mathbf{I}}_t(t) \tag{41a}$$

and satisfy (41b), where $\tilde{\mathbf{Y}}_{ci}$, $\tilde{\mathbf{Y}}_{ct}$ are respectively the TAM matrices right before and after the point of discontinuity:

$$\tilde{\mathbf{I}}_{i\xi}(t) = \tilde{\mathbf{Y}}_{ci}\tilde{\mathbf{V}}_i(t), \quad \tilde{\mathbf{I}}_r(t) = \tilde{\mathbf{Y}}_{ci}\tilde{\mathbf{V}}_r(t), \quad \tilde{\mathbf{I}}_t(t) = \tilde{\mathbf{Y}}_{ct}\tilde{\mathbf{V}}_t(t) \tag{41b}$$

##### 4.2.2.1.2 Characteristic Matrices

In the general case of incidence upon a *terminated junction* as in Fig. 2, i.e., a longitudinally infinitesimal TL point where three (3) or more TL sections intersect and passive lumped elements are as well connected in between one or more pairs of the TL conductors, covering thus also the simple cases of TL *termination* and TL *junction*, the infinitesimal longitudinal dimension of the junction point makes from (35b) the resultant TAM matrix *right after* the medium discontinuity equal to the *sum* of the TAM matrices of all *lumped* and *distributed* terminations, as:

$$\tilde{\mathbf{Y}}_{ct} = \hat{\mathbf{Y}}_0 + \sum_{i=1}^{n} \hat{\mathbf{Y}}_{ci} \tag{42}$$

The above expression stems directly from the TAM matrix definition, but is still quoted to assure full comprehension of the developed approach.

4.2.2.1.3 Line Discontinuity Matrices

The *travelling voltage reflection coefficient matrix*, or simply the *travelling reflection coefficient matrix* (TRCM) $\tilde{\rho}$, is defined from:

$$\tilde{\mathbf{V}}_r = \tilde{\rho}\tilde{\mathbf{V}}_{i\xi} \tag{43a}$$

The *travelling voltage transmission coefficient matrix*, or simply the *travelling transmission coefficient matrix* (TTCM) $\tilde{\mathbf{t}}$, is defined from:

$$\tilde{\mathbf{V}}_t = \tilde{\mathbf{t}}\tilde{\mathbf{V}}_{i\xi} \tag{43b}$$

Combination of (41) and (43) provides:

$$\tilde{\boldsymbol{\rho}} = \left(\tilde{\mathbf{Y}}_{ci} + \tilde{\mathbf{Y}}_{ct}\right)^{-1} \left(\tilde{\mathbf{Y}}_{ci} - \tilde{\mathbf{Y}}_{ct}\right) = \tilde{\boldsymbol{\rho}}^T \tag{44a}$$

$$\tilde{\mathbf{t}} = \mathbf{1}_\nu + \tilde{\rho} = 2\left(\tilde{\mathbf{Y}}_{ci} + \tilde{\mathbf{Y}}_{ct}\right)^{-1} \tilde{\mathbf{Y}}_{ci} = \tilde{\mathbf{t}}^T \tag{44b}$$

where the TAM matrices are supposed to be symmetric.

4.2.2.1.4 Modal Discontinuity Matrices

Since the TAM matrices from one to the other side of the medium discontinuity are not identical, in the general case, incidence causes generation of the whole set of line's modes, as:

$$\begin{gathered} \tilde{\mathbf{V}}_r(t) = \hat{\mathbf{T}}_{Vi}\tilde{\mathbf{V}}_{rm}(t), \quad \tilde{\mathbf{I}}_r(t) = \hat{\mathbf{T}}_{Ii}\tilde{\mathbf{I}}_{rm}(t), \\ \left[\tilde{\mathbf{I}}_{rm}(t)\right]_{\xi'} = \left[\hat{\mathbf{Y}}_{rm}\right]_{\xi'} \left[\tilde{\mathbf{V}}_{rm}(t)\right]_{\xi'}, \quad \xi' = 1, \ldots, \nu \end{gathered} \tag{45}$$

along the line of incidence, and

$$\begin{gathered} \tilde{\mathbf{V}}_t(t) = \hat{\mathbf{T}}_{Vti}\tilde{\mathbf{V}}_{tmi}(t), \quad \tilde{\mathbf{I}}_{ti}(t) = \hat{\mathbf{T}}_{Iti}\tilde{\mathbf{I}}_{tmi}(t), \\ \left[\tilde{\mathbf{I}}_{tmi}(t)\right]_{\xi'} = \left[\hat{\mathbf{Y}}_{tmi}\right]_{\xi'} \left[\tilde{\mathbf{V}}_{tmi}(t)\right]_{\xi'}, \quad \xi' = 1, \ldots, \nu \end{gathered} \tag{46}$$

along the $i$-th departure line as in Fig. 2, where $\hat{\mathbf{T}}_{Vi}$ and $\hat{\mathbf{T}}_{Ii}$ are respectively the voltage and current transformation matrices before the point of discontinuity, and $\hat{\mathbf{T}}_{Vti}$ , $\hat{\mathbf{T}}_{Iti}$ the homologous matrices of the $i$-th departure line.

The *modal travelling voltage reflection coefficient* or simply the *modal travelling reflection coefficient* (MTRC) $[\tilde{\boldsymbol{\rho}}_{mi}]_{\xi'\xi}$ from the $\xi$-th mode to the $\xi'$-th one is defined upon the medium discontinuity from:

$$\left[\tilde{\mathbf{V}}_{rm}(t)\right]_{\xi'} = [\tilde{\boldsymbol{\rho}}_m]_{\xi'\xi}\left[\tilde{\mathbf{V}}_{im}(t)\right]_{\xi}, \quad \xi' = 1, \ldots, \nu \tag{47a}$$

where $[\tilde{\mathbf{V}}_{im}(t)]_{\xi}$ the modal phasor of the incident line voltage path. The *modal travelling voltage transmission coefficient* or simply the *modal travelling transmission coefficient* (MTTC) $[\tilde{\mathbf{t}}_{mi}]_{\xi'\xi}$ from the $\xi$-th mode to the $\xi'$-th one along the $i$-th departure line is defined upon the medium discontinuity from:

$$\left[\tilde{\mathbf{V}}_{tmi}(t)\right]_{\xi'} = \left[\tilde{\mathbf{t}}_{mi}\right]_{\xi'\xi}\left[\tilde{\mathbf{V}}_{im}(t)\right]_{\xi}, \quad \xi' = 1, \ldots, \nu \tag{47b}$$

The $\nu \times \nu$ *modal travelling reflection coefficient matrix* (MTRCM) and the $\nu \times \nu$ *modal travelling transmission coefficient matrix* (MTTCM) are then extracted as:

$$\tilde{\boldsymbol{\rho}}_m = \left(\hat{\mathbf{T}}_{Vi}\right)^{-1}\tilde{\boldsymbol{\rho}}\hat{T}_{Vi} \tag{47c}$$

and

$$\tilde{\mathbf{t}}_{mi} = \left(\hat{\mathbf{T}}_{Vti}\right)^{-1}\tilde{\mathbf{t}}\hat{\mathbf{T}}_{Vi} \tag{47d}$$

respectively, combining (45), (46), and (44).

4.2.2.2 Transition to Line-multitude Discontinuity

4.2.2.2.1 Analytical TL Model Expansion

A medium discontinuity point in considered of $(n+1)$ intersecting TLs $(n \geq 1)$, each one in general with its own multitude of conductors as $(\nu_0 + 1)$ the line of incidence, $(\nu_1 + 1)$ the 1st departure line, $\ldots$, $(\nu_n + 1)$ the $n$-th departure line. Assuming (i) *continuity* of the *reference conductor* from one to the other side of the discontinuity, and (ii) numbering of all other conductors (excitation conductors) of the intersecting TLs according to the *homologous* in terms of *conduction continuity* excitation conductors of the line with the maximum multitude $\nu'$ as

$$\nu' = \max\{\nu_0, \nu_1, \ldots, \nu_n\} \tag{48}$$

the TAM matrix $(\tilde{\mathbf{Y}}_{ci})_{\nu_i\times\nu_i}$ of every $i$-th line $(i = 0, 1, \ldots, n)$ expands its dimension up to $\nu'$ as $(\tilde{\mathbf{Y}}_{ci}^{E})_{\nu'\times\nu'}$ setting by default equal to zero all additional lines and columns that correspond to non-existing conductors. Similarly, the vectorial phasors $[\tilde{\mathbf{V}}_{i\xi}(t)]_{\nu_0}$ and $[\tilde{\mathbf{I}}_{i\xi}(t)]_{\nu_0}$of the incident voltage and current paths are expanded as $[\tilde{\mathbf{V}}_{i\xi}^{E}(t)]_{\nu'}$, $[\tilde{\mathbf{I}}_{i\xi}^{E}(t)]_{\nu'}$ setting by default equal to zero all additional elements (voltage/current phasor quantities) that correspond to non-existing conductors.

Then, formulae (41) and (42) apply by default between the *expanded* matrices and vectors, whence the related $\nu' \times \nu'$ applicable TRCM and TTCM matrices $(\tilde{\boldsymbol{\rho}})_{\nu' \times \nu'}$, $(\tilde{\mathbf{t}})_{\nu' \times \nu'}$ are obtained from (44a) and (44b) respectively.

4.2.2.2.2 Modal Discontinuity Matrices

From the definition (43a), and since all expansion elements of $(\tilde{\mathbf{Y}}_{c0}^{E})_{\nu' \times \nu'}$, $[\tilde{\mathbf{V}}_{i\xi}^{E}(t)]_{\nu'}$, and $[\tilde{\mathbf{I}}_{i\xi}^{E}(t)]_{\nu'}$ are equal to zero, the $\nu_0 \times \nu_0$ TRCM matrix $(\tilde{\boldsymbol{\rho}}_{(i\to r)}t)_{\nu_0 \times \nu_0}$ relating the vectorial voltage phasor of the incident path with the resultant of all consequent reflected ones on the TL of incidence infinitesimally before the discontinuity as

$$\left[\tilde{\mathbf{V}}_{r}(t)\right]_{\nu_0} = \left(\tilde{\boldsymbol{\rho}}_{(i\to r)}\right)_{\nu_0 \times \nu_0} \left[\tilde{\mathbf{V}}_{i\xi}(t)\right]_{\nu_0} \tag{49a}$$

equals the sub-matrix remaining from the TRCM matrix $(\tilde{\boldsymbol{\rho}})_{\nu' \times \nu'}$ that comes out of (44a) after omitting all lines and columns of the conductors that do not exist along the line of incidence. The respective MTRCM matrix $(\tilde{\boldsymbol{\rho}}_{m(i\to r)})_{\nu_0 \times \nu_0}$ is then obtained from definition (47a) combined with (45), as:

$$\left(\tilde{\boldsymbol{\rho}}_{m(i\to r)}\right)_{\nu_0 \times \nu_0} = \left(\hat{\mathbf{T}}_{Vi}\right)^{-1}_{\nu_0 \times \nu_0} \left(\tilde{\boldsymbol{\rho}}_{(i\to r)}\right)_{\nu_0 \times \nu_0} \left(\hat{\mathbf{T}}_{Vi}\right)_{\nu_0 \times \nu_0} \tag{49b}$$

For any one of the departure lines, let it be the $i$-th one ($i = 1, \ldots, n$), and since all expansion elements of $(\tilde{\mathbf{Y}}_{ci}^{E})_{\nu' \times \nu'}$, $[\tilde{\mathbf{V}}_{i\xi}^{E}(t)]_{\nu'}$, and $\left[\tilde{\mathbf{I}}_{i\xi}^{E}(t)\right]_{\nu'}$ are by default equal to zero, the $\nu_i \times \nu_0$ applicable MTTCM matrix $(\tilde{\mathbf{t}}_{(0\to i)})_{\nu_i \times \nu_0}$ relating the vectorial voltage phasor of the incident path with the resultant of all consequent transmitted ones on the $i$-th departure line infinitesimally after the discontinuity as

$$\left[\tilde{\mathbf{V}}_{ti}(t)\right]_{\nu_i} = \left(\tilde{\mathbf{t}}_{(0\to i)}\right)_{\nu_i \times \nu_0} \left[\tilde{\mathbf{V}}_{i\xi}(t)\right]_{\nu_0} \tag{50a}$$

equals the sub-matrix remaining from the TTCM matrix $(\tilde{\mathbf{t}})_{\nu' \times \nu'}$ that comes out of (44b) after omitting all lines and columns of the conductors that do not exist along the $i$-th departure line. The respective MTTCM matrix $(\tilde{\mathbf{t}}_{m(0\to i)})_{\nu_i \times \nu_0}$ is then obtained from the definition (47b) combined with (46), as:

$$\left(\tilde{\mathbf{t}}_{m(0\to i)}\right)_{\nu_i \times \nu_0} = \left(\hat{\mathbf{T}}_{Vti}\right)^{-1}_{\nu_i \times \nu_i} \left(\tilde{\mathbf{t}}_{(0\to i)}\right)_{\nu_i \times \nu_0} \left(\hat{\mathbf{T}}_{Vi}\right)_{\nu_0 \times \nu_0} \tag{50b}$$

*4.2.3. Resultant Path Aggregation*

4.2.3.1 Generation Process

Incidence of any of the paths generated at the origin as soon as the excitation (23) is applied upon the end of the line generates the consequent reflected and transmitted paths, as described under Subsection 4.2.2. Incidence of any path travelling along a TL upon a horizontally infinitesimal discontinuity of the line (point of medium discontinuity) causes anew travelling-wave excitation, as also described in Subsection 4.2.2. As thus time elapses from the initial excitation (23) applied at the origin, the number of paths travelling over the TL network increases continuously with accelerating rate.

Denoted by $t_{\xi\kappa_\xi^\pm}$ the excitation time of the $\kappa_\xi^\pm$-th positively/negatively travelling path of the $\xi$-th mode, and numbering the paths consecutively on the basis of excitation sequence as:

$$\begin{aligned} &\kappa_\xi^\pm < \kappa'^\pm_\xi \Rightarrow t_{\xi\kappa_\xi^\pm} < t_{\xi\kappa'^\pm_\xi}, \quad \xi = 1, \ldots, \nu, \\ &\kappa_\xi^+, \kappa'^+_\xi \in \{0, 1, \ldots\}, \quad \kappa_\xi^-, \kappa'^-_\xi \in \{1, 2, \ldots\} \end{aligned} \tag{51}$$

the vectorial paths and phasors are represented as:

$$\left.\begin{aligned} \mathbf{V}^+_{\xi\kappa_\xi^+}(z,t) &\hat{=} \begin{bmatrix} V^+_{1\xi\kappa_\xi^+}(z,t) & \ldots & V^+_{\nu\xi\kappa_\xi^+}(z,t) \end{bmatrix}^T = \mathbf{V}^+_{\xi\kappa_\xi^+}(z,t) \\ &= \mathrm{Re}\left\{ \tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}\left[z, t - t_{\xi\kappa_\xi^+} - \tau_\xi(z)\right] e^{j2\pi f_c t} \right\} \\ \tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}\left[z, t - t_{\xi\kappa_\xi^+} - \tau_\xi(z)\right] &= e^{-\hat{\gamma}_\xi z}\tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}\left[0, t - t_{\xi\kappa_\xi^+} - \tau_\xi(z)\right], \\ \tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}\left[0, t - t_{\xi\kappa_\xi^+} - \tau_\xi(z)\right] &= \hat{\mathbf{t}}_{V\xi}\tilde{V}^+_{m\xi\kappa_\xi^+}\left[t - t_{\xi\kappa_\xi^+} - \tau_\xi(z)\right] \end{aligned}\right\} \tag{52a}$$

and

$$\left.\begin{aligned} &\mathbf{V}^-_{\xi\kappa_\xi^-}(z,t) \hat{=} \begin{bmatrix} V^-_{1\xi\kappa_\xi^-}(z,t) & \ldots & V^-_{\nu\xi\kappa_\xi^-}(z,t) \end{bmatrix}^T \\ &= \mathrm{Re}\left\{ \tilde{\mathbf{V}}^-_{\xi\kappa_\xi^-}\left[z, t - t_{\xi\kappa_\xi^-} - \tau_\xi(l-z)\right] e^{j2\pi f_c t} \right\} \\ &\tilde{\mathbf{V}}^-_{\xi\kappa_\xi^-}\left[z, t - t_{\xi\kappa_\xi^-} - \tau_\xi(l-z)\right] \\ &= e^{-\hat{\gamma}_\xi(l-z)} \cdot \tilde{\mathbf{V}}^-_{\xi\kappa_\xi^-}\left[l, t - t_{\xi\kappa_\xi^-} - \tau_\xi(l-z)\right], \\ &\tilde{\mathbf{V}}^-_{\xi\kappa_\xi^-}\left[l, t - t_{\xi\kappa_\xi^-} - \tau_\xi(z)\right] = \hat{\mathbf{t}}_{V\xi}\tilde{V}^-_{m\xi\kappa_\xi^-}\left[t - t_{\xi\kappa_\xi^-} - \tau_\xi(l-z)\right] \end{aligned}\right\} \tag{52b}$$

Formulae (51) and (52) hold also identically for respective current quantities and vectors, by substituting:

$$V, \mathbf{V} \leftrightarrow I, \mathbf{I} \quad , \quad \hat{\mathbf{T}}_V, \hat{\mathbf{t}}_{V\xi} \leftrightarrow \hat{\mathbf{T}}_I, \hat{\mathbf{t}}_{I\xi} \tag{53a}$$

and applying:

$$\tilde{I}^{\pm}_{m\xi\kappa^{\pm}_{\xi}}(t) = \hat{Y}_{m\xi}\tilde{V}^{\pm}_{m\xi\kappa^{\pm}_{\xi}}(t) \tag{53b}$$

4.2.3.2 End-to-end Aggregation

All the excited paths of (52) superpose along the TL to the following resultant response, of which the phasor (resultant complex envelope) indicates consecutive non-continuous step variations that are due to the above-described discrete path-generation process:

$$\begin{aligned}\mathbf{V}(z,t)=&\sum_{\xi=1}^{\nu}\sum_{\substack{\kappa^{+}_{\xi}\\ t\geq t_{\xi\kappa^{+}_{\xi}}+\tau_{\xi}(z)}}\mathbf{V}^{+}_{\xi\kappa^{+}_{\xi}}(z,t)+\sum_{\xi=1}^{\nu}\sum_{\substack{\kappa^{-}_{\xi}\\ t\geq t_{\xi\kappa^{-}_{\xi}}+\tau_{\xi}(l-z)}}\mathbf{V}^{-}_{\xi\kappa^{-}}(z,t)\\ =&\mathrm{Re}\left\{\left[\begin{array}{l}\displaystyle\sum_{\xi=1}^{\nu}\left[\sum_{\substack{\kappa^{+}_{\xi}\\ t\geq t_{\xi\kappa^{+}_{\xi}}+\tau_{\xi}(z)}}\tilde{\mathbf{V}}^{+}_{\xi\kappa^{+}_{\xi}}\left[z,t-t_{\xi\kappa^{-}_{\xi}}-\tau_{\xi}(z)\right]\right]\\ \displaystyle+\sum_{\xi=1}^{\nu}\left[\sum_{\substack{\kappa^{-}\\ t\geq t_{\xi\kappa^{-}_{\xi}}+\tau_{\xi}(z)}}\tilde{\mathbf{V}}^{-}_{\xi\kappa^{-}_{\xi}}\left[z,t-t_{\xi\kappa^{-}_{\xi}}-\tau_{\xi}(l-z)\right]\right]\end{array}\right]e^{j2\pi f_{c}t}\right\}\end{aligned} \tag{54a}$$

$$\begin{aligned}\mathbf{I}(z,t)=&\sum_{\xi=1}^{\nu}\sum_{\substack{\kappa^{+}_{\xi}\\ t\geq t_{\xi\kappa^{+}_{\xi}}+\tau_{\xi}(z)}}\mathbf{I}^{+}_{\xi\kappa^{+}_{\xi}}(z,t)+\sum_{\xi=1}^{\nu}\sum_{\substack{\kappa^{-}_{\xi}\\ t\geq t_{\xi\kappa^{-}_{\xi}}+\tau_{\xi}(l-z)}}\mathbf{I}^{-}_{\xi\kappa^{-}}(z,t)\\ =&\mathrm{Re}\left\{\left[\begin{array}{l}\displaystyle\sum_{\xi=1}^{\nu}\left[\sum_{\substack{\kappa^{+}_{\xi}\\ t\geq t_{\xi\kappa^{+}_{\xi}}+\tau_{\xi}(z)}}\tilde{\mathbf{I}}^{+}_{\xi\kappa^{+}_{\xi}}\left[z,t-t_{\xi\kappa^{-}_{\xi}}-\tau_{\xi}(z)\right]\right]\\ \displaystyle+\sum_{\xi=1}^{\nu}\left[\sum_{\substack{\kappa^{-}\\ t\geq t_{\xi\kappa^{-}_{\xi}}+\tau_{\xi}(z)}}\tilde{\mathbf{I}}^{-}_{\xi\kappa^{-}_{\xi}}\left[z,t-t_{\xi\kappa^{-}_{\xi}}-\tau_{\xi}(l-z)\right]\right]\end{array}\right]e^{j2\pi f_{c}t}\right\}\end{aligned} \tag{54b}$$

# 5. COINCIDENCE WITH FREQUENCY-DOMAIN ANALYSIS

## 5.1. Physical Interpretation

The FD analysis of a TL provides the response in terms of electric line quantities against harmonic excitation of line's origin according to (5a)–(5b), or equivalently after *infinite* time from application of the excitation defined in (23) with complex step phasor as:

$$\tilde{V}_{G_{kk'}}(t) = \tilde{V}_{G_{kk'}} u(t) \tag{55a}$$

so that the actual excitation applied is:

$$\begin{aligned} &V_{G_{kk'}}(t) \,\hat{=}\, \mathrm{Re}\left\{\hat{V}_{G_{kk'}}(t)\right\}, \\ &\hat{V}_{G_{kk'}}(t) = \tilde{V}_{G_{kk'}} u(t) e^{j2\pi f_c t} \rightleftarrows \hat{V}_{G_{kk'}} \left[\frac{\delta(f-f_c)}{2} + \frac{1}{j2\pi(f-f_c)}\right] \end{aligned} \tag{55b}$$

Despite the Fourier spectrum expression of (55b), whence the harmonic step $\tilde{V}_{G_{kk'}}(t)$ seems to lie infinitely along the FD, the excitation of (55a) is indeed narrowband in the sense of (23b), as (i) its spectrum fades rapidly when moving away from the centre (peak) value $f_c$, and (ii) assuming a narrowband modulation and transmission scheme, any residual spectral content outside the transmission band is normally expected to be suppressed.

Since correspondence between the TD response of any system and its respective TF in the FD is by default unique, in order to prove coincidence of the well established TL analysis in the FD as summarised in Section 3 above with the travelling-wave TD approach introduced herein, it is sufficient to demonstrate that the response against the harmonic excitation of (5a)–(5b) equals the response against the harmonic step of (55) for:

$$t \to \infty \tag{56a}$$

and

$$\tilde{V}_{G_{kk'}} = \hat{V}_{G_{kk'}}, \quad k = 1, \dots, \nu, \quad k' = 0, \dots, \nu, \quad k' \neq k \tag{56b}$$

## 5.2. Resulting Time-domain Response

Since (i) the phasor $\tilde{V}_{G_{kk'}}$ of the harmonic step excitation (55) is *constant* (time invariable), and (ii) the excitation terminal conditions (39) and the medium-discontinuity terminal conditions (41)–(44) involve all merely *linear* interdependence between the complex envelopes of the excitation applied and the consequent travelling line quantities, the *phasors* of all travelling modal and line quantities are as well *time-invariant*, and path aggregation along the line results in consecutive step variations of the complex envelope of the resultant response, as:

$$\begin{gathered}\mathbf{V}^{+}_{\xi\kappa_\xi^+}(z,t)=\mathrm{Re}\left\{\tilde{\mathbf{V}}^{+}_{\xi\kappa_\xi^+}(z)u\left[t-t_{\xi\kappa_\xi^+}-\tau_\xi(z)\right]e^{j2\pi f_c t}\right\},\\ \tilde{\mathbf{V}}^{+}_{\xi\kappa_\xi^+}(z)=e^{-\hat{\gamma}_\xi z}\tilde{\mathbf{V}}^{+}_{\xi\kappa_\xi^+}(0)\quad,\quad \tilde{\mathbf{V}}^{+}_{\xi\kappa_\xi^+}(0)=\hat{\mathbf{t}}_{V\xi}\tilde{V}^{+}_{m\xi\kappa_\xi^+}\end{gathered}\tag{57a}$$

$$\begin{gathered}\mathbf{V}^{-}_{\xi\kappa_\xi^-}(z,t)=\mathrm{Re}\left\{\tilde{\mathbf{V}}^{-}_{\xi\kappa_\xi^-}(z)u\left[t-t_{\xi\kappa_\xi^-}-\tau_\xi(l-z)\right]e^{j2\pi f_c t}\right\},\\ \tilde{\mathbf{V}}^{-}_{\xi\kappa_\xi^-}(z)=e^{-\hat{\gamma}_\xi (l-z)}\mathbf{V}^{-}_{\xi\kappa_\xi^-}(l),\quad \tilde{\mathbf{V}}^{-}_{\xi\kappa_\xi^-}(l)=\hat{\mathbf{t}}_{V\xi}\tilde{V}^{-}_{m\xi\kappa_\xi^-}\end{gathered}\tag{57b}$$

and analogously for the related current quantities via the substitutions of (53a) after setting:

$$\tilde{V}^{\pm}_{m\xi\kappa_\xi^\pm}(t)\leftrightarrow\tilde{V}^{\pm}_{m\xi\kappa_\xi^\pm},\quad \tilde{I}^{\pm}_{m\xi\kappa_\xi^\pm}(t)\leftrightarrow\tilde{I}^{\pm}_{m\xi\kappa_\xi^\pm}\tag{57c}$$

### 5.3. End-to-end Line Quantities

As time elapses towards infinite, the electric line quantities of (54) become harmonic, as:

$$\left.\begin{aligned}&\mathbf{V}_\infty(z,t)=\mathrm{Re}\left\{\tilde{\mathbf{V}}_\infty(z)\,e^{j2\pi f_c t}\right\},\quad \tilde{\mathbf{V}}_\infty(z)\hat{=}\tilde{\mathbf{V}}^{+}_\infty(z)+\tilde{\mathbf{V}}^{-}_\infty(z)\\ &\tilde{\mathbf{V}}^{\pm}_\infty(z)\hat{=}\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^\pm=^0_1}^{\infty}\tilde{\mathbf{V}}^{\pm}_{\xi\kappa^\pm}(z)=\hat{\mathbf{T}}_V e^{\mp\hat{\gamma}z}\tilde{\mathbf{V}}^{\pm}_{m\infty},\\ &\tilde{\mathbf{V}}^{\pm}_{m\infty}\hat{=}\left[\tilde{V}^{\pm}_{m1\infty}\quad\dots\quad\tilde{V}^{\pm}_{m\nu\infty}\right]^T,\quad \tilde{V}^{\pm}_{m\xi\infty}\hat{=}\sum_{\kappa_\xi^\pm=^0_1}^{\infty}\tilde{V}^{\pm}_{m\xi\kappa_\xi^\pm}\end{aligned}\right\}\tag{58a}$$

the line voltages, whereas respective line currents are similarly extracted as:

$$\left.\begin{aligned}&\mathbf{I}_\infty(z,t)=\mathrm{Re}\left\{\tilde{\mathbf{I}}_\infty(z)\,e^{j2\pi f_c t}\right\},\quad \tilde{\mathbf{I}}_\infty(z)\hat{=}\tilde{\mathbf{I}}^{+}_\infty(z)-\tilde{\mathbf{I}}^{-}_\infty(z)\\ &\tilde{\mathbf{I}}^{\pm}_\infty(z)\hat{=}\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^\pm=^0_1}^{\infty}\tilde{\mathbf{I}}^{\pm}_{\xi\kappa_\xi^\pm}(z)=\hat{\mathbf{T}}_I e^{\mp\hat{\gamma}z}\tilde{\mathbf{I}}^{\pm}_{m\infty},\\ &\tilde{\mathbf{I}}^{\pm}_{m\infty}\hat{=}\left[\tilde{I}^{\pm}_{m1\infty}\quad\dots\quad\tilde{I}^{\pm}_{m\nu\infty}\right]^T,\quad \tilde{I}^{\pm}_{m\xi\infty}\hat{=}\sum_{\kappa_\xi^\pm=^0_1}^{\infty}\tilde{V}^{\pm}_{m\xi\kappa_\xi^\pm}\end{aligned}\right\}\tag{58b}$$

and from (35b), (52), (53), (57), and (58) it results:

$$\tilde{\mathbf{I}}^{\pm}_\infty(z)=\hat{\mathbf{Y}}_c\tilde{\mathbf{V}}^{\pm}_\infty(z),\quad \tilde{I}^{\pm}_{m\xi\infty}=\hat{Y}_{m\xi}\tilde{V}^{\pm}_{m\xi\infty}\tag{59}$$

## 5.4. Terminal Conditions at Line's Origin

The phasors of the travelling excitation at line's origin determined in (38) satisfy the terminal conditions (39). From (57a), (57b), (35b), and (41b), the vectorial phasors $\tilde{\mathbf{V}}^{-}_{\xi\kappa_\xi^-}(0)$, $\tilde{\mathbf{I}}^{-}_{\xi\kappa_\xi^-}(0)$ of any $\kappa_\xi^-$-th negatively travelling path in line with the $\xi$-th mode (with no loss of generality) incident upon the TL origin excited according to (55), and the vectorial phasors $\tilde{\mathbf{V}}^{+}_{r\xi\kappa_\xi^-}(0)$, $\tilde{\mathbf{I}}^{+}_{r\xi\kappa_\xi^-}(0)$ of respective reflected paths as:

$$\begin{aligned}\tilde{\mathbf{V}}^{+}_{r\xi\kappa_\xi^-}(0) &= \sum_{\xi'=1}^{\nu} \tilde{\mathbf{V}}^{+}_{\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0),\\ \tilde{\mathbf{V}}^{+}_{\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) &= \left[\begin{array}{ccc} \tilde{V}^{+}_{1,\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) & \cdots & \tilde{V}^{+}_{\nu,\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)\end{array}\right]\end{aligned} \tag{60a}$$

$$\begin{aligned}\tilde{\mathbf{I}}^{+}_{r\xi\kappa_\xi^-}(0) &= \sum_{\xi'=1}^{\nu} \tilde{\mathbf{I}}^{+}_{\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0),\\ \tilde{\mathbf{I}}^{+}_{\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) &= \left[\begin{array}{ccc} \tilde{V}^{+}_{1,\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) & \cdots & \tilde{V}^{+}_{\nu,\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)\end{array}\right]\end{aligned} \tag{60b}$$

satisfy:

$$\tilde{\mathbf{I}}^{+}_{\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) = \hat{\mathbf{Y}}_c \tilde{\mathbf{V}}^{+}_{\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0), \quad \tilde{\mathbf{I}}^{+}_{r\xi\kappa_\xi^-}(0) = \hat{\mathbf{Y}}_c \tilde{\mathbf{V}}^{+}_{r\xi\kappa_\xi^-}(0) \tag{60c}$$

Combination of (43a) and (44a) gives:

$$\left.\begin{aligned}&\left[\sum_{\xi'=1}^{\nu} \tilde{I}^{+}_{k,\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) - \tilde{I}^{-}_{k\xi\kappa_\xi^-}(0)\right]\\ &= -\left(\sum_{\substack{k'=0\\k'\neq k}}^{\nu} \hat{Y}_{kk'}\right)\left[\sum_{\xi'=1}^{\nu} \tilde{V}^{+}_{k,\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) + \tilde{V}^{-}_{k\xi\kappa_\xi^-}(0)\right]\\ &\quad + \sum_{\substack{k'=1\\k'\neq k}}^{\nu} \hat{Y}_{kk'}\left[\sum_{\xi'=1}^{\nu} \tilde{V}^{+}_{k',\xi',\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) + \tilde{V}^{-}_{k'\xi\kappa_\xi^-}(0)\right]\\ &k = 1,\ldots,\nu\end{aligned}\right\} \tag{61}$$

As all positively travelling paths except the zero-indexed ones of (37) are generated from reflection following incidence of negatively travelling paths upon the excited origin, from (38a), (39), and (61), the terminal conditions of the harmonic electric line quantities of (58) at line's origin ($z=0$) become:

$$\left[\tilde{\mathbf{I}}_\infty(0)\right]_k = \hat{Y}_{k0}\left\{\hat{V}_{G_{k0}} - \left[\tilde{\mathbf{V}}_\infty(0)\right]_k\right\} - \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left\{\left[\tilde{\mathbf{V}}_\infty(0)\right]_k - \hat{V}_{G_{kk'}} - \left[\tilde{\mathbf{V}}_\infty(0)\right]_{k'}\right\}\hat{Y}_{kk'}, \quad k=1,\ldots,\nu \tag{62}$$

### 5.5. Terminal Conditions at Line's End

For the vectorial phasors $\tilde{\mathbf{V}}^+_{\xi\kappa^+_\xi}(l)$, $\tilde{\mathbf{I}}^+_{\xi\kappa^+_\xi}(l)$ of any positively-travelling $\kappa^+_\xi$-th path of the $\xi$-th mode (with no loss of generality) that arrives at the passively terminated end of the line (i.e., with no excitation source in between any pair of line conductors) and the related travelling excitation vectorial phasors $\tilde{\mathbf{V}}^-_{r\xi\kappa^+_\xi}(l)$, $\tilde{\mathbf{I}}^-_{r\xi\kappa^+_\xi}(l)$ of the reflected paths consequent upon incidence, as:

$$\begin{aligned}\tilde{\mathbf{V}}^-_{r\xi\kappa^+_\xi}(l) &= \sum_{\xi'=1}^{\nu}\tilde{\mathbf{V}}^-_{\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l),\\ \tilde{\mathbf{V}}^-_{\xi',\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l) &= \left[\tilde{V}^-_{1,\xi',\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l) \quad \cdots \quad \tilde{V}^-_{1,\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l)\right]\end{aligned} \tag{63a}$$

$$\begin{aligned}\tilde{\mathbf{I}}^-_{r\xi\kappa^+_\xi}(l) &= \sum_{\xi'=1}^{\nu}\tilde{\mathbf{I}}^-_{\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l),\\ \tilde{\mathbf{I}}^-_{\xi',\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l) &= \left[\tilde{V}^-_{1,\xi',\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l) \quad \cdots \quad \tilde{V}^-_{1,\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l)\right]\end{aligned} \tag{63b}$$

satisfying through (52a), (35b), and (41b) the conditions:

$$\tilde{\mathbf{I}}^-_{\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l) = \hat{\mathbf{Y}}_c\tilde{\mathbf{V}}^-_{\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l)\,, \quad \tilde{\mathbf{I}}^-_{r\xi\kappa^+_\xi}(l) = \hat{\mathbf{Y}}_c\tilde{\mathbf{V}}^-_{r\xi\kappa^+_\xi}(l) \tag{63c}$$

from (43a) and (44a) it holds:

$$\left.\begin{aligned}&\left[\tilde{I}^+_{k\xi\kappa^+_\xi}(l) - \sum_{\xi'=1}^{\nu}\tilde{I}^-_{k,\,\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l)\right]\\ &= \left(\sum_{\substack{k'=0\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\right)\left[\tilde{V}^+_{k\,\xi\kappa^+_\xi}(l) + \sum_{\xi'=1}^{\nu}\tilde{V}^-_{k,\,\xi',\,\kappa^-_{\xi'}(\xi,\kappa^+_\xi)}(l)\right]\\ &\quad - \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\left[\tilde{V}^+_{k'\xi\kappa^+_\xi}(l) + \sum_{\xi'=1}^{\nu}\tilde{V}^-_{k',\,\xi',\,\kappa^-_{\xi'}\left(\xi,\kappa^+_\xi\right)}(l)\right]\\ &k=1,\ldots,\nu\end{aligned}\right\} \tag{64}$$

As all negatively travelling paths are generated from reflection following incidence of positively travelling paths upon line's end, from (38a), (39), and (61), the terminal conditions of the harmonic electric line quantities in (58) at line's end ($z = l$) become:

$$\begin{aligned}&\tilde{\mathbf{I}}_\infty(l)=\hat{\mathbf{Y}}_0\tilde{\mathbf{V}}_\infty(l),\\ &\underset{k=1,\ldots,\nu}{\left[\tilde{\mathbf{I}}_\infty(l)\right]_k}=\hat{Y}_{k0}\left[\tilde{\mathbf{V}}_\infty(l)\right]_k+\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left\{\left[\tilde{\mathbf{V}}_\infty(l)\right]_k-\left[\tilde{\mathbf{V}}_\infty(l)\right]_{k'}\right\}\hat{Y}_{kk'}\end{aligned} \tag{65}$$

## 5.6. Resulting Coincidence

The response (58) against the asymptotically harmonic excitation deriving from (55) and (56a) is absolutely *equivalent* to the FD response set out in Section 3, since (i) all quantities are identically dependent one-to-one after substituting:

$$\left.\begin{pmatrix}\mathbf{V}(z,t)\leftrightarrow\mathbf{V}_\infty(z,t)\\ \hat{\mathbf{V}}(z)\leftrightarrow\tilde{\mathbf{V}}_\infty(z)\\ \hat{\mathbf{V}}^\pm(z)\leftrightarrow\tilde{\mathbf{V}}^\pm_\infty(z)\\ \hat{\mathbf{V}}^\pm_m\leftrightarrow\tilde{\mathbf{V}}^\pm_{m\infty}\\ \hat{V}^0_{G_{kk'}}\leftrightarrow\tilde{V}^0_{G_{kk'}}\end{pmatrix},\begin{pmatrix}\mathbf{I}(z,t)\leftrightarrow\mathbf{I}_\infty(z,t)\\ \tilde{\mathbf{I}}(z)\leftrightarrow\tilde{\mathbf{I}}_\infty(z)\\ \tilde{\mathbf{I}}^\pm(z)\leftrightarrow\tilde{\mathbf{I}}^\pm_\infty(z)\\ \tilde{\mathbf{I}}^\pm_m\leftrightarrow\tilde{\mathbf{I}}^\pm_{m\infty}\end{pmatrix}\right\} \tag{66}$$

and (ii) all homologous voltage and current phasors are identically associated along the line by virtue of (10), (8a), and (59), and (iii) from (5d), (62), and (65), fall under identical as well terminal conditions.

As a result, the travelling-wave TD approach developed herein is *identically equivalent* with the established TL theory in the FD.

# 6. IMPLEMENTATION GUIDELINES

## 6.1. Channel Response Estimation

The TD response against any narrowband excitation applied according to (23) at the origin of any TL is determined in the TD through the following sequence of steps:

1) Calculate the initial positively travelling paths generated as soon as excitation is applied, from the terminal conditions (39) and interrelation (37) between line and modal excitations.
2) Apply the proper form of the generic terminal conditions (49) and (50) every time an incident path arrives at a point of structural medium discontinuity.

3) Aggregate all paths existing at the point and time of observation according to (54).

If excitation is applied to *more* than one TL within the observed network, since *any uniform TL is a linear system*, the aforementioned calculation sequence is performed separately for every single excitation, and the overall response is the *resultant* of all several responses.

## 6.2. Structural Reductions

### *6.2.1. Open Circuit between Excitation Conductor Origins*

If no excitation or termination branches are connected except between the excitation conductors and the *reference* one, i.e.,

$$\hat{\mathbf{Y}}_0 = \mathrm{diag}\left[\hat{Y}_{k0}\right], \quad \hat{Y}_{k0} \in \mathbf{C}\ \forall\, k \in \{1, \ldots, \nu\} \tag{67a}$$

and

$$V_{G_{kk'}}(t) = 0, \qquad k, k' \in \{1, \ldots, \nu\} \tag{67b}$$

the voltage excitation phasors vector

$$\tilde{\mathbf{V}}_G(t) = \left[\begin{array}{ccc} \tilde{V}_{G_{10}}(t) & \ldots & \tilde{V}_{G_{\nu 0}}(t) \end{array}\right]^T \tag{68a}$$

is defined, and the line's origin excitation terminal conditions (39) become:

$$\tilde{\mathbf{I}}_0^+(0,t) = \hat{\mathbf{Y}}_0\left[\tilde{\mathbf{V}}_G(t) - \tilde{\mathbf{V}}_0^+(0,t)\right], \quad \tilde{\mathbf{I}}_0^+(0,t) = \hat{\mathbf{Y}}_c\,\tilde{\mathbf{V}}_0^+(0,t) \tag{68b}$$

### *6.2.2. Two-conductor Transmission Line*

In the case of a single *two-conductor* TL, the travelling-wave approach turns directly out *scalar*, as:

$$\left(\begin{array}{c} \tilde{\mathbf{V}}_{G,r,t} \leftrightarrow \tilde{V}_{G,r,t}, \tilde{\mathbf{I}}_{r,t} \leftrightarrow \tilde{I}_{r,t} \\ \sum\limits_{\xi=1}^{\nu} \mathbf{V}_0^+(0,t) \leftrightarrow V_0^+(0,t) \\ \sum\limits_{\xi=1}^{\nu} \mathbf{I}_0^+(0,t) \leftrightarrow I_0^+(0,t) \end{array}\right), \left(\begin{array}{c} \hat{\gamma} \leftrightarrow \hat{\gamma}_{\mathrm{II}} \\ \tilde{\mathbf{Y}}_c \leftrightarrow \tilde{Y}_c \\ \hat{\mathbf{Y}}_{c,0} \leftrightarrow \hat{Y}_{c,0} \\ \tilde{\boldsymbol{\rho}} \leftrightarrow \tilde{\rho} \\ \tilde{\mathbf{t}} \leftrightarrow \tilde{t} \end{array}\right) \tag{69}$$

From the scalar equivalent (69) of the terminal conditions (68b), the initial positively-travelling path is determined, which is raised as soon as excitation is applied at line's origin. The medium-discontinuity terminal conditions (41)–(44) are replaced by the scalar equivalent according to (69), and all paths raised travel in line with the same propagation constant $\hat{\gamma}_{\mathrm{II}}$ and the related propagation velocity $v_{\mathrm{II}}$, wavelength $\lambda_{\mathrm{II}}$, and propagation delay $\tau_{\mathrm{II}}(\Delta l)$, as:

$$v_{\mathrm{II}} \hat{=} \omega \Big/ \hat{\beta}_{\mathrm{II}}, \quad \lambda_{\mathrm{II}} \hat{=} v_{\mathrm{II}}/f_c, \quad \tau_{\mathrm{II}}(\Delta l) \hat{=} \Delta l / v_{\mathrm{II}} \tag{70}$$

### 6.3. Signalling Circuitry — Reference Transposition

Regarding PLC signalling over power cables, transmission is normally expected from an excitation conductor to the reference one, whereas reception is observed either in between the same pair of conductors to determine response of the transmission circuit (channel), or from a different excitation conductor to the common reference one, if interference over another channel (crosstalk) of the MTL structure examined is required. However, transmission and/or reception may also be respectively performed and observed in between a pair of *excitation* conductors, when *differential* TL excitation is applied, or if *interference* from Electromagnetic (EM) field(s) penetrating the TL is required. In such a case, *transposition* of the *reference conductor* is necessary in order to estimate the response on travelling-wave basis.

As regards reference transposition from the 0-th conductor of a $(\nu+1)$-conductor transmission line to the $k$-th one, the electric line quantities are by default transformed as follows:

$$\left.\begin{aligned} V'_{k'}(z,t) &= V_{k'}(z,t) - V_k(z,t), \quad k' \in \{1,\ldots,\nu\backslash k\} \\ V'_k(z,t) &= -V_k(z,t) \end{aligned}\right\} \tag{71a}$$

$$\left.\begin{aligned} I'_{k'}(z,t) &= I_{k'}(z,t), \quad k' \in \{1,\ \ldots\ ,\nu\backslash k\} \\ I'_k(z,t) &= -\sum_{k'=1}^{\nu} I_{k'}(z,t) \end{aligned}\right\} \tag{71b}$$

where the initial reference conductor is transposed with the final, and final quantities are indicated by the accentuation mark "′".

## 7. CONCLUSION

In this paper, travelling-wave analysis of uniform multi-conductor transmission line networks has been elaborated determining the time-domain response to narrowband excitation, developed in direct equivalence with the well established transmission line theory in the frequency domain. Applicability in respect of transmission-line structure has been clarified, and guidelines have been provided for computational implementation, along with further clarifications towards treatment of Power-Line Communication signalling, a well suitable field for application, whereas potential applicability is also extended to other related topics such as surge propagation resulting, e.g., from lightning or transient overvoltage.

## APPENDIX A. DERIVATION OF EQ. (28a)

$$
\begin{aligned}
V_k(z,t) &= \mathrm{Re}\left\{F^{-1}\left[\hat{H}_\xi(z)\,F\left[\left[\hat{\mathbf{T}}_V\right]_{k\xi}\tilde{V}^+_{m\xi}(t)e^{j2\pi f_c t}\right]\right]\right\}\\
&= \mathrm{Re}\left\{F^{-1}\left[F\left[e^{-\hat{a}_\xi z}e^{-j\hat{\beta}_\xi z}\left[\hat{\mathbf{T}}_V\right]_{k\xi}\tilde{V}^+_{m\xi}\left(t-\frac{z}{2\pi f_c/\hat{\beta}_\xi}\right)e^{j2\pi f_c t}\right]\right]\right\}\\
&= \mathrm{Re}\left\{F^{-1}\left[e^{-\hat{a}_\xi z}F\left[\left[\hat{\mathbf{T}}_V\right]_{k\xi}\tilde{V}^+_{m\xi}\left(t-\frac{z}{2\pi f_c/\hat{\beta}_\xi}\right)e^{j2\pi f_c\left(t-\frac{z}{2\pi f_c/\hat{\beta}_\xi}\right)}\right]\right]\right\}\\
&\Rightarrow \left.\begin{aligned} V_k(z,t) &= V_{k\xi}(z,t) \hat{=} \mathrm{Re}\left\{\tilde{V}_{k\xi}(z,t)e^{j2\pi f_c t}\right\}\\ \tilde{V}_{k\xi}(z,t) &= e^{-\hat{\gamma}_\xi z}\tilde{V}_{k\xi}\left(0,t-\tfrac{z}{2\pi f_c/\hat{\beta}_\xi}\right)\end{aligned}\right\},\quad k=1,\ldots,\nu
\end{aligned}
$$

## APPENDIX B. DERIVATION OF EQ. (58a)

$$
\begin{aligned}
&\mathbf{V}_\infty(z,t)\\
&=\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\mathbf{V}^+_{\xi\kappa_\xi^+}(z,t)+\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=0}^{\infty}\mathbf{V}^-_{\xi\kappa_\xi^-}(z,t)\\
&=\mathrm{Re}\left\{\left[\overbrace{\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}(z)}^{\hat{=}\tilde{\mathbf{V}}^+_\infty(z)}+\overbrace{\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=0}^{\infty}\tilde{\mathbf{V}}^-_{\xi\kappa_\xi^-}(z)}^{\hat{=}\tilde{\mathbf{V}}^-_\infty(z)}\right]e^{j2\pi f_c t}\right\}\\
&=\mathrm{Re}\left\{\underbrace{\left[\underbrace{\hat{\mathbf{T}}_V e^{-\hat{\gamma}z}\overbrace{\tilde{\mathbf{V}}^+_{m\infty}}^{\left[\tilde{V}^+_{m1\infty}\ \ldots\ \tilde{V}^+_{m\nu\infty}\right]^T}}_{=\tilde{\mathbf{V}}^+_\infty(z)}+\underbrace{\hat{\mathbf{T}}_V e^{\hat{\gamma}z}\overbrace{\tilde{\mathbf{V}}^-_{m\infty}}^{\left[\tilde{V}^-_{m1\infty}\ \ldots\ \tilde{V}^-_{m\nu\infty}\right]^T}}_{=\tilde{\mathbf{V}}^-_\infty(z)}\right]}_{\hat{=}\tilde{\mathbf{V}}_\infty(z)}e^{j2\pi f_c t}\right\}
\end{aligned}
$$

$$\Rightarrow \left.\begin{array}{l} \mathbf{V}_\infty(z,t) = \mathrm{Re}\left\{\tilde{\mathbf{V}}_\infty(z)\,e^{j2\pi f_c t}\right\}, \quad \tilde{\mathbf{V}}_\infty(z) \hat{=} \tilde{\mathbf{V}}^+_\infty(z) + \tilde{\mathbf{V}}^-_\infty(z) \\ \tilde{\mathbf{V}}^\pm_\infty(z) \hat{=} \sum\limits_{\xi=1}^{\nu} \sum\limits_{\kappa^\pm_\xi = {}^0_1}^{\infty} \tilde{\mathbf{V}}^\pm_{\xi\kappa^\pm}(z) = \hat{\mathbf{T}}_V e^{\mp\hat{\gamma}z} \tilde{\mathbf{V}}^\pm_{m\infty} \\ \tilde{\mathbf{V}}^\pm_{m\infty} \hat{=} \left[\begin{array}{ccc} \tilde{V}^\pm_{m1\infty} & \ldots & \tilde{V}^\pm_{m\nu\infty} \end{array}\right]^T, \quad \tilde{V}^\pm_{m\xi\infty} \hat{=} \sum\limits_{\kappa^\pm_\xi = {}^0_1}^{\infty} \tilde{V}^\pm_{m\xi\kappa^\pm_\xi} \end{array}\right\}$$

## APPENDIX C. DERIVATION OF EQ. (61)

$$\begin{aligned}
\tilde{\mathbf{V}}^+_{r\xi\kappa^-_\xi}(0) &= \left(\hat{\mathbf{Y}}_c + \hat{\mathbf{Y}}_0\right)^{-1} \left(\hat{\mathbf{Y}}_c - \hat{\mathbf{Y}}_0\right) \tilde{\mathbf{V}}^-_{\xi\kappa^-_\xi}(0) \\
&\Rightarrow \left(\hat{\mathbf{Y}}_c + \hat{\mathbf{Y}}_0\right) \tilde{\mathbf{V}}^+_{r\xi\kappa^-_\xi}(0) = \left(\hat{\mathbf{Y}}_c - \hat{\mathbf{Y}}_0\right) \tilde{\mathbf{V}}^-_{\xi\kappa^-_\xi}(0) \\
&\Rightarrow \hat{\mathbf{Y}}_c \left[\sum_{\xi'=1}^{\nu} \tilde{\mathbf{V}}^+_{\xi',\,\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) - \tilde{\mathbf{V}}^-_{\xi\kappa^-_\xi}(0)\right] \\
&= -\hat{\mathbf{Y}}_0 \left[\sum_{\xi'=1}^{\nu} \tilde{\mathbf{V}}^+_{\xi',\,\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) + \tilde{\mathbf{V}}^-_{\xi\kappa^-_\xi}(0)\right] \\
&\Rightarrow \left[\sum_{\xi'=1}^{\nu} \tilde{\mathbf{I}}^+_{\xi',\,\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) - \tilde{\mathbf{I}}^-_{\xi\kappa^-_\xi}(0)\right] \\
&= -\hat{\mathbf{Y}}_0 \left[\sum_{\xi'=1}^{\nu} \tilde{\mathbf{V}}^+_{\xi',\,\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) + \tilde{\mathbf{V}}^-_{\xi\kappa^-_\xi}(0)\right] \\
&\Rightarrow \left[\sum_{\xi'=1}^{\nu} \tilde{I}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) - \tilde{I}^-_{k\xi\kappa^-_\xi}(0)\right] \\
&= -\left(\sum_{\substack{k'=0 \\ k'\neq k}}^{\nu} \hat{Y}_{kk'}\right) \left[\sum_{\xi'=1}^{\nu} \tilde{V}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) + \tilde{V}^-_{k\xi\kappa^-_\xi}(0)\right] \\
&\quad + \sum_{\substack{k'=1 \\ k'\neq k}}^{\nu} \hat{Y}_{kk'} \left[\sum_{\xi'=1}^{\nu} \tilde{V}^+_{k',\,\xi',\kappa^+_{\xi'}(\xi,\kappa^-_\xi)}(0) + \tilde{V}^-_{k'\xi\kappa^-_\xi}(0)\right], \\
&k = 1, \ldots, \nu
\end{aligned}$$

## APPENDIX D. DERIVATION OF EQ. (62)

$$
\begin{aligned}
\left[\tilde{\mathbf{I}}_\infty(0)\right]_k =& \sum_{\xi=1}^{\nu} \tilde{I}^+_{k\xi 0}(0) + \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=1}^{\infty} \tilde{I}^+_{k\xi\kappa^+}(0) - \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty} \tilde{I}^-_{k\xi\kappa_\xi^-}(0) \\
=& \tilde{I}^+_{k0}(0) + \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\left[\sum_{\xi'=1}^{\nu}\tilde{I}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0) - \tilde{I}^-_{k\xi\kappa_\xi^-}(0)\right] \\
=& \hat{Y}_{k0}\left[\hat{V}_{G_{k0}} - \tilde{V}^+_{k0}(0)\right] - \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left[\tilde{V}^+_{k0}(0) - \hat{V}_{G_{kk'}} - \tilde{V}^+_{k'0}(0)\right]\hat{Y}_{kk'} \\
&-\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\left\{\left(\sum_{\substack{k'=0\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\right)\left[\sum_{\xi'=1}^{\nu}\tilde{V}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)+\tilde{V}^-_{k\xi\kappa_\xi^-}(0)\right]\right\} \\
&+\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\left\{\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\left[\sum_{\xi'=1}^{\nu}\tilde{V}^+_{k',\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)+\tilde{V}^-_{k'\xi\kappa_\xi^-}(0)\right]\right\} \\
=& \hat{Y}_{k0}\left\{\hat{V}_{G_{k0}} - \overbrace{\left[\tilde{V}^+_{k0}(0) + \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\left[\sum_{\xi'=1}^{\nu}\tilde{V}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)+\tilde{V}^-_{k\xi\kappa_\xi^-}(0)\right]\right]}^{=\left[\tilde{\mathbf{V}}_\infty(0)\right]_k}\right\} \\
&- \sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left\{\overbrace{\left[\tilde{V}^+_{k0}(0) + \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\left[\sum_{\xi'=1}^{\nu}\tilde{V}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)+\tilde{V}^-_{k\xi\kappa_\xi^-}(0)\right]\right]}^{=\left[\tilde{\mathbf{V}}_\infty(0)\right]_k}\right. \\
&\left. - \hat{V}_{G_{kk'}} - \overbrace{\left[\tilde{V}^+_{k'0}(0) + \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\left[\sum_{\xi'=1}^{\nu}\tilde{V}^+_{k,\,\xi',\,\kappa^+_{\xi'}(\xi,\kappa_\xi^-)}(0)+\tilde{V}^-_{k\xi\kappa_\xi^-}(0)\right]\right]}^{=\left[\tilde{\mathbf{V}}_\infty(0)\right]_{k'}}\right\}\hat{Y}_{kk'}
\end{aligned}
$$

$$\Rightarrow\left[\tilde{\mathbf{I}}_\infty(0)\right]_k = \hat{Y}_{k0}\left\{\hat{V}_{G_{k0}} - \left[\tilde{\mathbf{V}}_\infty(0)\right]_k\right\}$$
$$= -\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left\{\left[\tilde{\mathbf{V}}_\infty(0)\right]_k - \hat{V}_{G_{kk'}} - \left[\tilde{\mathbf{V}}_\infty(0)\right]_{k'}\right\}\hat{Y}_{kk'}, \quad k=1,\ldots,\nu$$

## APPENDIX E. DERIVATION OF EQ. (64)

$$\tilde{\mathbf{V}}^-_{r\xi\kappa_\xi^+}(l) = \left(\hat{\mathbf{Y}}_c + \hat{\mathbf{Y}}_0\right)^{-1}\left(\hat{\mathbf{Y}}_c - \hat{\mathbf{Y}}_0\right)\tilde{\mathbf{V}}^-_{\xi\kappa_\xi^+}(l)$$
$$\Rightarrow \hat{\mathbf{Y}}_c\left[\tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}(l) - \tilde{\mathbf{V}}^-_{r\xi\kappa_\xi^+}(l)\right] = \hat{\mathbf{Y}}_0\left[\tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}(l) + \tilde{\mathbf{V}}^-_{r\xi\kappa_\xi^+}(l)\right]$$
$$\Rightarrow \tilde{\mathbf{I}}^+_{\xi\kappa_\xi^+}(l) - \sum_{\xi'=1}^{\nu}\tilde{\mathbf{I}}^-_{\xi',\,\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l) = \hat{\mathbf{Y}}_0\left[\tilde{\mathbf{V}}^+_{\xi\kappa_\xi^+}(l) + \sum_{\xi'=1}^{\nu}\tilde{\mathbf{V}}^-_{\xi',\,\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l)\right]$$
$$\Rightarrow \left[\tilde{I}^+_{k\xi\kappa_\xi^+}(l) - \sum_{\xi'=1}^{\nu}\tilde{I}^-_{k,\,\xi',\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l)\right]$$
$$= \left(\sum_{\substack{k'=0\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\right)\left[\tilde{V}^+_{k\xi\kappa_\xi^+}(l) + \sum_{\xi'=1}^{\nu}\tilde{V}^-_{k,\,\xi',\,\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l)\right]$$
$$-\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\left[\tilde{V}^+_{k'\xi\kappa_\xi^+}(l) + \sum_{\xi'=1}^{\nu}\tilde{V}^-_{k',\xi',\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l)\right], \quad k=1,\ldots,\nu$$

## APPENDIX F. DERIVATION OF EQ. (65)

$$\left[\tilde{\mathbf{I}}_\infty(l)\right]_k = \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\tilde{I}^+_{k\xi\kappa_\xi^+}(l) - \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^-=1}^{\infty}\tilde{I}^+_{k\xi\kappa_{\xi^-}}(l)$$
$$= \sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\left\{\left(\sum_{\substack{k'=0\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\right)\left[\tilde{V}^+_{k\xi\kappa_\xi^+}(l) + \sum_{\xi'=1}^{\nu}\tilde{V}^-_{k,\xi',\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l)\right]\right\}$$

$$
-\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\left\{\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\hat{Y}_{kk'}\left[\tilde{V}^{+}_{k'\xi\kappa_\xi^+}(l)+\sum_{\xi'=1}^{\nu}\tilde{V}^{-}_{k',\xi',\kappa^-_{\xi'}\left(\xi,\kappa_\xi^+\right)}(l)\right]\right\}
$$

$$
=\hat{Y}_{k0}\overbrace{\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\left[\tilde{V}^{+}_{k\xi\kappa_\xi^+}(l)+\sum_{\xi'=1}^{\nu}\tilde{V}^{-}_{k,\xi',\kappa^-_{\xi'}\left(\xi,\kappa_\xi^+\right)}(l)\right]}^{=\left[\tilde{\mathbf{V}}_\infty(l)\right]_k}
$$

$$
+\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left\{\overbrace{\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\left[\tilde{V}^{+}_{k\xi\kappa_\xi^+}(l)+\sum_{\xi'=1}^{\nu}\tilde{V}^{-}_{k,\xi',\kappa_{\xi'}-\left(\xi,\kappa_\xi^+\right)}(l)\right]}^{=\left[\tilde{\mathbf{V}}_\infty(l)\right]_k}\right.
$$

$$
\left.-\overbrace{\sum_{\xi=1}^{\nu}\sum_{\kappa_\xi^+=0}^{\infty}\left[\tilde{V}^{+}_{k'\xi\kappa_\xi^+}(l)+\sum_{\xi'=1}^{\nu}\tilde{V}^{-}_{k',\xi',\kappa^-_{\xi'}(\xi,\kappa_\xi^+)}(l)\right]}^{=\left[\tilde{\mathbf{V}}_\infty(l)\right]_{k'}}\right\}\hat{Y}_{kk'}
$$

$$
\Rightarrow \tilde{\mathbf{I}}_\infty(l)=\hat{\mathbf{Y}}_0\tilde{\mathbf{V}}_\infty(l),
$$

$$
\underset{k=1,\dots,\nu}{\left[\tilde{\mathbf{I}}_\infty(l)\right]_k}=\hat{Y}_{k0}\left[\tilde{\mathbf{V}}_\infty(l)\right]_k+\sum_{\substack{k'=1\\k'\neq k}}^{\nu}\left\{\left[\tilde{\mathbf{V}}_\infty(l)\right]_k-\left[\tilde{\mathbf{V}}_\infty(l)\right]_{k'}\right\}\hat{Y}_{kk'}
$$

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