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A FAST TUNING METHOD FOR MICROWAVE FILTER USING VF-ASM TECHNOLOGY

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Abstract—A novel microwave filter tuning method based on vector fitting and aggressive space mapping (VF-ASM) technology is presented in this paper. The filter tuning is performed as a two step procedure. First, the equivalent circuit parameters are extracted through vector fitting method by a series of S-parameter measurements. Second, the optimal screw positions are calculated through ASM techniques. this novel tuning technique has been tested successfully with cross-coupled six-resonator and direct coupled eight-resonator filters.

1. INTRODUCTION

Microwave filters incorporating the generalized Chebyshev filtering functions have found wide applications in both satellite and terrestrial communication systems. A great deal of effort has been made over the past three decades in analytic synthesis of the filter coupling matrix according to an adequate topology with an optimal coarse model. The most recent representative work in this subject has been presented in Cameron's papers [1, 2]. Low-cost and High-Q microwave components are key components of many telecommunication systems. Large volume production and quick turnaround time have become important in the decision as to what kind of filter structures are most suitable to satisfy a range of specifications [3]. Since the traditional tuning skill of a human operator is mainly built up by years of tuning experience, the tuning process becomes very labor intensive and expensive, particularly for high-order filters with multiple cross-couplings. One of the difficulties associated with the traditional tuning

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is that it is not a deterministic process. In other word, there is no guarantee that each step of a tuning is always in the right direction.

The goal of the filter tuning is to find the optimal position of tuning elements (such as tuning screws) to satisfy the given filter specifications. From the filter specifications and based on standard filter synthesis, which provides the ideal filter parameters (resonant frequency, coupling between resonators, input/output couplings), a prototype response can be generated. For a de-tuned filter, the corresponding characteristic filter parameters can be extracted from S-parameter measurement through many techniques, such as Cauchy method [4–6]. Recently a great deal of effort has been made on computer aided tuning for microwave filter [7–20]. In this paper, a novel hybrid method are proposed. The equivalent circuit parameters are extracted through vector fitting technique. The increased position of the tuning element is then predicted by the ASM technique [21].

The paper is organized as follows. In Section 2, the vector fitting technique which is applied to equivalent circuit extraction is illustrated in detail. Then the ASM technique are used to predict the position of the tuning screws. The initialization of Broyden matrix here is different the traditional method. Two examples are demonstrated in Section 3. The first tuning example is a six-order cross-coupled filter with two finite transmission zeros; and the second tuning example is an eight-pole Chebyshev filter. Both of the example show the validity of the technique presented in this paper. Conclusions are then presented in Section 4.

2. BASIC THEORY

For the measured filter, the filter network model must be mapped onto the measured filter response at first. In this step, the coefficients of the network model are produced, then the parameter is extracted by the vector fitting technique [22], and the tuning screw positions are predicted by the ASM technique.

2.1. Parameter Extraction

In the parameter extraction process, if the result is not unique, it can lead to oscillation in the iterative process, which can be solved by the vector fitting technique. Following the formulation in [1], one can determine the two-port admittance matrix $[Y_N]$ for the general

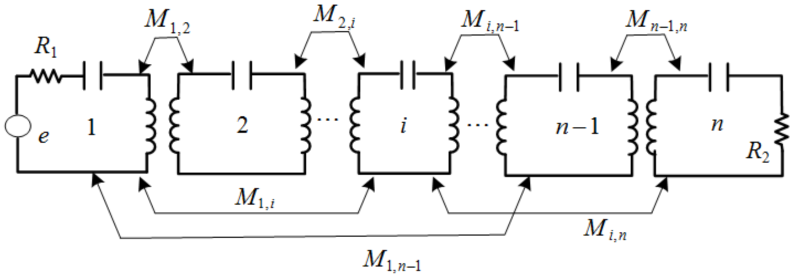


Figure 1. General two-port cross-coupled network.

cross-coupled network shown in Figure 1 as

$$[Y_N] = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix} = \sum_{k=1}^{N} \frac{1}{j\omega - j\lambda_k} \begin{bmatrix} r_{11k} & r_{12k} \\ r_{21k} & r_{22k} \end{bmatrix} \tag{1}$$

where λ_k are the eigen values of the coupling matrix, r_{ijk} are the residues of the Y-parameters.

Thus, if we can approximate the simulated Y-parameters by polynomials, $y_{ij,appx}(s)$, in the following form

$$[Y_{appx}] = \begin{bmatrix} y_{11,appx}(s) & y_{12,appx}(s) \\ y_{21,appx}(s) & y_{22,appx}(s) \end{bmatrix}$$
$$= \sum_{k=1}^{N} \frac{1}{j\omega - j\lambda_{k,appx}} \begin{bmatrix} r_{11k,appx} & r_{12k,appx} \\ r_{21k,appx} & r_{22k,appx} \end{bmatrix} \tag{2}$$

The vector fitting technique is applied to obtain the polynomials $y_{11,appx}(s)$ and $y_{21,appx}(s)$, and the detail is shown in reference [22]. Once the approximate Y-parameters are obtained, the equivalent circuit can be synthesized by the method in [1]. Then the parameter is applied to the iterations to obtain the position of the tuning screws.

2.2. Computation the Position of Tuning Screws

The aggressive space mapping (ASM) is a well known method. In the ASM technique, approximations to the matrix of first-order derivatives are updated by classic Broyden formula.

We refer to the coarse model parameters as x_c and the fine model parameters as x_f. The optimal coarse model design is denoted as x_c^*. We also denote the responses of the coarse model as $R_c(x_c)$ and that of the fine model as $R_f(x_f)$. Here, the coarse model is the equivalent circuit. First, we measure the filter when the tuning screw in the two different position states, then the coarse model parameters is extracted by the vector fitting technique. Second, use the ASM technique to predict the next better positions of the tuning screws. To illustrate the tuning procedure, a six-resonator bandpass filter with cross coupling between resonator 2-5 and eight-resonator Chebyshev bandpass filter are utilized. The goal is to find the best position of tuning screw p_i^{ideal}. The recipe is outlined in the following steps.

Step 1) Measure the filter twice with different tuning positions x_{f1}, x_{f2} to obtain the S-parameters, then obtain two groups equivalent circuit parameters x_{c1}, x_{c2}. Where, $x_f = [p_1 \quad p_2 \quad \ldots \quad p_i \quad \ldots]$ is the vector of tuning positions.

Step 2) Calculate the initial Broyden matrix B_0 through the equation $B_0 = \mathrm{diag}(\frac{x_{c2}-x_{c1}}{x_{f2}-x_{f1}})$. Here, the coarse model parameters and fine model parameters have different physical meanings, so the initial Broyden matrix B_0 is not the identity matrix.

Step 3) Evaluate the difference $f^{(1)} = x_c^{(1)} - x_c^*$. Stop if $\|f^{(1)}\| \leq \varepsilon$.

Step 4) Solve $B^{(j)} h^{(j)} = -f^{(j)}$ for the roots $h^{(j)}$. Where $h^{(j)}$ is the increasing value of the tuning screws.

Step 5) Set $x_f^{(j+1)} = x_f^{(j)} + h^{(j)}$.

Step 6) Measure the response $R_f(x_f^{(j+1)})$ when the tuning screws at the position $x_f^{(j+1)}$.

Step 7) Extract $x_c^{(j+1)}$ such that $R_c(x_c^{(j+1)}) \approx R_f(x_f^{(j+1)})$.

Step 8) Evaluate $f^{(j+1)} = x_c^{(j+1)} - x_c^*$. Stop if $\|f^{(j+1)}\| \leq \varepsilon$.

Step 9) Update $B^{(j+1)} = B^{(j)} + \frac{f^{(j+1)} h^{(j)T}}{h^{(j)T} h^{(j)}}$.

Step 10) Set $j = j + 1$; go to step 4.

To illustrate the proposed method, two examples are shown in Section 3.

3. EXPERIMENTAL RESULTS

3.1. Six-order Cross-coupled Coaxial Filter

The first example is a sixth order cross-coupled filter. The specifications of the filter are listed in the following table.

Table 1. Specifications of the cross-coupled six-resonator filter.

Center frequency f_0	Pass-band	Return loss	Normalized finite transmission zeros	Filter degree
2069.3 MHz	2015 MHz–2125 MHz	20 dB	$-2j$, $2j$	6

The coupling matrix M and external Q value can be easily obtained through the method in [1, 2]

$$M = \begin{bmatrix} 0 & 0.0445 & 0 & 0 & 0 & 0 \\ 0.0445 & 0 & 0.0320 & 0 & -0.0034 & 0 \\ 0 & 0.0320 & 0 & 0.0337 & 0 & 0 \\ 0 & 0 & 0.0337 & 0 & 0.0320 & 0 \\ 0 & -0.0034 & 0 & 0.0320 & 0 & 0.0445 \\ 0 & 0 & 0 & 0 & 0.0445 & 0 \end{bmatrix} \quad (3)$$

$$Q_{ext,S} = Q_{ext,L} = 18.829 \quad (4)$$

Because coupling matrix of the filter is symmetrical, the tuning screws which are shown in Figure 2 are also symmetrical. In order to help the readers, Figure 3 shows the resonating unit of the filter.

The tuning producer is shown as following steps:

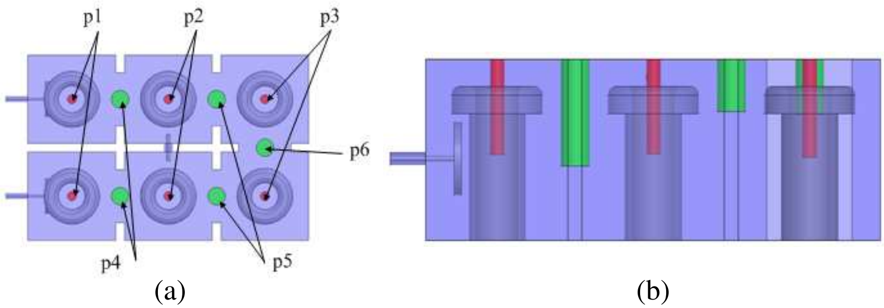


Figure 2. The model of six-resonator cross-coupled filter. (a) Top view, (b) side view.

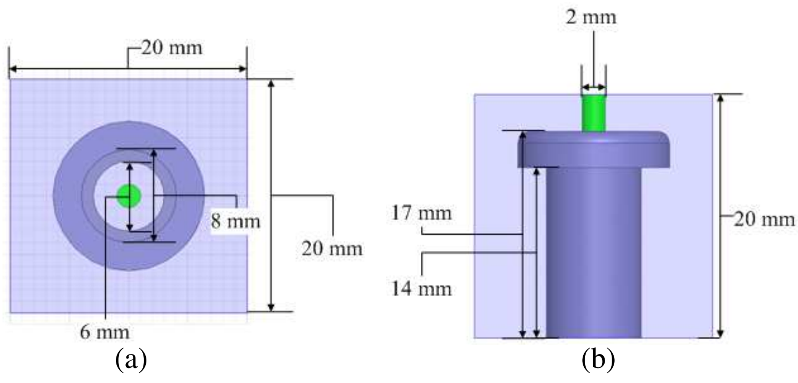


Figure 3. The size of single resonator. (a) Bottom view, (b) side view.

Step 1, obtain the optimal coarse model parameters

$$\begin{aligned} x_c^* &= [f_{01} \quad f_{02} \quad f_{03} \quad M_{12} \quad M_{23} \quad M_{34}] \\ &= [2.0693 \quad 2.0693 \quad 2.0693 \quad 0.0445 \quad 0.032 \quad 0.0337] \end{aligned} \tag{5}$$

Step 2, Measure the filter twice with different tuning positions x_{f1}, x_{f2} to obtain the S-parameters, then obtain two groups equivalent circuit parameters x_{c1}, x_{c2}. The initial position of tuning screws is

$$\begin{aligned} x_{f1} &= [p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5 \quad p_6] \\ &= [10.00 \quad 10.00 \quad 10.00 \quad 10.00 \quad 10.00 \quad 10.00]\,\text{mm} \end{aligned} \tag{6}$$

then use the vector fitting technique to obtain the equivalent circuit parameters x_{c1}

$$\begin{aligned} x_{c1} &= [f_1 \quad f_2 \quad f_3 \quad M_{12} \quad M_{23} \quad M_{34}] \\ &= [2.0936 \quad 2.0854 \quad 2.0887 \quad 0.04 \quad 0.043 \quad 0.0438] \end{aligned} \tag{7}$$

the second initial position of the tuning screw is decided by the comparison between x_{c1} and x_c.

if $x_{c1}(i) > x_c^*(i)$, $i = 1, 2, 3$, $x_{f2} = 1.01 * x_{f1}$; else $x_{f2} = 0.99 * x_{f1}$.
if $x_{c1}(i) > x_c^*(i)$, $i = 4, 5, 6$, $x_{f2} = 0.925 * x_{f1}$; else $x_{f2} = 1.025 * x_{f1}$.
So

$$\begin{aligned} x_{f2} &= [p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5 \quad p_6] \\ &= [10.10 \quad 10.10 \quad 10.10 \quad 10.25 \quad 9.25 \quad 9.25]\,\text{mm} \end{aligned} \tag{8}$$

then obtain the equivalent circuit parameters

$$\begin{aligned} x_{c2} &= [f_1 \quad f_2 \quad f_3 \quad M_{12} \quad M_{23} \quad M_{34}] \\ &= [2.0891 \quad 2.082 \quad 2.087 \quad 0.0406 \quad 0.041 \quad 0.0418] \end{aligned} \tag{9}$$

And the corresponding response is shown in Figure 4.

Step 3, Calculate the initial Broyden matrix B_0 through the equation $B_0 = \text{diag}(\frac{x_{c2}-x_{c1}}{x_{f2}-x_{f1}})$.

$$\begin{aligned} B^{(1)} = B_0 &= \text{diag}\left(\frac{x_{c2} - x_{c1}}{x_{f2} - x_{f1}}\right) \\ &= \text{diag}[-0.0456 \quad -0.034 \quad -0.0164 \quad 0.0025 \quad 0.0027 \quad 0.0026] \end{aligned} \tag{10}$$

Step 4, Start to iterate, Evaluate the difference

$$f^{(1)} = x_c^{(0)} - x_c^* = [0.0198 \;\; 0.0127 \;\; 0.0177 \;\; -0.0039 \;\; 0.009 \;\; 0.0081]^T \tag{11}$$

here $x_c^{(0)} = x_{c2}$. Stop if $\|f^{(1)}\| \leq \varepsilon$. where, ε is the convergence precision. Calculate the increased value of the tuning screw

$$\begin{aligned} h^{(1)} &= B^{(1)} \backslash (-f^{(1)}) \\ &= [0.4341 \;\; 0.3726 \;\; 1.0834 \;\; 1.5672 \;\; -3.2917 \;\; -3.1286]^T \end{aligned} \tag{12}$$

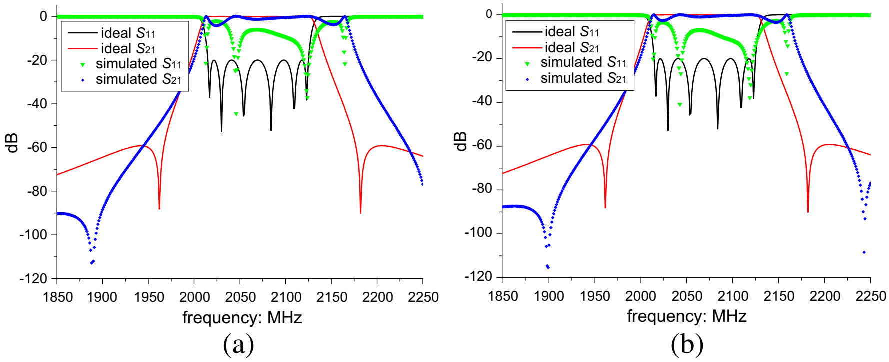


Figure 4. The initial Response of the six-order cross-coupled filter. (a) Initial one, (b) initial two.

And the position of the tuning screw is

$$x_f^{(1)} = x_{f2} + h^{(1)} = [10.53 \quad 10.47 \quad 11.18 \quad 11.82 \quad 5.96 \quad 6.12] \tag{13}$$

Extract the coarse model parameters

$$x_c^{(1)} = [2.068 \quad 2.0683 \quad 2.0528 \quad 0.0446 \quad 0.0321 \quad 0.0335] \tag{14}$$

Update the Broyden matrix

$$B^{(2)} = B^{(1)} + \left(f^{(1)} * h^{(1)^T}\right) \Big/ \left(h^{(1)^T} * h^{(1)}\right) \tag{15}$$

Step 5, second iteration. Evaluate the difference

$$f^{(2)} = x_c^{(1)} - x_c^* = [-0.0013 \quad -0.001 \quad -0.0165 \quad 0.0001 \quad 0.0001 \quad -0.0002]^T \tag{16}$$

Calculate the increased value of the tuning screw

$$h^{(2)} = B^{(2)} \backslash (-f^{(2)}) = [-0.0272 \quad -0.0283 \quad -0.9558 \quad -0.0240 \quad -0.0254 \quad 0.0736]^T \tag{17}$$

And the position of the tuning screw is

$$x_f^{(2)} = x_f^{(1)} + h^{(2)} = [10.51 \quad 10.44 \quad 10.23 \quad 11.79 \quad 5.93 \quad 6.19] \tag{18}$$

Extract the coarse model parameters

$$x_c^{(2)} = [2.07 \quad 2.07 \quad 2.0967 \quad 0.0446 \quad 0.0329 \quad 0.0341] \tag{19}$$

Step 6, the third and fourth iteration are similar as the second iteration. In order to reproduce the tuning methodology, the result of the two iterations are shown in the following

$$\begin{cases} x_c^{(3)} = [2.0693 \quad 2.0703 \quad 2.0694 \quad 0.0446 \quad 0.0321 \quad 0.0337] \\ x_f^{(3)} = [10.51 \quad 10.45 \quad 10.82 \quad 11.78 \quad 5.81 \quad 6.13] \\ x_c^{(4)} = [2.0694 \quad 2.0687 \quad 2.0694 \quad 0.0444 \quad 0.0319 \quad 0.0337] \\ x_f^{(4)} = [10.51 \quad 10.48 \quad 10.82 \quad 11.73 \quad 5.79 \quad 6.12] \end{cases} \tag{20}$$

The corresponding responses of the four iterations are shown in Figure 5.

From Figure 5, we can see that the final measured response agrees with the ideal response well. It shows the validity of this method in this paper.

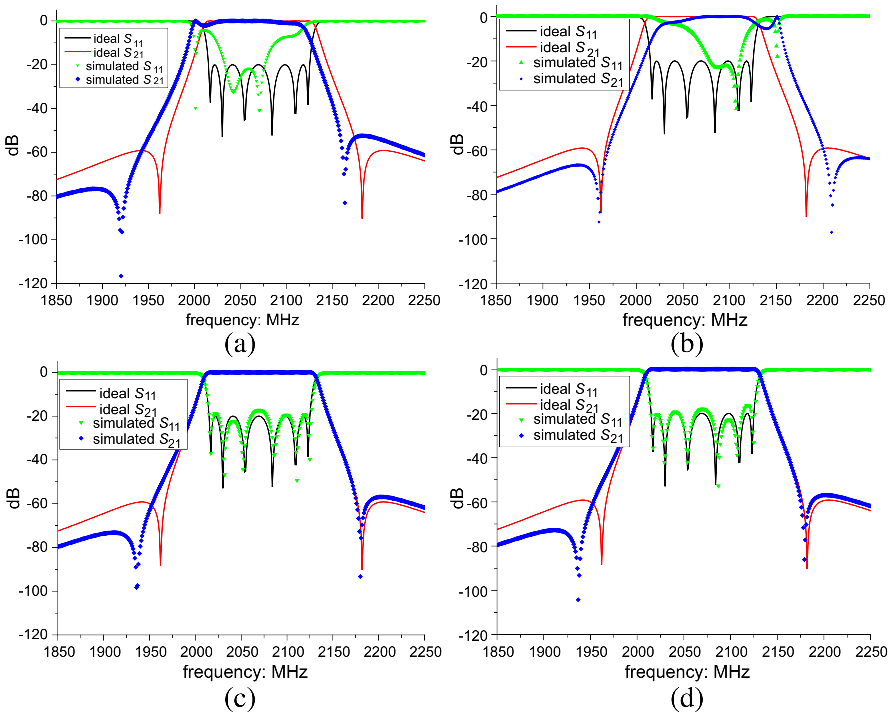


Figure 5. Response of the six-order cross-coupled filter. (a) Iteration one, (b) iteration two, (c) iteration three, (d) iteration four.

3.2. The Eight-pole Chebyshev Filter

In order to shown this method is validity for higher order filters, we applied the method to an eight pole Chebyshev filter. The specification of the filter is shown in Table 2.

The coupling matrix M and external Q value can be easily obtained through the method in [1, 2]. The results are shown as Equations (21) and (22)

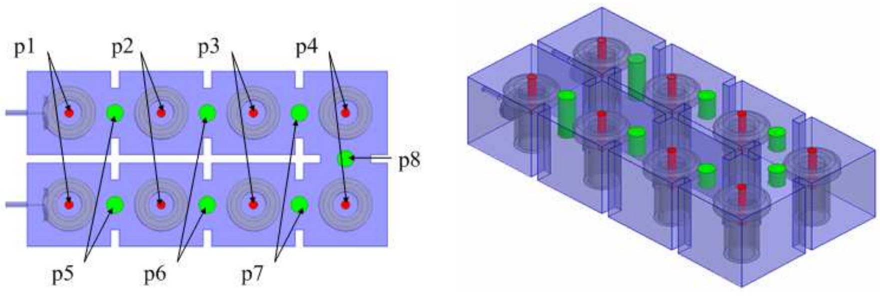
$$M = \begin{bmatrix} 0 & 0.0437 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0437 & 0 & 0.0314 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0314 & 0 & 0.0294 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0294 & 0 & 0.0290 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0290 & 0 & 0.0294 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0294 & 0 & 0.0314 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0314 & 0 & 0.0437 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0437 & 0 \end{bmatrix} \tag{21}$$

$$Q_{ext,S} = Q_{ext,L} = 19.1661 \tag{22}$$

The filter model is shown as Figure 6. There are eight tuning screws for the filter. First, Measure the filter twice with different tuning positions x_{f1}, x_{f2} to obtain the S-parameters, then obtain two groups

Table 2. Specifications of the eight-resonator Chebyshev filter.

Center frequency f_0	Pass-band	Return loss	Filter degree
2069.3 MHz	2015 MHz–2125 MHz	20 dB	8



(a) (b)

Figure 6. The model of eight-pole Chebyshev filter. (a) Top view, (b) full view.

equivalent circuit parameters x_{c1}, x_{c2}.

$$\begin{cases} x_{c1} = [f_1 \;\; f_2 \;\; f_3 \;\; f_4 \;\; M_{12} \;\; M_{23} \;\; M_{34} \;\; M_{45}] \\ \;\; = [2.0931 \;\; 2.0937 \;\; 2.0927 \;\; 2.0890 \;\; 0.0405 \;\; 0.0422 \;\; 0.0428 \;\; 0.0438] \\ x_{c2} = [f_1 \;\; f_2 \;\; f_3 \;\; f_4 \;\; M_{12} \;\; M_{23} \;\; M_{34} \;\; M_{45}] \\ \;\; = [2.0850 \;\; 2.0896 \;\; 2.0910 \;\; 2.0866 \;\; 0.0411 \;\; 0.0404 \;\; 0.0410 \;\; 0.0424] \end{cases} \tag{23}$$

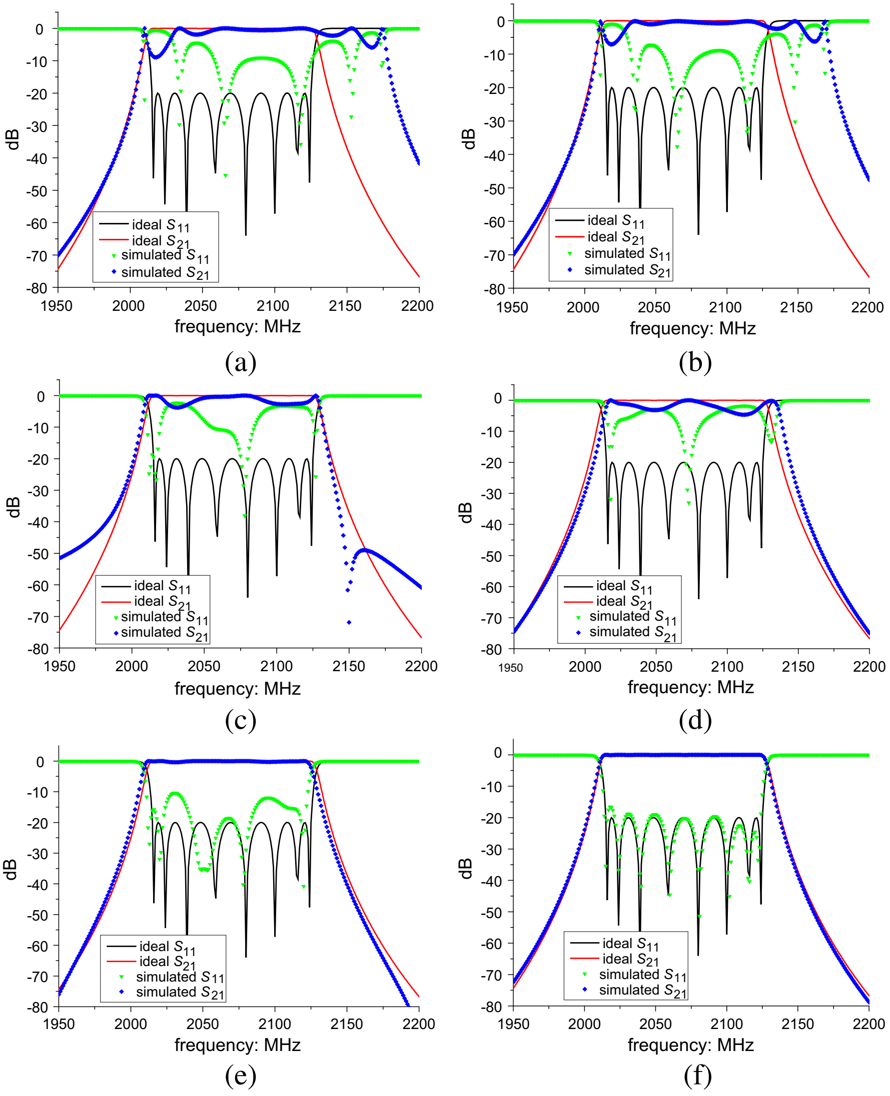


Figure 7. Response of the eight-pole Chebyshev filter. (a) Initial one, (b) initial two, (c) iteration one, (d) iteration two, (e) iteration three, (f) iteration four.

The initial Broyden matrix is

$$B_0 = \text{diag}[-0.0804\ \ -0.0412\ \ -0.0176\ \ -0.0239\ \ 0.0024\ \ 0.0024\ \ 0.0024\ \ 0.0027] \tag{24}$$

Once the initial Broyden matrix is obtained, the tuning process start to iterate. The insertion of the tuning screws is listed in Table 3. And the corresponding response is shown in Figure 7.

Table 3. The insertion of the tuning screws of the eight-pole filter (Unit: mm).

Screw position	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
Initial 1	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Initial 2	10.10	10.10	10.10	10.10	10.25	9.25	9.25	9.50
Iteration 1	10.29	10.59	11.33	10.82	11.35	5.59	4.49	4.52
Iteration 2	10.41	10.67	10.52	11.18	11.76	5.87	4.58	4.13
Iteration 3	10.44	10.65	11.13	11.06	11.64	5.78	4.59	4.16
Iteration 4	10.50	10.64	11.01	11.05	11.48	5.86	4.64	4.20

After four iterations, from Figure 7, we can see that the measured response agree with the required specifications well. It shows the validity of the new tuning technique.

4. CONCLUSION

A novel tuning method based on vector fitting and ASM technique is presented in this paper. The vector fitting is applied to extract the equivalent circuit model parameters from the measured S-parameters. And the ASM technique predicts the tuning directions and value of the tuning screws. The filter can be tuned well through less than four iterations. Two examples are used to verify this new method, and the results show the validity of this method.

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