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ELECTROMAGNETIC FIELDS EXCITATION BY A MULTIELEMENT VIBRATOR-SLOT STRUCTURES IN COUPLED ELECTRODYNAMICS VOLUMES

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Abstract—A problem of electromagnetic fields excitation by a system of finite-dimensional material bodies in two arbitrary electrodynamic volumes coupled by holes, cut in a common boundary of the volumes, is defined in a rigorous formulation. For the system containing two material bodies and one coupling hole, the problem is reduced to a system of two-dimensional integral equations relative to surface electric currents on the material bodies and equivalent magnetic current in the coupling hole. The resulting integral equations are correctly transformed to a system of one-dimensional equations for currents in a narrow slot and on thin impedance vibrators, which may have irregular electrophysical and geometrical parameters. The resulting equations system for a transverse slot in a broad wall of a rectangular waveguide and impedance vibrators with variable surface impedance is solved by a generalized method of induced electro-magneto-motive forces (EMMF) under assumption that interaction between the vibrators and the slot is absent. Calculated and experimental plots of electrodynamic characteristics for this vibrator-slot structure are presented.

1. INTRODUCTION

Linear vibrator-slot radiators are widely used now as stand-alone transceiver structures, elements of antenna systems, and devices in antenna feed lines [1–4]. The widespread use of combined vibrator-slot structures is a prerequisite for theoretical analysis of such systems. The vibrators can be located either in a half-space over an infinite perfectly conducting plane in which a hole is cut for coupling

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with another electrodynamic volume (a half-space over an infinite perfectly conducting plane, a waveguide, a resonator etc.) [5–8] or in waveguide transmission lines [9–13]. A special place among the combined vibrator-slot structures occupies multielement systems [14–20]. However, in above-mentioned and other publications on the subject authors have supposed that vibrators are perfectly conducting or are made of a metal with finite conductivity. To explore new possibilities to control characteristics of slotted-waveguide radiating elements there arises necessity to study vibrators with variable complex surface impedance.

A problem of electromagnetic waves scattering and radiation by a three-cell system, consisting of a transverse slot in a broad wall of a rectangular waveguide and two vibrators with variable surface impedance, was solved by generalized method of induced EMMF. The solution is based on analysis of a problem for multielement vibrator-slot structure, formulated in a general form. Axes of vibrators are located in a cross-section plane passing through a longitudinal axis of a slot perpendicular to a waveguide broad wall. Current distribution functions, obtained as analytical solutions of integral equations for currents in a single impedance vibrator and in a single slot by asymptotic averaging method, were used as basis functions.

2. PROBLEM FORMULATION AND INITIAL INTEGRAL EQUATIONS

Let us formulate the problem of electromagnetic fields excitation (scattering, radiation) by finite-size material bodies in two electrodynamic volumes coupled by holes cut in their common boundary. Suppose that there exists some arbitrary volume V_1, bounded by a perfectly conducting, impedance, or partially impedance surface S_1, some parts of which may be infinitely distant. The volume V_1 is coupled with another arbitrary volume V_2 through holes Σ_n $(n = 1, 2, \ldots, N)$, cut in the surface S_1. The boundary between the volumes V_1 and V_2 in the regions around the coupling holes has an infinitely small thickness. Permittivity and permeability of the medium filling volumes V_1 and V_2 are ε_1, μ_1 and ε_2, μ_2, respectively. Material bodies, enclosed in local volumes V_{m_1} $(m_1 = 1, 2, \ldots, M_1)$ and V_{m_2} $(m_2 = 1, 2, \ldots, M_2)$, bounded by smooth closed surfaces S_{m_1} and S_{m_2}, are allocated in the volumes V_1 and V_2, respectively. The bodies have homogeneous material parameters: permittivity ε_{m_1}, ε_{m_2}, permeability μ_{m_1}, μ_{m_2}, and conductivity σ_{m_1}, σ_{m_2}. The fields of extraneous sources can be specified as the electromagnetic wave fields, incident on the bodies and the holes (scattering problem), or as fields of electromotive forces, applied to the bodies (radiation problem), or as combination of these fields. Without loss of generality, we assume that electromagnetic fields of extraneous sources $\{\vec{E}_0(\vec{r}),\ \vec{H}_0(\vec{r})\}$ exist only in the volume V_1. The fields $\{\vec{E}_0(\vec{r}),\ \vec{H}_0(\vec{r})\}$ depend on the time t as $e^{i\omega t}$ ($\vec{r}$ is the radius vector of the observation point, $\omega = 2\pi f$ is an circular frequency and f is frequency, measured in Hertz). We seek the electromagnetic fields $\{\vec{E}_{V_1}(\vec{r}),\ \vec{H}_{V_1}(\vec{r})\}$ and $\{\vec{E}_{V_2}(\vec{r}),\ \vec{H}_{V_2}(\vec{r})\}$ in the volumes V_1 and V_2, satisfying Maxwell's equations and boundary conditions on the surfaces S_{m_1}, S_{m_2}, Σ_n, S_1 and S_2.

To solve the above-mentioned problem we express the electromagnetic fields in volumes V_1 and V_2 in terms of the tangential fields components on the surfaces S_{m_1}, S_{m_2} and Σ_n. In the Gaussian CGS system of units, the electromagnetic fields can be represented by the well-known Kirchhoff-Kotler integral equations [12]:

$$\begin{aligned}
\vec{E}_{V_1}(\vec{r}) = {} & \vec{E}_0(\vec{r}) + \frac{1}{4\pi ik\varepsilon_1}\left(\text{graddiv} + k_1^2\right) \\
& \sum_{m_1=1}^{M_1} \int\limits_{S_{m_1}} \hat{G}^e_{V_1}\left(\vec{r}, \vec{r}'_{m_1}\right)\left[\vec{n}_{m_1}, \vec{H}_{V_1}\left(\vec{r}'_{m_1}\right)\right] d\vec{r}'_{m_1} \\
& -\frac{1}{4\pi}\text{rot}\left\{\sum_{m_1=1}^{M_1} \int\limits_{S_{m_1}} \hat{G}^m_{V_1}\left(\vec{r}, \vec{r}'_{m_1}\right)\left[\vec{n}_{m_1}, \vec{E}_{V_1}\left(\vec{r}'_{m_1}\right)\right] d\vec{r}'_{m_1}\right. \\
& \left. + \sum_{n=1}^{N} \int\limits_{\Sigma_n} \hat{G}^m_{V_1}\left(\vec{r}, \vec{r}'_n\right)\left[\vec{n}_n, \vec{E}_{V_1}\left(\vec{r}'_n\right)\right] d\vec{r}'_n\right\}, \\
\vec{H}_{V_1}(\vec{r}) = {} & \vec{H}_0(\vec{r}) + \frac{1}{4\pi ik\mu_1}\left(\text{graddiv} + k_1^2\right) \\
& \left\{\begin{array}{l} \sum\limits_{m_1=1}^{M_1} \int\limits_{S_{m_1}} \hat{G}^m_{V_1}\left(\vec{r}, \vec{r}'_{m_1}\right)\left[\vec{n}_{m_1}, \vec{E}_{V_1}\left(\vec{r}'_{m_1}\right)\right] d\vec{r}'_{m_1} \\ + \sum\limits_{n=1}^{N} \int\limits_{\Sigma_n} \hat{G}^m_{V_1}\left(\vec{r}, \vec{r}'_n\right)\left[\vec{n}_n, \vec{E}_{V_1}\left(\vec{r}'_n\right)\right] d\vec{r}'_n \end{array}\right\} \\
& + \frac{1}{4\pi}\text{rot}\sum_{m_1=1}^{M_1} \int\limits_{S_{m_1}} \hat{G}^e_{V_1}\left(\vec{r}, \vec{r}'_{m_1}\right)\left[\vec{n}_{m_1}, \vec{H}_{V_1}\left(\vec{r}'_{m_1}\right)\right] d\vec{r}'_{m_1},
\end{aligned} \tag{1}$$

$$\vec{E}_{V_2}(\vec{r}) = \frac{1}{4\pi ik\varepsilon_2}\left(\text{graddiv} + k_2^2\right)\sum_{m_2=1}^{M_2} \int\limits_{S_{m_2}} \hat{G}^e_{V_2}\left(\vec{r}, \vec{r}'_{m_2}\right)\left[\vec{n}_{m_2}, \vec{H}_{V_2}\left(\vec{r}'_{m_2}\right)\right] d\vec{r}'_{m_2}$$

$$-\frac{1}{4\pi}\text{rot}\left\{\sum_{m_2=1}^{M_2}\int\limits_{S_{m_2}} \hat{G}^m_{V_2}\left(\vec{r},\vec{r}'_{m_2}\right)\left[\vec{n}_{m_2},\vec{E}_{V_2}\left(\vec{r}'_{m_2}\right)\right]d\vec{r}'_{m_2}\right.$$

$$\left.+\sum_{n=1}^{N}\int\limits_{\Sigma_n} \hat{G}^m_{V_2}\left(\vec{r},\vec{r}'_n\right)\left[\vec{n}_n,\vec{E}_{V_2}\left(\vec{r}'_n\right)\right]d\vec{r}'_n\right\},$$

$$\vec{H}_{V_2}(\vec{r})=\frac{1}{4\pi ik\mu_2}\left(\text{graddiv}+k_2^2\right)$$

$$\begin{Bmatrix}\sum\limits_{m_2=1}^{M_2}\int\limits_{S_{m_2}} \hat{G}^m_{V_2}\left(\vec{r},\vec{r}'_{m_2}\right)\left[\vec{n}_{m_2},\vec{E}_{V_2}\left(\vec{r}'_{m_2}\right)\right]d\vec{r}'_{m_2}\\ +\sum\limits_{n=1}^{N}\int\limits_{\Sigma_n} \hat{G}^m_{V_2}\left(\vec{r},\vec{r}'_n\right)\left[\vec{n}_n,\vec{E}_{V_2}\left(\vec{r}'_n\right)\right]d\vec{r}'_n\end{Bmatrix}$$

$$+\frac{1}{4\pi}\text{rot}\sum_{m_2=1}^{M_2}\int\limits_{S_{m_2}} \hat{G}^e_{V_2}\left(\vec{r},\vec{r}'_{m_2}\right)\left[\vec{n}_{m_2},\vec{H}_{V_2}\left(\vec{r}'_{m_2}\right)\right]d\vec{r}'_{m_2}.$$

Here $k = 2\pi/\lambda$ is the wave number, λ is the free space wavelength, $k_1 = k\sqrt{\varepsilon_1\mu_1}$ and $k_2 = k\sqrt{\varepsilon_2\mu_2}$ are wave numbers in the media filling the volumes V_1 and V_2, respectively; $\vec{r}'_{m_1,m_2,n}$ are radius-vectors of sources allocated at the surfaces S_{m_1}, S_{m_2} and Σ_n; $\vec{n}_{m1,m2,n}$ are unit vectors of external normals to the surfaces; $\hat{G}^e_{V_1,V_2}(\vec{r},\vec{r}')$ and $\hat{G}^m_{V_1,V_2}(\vec{r},\vec{r}')$ are the electric and magnetic tensor Green's functions for Hertz vector potentials in the coupled volumes satisfying the vector Helmholtz equation and the boundary conditions on surfaces S_1 and S_2. For the infinitely distant parts of surfaces S_1 or S_2 the boundary conditions for the Green's functions are transformed to the Sommerfeld's radiation condition.

Interpretation of the fields in the left-hand side of Equations (1) depends upon position of an observation point $\vec{r}$. If the observation point $\vec{r}$ belongs to the surfaces S_{m_1}, S_{m_2} or to the apertures Σ_n, the fields $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ represent the same field as in the integrals in the right-hand sides of Equations (1). In this case, Equations (1) is non-homogeneous linear integral Fredholm equations of the second kind, which are known to have the unique solution. If the observation point lies outside areas V_{m_1}, V_{m_2} and Σ_n, the Equations (1) becomes the equalities determining the total electromagnetic field by the field of specified extraneous sources. These equalities solve, in general terms, the problem of electromagnetic fields excitation by finite size obstacles if fields on the objects' surfaces are known. Certainly, to find these fields, the Fredholm integral equations should be solved beforehand.

The Equations (1) can be also used to solve electrodynamics problems if the fields on the material body surfaces can be defined by additional physical considerations. For example, if induced currents on well-conducting bodies ($\sigma \to \infty$) are concentrated near the body surface the skin layer thickness can be neglected and the well-known Leontovich-Shchukin approximate impedance boundary condition becomes applicable [4]

$$\left[\vec{n}, \vec{E}\left(\vec{r}\right)\right] = \bar{Z}_S\left(\vec{r}\right)\left[\vec{n}, \left[\vec{n}, \vec{H}\left(\vec{r}\right)\right]\right], \tag{2}$$

where $\bar{Z}_S(\vec{r}) = \bar{R}_S(\vec{r}) + i\bar{X}_S(\vec{r}) = Z_S(\vec{r})/Z_0$ is the distributed complex surface impedance, normalized to the characteristic free space impedance $Z_0 = 120\pi$ Ohm; the value of $\bar{Z}_S(\vec{r})$ may vary over the body surface. The impedance boundary condition (2) allows transition in the integral Equations (1) to the surface currents density. Without loss of generality, we will do such transition for configuration consisting of two material bodies in the volume V_1 coupled with the volume V_2 by a single hole. Placing the observation point on the surfaces S_{1_1} and S_{2_1} and using the continuity conditions for the tangential components of magnetic fields on the hole, we obtain the integral equations

$$\begin{aligned}
&Z_{S1}\left(\vec{r}_1\right)\vec{J}_1^e\left(\vec{r}_1\right) + \frac{k}{\omega}\mathrm{rot}\int\limits_{\Sigma}\hat{G}_{V_1}^m\left(\vec{r}, \vec{r}_3'\right)\vec{J}_3^m\left(\vec{r}_3'\right)d\vec{r}_3' \\
&= \vec{E}_0\left(\vec{r}\right) + \frac{1}{i\omega\varepsilon_1}\left(\mathrm{graddiv} + k_1^2\right)\left\{\int\limits_{S_{1_1}}\hat{G}_{V_1}^e\left(\vec{r}, \vec{r}_1'\right)\vec{J}_1^e\left(\vec{r}_1'\right)d\vec{r}_1' + \int\limits_{S_{2_1}}\hat{G}_{V_1}^e\left(\vec{r}, \vec{r}_2'\right)\vec{J}_2^e\left(\vec{r}_2'\right)d\vec{r}_2'\right\} \\
&+ \frac{1}{4\pi}\mathrm{rot}\left\{\int\limits_{S_{1_1}}\hat{G}_{V_1}^m(\vec{r}, \vec{r}_1')Z_{S1}\left(\vec{r}_1'\right)\left[\vec{n}_1,\ \vec{J}_1^e\left(\vec{r}_1'\right)\right]d\vec{r}_1'\right. \\
&\left. + \int\limits_{S_{2_1}}\hat{G}_{V_1}^m(\vec{r}, \vec{r}_2')Z_{S2}\left(\vec{r}_2'\right)\left[\vec{n}_2,\ \vec{J}_2^e\left(\vec{r}_2'\right)\right]d\vec{r}_2'\right\},
\end{aligned} \tag{3a}$$

$$\begin{aligned}
&Z_{S2}\left(\vec{r}_2\right)\vec{J}_2^e\left(\vec{r}_2\right) + \frac{k}{\omega}\mathrm{rot}\int\limits_{\Sigma}\hat{G}_{V_1}^m\left(\vec{r}, \vec{r}_3'\right)\vec{J}_3^m\left(\vec{r}_3'\right)d\vec{r}_3' \\
&= \vec{E}_0\left(\vec{r}\right) + \frac{1}{i\omega\varepsilon_1}\left(\mathrm{graddiv} + k_1^2\right)\left\{\int\limits_{S_{2_1}}\hat{G}_{V_1}^e\left(\vec{r}, \vec{r}_2'\right)\vec{J}_2^e\left(\vec{r}_2'\right)d\vec{r}_2' + \int\limits_{S_{1_1}}\hat{G}_{V_1}^e\left(\vec{r}, \vec{r}_1'\right)\vec{J}_1^e\left(\vec{r}_1'\right)d\vec{r}_1'\right\}
\end{aligned}$$

$$+\frac{1}{4\pi}\mathrm{rot}\left\{\int\limits_{S_{2_1}}\hat{G}^m_{V_1}\left(\vec{r},\vec{r}_2'\right)Z_{S2}\left(\vec{r}_2'\right)\left[\vec{n}_2,\ \vec{J}^e_2\left(\vec{r}_2'\right)\right]d\vec{r}_2'\right.$$

$$\left.+\int\limits_{S_{1_1}}\hat{G}^m_{V_1}\left(\vec{r},\vec{r}_1'\right)Z_{S1}\left(\vec{r}_1'\right)\left[\vec{n}_1,\ \vec{J}^e_1\left(\vec{r}_1'\right)\right]d\vec{r}_1'\right\}, \tag{3b}$$

$$\vec{H}_0\left(\vec{r}\right)+\frac{1}{i\omega\mu_1}\left(\mathrm{graddiv}+k_1^2\right)\int\limits_{\Sigma}\hat{G}^m_{V_1}\left(\vec{r},\vec{r}_3'\right)\vec{J}^m_3\left(\vec{r}_3'\right)d\vec{r}_3'$$

$$+\frac{1}{i\omega\mu_2}(\mathrm{graddiv}+k_2^2)\int\limits_{\Sigma}\hat{G}^m_{V_2}\left(\vec{r},\vec{r}_3'\right)\vec{J}^m_3\left(\vec{r}_3'\right)d\vec{r}_3'$$

$$=\frac{1}{i\omega\varepsilon_1}\left(\mathrm{graddiv}+k_1^2\right)\left\{\int\limits_{S_{1_1}}\hat{G}^m_{V_1}\left(\vec{r},\vec{r}_1'\right)Z_{S1}\left(\vec{r}_1'\right)\left[\vec{n}_1,\ \vec{J}^e_1\left(\vec{r}_1'\right)\right]d\vec{r}_1'\right.$$

$$\left.+\int\limits_{S_{2_1}}\hat{G}^m_{V_1}(\vec{r},\vec{r}_2')Z_{S2}\left(\vec{r}_2'\right)\left[\vec{n}_2,\ \vec{J}^e_2\left(\vec{r}_2'\right)\right]d\vec{r}_2'\right\}$$

$$-\frac{k}{\omega}\mathrm{rot}\left\{\int\limits_{S_{1_1}}\hat{G}^e_{V_1}\left(\vec{r},\vec{r}_1'\right)\vec{J}^e_1\left(\vec{r}_1'\right)d\vec{r}_1'+\int\limits_{S_{2_1}}\hat{G}^e_{V_1}\left(\vec{r},\vec{r}_2'\right)\vec{J}^e_2\left(\vec{r}_2'\right)d\vec{r}_2'\right\}. \tag{3c}$$

relative to the surface electrical currents $\vec{J}^e_{1,2}(\vec{r}_{1,2})$ on $S_{1_1,2_1}$ and equivalent magnetic current $\vec{J}^m_3(\vec{r}_3)$ on Σ

$$\vec{J}^e_{1,2}\left(\vec{r}_{1,2}\right)=\frac{c}{4\pi}\left[\vec{n}_{1,2},\ \vec{H}\left(\vec{r}_{1,2}\right)\right],\quad \vec{J}^m_3\left(\vec{r}_3\right)=\frac{c}{4\pi}\left[\vec{n}_3,\vec{E}\left(\vec{r}_3\right)\right], \tag{4}$$

where $c\approx 2.998\cdot 10^{10}\,\mathrm{cm/c}$ is the light velocity in vacuum.

3. INTEGRAL EQUATIONS FOR ELECTRIC AND MAGNETIC CURRENTS IN THIN VIBRATORS AND NARROW SLOTS

A solution of Equations (3) for material objects with complex surface shape and for the coupling hole Σ of arbitrary geometry encounters serious mathematical difficulties. However, if the cross-sectional perimeter of the impedance cylinder is small as compared to its length and to the wavelength in the ambient medium (thin vibrator approximation) and if the slot width satisfy analogues conditions (narrow slot approximation), the solution of Equations (3) may be simplified [12]. The approach used in [12] for the analysis of vibrator-slot system can be generalized for multielement systems. In addition, the boundary condition (2) can be extended to cylindrical vibrators with arbitrary distribution of the complex impedance, regardless of the exciting field structure and the electrical characteristics of the vibrator material [4].

Let us transform the Equations (3) for thin vibrators made of circular cylindrical wires and a narrow rectilinear slot that is, if the inequalities

$$\frac{r_{1,2}}{2L_{1,2}} \ll 1, \quad \frac{r_{1,2}}{\lambda_{1,2}} \ll 1, \quad \frac{d}{2L_3} \ll 1, \quad \frac{d}{\lambda_{1,2}} \ll 1 \tag{5}$$

hold. Here $\lambda_{1,2}$ are the wavelengths in corresponding media; $r_{1,2}$ and $2L_{1,2}$ are vibrator radii and lengths; d and $2L_3$ are the slot width and length. Then the electric currents induced in the vibrators and the equivalent magnetic current in the slot can be presented as

$$\begin{aligned} \vec{J}^{e}_{1(2)}\left(\vec{r}_{1(2)}\right) &= \vec{e}_{s_{1(2)}} J_{1(2)}\left(s_{1(2)}\right) \psi_{1(2)}\left(\rho_{1(2)}, \varphi_{1(2)}\right), \\ \vec{J}^{m}_{3}\left(\vec{r}_{3}\right) &= \vec{e}_{s_3} J_3(s_3) \chi\left(\xi\right), \end{aligned} \tag{6}$$

where $\vec{e}_{s_{1(2)}}$ and $\vec{e}_{s_3}$ are units vectors, oriented along the vibrators and slot axes, respectively; $s_{1(2)}$ and s_3 are local coordinates related to vibrators and slot axes; $\psi_{1(2)}(\rho_{1(2)}, \varphi_{1(2)})$ are functions of transverse ($\perp_{1(2)}$) polar coordinates $\rho_{1(2)}$, $\varphi_{1(2)}$ for vibrators; $\chi(\xi)$ is a function of transverse coordinate ξ for the slot. The functions $\psi_{1(2)}(\rho_{1(2)}, \varphi_{1(2)})$ and $\chi(\xi)$ satisfy the normalizing conditions

$$\int\limits_{\perp_{1(2)}} \psi_{1(2)}(\rho_{1(2)}, \varphi_{1(2)}) \rho_{1(2)} d\rho_{1(2)} d\varphi_{1(2)} = 1\,, \qquad \int\limits_{\xi} \chi(\xi) d\xi = 1\,, \tag{7}$$

and unknown currents (indexes "e" and "m" are omitted), $J_{1(2)}(s_{1(2)})$ and $J_3(s_3)$, satisfy the boundary conditions

$$J_{1(2)}(\pm L_{1(2)}) = 0\,, \quad J_3(\pm L_3) = 0\,. \tag{8}$$

The system of one-dimensional integral equations for currents, which takes into account mutual interaction between the vibrators and slot, can be derived by projecting the Equations (3a), (3b) on the vibrator axes, the Equation (3c) on the slot axis and by using inequalities $[\vec{n}_{1(2)}, \vec{J}_{1(2)}(\vec{r}_{1(2)})] \ll 1$ resulting from (5). This equations system may be presented as

$$\left(\frac{d^2}{ds_1^2}+k_1^2\right)\left\{\int_{-L_1}^{L_1}J_1(s_1')G_{s_1}^{V_1}(s_1,s_1')ds_1'+\int_{-L_2}^{L_2}J_2(s_2')G_{s_2}^{V_1}(s_1,s_2')ds_2'\right\}$$
$$-ik\vec{e}_{s_1}\mathrm{rot}\int_{-L_3}^{L_3}J_3(s_3')G_{s_3}^{V_1}(s_1,s_3')ds_3'=-i\omega\varepsilon_1\left[E_{0s_1}(s_1)-z_{i1}(s_1)J_1(s_1)\right],$$
$$\left(\frac{d^2}{ds_2^2}+k_1^2\right)\left\{\int_{-L_2}^{L_2}J_2(s_2')G_{s_2}^{V_1}(s_2,s_2')ds_2'+\int_{-L_1}^{L_1}J_1(s_1')G_{s_1}^{V_1}(s_2,s_1')ds_1'\right\} \quad (9)$$
$$-ik\vec{e}_{s_2}\mathrm{rot}\int_{-L_3}^{L_3}J_3(s_3')G_{s_3}^{V_1}(s_2,s_3')ds_3'=-i\omega\varepsilon_1\left[E_{0s_2}(s_2)-z_{i2}(s_2)J_2(s_2)\right],$$
$$\frac{1}{\mu_1}\left(\frac{d^2}{ds_3^2}+k_1^2\right)\int_{-L_3}^{L_3}J_3(s_3')G_{s_3}^{V_1}(s_3,s_3')ds_3'+\frac{1}{\mu_2}\left(\frac{d^2}{ds_3^2}+k_2^2\right)\int_{-L_3}^{L_3}J_3(s_3')G_{s_3}^{V_2}(s_3,s_3')ds_3'$$
$$+ik\vec{e}_{s_3}\mathrm{rot}\left\{\int_{-L_1}^{L_1}J_1(s_1')G_{s_1}^{V_1}(s_3,s_1')ds_1'+\int_{-L_2}^{L_2}J_2(s_2')G_{s_2}^{V_1}(s_3,s_2')ds_2'\right\}=-i\omega H_{0s_3}(s_3).$$

Here $z_{i1(2)}(s_{1(2)})$ are internal impedances per unit lengths of the vibrators ([Ohm/m]), $(Z_{S1(2)}(\vec{r}_{1(2)}) = 2\pi r_{1(2)}z_{i1(2)}(\vec{r}_{1(2)}))$, $E_{0s_{1(2)}}(s_{1(2)})$ and $H_{0s_3}(s_3)$ are projections of the fields induced by the extraneous sources on the vibrators and the slot axes, $G_{s_{1,2}}^{V_1}(s_{1,2,3},s_{1(2)}')$ and $G_{s_3}^{V_{1(2)}}(s_{1,2,3},s_3')$ are the tensor Green's functions components.

If the interactions between the vibrators and the slot are absent, the equations system (9) is simplified and can be written as

$$\left(\frac{d^2}{ds_1^2}+k_1^2\right)\left\{\int_{-L_1}^{L_1}J_1(s_1')G_{s_1}^{V_1}(s_1,s_1')ds_1'+\int_{-L_2}^{L_2}J_2(s_2')G_{s_2}^{V_1}(s_1,s_2')ds_2'\right\}$$
$$=-i\omega\varepsilon_1\left[E_{0s_1}(s_1)-z_{i1}(s_1)J_1(s_1)\right],$$
$$\left(\frac{d^2}{ds_2^2}+k_1^2\right)\left\{\int_{-L_2}^{L_2}J_2(s_2')G_{s_2}^{V_1}(s_2,s_2')ds_2'+\int_{-L_1}^{L_1}J_1(s_1')G_{s_1}^{V_1}(s_2,s_1')ds_1'\right\} \quad (10)$$
$$=-i\omega\varepsilon_1\left[E_{0s_2}(s_2)-z_{i2}(s_2)J_2(s_2)\right],$$

$$\frac{1}{\mu_1}\left(\frac{d^2}{ds_3^2}+k_1^2\right)\int\limits_{-L_3}^{L_3} J_3(s_3')G_{s_3}^{V_1}(s_3,s_3')ds_3'$$
$$+\frac{1}{\mu_2}\left(\frac{d^2}{ds_3^2}+k_2^2\right)\int\limits_{-L_3}^{L_3} J_3(s_3')G_{s_3}^{V_2}(s_3,s_3')ds_3' = -i\omega H_{0s_3}(s_3).$$

Since the Green's functions in Equations (9) and (10) are not specified in an explicit form, these equations are valid for any electrodynamic volumes, provided that the Green's functions are known or can be constructed.

4. MULTIELEMENT VIBRATOR-SLOT STRUCTURE IN A RECTANGULAR WAVEGUIDE

As an example, let us consider a problem of electromagnetic waves scattering by a narrow rectilinear transverse slot in a broad wall of a rectangular waveguide containing passive impedance vibrators and extraneous field sources.

Let a fundamental wave H_{10} propagates in a hollow ($\varepsilon_1=\mu_1=1$) infinite rectangular waveguide with perfectly conducting walls (index Wg) from the area $z=-\infty$. In a cross sectional plane of the waveguide several unbalanced thin impedance vibrators (monopoles) are placed and a narrow slot is cut in the broad wall of the waveguide symmetrically relative to its longitudinal axis. The slot radiates into the free half-space ($\varepsilon_2=\mu_2=1$) over an infinite perfectly conducting plane (index Hs). Since the axes of the vibrators and slot lie in $\{x0y\}$ plane, interaction between the vibrators and the slot over the waveguide cavity is absent (Figure 1).

The waveguide size is $\{a\times b\}$, the vibrator radii and lengths are $r_{1,2}$

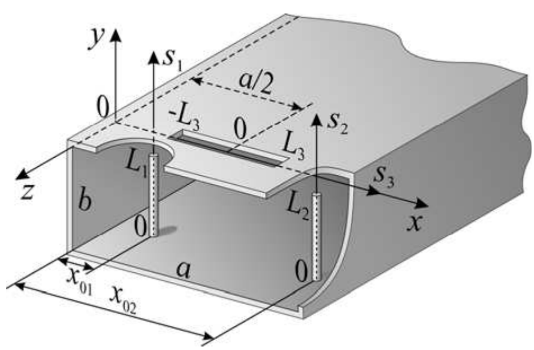

Figure 1. The geometry of the vibrator-slot structure and notations.

and $2L_{1,2}$; the thickness of the waveguide wall is h, the slot width and length are d and $2L_3$. The equations system (10) for this configuration may be reduced to

$$\left(\frac{d^2}{ds_1^2}+k^2\right)\left\{\int\limits_{-L_1}^{L_1} J_1(s_1')G_{s_1}^{Wg}(s_1,s_1')ds_1'+\int\limits_{-L_2}^{L_2} J_2(s_2')G_{s_2}^{Wg}(s_1,s_2')ds_2'\right\}$$
$$=-i\omega\left[E_{0s_1}(s_1)-z_{i1}(s_1)J_1(s_1)\right], \tag{11a}$$
$$\left(\frac{d^2}{ds_2^2}+k^2\right)\left\{\int\limits_{-L_2}^{L_2} J_2(s_2')G_{s_2}^{Wg}(s_2,s_2')ds_2'+\int\limits_{-L_1}^{L_1} J_1(s_1')G_{s_1}^{Wg}(s_2,s_1')ds_1'\right\}$$
$$=-i\omega\left[E_{0s_2}(s_2)-z_{i2}(s_2)J_2(s_2)\right], \tag{11b}$$
$$\left(\frac{d^2}{ds_3^2}+k_1^2\right)\int\limits_{-L_3}^{L_3} J_3(s_3')\left[G_{s_3}^{Wg}(s_3,s_3')+G_{s_3}^{Hs}(s_3,s_3')\right]ds_3'=-i\omega H_{0s_3}(s_3). \tag{11c}$$

Here $G_{s_{1,2}}^{Wg}(s_{1,2},s'_{1(2)})$ and $G_{s_3}^{Wg,Hs}(s_3,s_3')$ are the tensor Green's functions components for a rectangular waveguide and half-space above the plane [3, 4], $s_1=-L_1$ and $s_2=-L_2$ are coordinates of the vibrator mirror images, relative to the lower broad wall of the waveguide [4].

The equations system (11) was solved by the generalized method of induced EMMF [12, 13], using approximate expressions for the currents $J_{1(2)}(s_{1(2)})=J_{1(2)}^0 f_{1(2)}(s_{1(2)})$ and $J_3(s_3)=J_3^0 f_3(s_3)$. Here $J_{1(2)}^0$ and J_3^0 are unknown current amplitudes, $f_{1(2)}(s_{1(2)})$ and $f_3(s_3)$ are distribution functions, obtained by solving equations for currents in a single vibrator and a slot by averaging method [21, 22]. For the vibrator-slot structure excited by fundamental wave H_{10} according to [13], we have

$$f_1(s_1)=\cos\tilde{k}_1 s_1-\cos\tilde{k}_1 L_1, \tag{12a}$$
$$f_2(s_2)=\cos\tilde{k}_2 s_2-\cos\tilde{k}_2 L_2, \tag{12b}$$
$$f_3(s_3)=\cos k s_3-\cos k L_3, \tag{12c}$$

where $\tilde{k}_{1(2)}=k-\frac{i2\pi z_{i1(2)}^{av}}{Z_0\Omega_{1(2)}}$, $z_{i1(2)}^{av}=\frac{1}{2L_{1(2)}}\int\limits_{-L_{1(2)}}^{L_{1(2)}} z_{i1(2)}(s_{1(2)})ds_{1(2)}$ are length-average values of the vibrator internal impedance [4, 23], $\Omega_{1(2)}=2\ln(2L_{1(2)}/r_{1(2)})$.

According to the generalized method of induced EMMF, we multiply Equation (11a) by the function $f_1(s_1)$, Equation (11b) by the function $f_2(s_2)$, and (11c) by the function $f_3(s_3)$. Then we integrate the resulting equations over the vibrators and slot lengths and arrive to a system of linear algebraic equations relative to the current amplitudes $J_{1,2}^0$ and J_3^0

$$\begin{aligned}
&J_1^0\left[Z_{11}\left(kr_1,\tilde{k}_1L_1\right)+F_1^z\left(\tilde{k}_1r_1,\tilde{k}_1L_1\right)\right]+J_2^0Z_{12}\left(\tilde{k}_1L_1,\tilde{k}_2L_2\right)\\
&=-\frac{i\omega}{2k}E_1\left(\tilde{k}_1L_1\right),\\
&J_2^0\left[Z_{22}\left(kr_2,\tilde{k}_2L_2\right)+F_2^z\left(\tilde{k}_2r_2,\tilde{k}_2L_2\right)\right]+J_1^0Z_{21}\left(\tilde{k}_2L_2,\tilde{k}_1L_1\right)\\
&=-\frac{i\omega}{2k}E_2\left(\tilde{k}_2L_2\right),\\
&J_3^0\left[Z_{33}^{Wg}(kd_e,kL_3)+Z_{33}^{Hs}\left(kd_e,kL_3\right)\right]=-\frac{i\omega}{2k}H_3(kL_3).
\end{aligned}\qquad(13)$$

Here

$$\begin{aligned}
&Z_{11(22)}\left(kr_{1(2)},\tilde{k}_{1(2)}L_{1(2)}\right)\\
&=\frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n\left(k^2-k_y^2\right)\tilde{k}_{1(2)}^2}{kk_z\left(\tilde{k}_{1(2)}^2-k_y^2\right)^2}e^{-k_zr_{1(2)}}\sin^2k_xx_{01(02)}\\
&\times\left[\sin\tilde{k}_{1(2)}L_{1(2)}\cos k_yL_{1(2)}-(\tilde{k}_{1(2)}/k_y)\cos\tilde{k}_{1(2)}L_{1(2)}\sin k_yL_{1(2)}\right]^2,\\
&Z_{12(21)}\left(\tilde{k}_{1(2)}L_{1(2)},\tilde{k}_{2(1)}L_{2(1)}\right)\\
&=\frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_y^2)\tilde{k}_1\tilde{k}_2e^{-k_zr_{2(1)}}}{kk_z\left(\tilde{k}_1^2-k_y^2\right)\left(\tilde{k}_2^2-k_y^2\right)}\sin k_xx_{01}\sin k_xx_{02}\\
&\times\left[\sin\tilde{k}_1L_1\cos k_yL_1-\left(\tilde{k}_1/k_y\right)\cos\tilde{k}_1L_1\sin k_yL_1\right]\\
&\times\left[\sin\tilde{k}_2L_2\cos k_yL_2-\left(\tilde{k}_2/k_y\right)\cos\tilde{k}_2L_2\sin k_yL_2\right],\\
&Z_{33}^{Wg}(kd_e,kL_3)\\
&=\frac{8\pi}{ab}\sum_{m=1,3\ldots}^{\infty}\sum_{n=0,1\ldots}^{\infty}\frac{\varepsilon_nk}{k_z(k^2-k_x^2)}e^{-k_z\frac{d_e}{4}}\\
&\times\left[\sin kL_3\cos k_xL_3-(k/k_x)\cos kL_3\sin k_xL_3\right]^2,\\
&Z_{33}^{Hs}(kd_e,kL_3)\\
&=(\mathrm{Si}4kL_3-i\mathrm{Cin}4kL_3)-2\cos kL_3\\
&\times\left[2(\sin kL_3-kL_3\cos kL_3)\left(\ln\frac{16L_3}{d_e}-\mathrm{Cin}2kL_3\right.\right.\\
&\left.\left.-i\mathrm{Si}2kL_3\right)+\sin 2kL_3e^{-ikL_3}\right],
\end{aligned}\qquad(14)$$

$$E_{1(2)}(\tilde{k}_{1(2)}L_{1(2)})=2H_0\frac{k}{k_g\tilde{k}_{1(2)}}\sin\frac{\pi}{a}x_{01(02)}f(\tilde{k}_{1(2)}L_{1(2)}),$$
$$f(\tilde{k}_{1(2)}L_{1(2)})=\sin\tilde{k}_{1(2)}L_{1(2)}-\tilde{k}_{1(2)}L_{1(2)}\cos\tilde{k}_{1(2)}L_{1(2)},$$
$$H_3(kL_3)=2H_0\frac{1}{k}f(kL_3),$$
$$f(kL_3)=\sin kL_3-kL_3\cos kL_3,$$
$$F^z_{1(2)}\Big(\tilde{k}_{1(2)}r_{1(2)},\tilde{k}_{1(2)}L_{1(2)}\Big)=-\frac{i}{r_{1(2)}}\int\limits_0^{L_{1(2)}}f^2_{1(2)}(s_{1(2)})\bar{Z}_{S1(2)}(s_{1(2)})ds_{1(2)}. \quad (15)$$

In the formulas (14), (15) $\varepsilon_n=\left\{\begin{array}{l}1,\ n=0\\2,\ n\neq 0\end{array}\right.$, $k_x=\frac{m\pi}{a}$, $k_y=\frac{n\pi}{b}$, $k_z=\sqrt{k_x^2+k_y^2-k^2}$, m and n are integers, $k_g=2\pi/\lambda_g=\sqrt{k^2-(\pi/a)^2}$, λ_g is the waveguide wavelength; Si and Cin are the integral sine and cosine functions; $\bar{Z}_{S1(2)}(s_{1(2)})=\bar{R}_{S1(2)}+i\bar{X}_{S1(2)}\phi(s_{1(2)})$ are distributed complex surface impedances, $\phi(s_{1(2)})$ is the prescribed functions, $d_e=de^{-\frac{\pi h}{2d}}$ is equivalent slot width which takes into account a wall thickness h of the waveguide [3], H_0 is the amplitude of incident wave.

The analytical solution of equations system (13) can be written as

$$\begin{aligned}J_1^0&=-\frac{i\omega}{2k}\frac{E_1(Z_{22}+F_2^z)-E_2Z_{12}}{(Z_{11}+F_1^z)(Z_{22}+F_2^z)-Z_{21}Z_{12}}=-\frac{i\omega}{2k}\tilde{J}_1^0,\\J_2^0&=-\frac{i\omega}{2k}\frac{E_2(Z_{11}+F_1^z)-E_1Z_{21}}{(Z_{11}+F_1^z)(Z_{22}+F_2^z)-Z_{21}Z_{12}}=-\frac{i\omega}{2k}\tilde{J}_2^0,\\J_3^0&=-\frac{i\omega}{2k}\frac{H_3(kL_3)}{Z_{33}^{Wg}+Z_{33}^{Hs}}=-\frac{i\omega}{2k}\tilde{J}_3^0.\end{aligned} \quad (16)$$

Final expressions for the currents can be readily derived using (12) and (16) as

$$\begin{aligned}J_{1(2)}(s_{1(2)})&=-\frac{i\omega}{2k}\tilde{J}^0_{1(2)}(\cos\tilde{k}_{1(2)}s_{1(2)}-\cos\tilde{k}_{1(2)}L_{1(2)}),\\J_3(s_3)&=-\frac{i\omega}{2k}\tilde{J}_3^0(\cos ks_3-\cos kL_3).\end{aligned} \quad (17)$$

Energy characteristics of the vibrator-slot structure: reflection coefficient S_{11}, transmission coefficient S_{12}, and power radiation coefficient $|S_\Sigma|^2$ are defined by the following expressions

$$S_{11}=\frac{4\pi i}{abkk_g}\left\{\frac{2k_g^2}{k^2}\frac{f^2(kL_3)}{Z_{33}^{Wg}+Z_{33}^{Hs}}-\frac{k^2}{\tilde{k}_1}\tilde{J}_1^0\sin\left(\frac{\pi x_{01}}{a}\right)f(\tilde{k}_1L_1)\right.$$

$$-\frac{k^2}{\tilde{k}_2}\tilde{J}_2^0 \sin\left(\frac{\pi x_{02}}{a}\right) f(\tilde{k}_2 L_2)\Bigg\} e^{2ik_g z}, \tag{18}$$

$$S_{12} = 1 + \frac{4\pi i}{abkk_g}\left\{\frac{2k_g^2}{k^2}\frac{f^2(kL_3)}{Z_{33}^{Wg}+Z_{33}^{Hs}} + \frac{k^2}{\tilde{k}_1}\tilde{J}_1^0 \sin\left(\frac{\pi x_{01}}{a}\right) f(\tilde{k}_1 L_1)\right.$$

$$\left. +\frac{k^2}{\tilde{k}_2}\tilde{J}_2^0 \sin\left(\frac{\pi x_{02}}{a}\right) f(\tilde{k}_2 L_2)\right\}, \tag{19}$$

$$|S_\Sigma|^2 = 1 - |S_{11}|^2 - |S_{12}|^2. \tag{20}$$

5. NUMERICAL RESULTS

We perform a numerical analysis for a three-element vibrator-slot structure and compared its energy characteristics with that of a single radiating slot (without vibrators) and a structure consisting of a slot and a one vibrator. We have used three different distribution functions defining the imaginary part of vibrator impedance [4, 13]: $\phi_0(s_{1(2)}) = 1$, $\phi_1(s_{1(2)}) = 2[1 - (s_{1(2)}/L_{1(2)})]$ and $\phi_2(s_{1(2)}) = 2(s_{1(2)}/L_{1(2)})$.

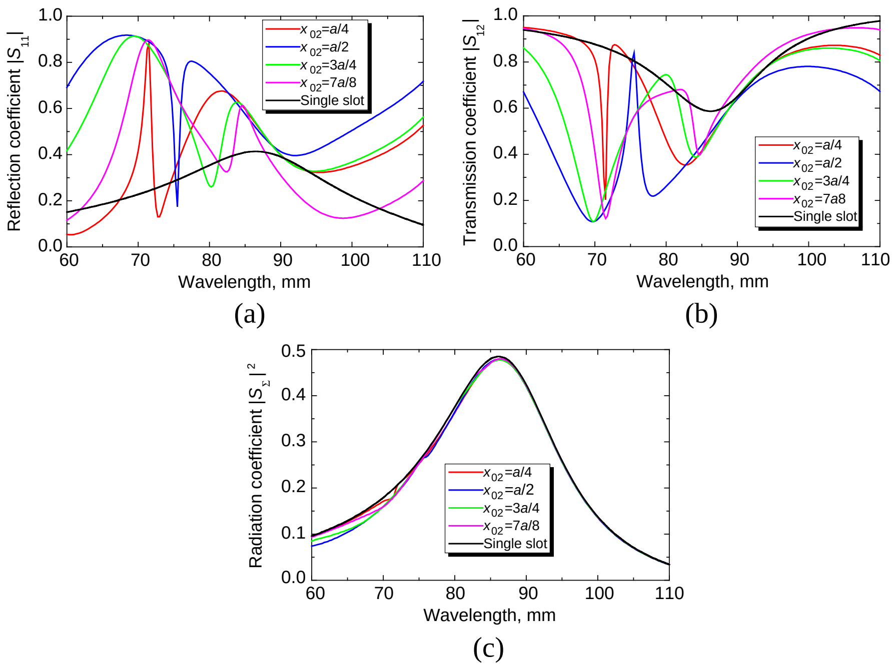

Figure 2. Energy characteristics of vibrators-slot system versus wavelength at $x_{01} = a/8$, $\bar{Z}_{S1} = 0$, $\bar{Z}_{S2} = ikr_2 \ln(4.0)$.

Analytical expressions for the functions $F^z_{1(2)}$, obtained by (**??**) for each distribution functions, are presented in [13].

Figures 2–5 show the wavelength dependences of the radiation coefficient, moduli of the reflection and transmission coefficients in the wavelength range of the waveguide single-mode regime, obtained using the following common parameters: $a = 58.0\,\mathrm{mm}$, $b = 25.0\,\mathrm{mm}$, $h = 0.5\,\mathrm{mm}$, $r_{1,2} = 2.0\,\mathrm{mm}$, $L_{1,2} = 15.0\,\mathrm{mm}$, $\bar{R}_{S1(2)} = 0$, $d = 4.0\,\mathrm{mm}$ and $2L_3 = 40.0\,\mathrm{mm}$. The choice of slot dimensions was stipulated by its natural resonance at the average wavelength of the waveguide frequency range $\lambda_3^{res} = 86.0\,\mathrm{mm}$. The dimensions of the vibrators have been selected so that their resonant wavelength was within the waveguide operating range. Here we present the results only for

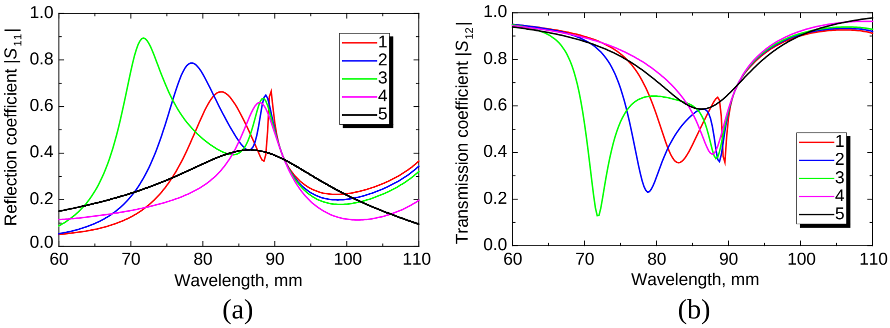

Figure 3. Energy characteristics of vibrators-slot system versus wavelength at $x_{01} = a/8$, $x_{02} = 7a/8$, $\bar{Z}_{S1}(s_1) = ikr_1 \ln(4.0)\phi_1(s_1)$: 1 — $\bar{Z}_{S2} = ikr_2 \ln(4.0)$; 2 — $\bar{Z}_{S2}(s_2) = ikr_2 \ln(4.0)\phi_2(s_2)$; 3 — $\bar{Z}_{S2} = 0$; 4 — slot and one vibrator; 5 — single slot.

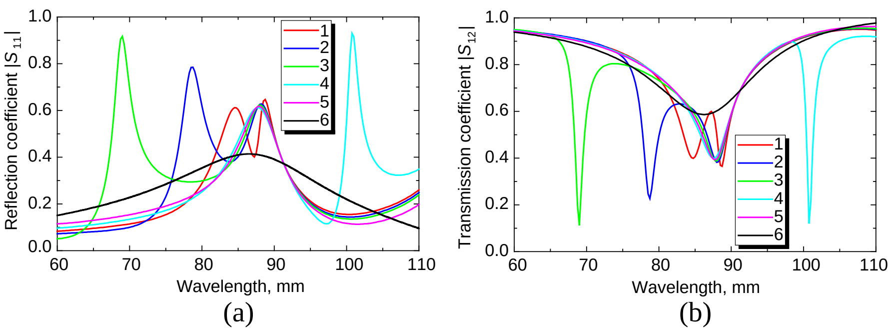

Figure 4. Energy characteristics of vibrators-slot system versus wavelength at $x_{01} = a/8$, $x_{02} = 15a/16$ (asymmetrical positions of vibrators), $\bar{Z}_{S1}(s_1) = ikr_1 \ln(4.0)\phi_1(s_1)$: 1 — $\bar{Z}_{S2} = ikr_2 \ln(4.0)$; 2 — $\bar{Z}_{S2}(s_2) = ikr_2 \ln(4.0)\phi_2(s_2)$; 3 — $\bar{Z}_{S2} = 0$; 4 — $\bar{Z}_{S2}(s_2) = ikr_2 \ln(8.0)\phi_1(s_2)$; 5— slot and one vibrator; 6 — single slot.

vibrators with inductive impedances ($\bar{X}_{S1(2)} > 0$), known to increase the vibrator electrical length, i.e., to increase $\lambda_{1,2}^{res}$ as compared to case $\bar{Z}_{S1(2)} = 0$, without decreasing a distance between the vibrators ends and the upper broad wall of the waveguide. This is very important for increasing the breakdown power for waveguide device as a whole.

As might be expected, the curves $|S_\Sigma|^2(\lambda)$ for different distances between the two vibrators and for single slot are practically coincide (Figure 2). That is, if the interaction between the slot and the vibrators is absent, the radiation coefficient of the slot-vibrator system is determined by the slot dimensions. Obviously, the polarization isolation between the slot and vibrators is realized and for other impedance distribution functions. Therefore, in Figures 3, 4 we present the curves $|S_{11}|(\lambda)$ and $|S_{12}|(\lambda)$ only inside the waveguide. As follows from the plots in Figures 2–4, the passive vibrators with variable surface impedance can significantly change $|S_{11}|(\lambda)$ and $|S_{12}|(\lambda)$ as compared to that for single slot and a slot with one passive vibrator. Therefore, there arises a possibility to optimize the waveguide-slot

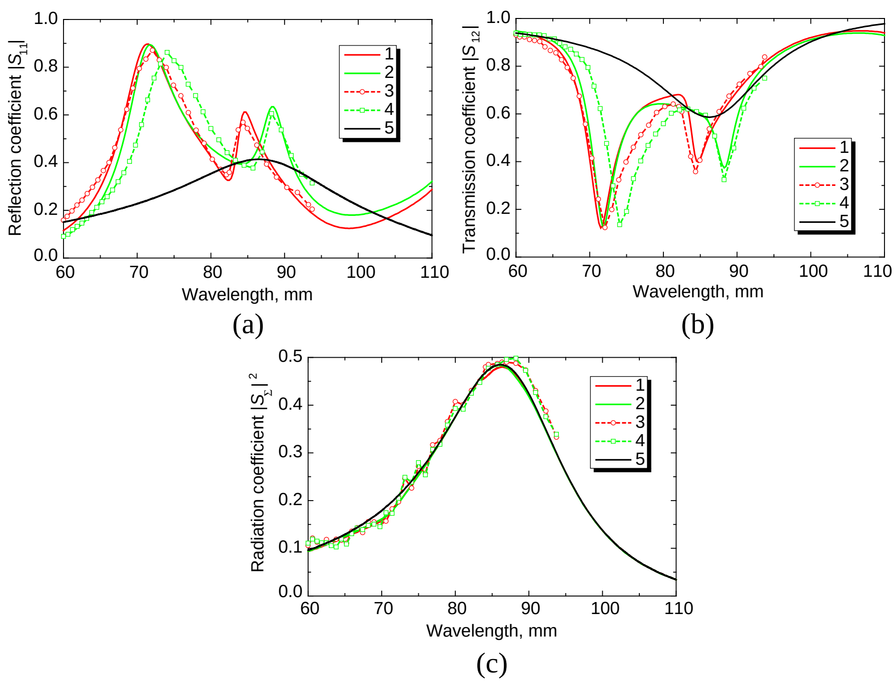

Figure 5. Energy characteristics of vibrators-slot system versus wavelength at $x_{01} = a/8$, $\bar{Z}_{S1} = 0$, $x_{02} = 7a/8$: 1 — $\bar{Z}_{S2} = ikr_2 \ln(4.0)$; 2 — $\bar{Z}_{S2}(s_2) = ikr_2 \ln(4.0)\phi_1(s_2)$; 3, 4 — experimental data (corrugated metallic conductors [12, 13]); 5 — single slot.

radiator matching at a given wavelength and to implement a one-way (input or output) signal filtering required for electromagnetic compatibility.

Comparison of the theoretical and experimental results shown in Figure 5 confirms reliability of the proposed method for analysis of multielement vibrator-slot structures and applicability of the generalized method of induced EMMF using approximation functions for the currents in a single impedance vibrator and a single slot obtained by averaging method.

6. CONCLUSION

The approach to the electrodynamic problem solution for the vibrator slot system which has been proven in [12, 13], was extended to multielement vibrator-slot structure. The problem of electromagnetic fields excitation in the two electrodynamic volumes by finite-dimensional material bodies was formulated in the most general form. For the three-element structure, consisting of the two material bodies and the hole, the general problem was reduced to the system of two-dimensional integral equations for the electric current on the material bodies and the equivalent magnetic current in the hole. In the assumption of the electrically thin structural elements, physically correct transition from two-dimensional integral equations to the system of one-dimensional equations for the currents in the narrow slot and on the impedance vibrators with irregular electrophysical and geometrical parameters was made. The system of one-dimensional equations was solved by a generalized method of induced EMMF. However, this method allows to find approximate analytical expressions for currents in a vibrator-slot structure with a small number of elements. The analytical solution of the equations system was obtained for the structure, consisting of vibrators with variable surface impedance and transverse slot, cut in the broad wall of the rectangular waveguide. Next, we examined the system where the vibrators are located in the waveguide cross section perpendicular to the slot axis. For this configuration, the equations system can be simplified since the interaction between the vibrators and the slot is absent and they are decoupled by polarization. The accuracy of the solution was confirmed by a good agreement between the calculated and experimental results for energy characteristics of the structure. Therefore, this numerical-analytical method may appear to be useful to study the properties of more complex multielement vibrator-slot structures. The possibility to control the reflection and transmission coefficients of vibrator-slot structure by the use of passive vibrators with variable surface impedance has been shown. This structure can be also used as a separate waveguide device.

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