

## COLLOCATED SIBC-FDTD METHOD FOR COATED CONDUCTORS AT OBLIQUE INCIDENCE

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**Abstract**—A collocated surface impedance boundary condition (SIBC) – finite difference time domain (FDTD) method is developed for conductors coated with lossy dielectric coatings at oblique incidence. The method is based on the collocated electric and magnetic field components on the planar interface between two media, and rational approximation for tangent function of surface impedance formulation is adopted. In contrast to the traditional SIBC-FDTD implementation which is approximated with the magnetic field component on the boundary located at half-cell distance from the interface and half time step earlier in time, the collocation approach is more accurate for both magnitude and phase of reflection coefficient. By the comparison with exact results, the proposed model is numerically verified in the frequency domain for both parallel polarization plane wave and vertical polarization plane wave at varying oblique angles of incidence.

### 1. INTRODUCTION

With the development of modern science and technology, the electromagnetic scattering of coated targets becomes more and more important in military and technology. For instance, the absorbing material covering on the strong scattering source of military objects including the vehicle, missile, ship and tanks can be used to reduce the RCS of the objects. The absorbing material is employed to

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build up a microwave anechoic chamber without reflection. The thin dielectric backplate does exist in the design of a microstrip circuit and microstrip antenna. The research on the electromagnetic scattering by a coated target [1–4] is of great importance in both theory and practical applications.

Finite-Difference Time-Domain (FDTD) [5, 6] method is widely applied in various electromagnetic problems for its particular advantage in dealing with inhomogeneous and complex-shaped dielectrics. For the calculation of electromagnetic scattering of coated targets, the computational domain of conventional FDTD technique needs to include the two-region media of the interface. Since the wavelength of electromagnetic waves in the modeled medium is very small compared with the wavelength in free space, the extremely fine mesh inside the medium is required. This straightforward fine mesh dissection method may largely increase the memory requirements and computation time. Surface impedance boundary conditions (SIBC) [7–11] are introduced into the FDTD method, which let us to ignore the coated targets under consideration from computation space and merely discretize the fields of the surrounding space.

FDTD models for conductors coated with lossy dielectric coatings using the SIBC technique can be found in [12]. A more general FDTD model for dispersive coatings on conductor surfaces is developed in [13]. However, conventional FDTD implementation, assuming the magnetic field component located at half-cell size distance away from the interface and half time step earlier in time equal to the magnetic field component on the interface, have been adopted in the above-mentioned literatures. In many cases the approximate method may result in errors and instabilities. A more accurate SIBC method for coated conductors, utilizing collocated electric and magnetic field components on the interface in FDTD method [14] has been presented but without taking into account the incidence angle and polarization of plane wave. Based on collocated SIBC, the FDTD models of lossy dielectric coatings on perfect conductors for parallel polarization and vertical polarization plane wave at oblique incidence are developed in this article.

This paper proceeds as follows. In Section 2, taking TM plane wave at oblique incidence as example, we begin with the analytical impedance boundary formula simulating coated materials in the frequency domain. Utilizing rational approximation of tangent function and inverse Laplace transform, the impedance boundary conditions in the time domain are derived in Section 3. Next, based on the collocated electric and magnetic field components on the interface and Recursive Convolution (RC) method [15], the associated discrete

SIBC-FDTD expression is provided in Section 4. In Section 5, the formula is verified by the comparison with the exact results and it is shown that the collocation approach is more accurate compared with  $H$ -node shifting implementation for both magnitude and phase of reflection coefficient. Section 6 is devoted to the conclusion.

## 2. SIBC FOR A COATING ON CONDUCTOR SURFACE AT OBLIQUE INCIDENCE

The total tangential electric field and total tangential magnetic field on the interface of two media is defined by [10]

$$\mathbf{E}_{\text{tan}} = Z_s(\omega) (\hat{\mathbf{n}} \times \mathbf{H}_{\text{tan}}) \quad (1)$$

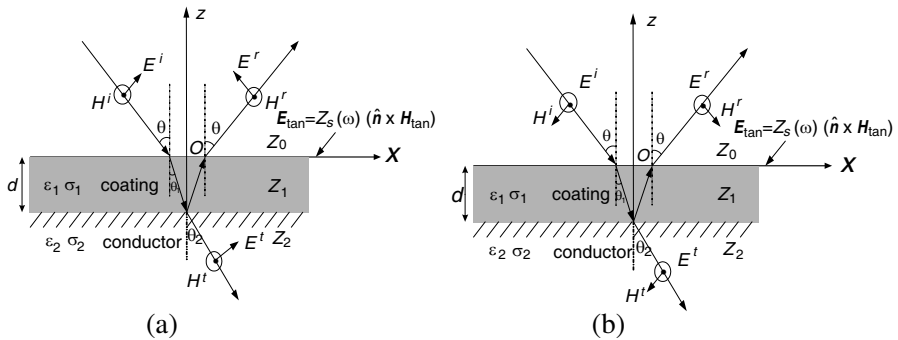
where  $Z_s(\omega)$  is a surface impedance,  $\hat{\mathbf{n}}$  is the unit surface normal pointing outwards from the interface.

Here a model that a conductor is covered with a lossy dielectric coating of thickness  $d$  will be considered. The complex permittivity of the coating is of the form

$$\varepsilon_1 = \varepsilon'_{1,r} \varepsilon_0 - j \frac{\sigma_1}{\omega} \quad (2)$$

where  $\varepsilon'_{1,r}$ ,  $\sigma_1$  and  $\varepsilon_0$  are relative dielectric constant, electric conductivity and free space dielectric constant, respectively. The relative permeability  $\mu_r$  of the coating is supposed to be a constant.

When the parallel polarization(TM) and the vertical polarization (TE) plane wave incident at angle  $\theta$  on the coated conductor, the surface impedance models are shown in Fig. 1. Take TM plane wave



**Figure 1.** The equivalent surface impedance model for plane wave from a coated conductor at oblique incidence. (a) Parallel polarization (TM). (b) Vertical polarization (TE).

as example. Based on the simple transmission line model, the input impedance at a distance of  $d$  from the conductor plate is the analytical surface impedance, and we get

$$Z_s(\omega) = Z_1 \cos \theta_1 \frac{Z_2 \cos \theta_2 + j Z_1 \cos \theta_1 \tan(k_1 d \cos \theta_1)}{Z_1 \cos \theta_1 + j Z_2 \cos \theta_2 \tan(k_1 d \cos \theta_1)} \quad (3)$$

where  $Z_1 = \sqrt{\mu_1/\varepsilon_1}$  is the characteristic wave impedance of the coating;  $Z_2 = \sqrt{\mu_2/\varepsilon_2}$  is the characteristic wave impedance of the material under the coating;  $k_1 = \omega\sqrt{\varepsilon_1\mu_1}$  is the wave number inside the coating;  $\theta_1$  and  $\theta_2$  are angle of transmission in the coating and conductor, respectively;  $d$  is thickness of the coating.

According to Snell's law, we have

$$\cos \theta_1 = \left[ (1 - \alpha \sin^2 \theta) \frac{j\omega + \beta'}{j\omega + \beta} \right]^{1/2} \quad (4)$$

where  $\alpha = \frac{\mu_0}{\mu_1 \varepsilon_{1,r}'} , \beta = \frac{\sigma_1}{\varepsilon_0 \varepsilon_{1,r}'}, \beta' = \frac{\beta}{1 - \alpha \sin^2 \theta}$ .

When the background material is the ideal conductor, the characteristic wave impedance satisfies  $Z_2 \approx 0$ , then the surface impedance formula (3) becomes

$$Z_s(\omega) = j Z_1 \cos \theta_1 \tan(k_1 d \cos \theta_1) \quad (5)$$

### 3. DERIVATION OF THE SIBC IN THE TIME DOMAIN

In Fig. 1, we assume the unit surface normal vector of the interface parallel to the  $z$ -axis:  $\hat{n} = \hat{z}$ , the impedance boundary condition in (1) can be represented respectively as

$$E_x = -Z_s(\omega) H_y \quad (6)$$

$$E_y = Z_s(\omega) H_x \quad (7)$$

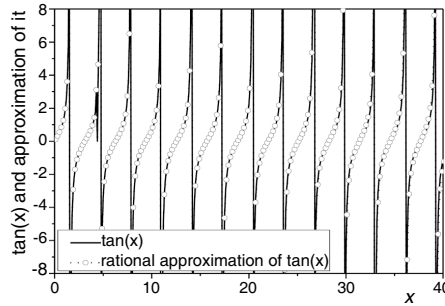
In this paper, we will derive Equation (6) as example.

For the convenience of calculation, the tangent function in surface impedance formula (5) is represented as continuous rational approximations according to [13]

$$\tan(x) \approx f(x) = \sum_{s=1}^M \frac{a_s x}{1 - q_s x^2} \quad (8)$$

where  $q_s = 4/(2s-1)^2/\pi^2$  are to correctly model the singularities of the tangent function corresponding to the thickness resonances of the coating and the value of  $a_s$  have to guarantee the rational approximation zero point be equal to the tangent function zero point.

By setting the value of  $M$ , we get the same number of singularities:  $q_s$ ,  $s = 1, \dots, M$ . And zeros  $z_s = (s - 1)\pi$ ,  $s = 1, \dots, M$ . The tangent function and the proposed rational approximation ( $M = 20$ ,  $x_0 = \pi/4$ ) are shown in Fig. 2.



**Figure 2.** Tangent function and an approximation of it.

Substituting Equations (5) and (8) into (6), we get the sum form of Equation (6):

$$E_x = \sum_{s=1}^M \frac{-jZ_1 \cos^2 \theta_1 k_1 da_s}{1 - q_s \cos^2 \theta_1 k_1^2 d^2} H_y = \sum_{s=1}^M E_s \quad (9)$$

with

$$E_s = \frac{-jZ_1 \cos^2 \theta_1 k_1 da_s}{1 - q_s \cos^2 \theta_1 k_1^2 d^2} H_y \quad (10)$$

Equation (10) is the basis for the rest of this paper.

We substitute  $Z_1$ ,  $k_1$  and  $\cos \theta_1$  into (10). By simplifying, the following equation is obtained

$$\begin{aligned} & \left[ 1 - \omega^2 q_s \mu_1 \varepsilon_1' d^2 (1 - \alpha \sin^2 \theta) \frac{j\omega + \beta'}{j\omega + \beta} + j\omega q_s \mu_1 \sigma_1 d^2 (1 - \alpha \sin^2 \theta) \frac{j\omega + \beta'}{j\omega + \beta} \right] E_s \\ &= -j\omega \mu_1 da_s (1 - \alpha \sin^2 \theta) \frac{j\omega + \beta'}{j\omega + \beta} H_y \end{aligned} \quad (11)$$

With the Laplace transform variable  $s = j\omega$ , we have

$$\begin{aligned} & \left[ 1 + s^2 q_s \mu_1 \varepsilon_1' d^2 (1 - \alpha \sin^2 \theta) \frac{s + \beta'}{s + \beta} + s q_s \mu_1 \sigma_1 d^2 (1 - \alpha \sin^2 \theta) \frac{s + \beta'}{s + \beta} \right] E_s \\ &= -s \mu_1 da_s (1 - \alpha \sin^2 \theta) \frac{s + \beta'}{s + \beta} H_y \end{aligned} \quad (12)$$

Setting  $F(s) = \frac{s+\beta'}{s+\beta} = 1 + \frac{\beta'-\beta}{s+\beta}$ , we can get the following formula by applying the inverse Lapalace transform:

$$L^{-1}\{F(s)\} = \delta(t) + (\beta' - \beta) e^{-\beta t} \quad (13)$$

Substituting (13) into (12) and using the corresponding relationship of time domain with frequency domain  $j\omega \leftrightarrow \frac{\partial}{\partial t}$ , (12) is transformed into the time domain

$$\begin{aligned} & E_s + A_1 \frac{\partial^2 E_s}{\partial t^2} + A_1 \int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial^2 E_s}{\partial (t-\tau)^2} d\tau \\ & + A_2 \frac{\partial E_s}{\partial t} + A_2 \int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial E_s}{\partial (t-\tau)} d\tau \\ & = -A_3 \frac{\partial H_y}{\partial t} - A_3 \int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial H_y}{\partial (t-\tau)} d\tau \end{aligned} \quad (14)$$

where  $A_1 = \varepsilon'_1 A_0$ ,  $A_2 = \sigma_1 A_0$ ,  $A_3 = a_s A_0 / q_1 d$ ,  $A_0 = q_s \mu_1 d^2 (1 - \alpha \sin^2 \theta)$ .

#### 4. IMPLEMENT OF SIBC IN FDTD

According to Equation (14), the tangential electric field components on the interface between free space and the coated conductor is related to the tangential magnetic field components at the same position. However, the electric and magnetic field components are not collocated in space and have half time step difference in time in FDTD method. Traditional SIBC implementation is approximated with the magnetic field component located at half-cell size distance away from the interface and half time step earlier in time:  $H_y|_0^n \approx H_y|_{1/2}^{n-1/2}$ . In many cases the approximate method may results in errors and instabilities.

In this paper, Equation (14) is dealt with collocated magnetic and electric components on the boundary according to the Reference [14]. At time step  $t = n\Delta t$ , the finite difference expression of the SIBC fields and their derivatives are as follows:

$$\begin{aligned} X(n\Delta t) &= \alpha X|_0^n + \frac{\beta}{2} (X|_0^{n+1} + X|_0^{n-1}) \\ \frac{\partial X(n\Delta t)}{\partial t} &= \frac{X|_0^{n+1} - X|_0^{n-1}}{2\Delta t}, \quad \frac{\partial^2 X(n\Delta t)}{\partial t^2} = \frac{X|_0^{n+1} - 2X|_0^n + X|_0^{n-1}}{\Delta t^2} \end{aligned} \quad (15)$$

where  $X \in \{E_s, H_y\}$ ,  $\alpha + \beta = 1$  ( $0 \leq \alpha, \beta \leq 1$ ),  $\alpha = 0.5$  can guarantee the stabilities of the collocation method.

In FDTD method, electric field and magnetic field component in each time step can be assumed to be constant value. Then the

convolution integrals in (13) are expressed as sums of the form

$$\int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial^2 E_s}{\partial(t-\tau)^2} d\tau = \sum_{m=0}^n \frac{E_s|_0^{n-m+1} - 2E_s|_0^{n-m} + E_s|_0^{n-m-1}}{\Delta t^2} \chi^m \quad (16)$$

where

$$\chi^m = \int_{m\Delta t}^{(m+1)\Delta t} (\beta' - \beta) e^{-\beta\tau} d\tau \quad (17)$$

Since  $\chi^m = e^{-\beta\Delta t} \chi^{m-1}$ , (16) can be represented as

$$\int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial^2 E_s}{\partial(t-\tau)^2} d\tau = \frac{E_s|_0^{n+1} - 2E_s|_0^n + E_s|_0^{n-1}}{\Delta t^2} \chi^0 + \varphi_1^n \quad (18)$$

where

$$\varphi_1^n = \frac{E_s|_0^n - 2E_s|_0^{n-1} + E_s|_0^{n-2}}{\Delta t^2} \chi^1 + e^{-\beta\Delta t} \varphi_1^{n-1} \quad (19)$$

In a similar way, the other two convolution integrals in Equation (14) can be expressed as

$$\int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial E_s}{\partial(t-\tau)} d\tau = \frac{E_s|_0^{n+1} - E_s|_0^{n-1}}{2\Delta t} \chi^0 + \varphi_2^n \quad (20)$$

where

$$\varphi_2^n = \frac{E_s|_0^n - E_s|_0^{n-2}}{2\Delta t} \chi^1 + e^{-\beta\Delta t} \varphi_2^{n-1} \quad (21)$$

and

$$\int_0^t (\beta' - \beta) e^{-\beta\tau} \frac{\partial H_y}{\partial(t-\tau)} d\tau = \frac{H_y|_0^{n+1} - H_y|_0^{n-1}}{2\Delta t} \chi^0 + \varphi_3^n \quad (22)$$

where

$$\varphi_3^n = \frac{H_y|_0^n - H_y|_0^{n-2}}{2\Delta t} \chi^1 + e^{-\beta\Delta t} \varphi_3^{n-1} \quad (23)$$

In this way, the SIBCs differential Equation (14) is discretized as

$$\begin{aligned} & E_s|_0^{n+1} CA + H_y|_0^{n+1} CB \\ & = E_s|_0^n CA_1 + E_s|_0^{n-1} CA_2 + H_y|_0^{n-1} CB - A_1 \varphi_1^n - A_2 \varphi_2^n - A_3 \varphi_3^n \end{aligned} \quad (24)$$

where

$$\begin{aligned} CA &= \frac{\beta}{2} + \frac{A_1}{\Delta t^2} (1 + \chi^0) + \frac{A_2}{2\Delta t} (1 + \chi^0), \quad CB = \frac{2A_1}{\Delta t^2} (1 + \chi^0) \\ CA_1 &= -\alpha + \frac{2A_1}{\Delta t^2} (1 + \chi^0), \quad CA_2 = -\frac{\beta}{2} - \frac{A_1}{\Delta t^2} (1 + \chi^0) + \frac{A_2}{2\Delta t} (1 + \chi^0) \end{aligned} \quad (25)$$

Setting

$$H_s = E_s|_0^n CA_1 + E_s|_0^{n-1} CA_2 - Hy|_0^{n-1} CB - A_1\varphi_1^n - A_2\varphi_2^n - A_3\varphi_3^n \quad (26)$$

In light of Equations (9), (24) and (26), it is easy to obtain

$$E_x|_0^{n+1} + Hy|_0^{n+1} \cdot KK = \sum_{s=1}^{20} H_s \quad (27)$$

where  $KK = \sum_{s=1}^{20} \frac{CB}{CA}$ .

Utilizing the one-dimensional FDTD formula of Faraday's law [6, 14]

$$E_x|_0^{n+1} = E_x|_0^n - \frac{2}{\Delta z} \cdot \frac{\Delta t}{\varepsilon_0} \left( Hy|_{1/2}^{n+1/2} - \frac{1}{2} Hy|_0^{n+1} - \frac{1}{2} Hy|_0^n \right) \quad (28)$$

and combining Equations (27) and (28), we can obtain the following expressions

$$\begin{aligned} & \left( 1 + \frac{\Delta t}{\Delta x \cdot \varepsilon_0 \cdot KK} \right) E_x|_0^{n+1} \\ &= E_x|_0^n - \frac{2}{\Delta z} \cdot \frac{\Delta t}{\varepsilon_0} \left( Hy|_{1/2}^{n+1/2} - \frac{1}{2KK} \sum_{s=1}^{20} H_s - \frac{1}{2} Hy|_0^n \right) \quad (29) \\ & Hy|_0^{n+1} = \frac{1}{KK} \left( \sum_{s=1}^{20} H_s - E_x|_0^{n+1} \right) \quad (30) \end{aligned}$$

(29) and (30) are one-dimensional update formulas of tangential electric and magnetic field components on the interface, respectively.

## 5. NUMERICAL VERIFICATION

In this section, the numerical examples of the proposed collocated SIBC-FDTD method will be presented. The incident field is a differentiated Gaussian pulse of the form

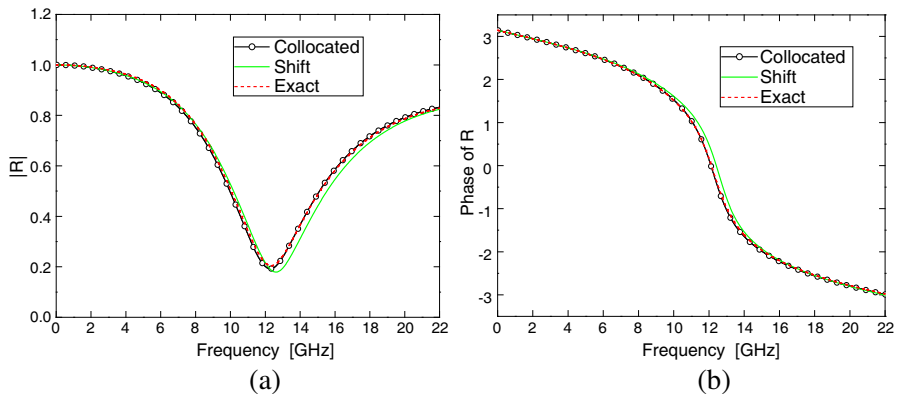
$$E(t) = (t - \tau_1) e^{-\left(\frac{t - \tau_1}{\tau_2}\right)^2} \quad (31)$$

### 5.1. Reflection of TM Plane Wave from a Coated Conductor at Oblique Incidence

Firstly, let us analyze the case that a TM plane wave is incident at the angle  $\theta = 30^\circ$  on a coated ideal conductor. The thickness of the

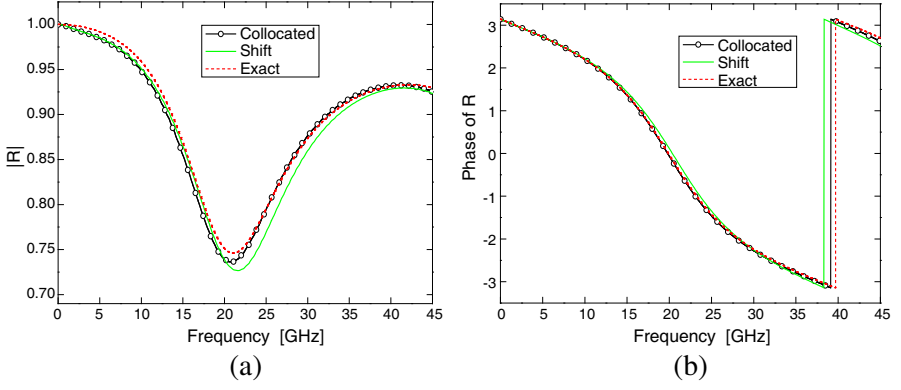


coating is  $d = 2$  mm, the relative dielectric constant of the coating is  $\varepsilon'_{1,r} = 10$ , and the conductivity of the coating is  $\sigma_1 = 2$  S/m. The relative permeability of coating is equal to 1. The FDTD spatial step is  $\Delta x = 2$  mm, and the time step is  $\Delta t = \Delta x/2c_0$ . The parameters of the pulse are  $\tau_1 = 40\Delta t$ ,  $\tau_2 = 12\Delta t$ . To reduce the reflection from absorbing boundary, a modified Mur absorbing boundary is used in the oblique incident situations [16]. Fig. 3(a) shows the magnitude of reflection coefficients simulated with the collocated SIBC-FDTD method and the traditional  $H$ -node shifts SIBC-FDTD method. The exact reflection coefficients in the same conditions are also presented. We can see that both of the two simulated results agree quite well with the exact values for lower frequencies from 0 GHz to 12 GHz. However, in the frequency range from 7 GHz to 20 GHz, it is clearly seen that the collocation approach solution is considerably more accurate as compared to the method with time and space shift for magnetic component. In Fig. 3(b), the phase of the reflection coefficient simulated with the proposed method is also better than results simulated by the traditional approximation method.

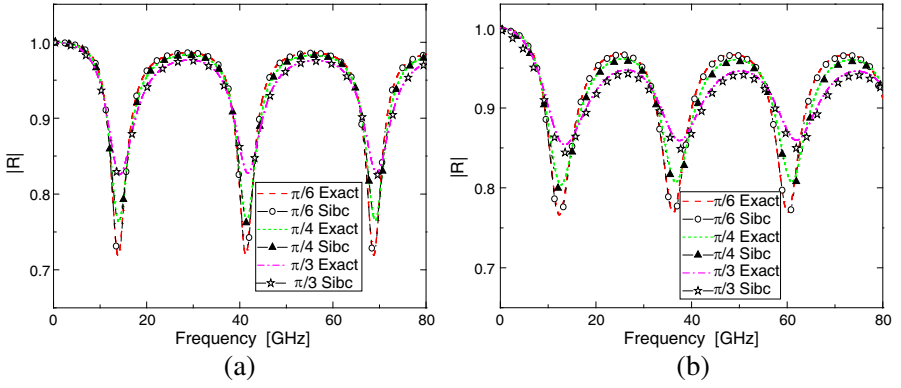


**Figure 3.** The reflection coefficient of TM plane wave from a coated conductor at the angle of incidence  $\theta = 30^\circ$ . Parameters of the coating:  $d = 2$  mm,  $\varepsilon'_{1,r} = 10$ ,  $\sigma_1 = 2$  S/m. (a) Magnitude. (b) Phase.

The parameters of the case in Fig. 4 are as follows: the incident angle of TM plane wave is  $\theta = 60^\circ$ , the thickness of the coating is  $d = 1$  mm, the relative dielectric constant of the coating is  $\varepsilon'_{1,r} = 15$ , and the conductivity of the coating is  $\sigma_1 = 1.5$  S/m. The FDTD spatial step is  $\Delta x = 1$  mm, and the time step is  $\Delta t = \Delta x/2c_0$ . Compared to the results simulated by traditional shift SIBC-FDTD method, we can also see that the magnitude and the phase of the reflection



**Figure 4.** The reflection coefficient of TM plane wave from a coated conductor at the angle of incidence  $\theta = 60^\circ$ . Parameters of the coating:  $d = 1$  mm,  $\varepsilon'_{1,r} = 15$ ,  $\sigma_1 = 1.5$  S/m. (a) Magnitude. (b) Phase.



**Figure 5.** Magnitude of the reflection coefficient of TM plane wave at varying oblique angles. Parameters of the coating: (a)  $d = 1$  mm,  $\varepsilon'_{1,r} = 30$ ,  $\sigma_1 = 1$  S/m. (b)  $d = 2$  mm,  $\varepsilon'_{1,r} = 10$ ,  $\sigma_1 = 0.4$  S/m.

coefficient calculated by collocated SIBC-FDTD method agrees better with the exact results. The minor errors appearing in Figs. 3 and 4 are acceptable, since the rational approximation of the tangent function has a limited range of good accuracy.

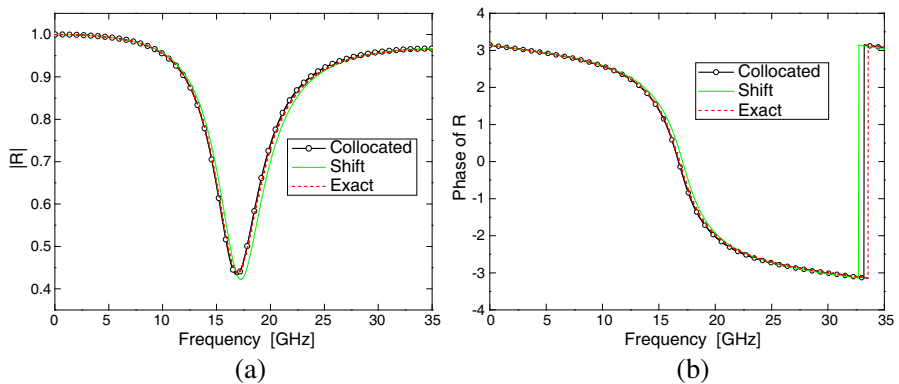
Figure 5 below shows the reflection coefficients calculated by collocated SIBC-FDTD method for the TM plane wave with varying oblique incident angles. The incident angle of the pulse is  $\frac{\pi}{6}$ ,  $\frac{\pi}{4}$  and  $\frac{\pi}{3}$ , respectively. The following parameters were used for the coatings in Fig. 5(a):  $d = 1$  mm,  $\varepsilon'_{1,r} = 30$ ,  $\sigma_1 = 1$  S/m. The coating in

Fig. 5(b) has parameters:  $d = 2 \text{ mm}$ ,  $\varepsilon'_{1,r} = 10$ ,  $\sigma_1 = 0.4 \text{ S/m}$ . The relative permeability of coating is equal to 1. The FDTD spatial step is  $\Delta x = 0.2 \text{ mm}$ , and the time step is  $\Delta t = \Delta x/2c_0$ . As shown in Fig. 5, the simulated reflection coefficients agree quite well with the analytical results in the entire frequency range from 0 GHz to 80 GHz.

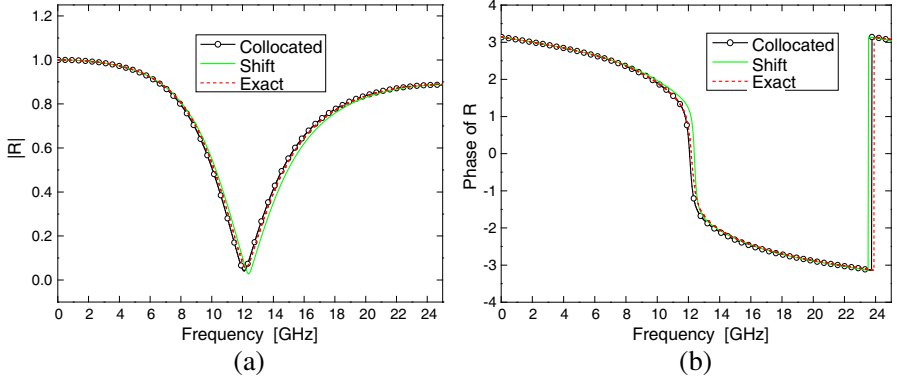
## 5.2. Reflection of TE Plane Wave from a Coated Conductor at Oblique Incidence

Next, let us consider the problem of TE plane wave incident obliquely on the coated conductors. In Fig. 6, the incident angle of the pulse is  $\theta = 15^\circ$ . The parameters of the coating is:  $d = 1 \text{ mm}$ ,  $\varepsilon'_{1,r} = 20$ ,  $\sigma_1 = 2 \text{ S/m}$ . In Fig. 7, the incident angle of the pulse is  $\theta = 35^\circ$ . The parameters of the coating is:  $d = 2 \text{ mm}$ ,  $\varepsilon'_{1,r} = 10$ ,  $\sigma_1 = 2 \text{ S/m}$ . Compared to the results simulated by traditional shift SIBC-FDTD method, it is also clearly seen that the approach with collocated fields is considerably more accurate for the reflection coefficient, in both magnitude and phase.

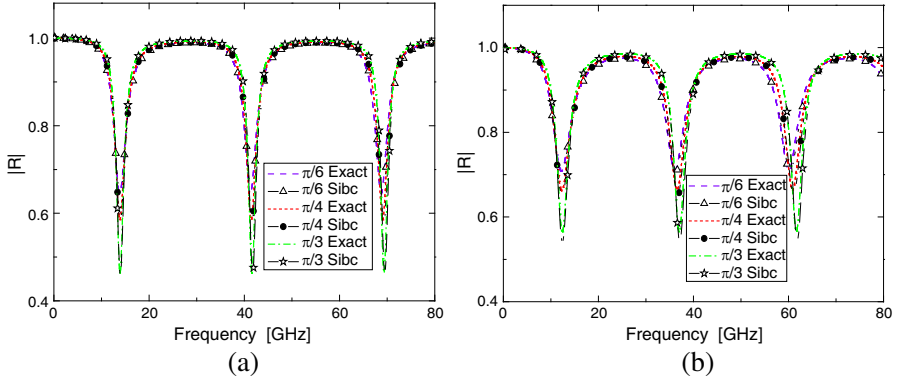
The magnitude of reflection coefficient calculated with collocated SIBC-FDTD method for TE plane wave at oblique incident are shown in Fig. 8. The incident angle of the pulse is  $\frac{\pi}{6}$ ,  $\frac{\pi}{4}$  and  $\frac{\pi}{3}$ , respectively. The parameters of the coatings are the same as Fig. 5. The simulated reflection coefficients and the analytical results are also in good agreement in the frequency range from 0 GHz to 80 GHz. It indicates the validation of the proposed method for TE wave at oblique incidence situation.



**Figure 6.** The reflection coefficient of TE plane wave from a coated conductor at the angle of incidence  $\theta = 15^\circ$ . Parameters of the coating:  $d = 1 \text{ mm}$ ,  $\varepsilon'_{1,r} = 20$ ,  $\sigma_1 = 2 \text{ S/m}$ . (a) Magnitude. (b) Phase.



**Figure 7.** The reflection coefficient of TE plane wave from a coated conductor at the angle of incidence  $\theta = 35^\circ$ . Parameters of the coating:  $d = 2$  mm,  $\varepsilon'_{1,r} = 10$ ,  $\sigma_1 = 2$  S/m. (a) Magnitude. (b) Phase.



**Figure 8.** Magnitude of the reflection coefficient of TE plane wave at varying oblique angles. Parameters of the coating: (a)  $d = 1$  mm,  $\varepsilon'_{1,r} = 30$ ,  $\sigma_1 = 1$  S/m. (b)  $d = 2$  mm,  $\varepsilon'_{1,r} = 10$ ,  $\sigma_1 = 0.4$  S/m.

## 6. CONCLUSION

In this paper, a collocated surface impedance boundary condition (SIBC) — finite difference time domain (FDTD) method for conductors coated with lossy dielectric coatings at oblique incidence is implemented. The proposed method is numerically verified by comparison with the analytical results for both parallel polarization plane wave and vertical polarization plane wave at varying oblique angles of incidence in 1D. The numerical results shows that the collocation approach is more accurate for both magnitude and

phase of reflection coefficient compared with the traditional SIBC-FDTD implementation which is approximated with the magnetic field component on the boundary located at half-cell distance from the interface and half time step earlier in time.

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