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DEGREE OF DEPOLARIZATION OF QUANTIZATION HERMITE-GAUSSIAN BEAM IN A TURBULENT ATMOSPHERE

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Abstract—Based on quantum Stokes operators and non-Kolmogorov spectrum model of index-of-refraction fluctuation, the analytical formulas for the quantum degree of depolarization of quantization Hermite-Gaussian (QHG) beams propagating in a turbulent atmosphere slant channel are derived. The nonclassical polarization properties of QHG beams propagating in turbulent atmosphere are studied numerically. It is found that the polarization fluctuations of QHG beams are dependent of the turbulence factors such as spectrum power-law exponent, refractive index structure parameter at the ground and zenith angle. The degree of depolarization of QHG beams has a saltation and reaches the minimum value at spectrum power-law exponent $\alpha = 11/3$, the refractive index structure parameter at the ground of the turbulent atmosphere slightly affects the polarization degree of QHG beams which have travelled a long distance, and the change of polarization degree decreases with the increasing zenith angle. Furthermore, the numerical simulations show that QHG beams with higher photon-number level, lower beam order, shorter wavelength are less affected by the turbulence. These results indicate that One can choose low-order QHG beams with wavelength $\lambda = 690\,\text{nm}$ as optical carrier, increase photon number, set the size of transmitting aperture w_0 as about 0.1 m, and detect communication signals at the central region of beams to improve the performance of a polarization-encoded free-space quantum communication system.

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1. INTRODUCTION

Polarization properties of quantum light propagation through the turbulent atmosphere are required in the context of implementations of quantum cryptography for communication channels between earth-based stations and between satellites and earth-based stations. The effects of turbulent atmosphere on the polarization of linear polarized single photons and the transfer of polarization entanglement have attracted a great attention since recent experiments [1–5], which have demonstrated the feasibility of quantum key distribution and secure key exchange in free-space channels. In this connection the question arises whether nonclassical properties of polarized light can be preserved during its propagation in fluctuating media. Recently, the experimental study for the polarization fluctuation [1–4, 6, 7, 9, 10], the transfer of polarization entanglement [5, 8, 11–15] of quantum light through the turbulent atmosphere have been reported. But, the theoretical model for the polarization fluctuation of quantum light propagation through the turbulent atmosphere has little study.

Hermite-Gaussian beams are a family of structurally stable laser modes which have rectangular symmetry along the propagation axis, and they have important applications in nonlinear optics, electron acceleration, atom trapping and free-space optical communications [16–19]. Hermite-Gaussian beams can be generated using a graded phase-mirror or infiber-coupled laser-diode end-pumped lasers [20, 21]. Propagation properties of a Hermite-Gaussian beam through free space, a paraxial optical system and turbulent atmosphere have been studied in [22–25]. Moreover, Propagation properties of a similar family of beams, for example, cosh-Gaussian beams have been investigated in detail [26–30]. Recently, because of the requirement of atmospheric communication, detection and remote sensing, the characteristics of the laser beam propagation on slant path become very important [31, 32]. To the best of our knowledge, there has been no report on the quantum polarization fluctuations of QHG beams propagating in a turbulent atmosphere slant channel. In this paper, our aim is to develop a theoretical model for the quantum polarization fluctuations of the QHG beams propagating in a turbulent atmosphere slant channel by applying quantum Stokes operators and non-Kolmogorov spectrum model of index-of-refraction fluctuation, and to explore the advantage of such beam for application in free-space optical communications.

The paper is organized as follows. In Section 2 we summarize the needed basic principles of the quantum Stokes operators and the degree of depolarization. In Section 3 the quantum Stokes operators in a turbulent atmosphere are studied and the formulae for the Stokes parameter of the turbulent atmosphere ensemble average and the degree of depolarization of QHG beams in a turbulent atmosphere are obtained. Some numerical results are illustrated in Section 4. Finally, some important conclusions are given in last section.

2. THE QUANTUM STOKES OPERATORS AND THE DEGREE OF DEPOLARIZATION

The depolarization properties of photons in free-space are conveniently addressed by means of the Hermitian Stokes operators [33]

$$\begin{aligned} \hat{S}_0 &= \hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2, \\ \hat{S}_1 &= \hat{a}_1^+\hat{a}_1 - \hat{a}_2^+\hat{a}_2, \\ \hat{S}_2 &= \hat{a}_1^+\hat{a}_2 + \hat{a}_2^+\hat{a}_1, \\ \hat{S}_3 &= \mathrm{i}\left(\hat{a}_2^+\hat{a}_1 - \hat{a}_1^+\hat{a}_2\right), \end{aligned} \tag{1}$$

where $\hat{a}_{1,\ 2}$ ($\hat{a}_{1,\ 2}^+$) are the photon annihilation (creation) operators in the model ($\mathbf{q}$, j), respectively. $\mathbf{q}$ is the momentum of a photon, and $j\,(j = 1, 2)$ is its polarization. They obey the bosonic commutation relations:

$$[\hat{a}_j,\ \hat{a}_k^+] = \delta_{jk}, \quad j, k \in 1, 2. \tag{2}$$

So the Stokes operators satisfy the SU(2)-like commutation relations:

$$\left[\hat{S}_0,\ \hat{S}_j\right] = 0,\ \left[\hat{S}_j,\ \hat{S}_k\right] = 2\mathrm{i}\sum_l \varepsilon_{jkl}\hat{S}_l, \quad (j, k, l = 1, 2, 3), \tag{3}$$

and their mean values correspond to the Stokes parameters ($\langle\hat{S}_0\rangle$, $\langle\hat{\mathbf{S}}\rangle$), where $\hat{\mathbf{S}} = \sum_{j=1}^3 \hat{S}_j\mathbf{e}_j$, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are mutually orthogonal unit vectors along the corresponding axes of coordinates in the polarization space.

The changes of the polarization of initially polarized light when it propagates through a turbulent atmosphere can be easily accounted for with the use of the degree of depolarization [34]

$$DP = \left\{1 - \sum_{j=1}^{3}\left\langle\hat{S}_j\right\rangle^2 \Big/ \sum_{j=1}^{3}\left\langle\hat{S}_j^2\right\rangle\right\}^{1/2}, \tag{4}$$

where $\sum_{j=1}^{3}\hat{S}_j^2 = \hat{S}_0(\hat{S}_0 + 2)$.

For linearly polarized wave, that is $\hat{a}_2 = 0$, one can obtain

$$\hat{S}_0 = \hat{S}_1 = \hat{a}_1^+\hat{a}_1, \quad \hat{S}_2 = \hat{S}_3 = 0, \quad \left|\hat{\mathbf{S}}\right| = \hat{S}_0. \tag{5}$$

Then the degree of depolarization can be expressed as

$$DP = \left\{ \frac{2}{\left\langle \hat{S}_0 \right\rangle + 2} \right\}^{1/2}. \tag{6}$$

3. THE QUANTUM STOKES OPERATORS IN A TURBULENT ATMOSPHERE

Considering symmetry of the Hermite-Gaussian beam, we only research the quantum polarization fluctuation of one-dimensional Hermite-Gaussian beam. In this case, the propagated quantization Hermite-Gaussian beam in the paraxial turbulent atmosphere channel is given by [34]

$$\begin{aligned} \hat{E}_j^{+}(x,z) &= -\frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}kz}}{2\pi z}\sqrt{\tau}\int \mathrm{d}x' \exp\left[\frac{\mathrm{i}k}{2z}\left(x-x'\right)^2\right] \\ &\quad \times \exp\left[\psi\left(x,x',z\right)\right]\hat{E}_j^{+}\left(x',0\right), \end{aligned} \tag{7}$$

where $\tau = \tau_d\tau_e\tau_a\tau_{ro}\tau_p$ is transmittance of channel, τ_e is the efficiency transmitting optics, τ_d is the detector quantum efficiency, τ_{ro} is efficiency of receiving transmission, τ_a is the one-way atmospheric transmission and τ_p is the attenuation efficiency of point errors, $k = 2\pi/\lambda$ is the wave number of light, x and x' denote transverse coordinates of the photon at the z and source plane respectively, the function $\psi(x,x',z) = \chi(x,x',z) + \mathrm{i}s(x,x',z)$ describes the effects of the atmospheric turbulence on the propagation of a spherical wave, here terms $\chi(x,x',z)$ and $s(x,\, x',\, z)$ account for the stochastic log-amplitude and phase fluctuations, respectively, imposed by atmospheric turbulence. For the approximation of the passive atmospheric medium [35], which implies that the self-radiation of the medium is not taken into account, the field at $z = 0$ is $\hat{E}_j\,(x',\,0)$ can be given by [36]

$$\hat{E}_j^{+}\left(x',0\right) = \frac{1}{2\pi}\int \mathrm{d}q\hat{a}_j\left(q,x'\right)\mathrm{e}^{\mathrm{i}qx'}, \tag{8}$$

here $\hat{a}_j(q,x') = \hat{a}_{0j}(q)u(x',0)$ is the effective photon annihilation operator, $u(x',\,0)$ is the transverse beam amplitude function for the beam modes at the plane $z = 0$, $\hat{n}_{0j}(q) = \hat{a}_{0j}^{+}(q)\hat{a}_{0j}(q)$ is the initial number operator, $\hat{n}_{0j}(q)|\zeta_j\rangle = n_{0j}$.

Substituting (8) into (7), we have

$$\begin{aligned}\hat{E}_j^+(x,z) &= -\frac{\mathrm{i}ke^{ikz}}{4\pi^2 z}\sqrt{\tau}\int \mathrm{d}q \int \mathrm{d}x' e^{iqx'}\hat{a}_{0j}(q)\,u(x')\\ &\quad\times\exp\left[\frac{\mathrm{i}k}{2z}(x-x')^2+\psi(x,x',z)\right]\\ &= \frac{1}{2\pi}\int \mathrm{d}q\hat{a}_j(q,x,z)\,e^{\mathrm{i}qx}, \qquad (9)\end{aligned}$$

where

$$\begin{aligned}\hat{a}_j(q,x,z) &= -\frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}kz}}{2\pi z}\sqrt{\tau}\int \mathrm{d}x'\hat{a}_{0j}(q)\,u(x')\,\mathrm{e}^{\mathrm{i}q(x'-x)}\\ &\quad\times\exp\left[\frac{\mathrm{i}k}{2z}(x-x')^2+\psi(x,x',z)\right]. \qquad (10)\end{aligned}$$

By Eq. (10), we can obtain the average Stokes parameter $\langle\hat{S}_0\rangle$ of turbulent ensemble in z plane of a beam propagation through atmosphere

$$\begin{aligned}\langle\hat{S}_0\rangle &= \left\langle\hat{a}_j^+(q,x,z)\,\hat{a}_j(q,x,z)\right\rangle_m\\ &= n_{11}\left(\frac{k}{2\pi z}\right)^2\int\int \mathrm{d}x'\mathrm{d}x''u^*(x')\,u(x'')\,\mathrm{e}^{-\mathrm{i}q(x'-x'')}\\ &\quad\times\langle\exp\left[\psi^*(x,x',z)+\psi(x,x'',z)\right]\rangle_m\\ &\quad\times\exp\left[-\frac{\mathrm{i}k}{2z}\left[x'^2-x''^2-2x(x'-x'')\right]\right], \qquad (11)\end{aligned}$$

where $n_{11} = n_0\tau$ is the number operator of transmissive photon, and $\langle\ldots\rangle_m$ denotes average over the ensemble of the turbulent atmosphere.

Taking into account the quadratic approximation for Rytov's phase structure function [37, 38], we have

$$\begin{aligned}\langle\exp\left[\psi^*(x,x',z)+\psi(x,x'',z)\right]\rangle_m &= \exp\left[-\frac{1}{2}D_\psi(x'-x'',z)\right]\\ &= \exp\left[-\frac{(x'-x'')^2}{\rho_0^2}\right], \qquad (12)\end{aligned}$$

here $D_\psi(x'-x'',\,z)$ is the phase structure function in Rytov's representation, ρ_0 is the spatial radius of a spherical wave propagation in turbulent atmosphere.

For the non-Kolmogorov channel, the spectrum of atmospheric turbulence is represented by [39]

$$\phi_n(\alpha,\ \kappa) = A(\alpha)C_n^2(z,\ \alpha)\kappa^{-\alpha}, \quad 0 \le \kappa < \infty,\ 3 < \alpha < 5, \qquad (13)$$

where κ is magnitude of the spatial frequency, $A(\alpha) = \Gamma(\alpha - 1)\cos(\alpha\pi/2)/4\pi^2$, $\Gamma(\cdot)$ the Gamma function, α the power-law exponent of the non-Kolmogorov spectrum, and $C_n^2(z,\ \alpha)$ a generalized refractive-index structure parameter with unit $\mathrm{m}^{3-\alpha}$, which is altitude dependent and is given by [34]

$$\begin{aligned} C_n^2(z,\ \alpha) \ = \ & 0.033\left(\sqrt{kz}\right)^{(\alpha-11/3)}\Big[0.00594(v/27)^2 \\ & \times\left(z\cos\theta\times 10^{-5}\right)^{10}\exp(-z\cos\theta/1000) \\ & +2.7\times 10^{-16}\exp(-z\cos\theta/1500) \\ & +C_n^2(0)\exp(-z\cos\theta/100)\Big]/A(\alpha), \end{aligned} \tag{14}$$

where $z\cos\theta = h$ is altitude, $v = 21\,\mathrm{m/s}$ the rms wind speed, $C_n^2(0)$ the refractive index structure parameter at the ground, and θ the zenith angle of communication channel.

By the non-Kolmogorov spectrum Eq. (13), the spatial coherence radius of a spherical wave is given by

$$\rho_0(\alpha) = \left\{\frac{2\Gamma\left(\frac{3-\alpha}{2}\right)}{\pi^{1/2}k^2\Gamma\left(\frac{2-\alpha}{2}\right)z\int_0^1 C_n^2(\xi z,\ \alpha)(1-\xi)^{\alpha-2}d\xi}\right\}^{1/(\alpha-2)}, \quad 3<\alpha<4. \tag{15}$$

Substituting (12) into (11), the turbulent ensemble average number operator is given by the expression

$$\begin{aligned} & \left\langle \hat{a}_j^+(q,x,z)\,\hat{a}_j(q,x,z)\right\rangle_m \\ & = n_{11}\left(\frac{k}{2\pi z}\right)^2\int\int \mathrm{d}x'\mathrm{d}x''u^*\left(x'\right)u\left(x''\right)\exp\left[-\frac{\left(x'-x''\right)^2}{\rho_0^2}\right] \\ & \quad\times\mathrm{e}^{-\mathrm{i}q(x'-x'')}\exp\left\{-\frac{\mathrm{i}k}{2z}\left[x'^2-x''^2-2x\left(x'-x''\right)\right]\right\}. \end{aligned} \tag{16}$$

For QHG beam, the field $u(x',\,0)$ at $z = 0$ can be represented as [36]

$$u\left(x',0\right) = \frac{1}{\sqrt{2^{m-1}m!\pi w_0}}\exp\left(-\frac{x'^2}{w_0^2}\right)H_m\left(\frac{\sqrt{2}x'}{w_0}\right), \tag{17}$$

where w_0 are the waist width of Hermite-Gaussian beams, $H_m(\cdot)$ is a Hermite polynomial.

Substituting (17) into (16), we have

$$\begin{aligned}\left\langle\hat{S}_0\right\rangle_{at} &= \left\langle\hat{a}_j^+(q,x,z)\hat{a}_j(q,x,z)\right\rangle_m \\ &= \left(\frac{k}{2\pi z}\right)^2\frac{n_{11}}{2^{m-1}\pi m!w_0}\int\int \mathrm{d}x'\mathrm{d}x''\exp\left(-\frac{x'^2}{w_0^2}-\frac{x''^2}{w_0^2}\right) \\ &\quad\times H_m^*\left(\frac{\sqrt{2}x'}{w_0}\right)H_m\left(\frac{\sqrt{2}x''}{w_0}\right)\exp\left[-\frac{(x'-x'')^2}{\rho_0^2}\right] \\ &\quad\times \mathrm{e}^{-\mathrm{i}q(x'-x'')}\exp\left\{-\frac{\mathrm{i}k}{2z}\left[x'^2-x''^2-2x\left(x'-x''\right)\right]\right\}. \qquad (18)\end{aligned}$$

Recalling following integral and expansion formulae [40]

$$\int_{-\infty}^{\infty}\exp\left[-(x-y)^2\right]H_n(ax)dx=\sqrt{\pi}\left(1-a^2\right)^{n/2}H_n\left[\frac{ay}{(1-a^2)^{1/2}}\right],$$

$$\int_{-\infty}^{\infty}\exp\left[-(x-y)^2\right]H_m(ax)H_n(bx)dx$$

$$\begin{aligned}&= \left(\frac{b}{a}\right)^n\sqrt{\pi}\sum_{k=0}^{[n/2]}\sum_{l=0}^{\min(m,n-2k)}\binom{m}{l}\frac{n!}{k!(n-2k-l)!}2^l \\ &\quad\times\left[1-\frac{1}{(b/a)^2}\right]^k\left(1-a^2\right)^{\frac{m+n}{2}-k-l}H_{m+n-2k-2l}\left[\frac{ay}{(1-a^2)^{1/2}}\right],\end{aligned}$$

$$H_m(x+y)=\frac{1}{2^{m/2}}\sum_{n=0}^{m}\binom{m}{n}H_n(\sqrt{2}x)H_{m-n}(\sqrt{2}y),$$

after very tedious integral calculations, we obtain the turbulent atmosphere ensemble average of the Stokes parameter

$$\begin{aligned}\langle\hat{S}_0\rangle_{at} &= \left(\frac{k}{2\pi z}\right)^2\frac{n_{11}}{2^{m-1}\pi m!w_0}\sqrt{\frac{\pi^2}{AB}}\left(\frac{D}{2}\right)^{m/2} \\ &\quad\times\exp\left[-\left(\frac{1}{4A}-\frac{C^2}{4B}\right)\left(\frac{k}{z}x-q\right)^2\right] \\ &\quad\times\sum_{n=0}^{m}\sum_{p=0}^{[n/2]}\sum_{l=0}^{n-2p}\binom{m}{n}\binom{m}{l}\left(\frac{\sqrt{2}}{A\rho_0^2D^{1/2}}\right)^n\frac{n!2^l}{p!(n-2p-l)!} \\ &\quad\times\left(1-\frac{A^2\rho_0^4D}{2}\right)^pE^{\frac{m+n}{2}-p-l}H_{m-n}\left[\frac{i}{Aw_0D^{1/2}}\left(\frac{k}{z}x-q\right)\right]\end{aligned}$$

$$\times H_{m+n-2p-2l}\left[\frac{i\sqrt{2}C}{2Bw_0E^{1/2}}\left(\frac{k}{z}x-q\right)\right], \tag{19}$$

where

$$A = \frac{1}{\rho_0^2}+\frac{1}{w_0^2}+\frac{\mathrm{i}k}{2z}, \tag{20}$$

$$B = \frac{1}{\rho_0^2}+\frac{1}{w_0^2}-\frac{\mathrm{i}k}{2z}-\frac{1}{A\rho_0^4}, \tag{21}$$

$$C = \frac{1}{A\rho_0^2}-1, \tag{22}$$

$$D = 1-\frac{2}{Aw_0^2}, \tag{23}$$

$$E = 1-\frac{2}{Bw_0^2}. \tag{24}$$

For linearly polarized quantum light, substituting (22) into (6), we can obtain the formula for the degree of depolarization of QHG beams propagating through a turbulent atmosphere slant channel

$$DP(x,z)=\left\{\frac{2}{T_0T_m\exp\left[-\left(\frac{1}{4A}-\frac{C^2}{4B}\right)\left(\frac{k}{z}x-q\right)^2\right]+2}\right\}^{1/2}, \tag{25}$$

where

$$T_0=\left(\frac{k}{2\pi z}\right)^2\frac{n_{11}}{2^{m-1}\pi m!w_0}\sqrt{\frac{\pi^2}{AB}}\left(\frac{D}{2}\right)^{m/2},$$

$$T_m=\sum_{n=0}^{m}\sum_{p=0}^{[n/2]}\sum_{l=0}^{n-2p}\binom{m}{n}\binom{m}{l}\left(\frac{\sqrt{2}}{A\rho_0^2D^{1/2}}\right)^n\frac{n!2^l}{p!(n-2p-l)!}$$
$$\times\left(1-\frac{A^2\rho_0^4D}{2}\right)^pE^{\frac{m+n}{2}-p-l}H_{m-n}\left[\frac{i}{Aw_0D^{1/2}}\left(\frac{k}{z}x-q\right)\right]$$
$$\times H_{m+n-2p-2l}\left[\frac{i\sqrt{2}C}{2Bw_0E^{1/2}}\left(\frac{k}{z}x-q\right)\right].$$

4. NUMERICAL RESULTS

Now we study the numerical results of the influence of turbulence on the depolarization degree of QHG beams by using the formulae derived in the above section.

Figure 1 gives the degree of depolarization $DP(x, z)$ at different propagation distance in turbulent atmosphere and in free space. It can be seen from Fig. 1(b) that the shapes of $DP(x, z)$ at different propagation distance are similar and $DP(x, z)$ becomes bigger with increasing propagation distance. However, QHG beams propagating in turbulent atmosphere experience more disturbance beyond diffraction than those in free space. So we can see from Fig. 1(a) that the profile of $DP(x, z)$ does not remain invariant on propagation, but turns into a Gaussian shape gradually with increasing propagation distance, and the effect of depolarization at the point $x = 0$ becomes stronger when the QHG beam travels a further distance.

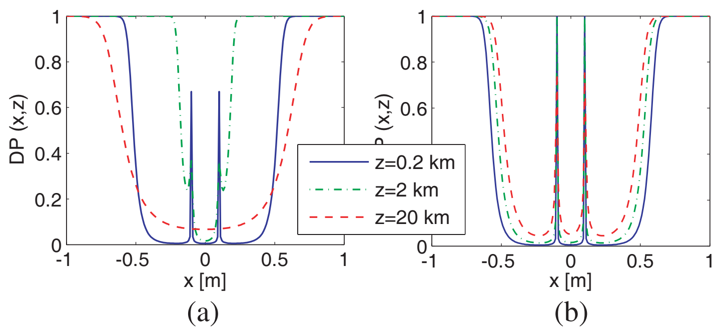

Figure 1. Degree of depolarization $DP(x, z)$ for different values of the propagation distance $z = 0.2$, 2 and 20 km. The calculation parameters are: $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $\lambda = 1.55\,\mu\mathrm{m}$, $n_{11} = 15$, $m = 2$. (a) In turbulent channel: $C_n^2(0) = 10^{-15}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $\theta = \pi/6$, (b) in free space.

Figure 2 compares the distributions of the degree of depolarization $DP(x, z)$ of Hermite-Gaussian beams and Gaussian beams ($m = 0$) at different propagation distance. It is seen from Fig. 2 that there exists distinct difference between the distributions of the degree of depolarization of Hermite-Gaussian beams and Gaussian beams at first and the difference disappears gradually with increasing propagation distance. By comparison, the effect of depolarization of Hermite-Gaussian beams at the point $x = 0$ is stronger than that of Gaussian beams.

Figures 3 and 4 show the change in the degree of depolarization $DP(x, z)$ versus propagation distance z at different positions. One can find from Fig. 3 (a) that the degree of depolarization $DP(x, z)$ of QHG beams propagating in turbulent atmosphere has an oscillatory behavior at first, and then it arrives at a limiting value near 0 with increasing

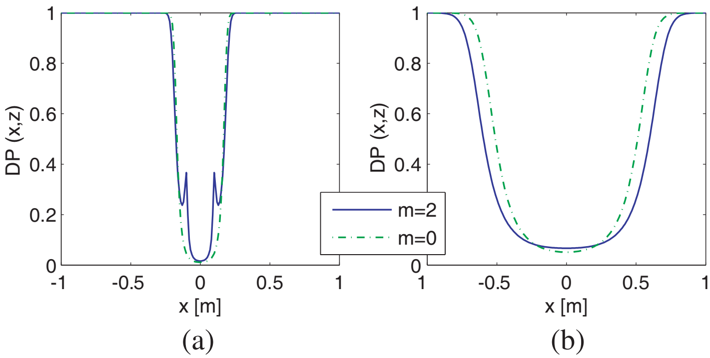

Figure 2. Degree of depolarization $DP(x, z)$ for different values of beam order $m = 2, 0$. The calculation parameters are: $C_n^2(0) = 10^{-15}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $\lambda = 1.55\,\mu\mathrm{m}$, $n_{11} = 15$, $\theta = \pi/6$. (a) $z = 2\,\mathrm{km}$, (b) $z = 20\,\mathrm{km}$.

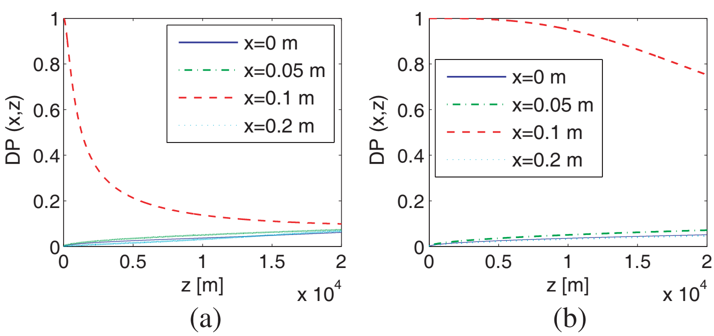

Figure 3. Degree of depolarization $DP(x, z)$ at different positions. The calculation parameters are: $\lambda = 1.55\,\mu\mathrm{m}$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $n_{11} = 15$, $m = 2$. (a) In turbulent channel: $C_n^2(0) = 10^{-15}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $\theta = \pi/6$, (b) in free space.

propagation distance. In free space, the degree of depolarization $DP(x, z)$ of QHG beams still keeps away from 0 ever after travelling 20 km. It also can be found that the effect of depolarization at the point $x = 0$ is weaker than at the off-axis points at the plane $z = 20\,\mathrm{km}$, that is to say, $x = 0$ is a optimal detecting point for second order QHG beams. In Fig. 4, we can see that the degree of depolarization of Gaussian beams ($m = 0$) increases monotonously with the increasing propagation distance at any detecting point. However, there exists different optimal detecting point for QHG beams with different beam

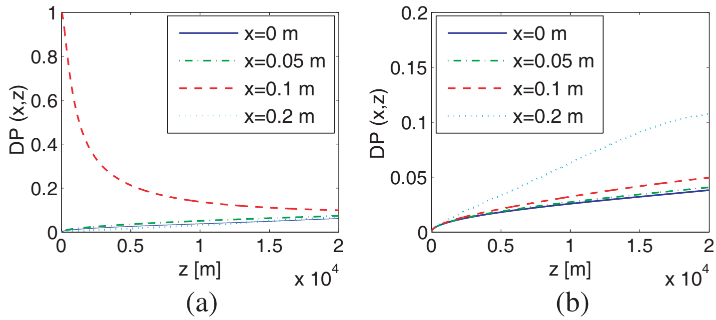

Figure 4. Degree of depolarization $DP(x,\, z)$ for different positions. The calculation parameters are: $C_n^2(0) = 10^{-15}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $\lambda = 1.55\,\mu\mathrm{m}$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $n_{11} = 15$, $\theta = \pi/6$. (a) $m = 2$, (b) $m = 0$.

order.

In Fig. 5, we examine the effects of spectrum power-law exponent α on the degree of depolarization $DP(x,\, z)$ of QHG beams. One can find that the polarization fluctuation is strong when the spectrum power-law exponent $\alpha < 11/3$, and $DP(x,\, z)$ has a saltation and reaches the minimum value at $\alpha = 11/3$, and it increases slowly when $\alpha > 11/3$. Furthermore, It can be seen from Fig. 5 that the influence of beam order and zenith angle on the degree of depolarization is slight, but the influence of detection photon numbers and wavelength on the degree of depolarization is obvious and the QHG beams with larger detection photon numbers and shorter wavelength cause smaller change in the degree of depolarization of QHG beams.

Figure 6 presents the degree of depolarization $DP(x,\, z)$ of QHG beams with different $C_n^2(0)$ in turbulent atmosphere. It is seen from Fig. 6 that the degree of depolarization of QHG beams which propagate through atmosphere has an oscillatory behavior and the stronger strength of turbulence causes stronger polarization fluctuation when $z < 15\,\mathrm{km}$, and it tends to a limiting value with increasing propagation distance and the influence of $C_n^2(0)$ on the degree of depolarization of QHG beams seems slight at the plane $z = 20\,\mathrm{km}$.

Figure 7 shows the degree of depolarization $DP(x,\, z)$ of QHG beams versus zenith angle θ. One can find from Fig. 7 that $DP(x,\, z)$ deceases slowly as θ increases, that is, the influence of zenith angle on in the polarization of QHG beams is slight. Moreover, it also can be found that lower beam order, larger detection photon numbers, shorter wavelength cause small change in the polarization of QHG beams.

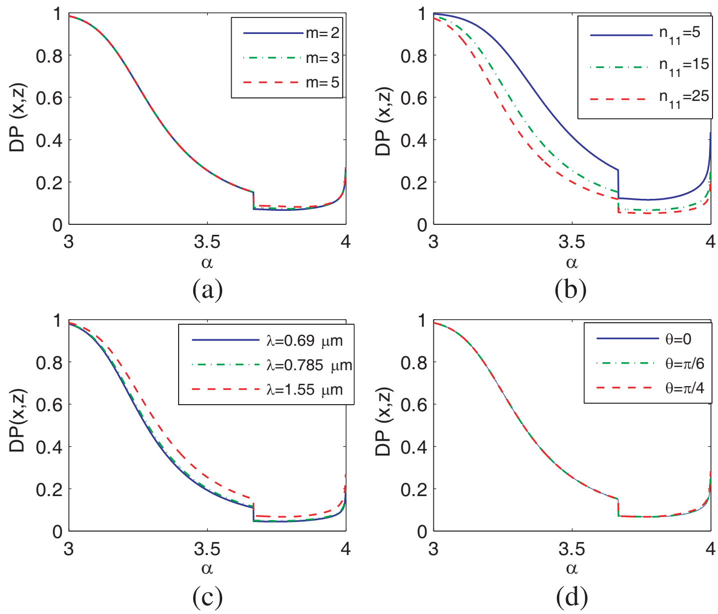

Figure 5. Degree of depolarization $DP(x, z)$ versus spectrum power-law exponent α. The calculation parameters are: $C_n^2(0) = 10^{-13}$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $x = 0\,\mathrm{m}$, $z = 20\,\mathrm{km}$, $n_{11} = 15$, $m = 2$, $\lambda = 1.55\,\mu\mathrm{m}$, $\theta = \pi/6$. (a) For different beam order $m = 2, 3, 5$, (b) for different photon number $n_{11} = 5, 15, 25$, (c) for different wavelength $\lambda = 0.69, 0.785, 1.55\,\mu\mathrm{m}$, (d) for different zenith angle $\theta = 0, \pi/6, \pi/4$.

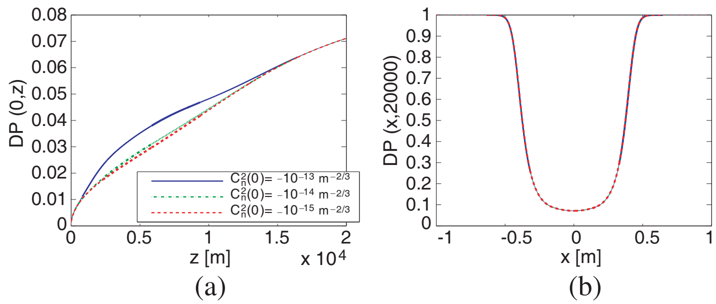

Figure 6. Degree of depolarization $DP(x, z)$ for different values of the refractive index structure parameter at the ground $C_n^2(0) = 10^{-13}$, 10^{-14} and $10^{-15}\,\mathrm{m}^{-2/3}$. The calculation parameters are: $\alpha = 3.67$, $\lambda = 1.55\,\mu\mathrm{m}$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $n_{11} = 15$, $m = 2$, $\theta = \pi/6$. (a) $x = 0\,\mathrm{m}$, (b) $z = 20\,\mathrm{km}$.

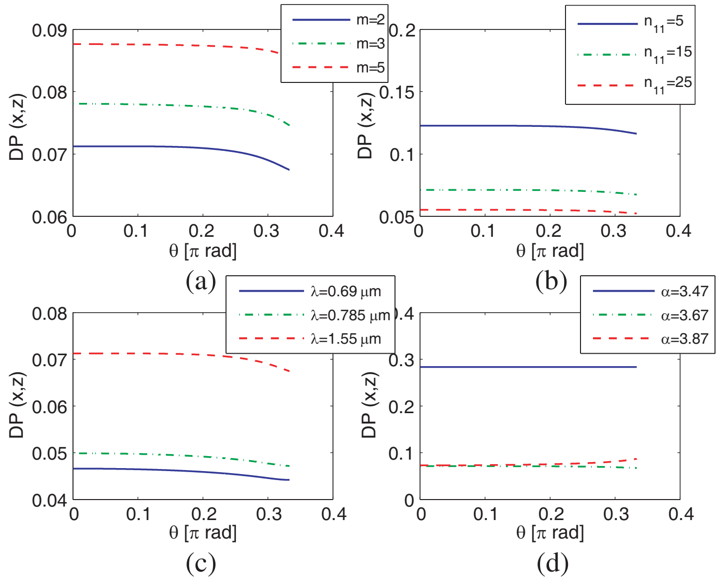

Figure 7. Degree of depolarization $DP(x, z)$ versus zenith angle θ. The calculation parameters are: $C_n^2(0) = 10^{-13}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $x = 0\,\mathrm{m}$, $z = 20\,\mathrm{km}$, $n_{11} = 15$, $m = 2$, $\lambda = 1.55\,\mu\mathrm{m}$. (a) For different beam order $m = 2, 3, 5$, (b) for different photon number $n_{11} = 5, 15, 25$, (c) for different wavelength $\lambda = 0.69, 0.785, 1.55\,\mu\mathrm{m}$, (d) for different spectrum power-law exponent $\alpha = 3.47, 3.67, 3.87$.

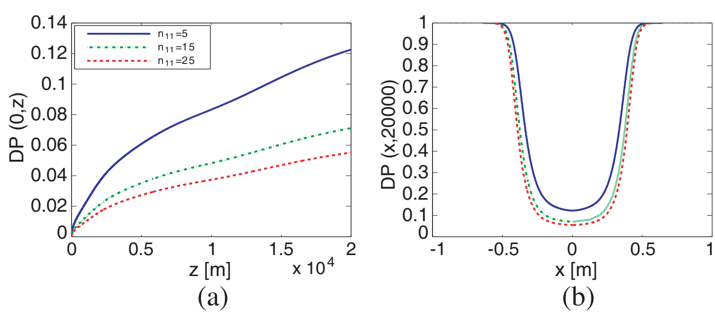

Figure 8. Degree of depolarization $DP(x, z)$ for different values of the photon-number $n_{11} = 5$, 15 and 25. The calculation parameters are: $C_n^2(0) = 10^{-13}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $\lambda = 1.55\,\mu\mathrm{m}$, $m = 2$, $\theta = \pi/6$. (a) $x = 0\,\mathrm{m}$, (b) $z = 20\,\mathrm{m}$.

Figure 8 presents the degree of depolarization $DP(x,\, z)$ of QHG beams for different detection photon numbers n_{11}. It can be found from Fig. 8 that the degree of depolarization decreases with the increasing detection photon number. In comparison with classical theory, the degree of depolarization has no changes in their traveling; the quantum light tends to classical light with the detection photon numbers increase, so we should choose large detection photon numbers when we take the robustness of polarization as the carrier in single-photon communications.

Figure 9 gives the change in the degree of depolarization of QHG beams for different values of beam order m. It can be seen that the the

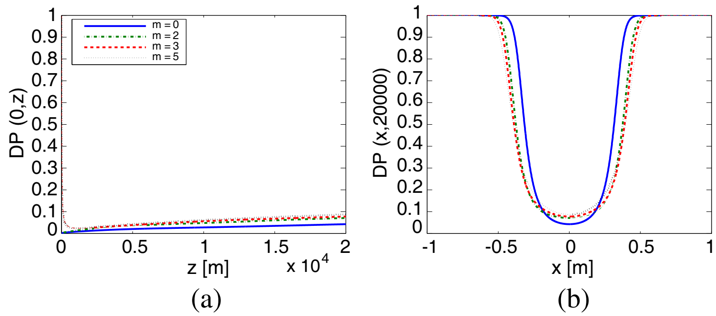

Figure 9. Degree of depolarization $DP(x,\, z)$ for different values of the beam order $m = 0$, 2, 3 and 5. The calculation parameters are: $C_n^2(0) = 10^{-13}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25\times10^{-28}$, $n_{11} = 15$, $\lambda = 1.55\,\mu\mathrm{m}$, $\theta = \pi/6$. (a) $x = 0\,\mathrm{m}$, (b) $z = 20\,\mathrm{km}$.

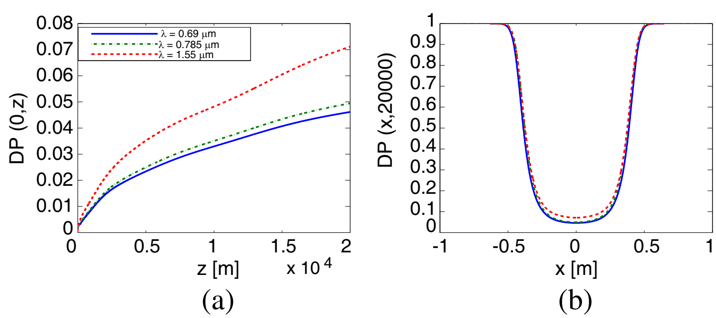

Figure 10. Degree of depolarization $DP(x,\, z)$ for different values of the beam wavelength $\lambda = 0.69$, 0.785 and $1.55\,\mu\mathrm{m}$. The calculation parameters are: $C_n^2(0) = 10^{-13}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $w_0 = 0.2\,\mathrm{m}$, $q = 6.25 \times 10^{-28}$, $n_{11} = 15$, $m = 2$, $\theta = \pi/6$. (a) $x = 0\,\mathrm{m}$, (b) $z = 20\,\mathrm{km}$.

change of magnitude of the limiting value of $DP(x,\, z)$ for QHG beams with higher beam order in turbulence is greater than that with lower beam order. So when we take the robustness of polarization as the carrier in single-photon communications, lower beam order is better than higher ones.

Figure 10 shows the change in the degree of depolarization of QHG beams for different wavelength λ. It is clearly that the change in the degree of depolarization of QHG beams with longer wavelength is greater than that with shorter wavelength. The physical reason is that the scattering of quantum light is more obvious of short wavelength than long wavelength so the degree of depolarization increases with the wavelength decrease. When we take the robustness of polarization as the carrier in single-photon communications we should choose shorter wavelength because with short wavelength the degree of depolarization changes into small.

In Fig. 11, we examine the effects of waist width w_0 on the degree

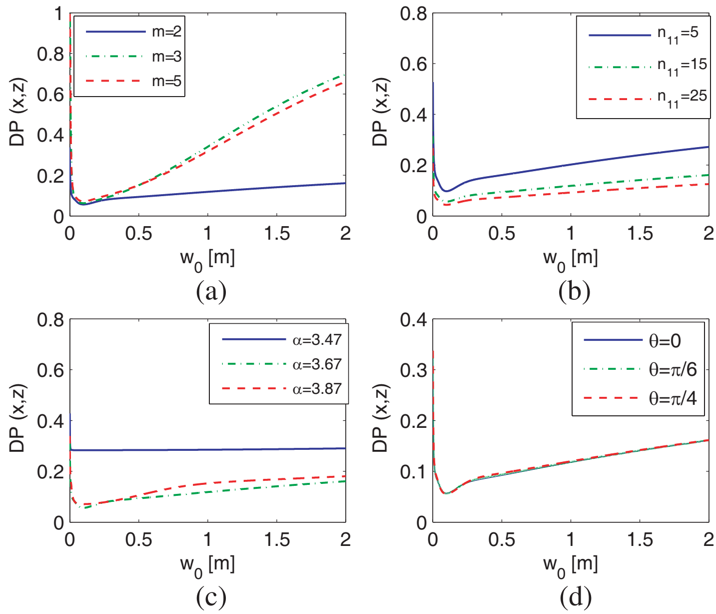

Figure 11. Degree of depolarization $DP(x,\, z)$ versus w_0. The calculation parameters are: $C_n^2(0) = 10^{-13}\,\mathrm{m}^{-2/3}$, $\alpha = 3.67$, $q = 6.25 \times 10^{-28}$, $x = 0\,\mathrm{m}$, $z = 20\,\mathrm{km}$, $n_{11} = 15$, $m = 2$, $\lambda = 1.55\,\mu\mathrm{m}$, $\theta = \pi/6$. (a) For different beam order $m = 2,\, 3,\, 5$, (b) for different photon number $n_{11} = 5,\ 15,\ 25$, (c) for different spectrum power-law exponent $\alpha = 3.47,\ 3.67,\ 3.87$, (d) for different zenith angle $\theta = 0$, $\pi/6$, $\pi/4$.

of depolarization $DP(x, z)$. The results demonstrate that the degree of depolarization $DP(x, z)$ arrives its minimum at about $w_0 = 0.1\,\mathrm{m}$, and too small or too big w_0 will cause the big change in the polarization of QHG beams.

5. CONCLUSIONS

In this paper, the effects of atmospheric turbulence on the quantum polarization fluctuations of QHG beams are studied and the analytical formulae for the degree of depolarization of QHG beams propagating in a turbulence atmosphere slant channel are obtained. It is found from numerical results that the polarization fluctuations of QHG beams are dependent of the turbulence factors such as spectrum power-law exponent, refractive index structure parameter at the ground and zenith angle. The degree of depolarization of QHG beams has a saltation and reaches the minimum value at spectrum power-law exponent $\alpha = 11/3$, the refractive index structure parameter at the ground of the turbulent atmosphere $(C_n^2(0))$ slightly affects the polarization degree of QHG beams which have travelled a long distance, and the change of polarization degree decreases with the increasing zenith angle. We also discuss the effects of different detection photon number on the degree of depolarization of QHG beams, and find that, for small detection quantum number, the polarization fluctuations of quantum light propagation through the turbulent atmosphere can no longer be negligible as the case of classical light propagation in the turbulent atmosphere. Furthermore, the effects of different beam order and the wavelength are also examined, and find that shorter wavelength and lower beam order cause smaller change in the polarization of QHG beams. So when we take the robustness of polarization as the carrier in quantum optical communications, we should choose the QHG beams with shorter wavelength, lower beam order, and larger detection photon numbers. These results are likely to be useful for remote sensing and optical communications.

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