

TIME-DOMAIN DISTRIBUTED PARAMETERS TRANSMISSION LINE MODEL FOR TRANSIENT ANALYSIS

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Abstract—This article describes a time-domain transmission line model based on distributed parameters for transient analysis. This model is based directly on the differential equations for the basic transmission line without any previous simplification. The solution presented here for these differential equations results in a more detailed time-domain model than others models currently in use, and with certain structural similarities with the distributed parameter frequency-domain model for long transmission lines. The deduction of a general time-domain transmission line model for fundamental frequencies parameters and single-phase line are presented in this article, but the model can also be extended to cases with multiconductor and frequency-depended parameters. In order to validate the model, a comparative test is presented to facilitate the analysis about the main similarities and differences between this and other models.

1. INTRODUCTION

The time-domain transmission lines models are of great importance in the electromagnetic transient simulation of power systems. Currently, time-domain line models are based on simplifications of distributed parameters, or on lumped parameters. Models based directly on distributed parameters are most commonly used in frequency-domain, because frequency-domain facilitates the manipulation of the differential equations that describe the behavior of transmission lines.

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There are many ways available to obtain a line model. Each has its own advantages and disadvantages relative to the others, depending on the particular transmission system under analysis and the nature of the signals being considered. The base for most the widely used time-domain models for transient analysis was proposed by Dommel in 1969 [1] and is based in solving line differential equations using the method of characteristics. The method of characteristics is a mathematical method developed in the early twentieth century. One of its greatest promoters was Bergeron in 1928 [2] where it was used as a graphical method for calculating transients in penstocks. Its applications range over a wide variety of topics including the propagation of surges on electrical transmission lines. For this reason, the model is called the Bergeron's model. A survey of these and other models will be addressed in Section 2.

The Bergeron's model found application in a power system transients program by Dommel. He solved the line differential equations by neglecting the losses and then giving a lumped losses element representation in the model. This results in a very simple and robust model that is able to simulate the behavior of the transmission line with acceptable accuracy.

For more than 40 years Bergeron's model has provided a widely accepted solution for fundamental frequencies line modeling, but in the last 40 years electrical systems have changed greatly. The aperture of the markets and the introduction of renewable energy have led systems to operate at the limit of safety. Under these conditions it is increasingly important the accurate prediction of the system's behavior. This justifies the search of more accurate line models.

This article presents a different time-domain transmission line model for transient analysis. The model is based directly on the transmission line differential equations represented as the distributed parameter model without any approximation. The solution presented here for these differential equations results in a model with certain parallels with the frequency-domain model for long transmission lines, and also with many similarities with the Bergeron's model but in a more detailed way. In order to validate the model, a comparative test is performed between the presented model and other models.

2. LINE MODEL FOUNDATIONS

Power transmission lines are formed by two or more isolated conductors, in order to allow power to flow only in the longitudinal direction of the line (here called X direction) with minimal losses, and to prevent the flow in other directions as much as possible. In this

article, the model will be deduced for a single line, but could also be deduced for multiconductor lines.

Transmission lines have four primary parameters, which are a series resistance R , a series inductance L , a parallel capacitance C and a parallel conductance G . These parameters are distributed along the entire line and are used to model the behavior of the voltage (V) and current (I) signals as they travel throughout the line, as represented in Fig. 1. In this article the model will be deduced for the fundamental frequency parameters, but the model could also be expanded in order to incorporated frequency-dependent parameters [3, 4].

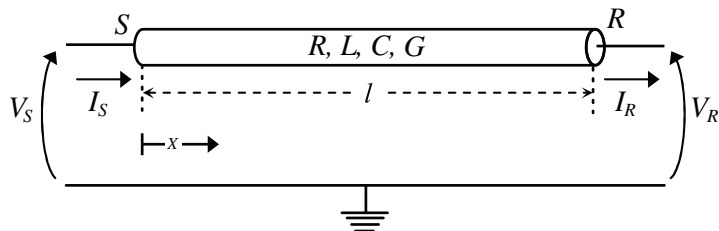


Figure 1. General line representation.

The differential equations of a transmission line are obtained focusing attention on an infinitesimal section of Δx line length, which is located at the coordinate X of the line and far from the line ends. This section of the line has a total resistance $R\Delta x$, a total inductance $L\Delta x$, a total capacitance $C\Delta x$, and a total conductance $G\Delta x$. Its equivalent circuit in a quadrupole form can be represented incorporating these circuit elements in many ways. One of these is shown in Fig. 2.

Using the instantaneous quantities of Fig. 2, these relationships

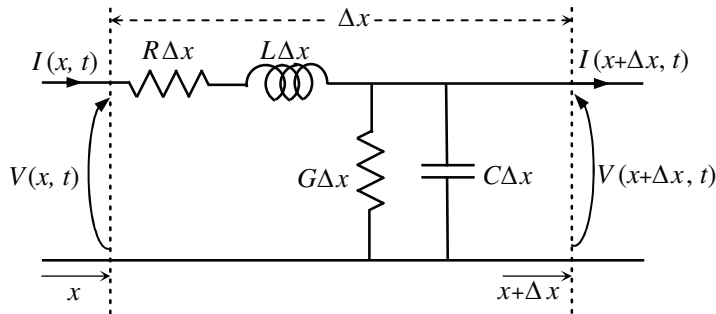


Figure 2. Line section of Δx length.

can be expressed by the following equations, applying Kirchhoff's laws:

$$V(x+\Delta x, t) - V(x, t) = \Delta V(x, t) = -R\Delta x I(x, t) - L\Delta x \frac{\partial I(x, t)}{\partial t} \quad (1)$$

$$I(x+\Delta x, t) - I(x, t) = \Delta I(x, t) = -G\Delta x V(x, t) - C\Delta x \frac{\partial V(x, t)}{\partial t} \quad (2)$$

Both V and I are function of the distance x and the time t . Then, dividing by Δx and making Δx tend to zero, leads to the following partial differential equations:

$$\frac{\partial V(x, t)}{\partial x} = -RI(x, t) - L\frac{\partial I(x, t)}{\partial t} \quad (3)$$

$$\frac{\partial I(x, t)}{\partial x} = -GV(x, t) - C\frac{\partial V(x, t)}{\partial t} \quad (4)$$

Voltage $V(x, t)$ and current $I(x, t)$ will be denoted only as V and I respectively, but knowing that they are always functions of x and t . With (3) and (4), expressions for V and I can be found as functions of x and t , and subject to the boundary conditions determined by the nature of the devices connected to the line ends, i.e., sources in $x = 0$ and load circuits in $x = l$. Usually, the first step to try to solve these equation systems is by removing one variable. To solve (3) and (4) is required to apply partial differential, with respect to t on (3), and with respect to x on (4):

$$\frac{\partial^2 V}{\partial t \partial x} = -R\frac{\partial I}{\partial t} - L\frac{\partial^2 I}{\partial t^2} \quad (5)$$

$$\frac{\partial^2 I}{\partial x^2} = -G\frac{\partial V}{\partial x} - C\frac{\partial^2 V}{\partial t \partial x} \quad (6)$$

and substituting (3) and (5) in (6), is obtained a differential equation for I .

$$LC\frac{\partial^2 I}{\partial t^2} + (RC + LG)\frac{\partial I}{\partial t} + RGI - \frac{\partial^2 I}{\partial x^2} = 0 \quad (7)$$

In the same manner, a differential equation for V is obtained.

These equations are a complete description of the possible interrelationships between voltage and current, and its derivatives at any time (t) and at any point in the line(x).

Equation (7) is a second order linear partial differential equation with a coordinate of space and other of time, making it difficult to obtain a complete solution for them in terms of V and I as functions of x and t , respectively. To find an accurate solution to this equation is the main problem of transmission line modeling.

In the literature is found a lot of information about a solution for (7) when is changed from time-domain to frequency-domain [5, 6].

Thus the partial derivative components with respect to time become constant, and the Equations (5) and (6) are solved as standard second order differential equations.

Directly in time-domain, some solutions have been developed by reducing the generality of the specifications, i.e., assuming that some of the parameters R, L, G and C are small enough to be negligible. This simplifies (7), and solutions for certain border conditions can be found. These reductions represent specific applications of the transmission line.

One of these reductions is $L = 0$ and $G = 0$ to describe underground cables. This type of approach is widely used to model systems of low frequency or DC [7]. Another solution is given in [1, 8] which reduces Equations (5) and (6) for the case of a lossless line, where $R = 0$ and $G = 0$, thus the equations can be integrated directly. Then lumped resistance is added in order to compensate the attenuation effect produced by losses. This model is called the Bergeron's model, which is the most basic time-domain model and is used as foundation for the most widely used models in power systems simulations. Some of this models is [3, 9], which retains the basic idea behind Bergeron's method but is based on starting with the frequency-domain model to see the effect that different frequencies have in the line parameters and transfer it to time-domain doing some approximations. This approach results in a time domain model equivalent to [1, 8] that can be used for studies that require representation of non-fundamental frequencies parameters.

Works for transmission line modeling in time-domain that do not apply simplifications to (7) have also been proposed, in order to find a direct solution to the problem. These works propose the use of different numerical methods for solving the differential equation. Like using π -circuits [10], the modal method [11], the finite difference method (FDTD) [12], the finite element method [13], the time-step integration method [14], and others [4]. All these works bring useful transmission lines models, but all these resolution methods require, in a greater or lesser extent, numerical approximations to solve the differential equation. This compromises in one way or another the ability of models to accurately describe the behavior of the transmission lines.

In the following deduction, is presented a model for transmission lines in time-domain, based directly on (7) that does not take into account any of the previous approximations but uses the entire equation. In order to facilitate the explanation of the model, the deduction is presented to a single line and for fundamental frequency parameters.

3. LINE MODEL DEDUCTION

The presented solution begins by dividing the partial differential equation of (7) into two equations: one in time-domain and the other in space-domain. Then, the general solutions of each one compensate each other, in order to describe the complete behavior of the line. Therefore, the general solution for (7) is given by the general solutions of its components in time and space, as shown in the auxiliary equations:

Time-domain:

$$LCS_t^2 + (RC + LG)S_t = 0 \begin{cases} S_{t1} = 0 \\ S_{t2} = -\left(\frac{R}{L} + \frac{G}{C}\right) \end{cases} \quad (8)$$

Space-domain:

$$-S_x^2 + RG = 0 \begin{cases} S_{x1} = \sqrt{RG} \\ S_{x2} = -\sqrt{RG} \end{cases} \quad (9)$$

The general solution of the entire equation is:

$$I = K_a e^{-\left[\left(\frac{R}{L} + \frac{G}{C}\right)t + \sqrt{RG}x\right]} + K_b e^{-\left[\left(\frac{R}{L} + \frac{G}{C}\right)t - \sqrt{RG}x\right]} + K_c e^{\sqrt{RG}x} + K_d e^{-\sqrt{RG}x} \quad (10)$$

In this deduction are not used approximations to solve the differential equations, instead a different approach is used where a relative weight of importance is given to t and x in the partial derivatives, then each variable is solved separately as a first-order differential equation.

Now, (10) is differentiated with respect to t in order to obtain the general solution for V , and substituting in (2), is obtained a differential equation for V as function of x and t :

$$\begin{aligned} \frac{\partial V}{\partial x} = & \frac{LG}{C} K_a e^{-\left[\left(\frac{R}{L} + \frac{G}{C}\right)t + \sqrt{RG}x\right]} + \frac{LG}{C} K_b e^{-\left[\left(\frac{R}{L} + \frac{G}{C}\right)t - \sqrt{RG}x\right]} \\ & - RK_c e^{\sqrt{RG}x} - RK_d e^{-\sqrt{RG}x} \end{aligned} \quad (11)$$

If (11) is integrated to find the expression for V . Then, the general solution for V is obtained:

$$\begin{aligned} V = & -\frac{L}{C} \sqrt{\frac{G}{R}} K_a e^{-\left[\left(\frac{R}{L} + \frac{G}{C}\right)t + \sqrt{RG}x\right]} + \frac{L}{C} \sqrt{\frac{G}{R}} K_b e^{-\left[\left(\frac{R}{L} + \frac{G}{C}\right)t - \sqrt{RG}x\right]} \\ & - \sqrt{\frac{R}{G}} K_c e^{\sqrt{RG}x} + \sqrt{\frac{R}{G}} K_d e^{-\sqrt{RG}x} \end{aligned} \quad (12)$$

3.1. Boundary Conditions

Values for K_a , K_b , K_c and K_d can be determined, establishing boundary conditions at one line end, e.g., the voltage V_1 and current I_1 when $x = 0$ and $t = t_1$, and V_2 and I_2 when $x = 0$ and $t = t_2$.

$$\begin{aligned}
 I_1 &= (K_a + K_b)e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1} + K_c + K_d \\
 V_1 &= \frac{L}{C}\sqrt{\frac{G}{R}}(-K_a + K_b)e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1} + \sqrt{\frac{R}{G}}(-K_c + K_d) \\
 I_2 &= (K_a + K_b)e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} + K_c + K_d \\
 V_2 &= \frac{L}{C}\sqrt{\frac{G}{R}}(-K_a + K_b)e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} + \sqrt{\frac{R}{G}}(-K_c + K_d) \quad (13)
 \end{aligned}$$

With this four equations system, the expressions for K_a , K_b , K_c and K_d are obtained:

$$\begin{aligned}
 K_a &= \frac{1}{2} \left[(I_2 - I_1) - \frac{(V_2 - V_1)}{\frac{L}{C}\sqrt{\frac{G}{R}}} \right] \frac{1}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \\
 K_b &= \frac{1}{2} \left[(I_2 - I_1) + \frac{(V_2 - V_1)}{\frac{L}{C}\sqrt{\frac{G}{R}}} \right] \frac{1}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \\
 K_c &= \frac{1}{2} \left[I_2 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1} - I_1 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} \right. \\
 &\quad \left. - \frac{V_2 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1} - V_1 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2}}{\sqrt{\frac{R}{G}}} \right] \frac{1}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \\
 K_d &= \frac{1}{2} \left[I_2 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1} - I_1 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} \right. \\
 &\quad \left. + \frac{V_2 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1} - V_1 e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2}}{\sqrt{\frac{R}{G}}} \right] \frac{1}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \quad (14)
 \end{aligned}$$

Substituting these values in (10) and (12), the expressions for current I and voltage V at any time t and place x of the line are

obtained.

$$\begin{aligned}
 V = & V_2 \left[\frac{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} + e^{-\sqrt{RG}x}}{2} \right] \\
 & - V_1 \left[\frac{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} + e^{-\sqrt{RG}x}}{2} \right] \\
 & + I_2 \sqrt{\frac{R}{G}} \left[\frac{\frac{LG}{RC} e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} + e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} - e^{-\sqrt{RG}x}}{2} \right] \\
 & - I_1 \sqrt{\frac{R}{G}} \left[\frac{\frac{LG}{RC} e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} + e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} - e^{-\sqrt{RG}x}}{2} \right] \quad (15)
 \end{aligned}$$

$$\begin{aligned}
 I = & \frac{V_2}{\sqrt{\frac{R}{G}}} \left[\frac{\frac{RC}{LG} e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} + e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} - e^{-\sqrt{RG}x}}{2} \right] \\
 & - \frac{V_1}{\sqrt{\frac{R}{G}}} \left[\frac{\frac{RC}{LG} e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} - e^{-\sqrt{RG}x}}{2} \right] \\
 & + I_2 \left[\frac{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} - e^{-\sqrt{RG}x}}{2} \right] \\
 & - I_1 \left[\frac{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2}}{e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_2} - e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t_1}} \frac{e^{\sqrt{RG}x} - e^{-\sqrt{RG}x}}{2} \right] \quad (16)
 \end{aligned}$$

From $\sinh(\beta) = \frac{e^\beta - e^{-\beta}}{2}$ and $\cosh(\beta) = \frac{e^\beta + e^{-\beta}}{2}$. And also, taking a similar definition to propagation coefficient (γ) and characteristic impedance (Z_0) from the frequency domain model [5, 6]: $\gamma_{RG} = \sqrt{RG}$, $Z_{RG} = \sqrt{\frac{R}{G}}$ and $Z_{LC} = \sqrt{\frac{L}{C}}$, where RG and LC denote the line parameters that compose them. Then, if the values at the line origins are known, the values for voltage and current at any point of the line

can be expressed by:

$$\begin{aligned}
 V = & V_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} \cosh(x\gamma_{RG}) \\
 & - I_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_2} + \frac{LG}{RC} e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} Z_{RG} \sinh(x\gamma_{RG}) \\
 & - V_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_1} - e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} \cosh(x\gamma_{RG}) \\
 & + I_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_1} + \frac{LG}{RC} e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} Z_{RG} \sinh(x\gamma_{RG}) \quad (17)
 \end{aligned}$$

$$\begin{aligned}
 I = & -V_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_2} + \frac{RC}{LG} e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\
 & + I_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} \cosh(x\gamma_{RG}) \\
 & + V_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_1} + \frac{RC}{LG} e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\
 & - I_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t_1} - e^{-(\frac{R}{L} + \frac{G}{C})t}}{e^{-(\frac{R}{L} + \frac{G}{C})t_2} - e^{-(\frac{R}{L} + \frac{G}{C})t_1}} \cosh(x\gamma_{RG}) \quad (18)
 \end{aligned}$$

The signals V and I in a time t at any point x of the line, are formed by the sum of a series of traveling waves. In this deduction, V_1 and I_1 are the signals measured at time t_1 and V_2 and I_2 are the signals measured at time t_2 . t_1 and t_2 are two arbitrary values, but because the model follows the form proposed by d'Alembert [17], the results are actually more precise if $t - t_1 = t_2 - t$ where t is the time where the values of V and I want to be found. The choice of t_1 and t_2 is also related to the distance between the point of measurement and the point where V and I want to be found. For example, if the values of V and I want to be known in point x of the line shown in Fig. 3, the result will be more precise if the condition $t - t_1 = t_2 - t = x/v_t$ is met, where v_t is the wave velocity and is $v_t = 1/\sqrt{LC}$.

The time that is taken by both waves to travel between the sender terminal and the point x are equal, and are expressed by x/v_t . Therefore, the times t_1 and t_2 measured at the sender terminal are related to the travel time and with t by $t_1 = t - x\sqrt{LC}$, and $t_2 = t + x\sqrt{LC}$. So, substituting these expressions of t_1 and t_2 into

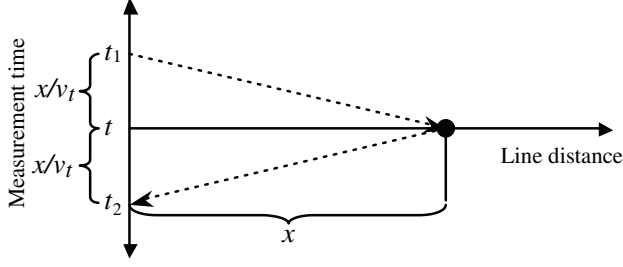


Figure 3. Criteria by which t_1 and t_2 were chosen that provide greater accuracy.

(17) and (18) are obtained:

$$\begin{aligned}
 V = & V_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - 1 \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} \cosh(x\gamma_{RG}) \\
 & - I_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} + \frac{LG}{RC} \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} Z_{RG} \sinh(x\gamma_{RG}) \\
 & - V_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - 1 \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} \cosh(x\gamma_{RG}) \\
 & + I_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} + \frac{LG}{RC} \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} Z_{RG} \sinh(x\gamma_{RG}) \quad (19)
 \end{aligned}$$

$$\begin{aligned}
 I = & -V_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} + \frac{RC}{LG} \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\
 & + I_1 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - 1 \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} \cosh(x\gamma_{RG}) \\
 & + V_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} + \frac{RC}{LG} \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\
 & - I_2 \frac{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - 1 \right)}{e^{-(\frac{R}{L} + \frac{G}{C})t} \left(e^{-(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} - e^{(\frac{R}{L} + \frac{G}{C})x\sqrt{LC}} \right)} \cosh(x\gamma_{RG}) \quad (20)
 \end{aligned}$$

The expression $e^{-\left(\frac{R}{L}+\frac{G}{C}\right)t}$ is canceled, and the equation's dividend can be expressed as follows:

$$e^{-\left(\frac{R}{L}+\frac{G}{C}\right)x\sqrt{LC}} - e^{\left(\frac{R}{L}+\frac{G}{C}\right)x\sqrt{LC}} = -2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)$$

So, (19) and (20) are simplified as follows:

$$\begin{aligned} V = & -V_1 \frac{e^{-\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} - 1}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} \cosh(x\gamma_{RG}) \\ & + I_1 \frac{e^{-\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} + \frac{LG}{RC}}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} Z_{RG} \sinh(x\gamma_{RG}) \\ & + V_2 \frac{e^{\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} - 1}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} \cosh(x\gamma_{RG}) \\ & - I_2 \frac{e^{\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} + \frac{LG}{RC}}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} Z_{RG} \sinh(x\gamma_{RG}) \end{aligned} \quad (21)$$

$$\begin{aligned} I = & V_1 \frac{e^{-\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} + \frac{RC}{LG}}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\ & - I_1 \frac{e^{-\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} - 1}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} \cosh(x\gamma_{RG}) \\ & - V_2 \frac{e^{\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} + \frac{RC}{LG}}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\ & + I_2 \frac{e^{\left(\frac{R}{Z_{LC}}+GZ_{LC}\right)x} - 1}{2 \sinh \left(\left(\frac{R}{Z_{LC}} + GZ_{LC} \right) x \right)} \cosh(x\gamma_{RG}) \end{aligned} \quad (22)$$

Finally, the solutions in (21) and (22) can be expressed in matrix form:

$$\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} A_1 & B_1 & \vdots & A_2 & B_2 \\ C_1 & D_1 & \vdots & C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \\ V_2 \\ I_2 \end{bmatrix} \quad (23)$$

where $A_1, B_1, C_1, D_1, A_2, B_2, C_2$, and D_2 are:

$$\begin{aligned}
 A_1 &= -\frac{e^{-\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} - 1}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} \cosh(x\gamma_{RG}) \\
 B_1 &= \frac{e^{-\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} + \frac{LG}{RC}}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} Z_{RG} \sinh(x\gamma_{RG}) \\
 C_1 &= \frac{e^{-\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} + \frac{RC}{LG}}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\
 D_1 &= -\frac{e^{-\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} - 1}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} \cosh(x\gamma_{RG}) \\
 A_2 &= \frac{e^{\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} - 1}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} \cosh(x\gamma_{RG}) \\
 B_2 &= -\frac{e^{\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} + \frac{LG}{RC}}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} Z_{RG} \sinh(x\gamma_{RG}) \\
 C_2 &= -\frac{e^{\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} + \frac{RC}{LG}}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} \frac{\sinh(x\gamma_{RG})}{Z_{RG}} \\
 D_2 &= \frac{e^{\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x} - 1}{2 \sinh\left(\left(\frac{R}{Z_{LC}} + GZ_{LC}\right)x\right)} \cosh(x\gamma_{RG})
 \end{aligned}$$

3.2. Model Analysis

As (23) shows, the results for (3) and (4) have the form $f(x, t) = f_1(x, t_1) + f_2(x, t_2)$ which are exactly the results predicted by d'Alembert's method [17], who first provides a solution to the wave propagation, and then was applied for transient phenomena transmission line analysis [1, 8, 9, 10].

At first glance it seems that the elements $A_1, B_1, C_1, D_1, A_2, B_2, C_2$ and D_2 are not time-dependent because of the cancellation of $e^{-\left(\frac{R}{L} + \frac{G}{C}\right)t}$ in (19) and (20), but is necessary to remember that $x\sqrt{LC}$

is a time measurement and describes the time that takes the waves to travel from one terminal to the point x throughout the line.

The model deduced here has a structure similar to other well known time-domain models such as the Bergeron's model, but in a more complex way that provides greater accuracy. This is because approximations are not used to solve differential equations.

Several analogies can be found between this model and the distributed parameter model in frequency-domain, e.g., in constants K_a , K_b , K_c and K_d obtained from the boundary conditions, which have the same structure as those obtained for other models in frequency domain [5, 6, 15]. The same applies to the elements A_1 , B_1 , C_1 , D_1 , A_2 , B_2 , C_2 , and D_2 that have hyperbolic functions. But also, they have other exponential and hyperbolic terms that describe the attenuation of voltage and current waves as they travel throughout the line, which was expected in a time-domain model [16].

In this model are also factors similar to the propagation coefficient and characteristic impedance that are equivalent terms to the model in the frequency domain [5, 6]. In the frequency domain exists only one expression for the characteristic impedance because the frequency domain provides the way to encompass the various line parameters as one impedance or admittance. But in this time-domain model are two characteristic impedances because the primary parameters come from different physical properties of the line. Since R and G describe the energy dissipation or line losses, they are included in a characteristic impedance, while L and C describe the storage of energy in the fields around the line, they are encompassed by a second characteristic impedance.

In order to illustrate the previous idea, the particular case of DC systems can be analyzed using (23). Thus, the terms where Z_{LC} appears tend to decrease for long time intervals since it exists a limit to the energy stored in the fields of the line. Also, as the line losses are always present, the terms where Z_{RG} appears do not decrease over time. This condition would be maintained as long as the voltages and currents do not vary. When variations occur in the system, the energy stored in the fields also changes and the terms related to Z_{LC} reappear, until the system is stabilized again in another operational point. Thus, the terms where Z_{LC} appears can be associated with states where there are variations of voltage and current, while the terms where Z_{RG} appears can be associated with the states where the voltage and current are constant.

Other line models which are entirely deduced from time-domain, have also one of these two characteristic impedances, but no previous model has both terms. The presence of one or other characteristic

impedance depends on the type of simplification used in the deduction of each different model. This model is able to take into account the full range of characteristic impedances and propagation coefficient, since Z_{LC} , Z_{RG} , and γ_{RG} are not approximated values of characteristic impedances and propagation coefficient in frequency-domain; instead, they are time-domain decoupled representations of the characteristic impedances and propagation coefficient in frequency-domain.

In this article, several sections will be dedicated to a more detailed analysis of the model in order to validate its behavior.

4. PRACTICAL EXAMPLES AND NUMERICAL RESULTS

In the next section, real transient-state records of different transmission lines will be analyzed with the model deduced here in order to validate the model's behavior. The results will be compared to the Bergeron's model which is the line model most widely used. Tests with records of AC and DC systems were carried out in order to show that the model is applicable to different types of lines.

The tests consist in taking the voltage and current from transient event records of the sender line end, and with this information try to predict what is happening at the receiver end. This is done both with the model deduced here and with the Bergeron's model. Then, the results are compared with real records of the receiver end in order to show which of the two models can predict more accurately the actual behavior of the lines.

4.1. AC Line Tests

In order to use real transient event records, two faults in actual AC lines were selected for this test. One fault in a long 765 kV line with 153 km, and another fault in a short 230 kV line with 44 km. The faults are single-phase and they occur outside the lines analyzed, i.e., faults occurred on the receiver end of parallel lines, so they are not fault records but contribution records to an external fault. The reason for selecting these records is to avoid the change in power flow that occurs in the receiver end. If the fault was within the line, this flow change would prevent the calculation of receiver end signals during the fault and could not observe the model's accuracy during different system conditions (pre-fault, fault and post-fault). For AC systems testing, the signals instantaneous values were used.

There are some circumstances that have not been taken into account and that could affect the model's accuracy in a real faults

analysis of an AC system. One is that the most common problems in real faults analysis are possible inaccuracies in the line parameters. In this test, any problems of this type would affect in the same way both models and would not affect the final results because this test seeks to compare these results to each other.

On the other hand, the model deduced here is based on a single-phase line, while AC systems are triphasic, and there are mutual effects from other phases; this problem is corrected using decoupled parameters, in other words, in order to implement this model in three phase systems, the phase domain signals are first decomposed into their modal components [18]. In this study, all line models are assumed to be fully transposed and, therefore, a transformation matrix is used.

$$S_{\text{mode}} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \end{bmatrix} S_{\text{phase}} \quad (24)$$

where S_{phase} and S_{mode} are the phase signal and mode signal components, respectively. Simulated records phase signals are first transformed into their modal components and the mode 2 is taken for analysis. The second mode (mode 2), also known as the aerial mode, is the most common mode used in this type of analysis since is present for any kind of fault.

Tables 1 and 2 show the errors percentage obtained when comparing the results of both models with actual measured values. These errors are in percentages (%) based on the nominal values of

Table 1. Test average errors with 765 kV line records.

	Voltage Error (%)		Current Error (%)	
	BM	DM	BM	DM
Pre-fault	12.541	2.637	5.298	2.470
Fault	14.506	3.061	6.502	3.960
Post-fault	12.072	3.695	5.788	3.333

Table 2. Test average errors with 230 kV line records.

	Voltage Error (%)		Current Error (%)	
	BM	DM	BM	DM
Pre-fault	1.492	1.480	3.139	3.106
Fault	0.730	0.727	2.771	2.693
Post-fault	1.360	1.345	3.079	3.041

each line. They are an average of the instantaneous values of each stage that make up the records.

Also, in order to illustrate the tests, Figs. 4 and 5 are the results of voltage and current in the faulted phase of the records analyzed, these graphics show the results obtained with both models superimposed on the receiver end of the actual records.

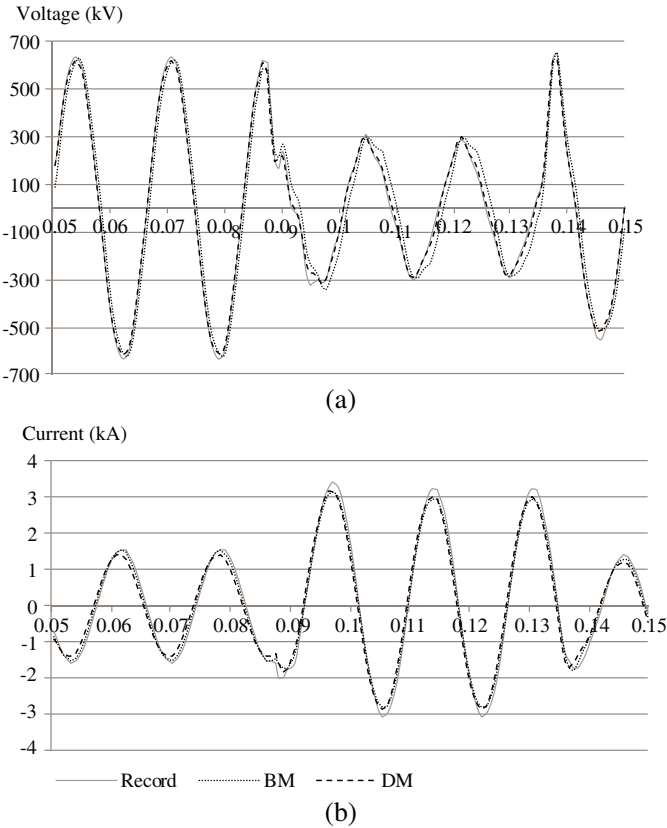


Figure 4. 765 kV line actual record compared with Bergeron's model (BM) and with the developed model (DM). (a) Voltage, (b) current.

The largest errors were obtained in long line tests using the Bergeron's model. In this test, the model presented here showed to be more accurate because it is better adapted for the modeling of long lines.

In the short line test, the results obtained with both models are similar and they are more precise than in the long line test. This is because the approximations used in Bergeron's model apply better to

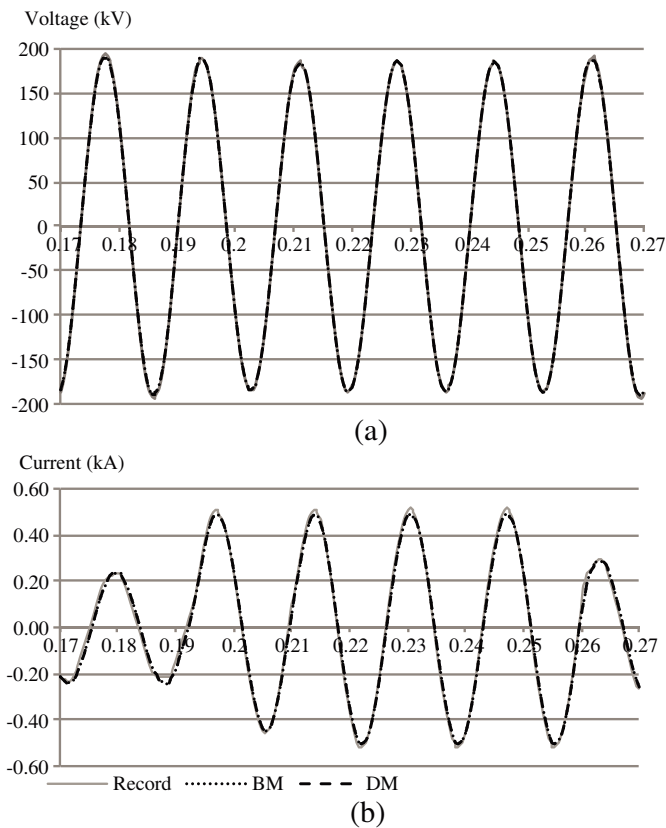


Figure 5. 230 kV line actual record compared with Bergeron's model (BM) and with the developed model (DM). (a) Voltage, (b) current.

this type of line than with long lines. Even so, the model deduced here was slightly more accurate than the Bergeron's model.

4.2. DC Line Tests

In the following item, simulated records of different DC lines are analyzed with the deduced model. The results are compared with the Bergeron's model. Like in the previous item, the tests consist in taking the voltage and current records of the sender-end line, and with this information try to predict what is happening at the receiver-end. The results of both models are compared with the records produced to the receiver-end in order to see which model can predict more accurately the actual behavior of the line.

For this test, were simulated transient events of two DC lines. A long line with 900 km and a short line with 150 km. The transient event used was the line being energized. The energizing was simulated without any control so that the lines pass from zero to 1 p.u. (sender end) in a natural way. This gives the opportunity to analyze two different system conditions: first, a transitional period when the line is energized, and then a steady-state period when the line reaches a value close to 1 p.u. (on the receiver end).

Tables 3 and 4 show the percentage of errors obtained when comparing the results of both models with the receiver end values. They are an average of the instantaneous values of each period that make up the records.

Also, in Figs. 6 and 7 are presented the results for voltage and current for the lines energizing process, these graphics show the results obtained with both models superimposed on the receiver end records.

The errors shown in Tables 3 and 4 are much smaller than those

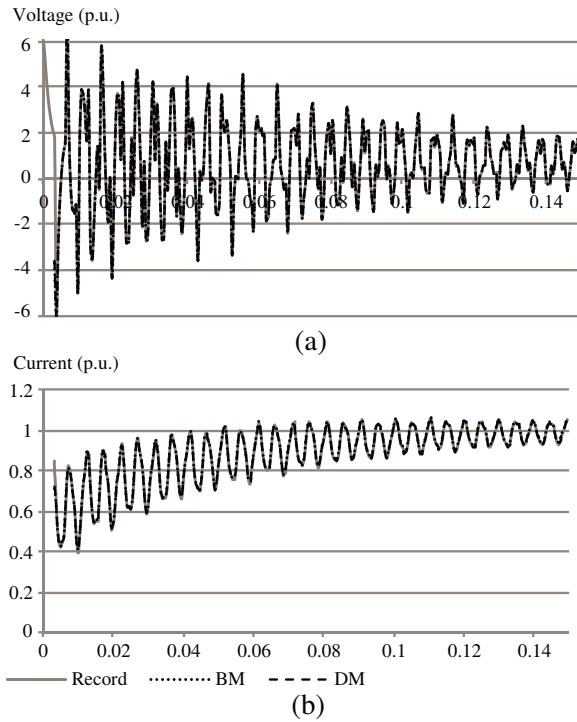


Figure 6. Long DC line record compared with Bergeron's model (BM) and with the developed model (DM). (a) Voltage, (b) current.

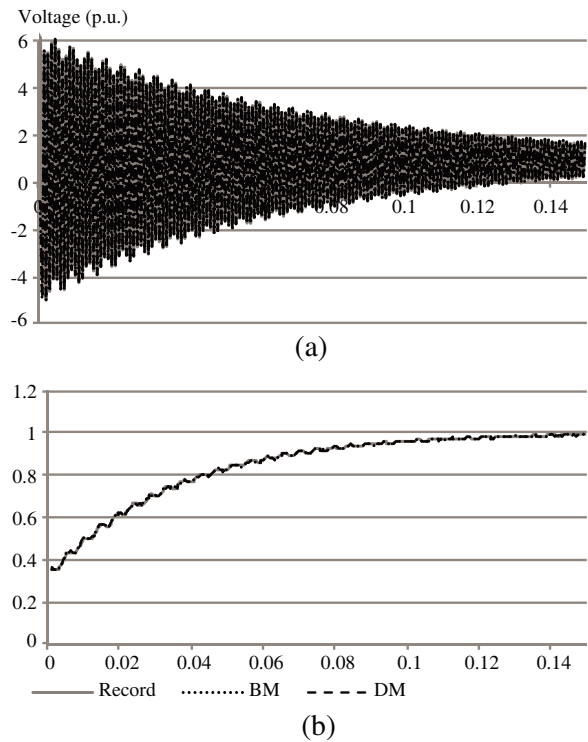


Figure 7. Short DC line record compared with Bergeron’s model (BM) and with the developed model (DM). (a) Voltage, (b) current.

Table 3. Tests’ average errors with the long line records.

	Voltage Error (%)		Current Error (%)	
	BM	DM	BM	DM
Transient	0.789	0.662	0.820	0.558
Steady-state	0.437	0.426	0.410	0.140

Table 4. Tests’ average errors with the short line records.

	Voltage Error (%)		Current Error (%)	
	BM	DM	BM	DM
Transient	0.421	0.421	0.178	0.175
Steady-state	0.045	0.045	0.044	0.017

of Tables 1 and 2, because the tests with simulated data have a better control of the accuracy of the line parameters.

In the long-line analysis (Table 3), is possible to see that the model presented here is more accurate than Bergeron's model. While in the short-line analysis (Table 4), the results of both models are virtually identical in most cases. Like in the other test, this is because the approximations used in Bergeron's model apply better to short lines than long lines.

5. CONCLUSIONS

This article presented the basics of a general transmission line and the development of the time-domain model for a single-phase line. Also, there are presented some analysis using this time-domain transmission line model. This analysis increases the credibility of the presented model. This article also proves that this model is more accurate than Bergerons model in describing the behaviour of different transmission lines. Since the most widely used models, including models with frequency-dependent parameters, use an approach which retains the basic idea behind Bergeron's method, this more detailed model could be used to improve other models.

The added value of this model is the adoption of the distributed characteristics of all line parameters, which allows a more accurate description of the transmission line behavior in time-domain.

These are some initial tests that were performed to validate the model. This article shows the deduction of a model for fundamental frequency parameters and single-phase, but the model can also be extended for cases with multiconductor and frequency depended parameters. In this regard, the model has potential for improvements in areas such as electromagnetic transient simulation or any other analysis that requires a high accuracy in predicting the transmission lines behavior in time-domain.

APPENDIX A. EQUIVALENT SYSTEM DATA

Equivalent system data used in the simulated record test of Section 4 is shown here.

For the simulation of the energizing process of two DC lines, it was used the SimPowerSystem module of the Simulink/MatLab software. The sample time intervals are of 0.5 ms. The system parameters are shown below:

Table A1. Transmission lines parameters.

	Long Line	Short Line
Length (km)	900	150
Resistance (Ω/km)	0.011	0.011
Inductance (mH/km)	0.832	0.832
Capacitance (pF/km)	13.41	13.41
Conductance (pS/km)	27.668	27.668

Table A2. Equivalent system parameters.

	Sender end	Receiver end
Resistance (Ω)	0.001	30
Inductance (H)	0.5	0.5

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