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MUTUAL COUPLING CALIBRATION FOR ELECTROMAGNETIC VECTOR SENSOR

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Abstract—A subspace self-calibration ESPRIT algorithm for mutual coupling across an electromagnetic vector sensor is proposed in this paper. By introducing an auxiliary array element, the mutual coupling is calibrated. The whole array's mutual coupling matrix can be obtained simultaneously. A mathematic model for mutual coupling across the six collocated antennas of an electromagnetic vector sensor is established. And the solution of mutual coupling matrix was transformed into the solution of several matrix elements. The Cramer-Rao Bound (CRB) is also derived in the end of this paper to verify the efficacy of the proposed algorithm. The simulation results demonstrate that this approach is correct and effective.

1. INTRODUCTION

1.1. Basic Principles Underlying the New Algorithm

1.1.1. Electromagnetic Vector Sensor

A six-component electromagnetic vector sensor (EMVS) consists of three spatially co-located identical but orthogonally oriented, electrically short dipoles and magnetically small loops separately measuring all three electric-field components and three magnetic-field components of incident signals. Such an electromagnetic vector sensor can exploit any polarization diversity among the impinging sources. Diverse six-component arrays have been exploited in a number of direction finding algorithms. The first direction finding algorithms explicitly exploiting all six electromagnetic components seem to have been developed separately by Nehorai & Paldi [1, 2]

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and Li [3]. Nehorai & Paldi [1, 2], who coined the term "vector sensor", pioneered the simple but novel idea that directly estimated the two-dimensional radial direction of a source by using the vector cross-product of electric- and magnetic-field vector estimates. The vector cross-product estimator was first adapted to ESPRIT by Wong & Zoltowski [4, 12] in their extended-aperture vector-sensor ESPRIT algorithm that extended array aperture for sparsely but arbitrarily spaced vector sensors. The DOA and polarization estimation of polynomial-phase signal was studied by Yuan [15, 16]. Many other direction finding methods using a single vector sensor or arbitrarily spaced vector sensor arrays were proposed in [5–17, 34, 35, 37]. Such a six-component electromagnetic vector sensor has many advantages of direction-of-arrival (DOA) estimates, which were summarized by Wong & Yuan [17]. Besides, an "incomplete" EMVS such as tripole and crossed dipole comprises only a subset of the above-mentioned six EMVS and is of great interest in some practical applications [28, 29, 31–33, 36, 38]. Nehorai and Paldi have worked out the Cramer-Rao bound (CRB) on DOA estimation of stochastic sources [2], and Yuan has derived the closed-form CRB on DOA and polarization estimation of six-component non-collocating and three-component collocating EMVS [6, 30].

1.1.2. Mutual Coupling Across the Six Collocated Antennas of an EMVS

In array signal processing, mutual coupling between antenna elements is well known as an undesired effect, which degrades the performance of array signal processing algorithms. The compensation of such an undesired effect has been a popular research topic throughout the years. Various approaches for scalar antenna arrays' mutual-coupling calibration have been developed, and they can easily be found in the open literature [18–25]. Wong & Yuan used spatially noncollocating six-component EMVS to decreases the mutual coupling across the six collocated antennas [17]. However, there are few literatures on mutual coupling calibration methods for vector sensor or vector sensor array. It is well known that mutual coupling matrix of a scalar array always has Toeplitz matrix, cyclic matrix or other special matrix structure [26, 27]. For the vector sensor, the mutual coupling among vector sensor is same as that of scalar antenna arrays, but mutual coupling across the six collocated antennas is different from scalar antenna arrays. The methods that developed to calibrate mutual coupling of scalar array do not adapt to a vector sensor array. It is necessary to conduct research on mutual coupling calibration basing on the special space structure of the six-component electromagnetic vector sensor. The mutual coupling among the vector sensors is ignored, which is based on the following three reasons: (1) it has the same structure as scalar array. (2) The interval between vector sensors is always far larger than half wavelength. (3) Many applications are needs arbitrarily spaced electromagnetic vector sensor at unknown location.

1.2. The Main Contribution of This Paper

In this paper, a mathematic model of mutual coupling across the six collocated antennas of an electromagnetic vector sensor is established. A subspace self-calibration ESPRIT algorithm for mutual coupling across an electromagnetic vector sensor is proposed in this paper. This proposed algorithm offers closed-form solutions of mutual coupling, whereas most other algorithms for scalar array would require iterative searches (such as the MUSIC algorithm). However, this proposed algorithm: 1) is computationally less intensive than many open-form search methods; 2) produces closed-form unambiguous mutual coupling estimation with no extra computation needed for disambiguation; 3) gives a mutual coupling matrix model for a vector sensor based on the basic principles of antennas array and electromagnetics.

2. PROBLEM FORMULATION

An ideal electromagnetic vector sensor called auxiliary element was introduced in this paper. Receiving array is an arbitrarily spaced three-dimensional array consisting of the auxiliary element and $N-1$ EMVS with mutual coupling. Assume that a narrow-band, transverse electromagnetic (TEM) plane-wave signal having traveled through a non-conductive homogenous medium impinge upon the receiving array.

The auxiliary element is idealized, by overlooking all mutual coupling among its six collocated constituent antennas, as shown in Fig. 1. Such an idealized vector sensor's array manifold (also called unit-power electromagnetic field vector) would be a concatenation of the 3×1 electric-field vector $\mathbf{e}$ with the 3×1 magnetic-field vector $\mathbf{h}$, to be [4]:

$$\mathbf{a}(\theta,\phi,\gamma,\eta) = \begin{bmatrix} e_x \\ e_y \\ e_z \\ h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & -\sin\phi \\ \cos\theta\sin\phi & \cos\phi \\ -\sin\theta & 0 \\ -\sin\phi & -\cos\theta\cos\phi \\ \cos\phi & -\cos\theta\sin\phi \\ 0 & \sin\theta \end{bmatrix} \begin{bmatrix} \sin\gamma e^{j\eta} \\ \cos\gamma \end{bmatrix} \quad (1)$$

where $\theta \in [0, \pi]$ is the signal's elevation angle measured from the positive z-axis; $\phi \in [0, 2\pi]$ denotes the azimuth angle measured form the positive x-axis; $\gamma \in [0, \pi/2]$ represents the auxiliary polarization angle; $\eta \in [-\pi, \pi]$ symbolizes the polarization phase difference. Mutual coupling depends on the shape of antennas and the relative position between antennas. As shown in Fig. 1, the three dipoles No. 1, 2, 3 are identical but orthogonally oriented and the three loops No. 4, 5, 6 are also identical but orthogonally oriented.

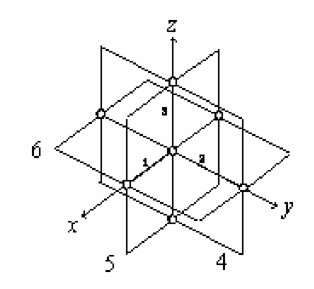

Figure 1. A six-component electromagnetic vector sensor.

Suppose:

(a) There is no amplitude and phase error among the six elements of an electromagnetic vector sensor.

(b) Electromagnetic vector sensor is used as a receiving array so the dipole antennas are considered as electric dipoles and the little loops as magnetic dipoles.

Electromagnetic vector sensor is composed of three orthogonally oriented short dipoles and three orthogonally oriented magnetic loops, all co-located in space [4, 12], measuring the full six components of electric field and magnetic field at one point in the space [11, 39]. The coupling model of this paper is established on the basis of the EMVS model of literature [4, 28]. The intensity of the electric field and the intensity of the magnetic field are usually not the same for dipoles and loops, but it have no use for collocating EMVS because of the very limit antenna interface space for the channels of the receiver [11, 39].

Under above assumption, the electromagnetic field has the form of

$$E_i(r) \approx H_i(r) \approx \frac{c}{r^3}$$

Based on the basic principles of antenna array and electromagnetics, the mutual coupling among the six elements have the following relationship:

$$\begin{aligned} M_{12} &= M_{13} = M_{23} = C, \\ M_{45} &= M_{46} = M_{56} = C, \quad M_{14} = M_{25} = M_{36} = B; \end{aligned}$$

$$M_{15} = M_{16} = M_{24} = M_{26} = M_{34} = M_{35} = D;$$
$$M_{11} = M_{22} = M_{33} = M_{44} = M_{55} = M_{66} = A$$

Based on reciprocity principle, we get

$$\begin{aligned} M_{21} &= M_{31} = M_{32} = C, \quad M_{54} = M_{64} = M_{65} = C, \\ M_{41} &= M_{52} = M_{63} = B; \\ M_{51} &= M_{61} = M_{42} = M_{62} = M_{43} = M_{53} = D; \end{aligned}$$

According to the basic principles of antenna array and electromagnetics and reciprocity principle, the mutual coupling matrix of the l-th ($l = 1, \ldots, N-1$) element can be expressed as:

$$\mathbf{A}_{rl} = \begin{bmatrix} A_l & C_l & C_l & B_l & D_l & D_l \\ C_l & A_l & C_l & D_l & B_l & D_l \\ C_l & C_l & A_l & D_l & D_l & B_l \\ B_l & D_l & D_l & A_l & C_l & C_l \\ D_l & B_l & D_l & C_l & A_l & C_l \\ D_l & D_l & B_l & C_l & C_l & A_l \end{bmatrix} \tag{2}$$

where A_l, B_l, C_l, D_l symbolize different mutual coupling variables. As mutual coupling across six collocated constituent antennas is taken into account, the unit-power electromagnetic field vector receiving by the l-th sensor is no longer $\mathbf{a}$ (which has the form of formula (1)) but $\mathbf{a}'$, their relation can be expressed as:

$$\mathbf{a}' = \begin{bmatrix} A_l & C_l & C_l & B_l & D_l & D_l \\ C_l & A_l & C_l & D_l & B_l & D_l \\ C_l & C_l & A_l & D_l & D_l & B_l \\ B_l & D_l & D_l & A_l & C_l & C_l \\ D_l & B_l & D_l & C_l & A_l & C_l \\ D_l & D_l & B_l & C_l & C_l & A_l \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} \tag{3}$$

According to matrix theory, Equation (3) can be rewritten as:

$$\mathbf{a}' = \underbrace{\begin{bmatrix} e_1 & h_1 & e_2 + e_3 & h_2 + h_3 \\ e_2 & h_2 & e_1 + e_3 & h_1 + h_3 \\ e_3 & h_3 & e_1 + e_2 & h_1 + h_2 \\ h_1 & e_1 & h_2 + h_3 & e_2 + e_3 \\ h_2 & e_2 & h_1 + h_3 & e_1 + e_3 \\ h_3 & e_3 & h_1 + h_2 & e_1 + e_2 \end{bmatrix}}_{\stackrel{def}{=} \mathbf{C}} \underbrace{\begin{bmatrix} A_l \\ B_l \\ C_l \\ D_l \end{bmatrix}}_{\stackrel{def}{=} \mathbf{g}_l} \tag{4}$$

Eigen-structure-based direction finding techniques such as MUSIC or ESPRIT have superior performance compared to conventional direction finding methods. However, the performance depends strongly on the accuracy of the array manifold. There is an urgent need to develop techniques for electromagnetic vector sensor mutual coupling calibration.

3. MUTUAL COUPLING CALIBRATION ALGORITHM

3.1. Mathematic Model

Let us define the space phase factor for a vector sensor centered at location (x_l, y_l, z_l) relative to the coordinate origin:

$$q_l = e^{j\frac{2\pi}{\lambda}(x_l \sin\theta\cos\phi + y_l \sin\theta\sin\phi + z_l \cos\theta)} \tag{5}$$

Two time-delayed sets of data collected from the vector sensor, that is, a monochromatic signal characterized by $(\theta, \phi, \gamma, \eta, f)$ impinging upon the vector sensor would contribute toward two $6 \times M$ data sets

$$X_l(t_n) = \mathbf{a}s(t_n)\,q_l \quad n = 1, \ldots M \quad s(t_n) = Ee^{j(2\pi f t_n + \varphi)} \tag{6}$$

$$X_l(t_n + \nabla T) = \mathbf{a}s(t_n + \nabla T)\,q_l$$

$$s(t_n + \nabla T) = Ee^{j(2\pi f(t_n + \nabla T) + \varphi)} = e^{j2\pi f \nabla T} s(t_n) \tag{7}$$

With E the amplitude and f the frequency of the signal, φ the carrier phase of the signal at the coordinate origin at $t = 0$.

Assume that K such signals are incident on the array. A thermal noise voltage vector $\mathbf{n}_i(t)$ is present on each EMVS. The $\mathbf{n}_i(t)$ are assumed to be zero mean, complex Gaussian processes statistically independent of each other with covariance $\sigma^2\mathbf{I}$, where $\mathbf{I}$ denotes the identity matrix. The $\mathbf{n}_i(t)$ is also assumed to be statistically independent of the incident signals. Under these assumptions, the total signal vector received by the vector sensor array has the flowing form:

$$\mathbf{Z}(t) = \mathbf{bs}(t) + \mathbf{n}(t) \tag{8}$$

$\mathbf{Z}(t), \mathbf{s}(t)$ and $\mathbf{n}(t)$ be column vectors containing the received signals, incident signal, and noise, respectively, i.e.,

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{z}_0(t) \\ \mathbf{z}_1(t) \\ \vdots \\ \mathbf{z}_{N-1}(t) \end{bmatrix}, \ \mathbf{n}(t) = \begin{bmatrix} \mathbf{n}_0(t) \\ \mathbf{n}_1(t) \\ \vdots \\ \mathbf{n}_{N-1}(t) \end{bmatrix}, \ \mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_K(t) \end{bmatrix} \tag{9}$$

$\mathbf{b}$ is the $12N \times K$ array manifold for the entire N-element electromagnetic vector-sensor array:

$$\mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} \quad \mathbf{b}_2 = \mathbf{b}_1\mathbf{\Phi} \tag{10}$$

$$\mathbf{b}_1 = \begin{bmatrix} \mathbf{a} \\ \mathbf{a}'_1 q_1 \\ \vdots \\ \mathbf{a}'_{N-1} q_{N-1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{r0} & & & \\ & \mathbf{A}_{r1} & & \\ & & \ddots & \\ & & & \mathrm{A}_{rN-1} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{a}q_1 \\ \vdots \\ \mathbf{a}q_{N-1} \end{bmatrix} \tag{11}$$

where $\mathbf{A}_{r0}$ is the coupling matrix of the auxiliary element, it is a 6×6 identity matrix, $\mathbf{A}_{rl}$ is mutual coupling matrix of the l-th array element. where $\mathbf{\Phi} = \mathrm{diag}\left(\left[\exp\left(j\left(2\pi f_1 \nabla T\right)\right), \ldots, \exp\left(j\left(2\pi f_K \nabla T\right)\right)\right]\right)$ is full-rank matrix.

3.2. Estimation of Mutual Coupling Variables

The procedure of mutual coupling variable estimation is fall into 4 steps:

Step 1: calculate the correlation matrix of overall data $\mathbf{Z}(t)$.

$$\mathbf{R}_{ZZ} = \mathbf{E}\left[\mathbf{Z}(t)\,\mathbf{Z}^H(t)\right] = \mathbf{b}R_s\mathbf{b}^H + \sigma^2\mathbf{I} \tag{12}$$

with $\mathbf{E}[\cdot]$ symbolizing the statistical mean, $(\cdot)^H$ denoting the complex conjugate transpose and $R_s = \mathbf{E}\left[\mathbf{s}(t_1)\,\mathbf{s}^H(t_1)\right]$ representing the source covariance matrix.

Step 2: eigen-decompose the overall data correlation matrix $\mathbf{R_{ZZ}}$. In eigen-structure direction finding methods such as ESPRIT, the overall data correlation matrix is decomposed into a signal subspace and a noise subspace.

$$\mathbf{R}_{ZZ} = \sum_{i=1}^{6N} \lambda_i \mathbf{V}_i \mathbf{V}_i^H \tag{13}$$

where $\mathbf{R_{ZZ}}$ is a $12N \times 12N$ data correlation matrix, $\lambda_1 > \lambda_2 \ldots \lambda_K > \lambda_{K+1} = \ldots = \lambda_{12N}$ are the eigenvalues of $\mathbf{R_{ZZ}}$. Let $\mathbf{E}_s$ denotes the signal subspace whose columns are the eigenvectors corresponding to the K large eigenvalues of $\mathbf{R_{ZZ}}$. Then there exists a unique nonsingular $\mathbf{T}$ such that $\mathbf{E}_s = \mathbf{bT}$.

Step 3: estimate signal's electric-field and magnetic-field. According to the subspace theory:

$$\mathbf{E}_s = \mathbf{bT} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} \mathbf{T} \quad \mathbf{E}_1 = \mathbf{b}_1\mathbf{T} \quad \mathbf{E}_2 = \mathbf{b}_2\mathbf{T} = \mathbf{b}_1\mathbf{\Phi}\mathbf{T} \tag{14}$$

$$\left(\mathbf{E}_1^H\mathbf{E}_1\right)^{-1}\mathbf{E}_1^H\mathbf{E}_2\mathbf{T}^{-1} = \mathbf{\Phi}\mathbf{T}^{-1} \tag{15}$$

From formula (15) we know that eigenvectors of $\left(\mathbf{E}_1^H\mathbf{E}_1\right)^{-1}\mathbf{E}_1^H\mathbf{E}_2$ constitute the columns of $\mathbf{T}^{-1}$. Thus array manifolds may be estimated as:

$$\hat{\mathbf{b}}_1 = \mathbf{E}_1\mathbf{T}^{-1} \tag{16}$$

Any column of $\hat{\mathbf{b}}_1$ can be used to estimate electromagnetic-field. For generally we choose the first column. Thus, the first signal's electric-field and magnetic-field can be estimated as:

$$\hat{\mathbf{a}}(1:3,1) = \frac{\mathbf{b}_1(1:3,1)}{\|\mathbf{b}_1(1:3,1)\|} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$
$$\hat{\mathbf{a}}(4:6,1) = \frac{\mathbf{b}_1(4:6,1)}{\|\mathbf{b}_1(4:6,1)\|} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \quad (17)$$

Step 4: estimate the mutual coupling variables of the $N-1$ array elements.

From Equations (4) and (17), we obtain matrix $\mathbf{C}$. Substituting Equation (17) into Equation (10) yields

$$\hat{\mathbf{b}}_1(:,1) = \begin{bmatrix} \hat{\mathbf{a}} \\ \mathbf{A}_{r1}\hat{\mathbf{a}}q_1 \\ \vdots \\ \mathbf{A}_{rN-1}\hat{\mathbf{a}}q_{N-1} \end{bmatrix} = \begin{bmatrix} q_0\mathbf{C}\mathbf{g}_0 \\ q_1\mathbf{C}\mathbf{g}_1 \\ \vdots \\ q_{N-1}\mathbf{C}\mathbf{g}_{N-1} \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{P}_0 & & \\ & \ddots & \\ & & \mathbf{P}_{N-1} \end{bmatrix}}_{\overset{def}{=}\mathbf{P}} \underbrace{\begin{bmatrix} \mathbf{g}_0 \\ \vdots \\ \mathbf{g}_{N-1} \end{bmatrix}}_{\overset{def}{=}\mathbf{G}} \quad (18)$$

where $\mathbf{P}_l = q_l\mathbf{C}$, $\mathbf{g}_l = [A_l, B_l, C_l, D_l]^T$, $l = 0, \dots, N-1$, $q_0 = 1$, $\mathbf{g}_0 = [1, 0, 0, 0]^T$.

It is shown in (18) that $\mathbf{G}$ can be expressed explicitly as

$$\mathbf{G} = \left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\hat{\mathbf{b}}_1(:,1) \quad (19)$$

where $\mathbf{G}$ is the least square solution of Equation (18). Normalizing $\mathbf{G}$ yields mutual coupling variables:

$$\hat{\mathbf{G}} = \frac{\mathbf{G}}{\mathbf{g}_1(1)} = [1, 0, 0, 0, A_1, B_1, C_1, D_1, \dots, A_{N-1}, B_{N-1}, C_{N-1}, D_{N-1}]^T \quad (20)$$

Substituting the mutual coupling variables estimates into formula (2) yields $\mathbf{A}_{rl}$. Having identified the true mutual coupling of each vector-sensor by the foregoing calibration algorithm, the whole mutual coupling matrix can be got as

$$\mathbf{A}_r = \begin{bmatrix} \mathbf{A}_{r0} & & \\ & \ddots & \\ & & \mathrm{A}_{rN-1} \end{bmatrix} \quad (21)$$

3.3. Calibration for Vector-sensor Mutual Coupling

The eigen-structure-based direction finding algorithm based on ideal electromagnetic vector sensor array may be modified as follows to accommodate mutual coupling among vector sensors:

$$\tilde{\mathbf{Z}}(t) = \mathbf{A}_r^{-1}\mathbf{Z}(t) \tag{22}$$

After mutual coupling calibration, $\tilde{\mathbf{Z}}(t)$ can be considered as receiving data of ideal EMVS.

4. REFERENCE SIGNAL SELECTION

In order to ensure that Equation (19) exists, $\mathbf{P}$ must be full-rank $\text{rank}(\mathbf{P}) = 4N$. If $\mathbf{C}$ is full-rank, i.e., $\text{rank}(\mathbf{C}) = 4$, then $\mathbf{P}$ is full-rank. The following six signals cannot be choose as the reference signals: $(\theta_1, \phi_1, \gamma_1, \eta_1) = (45°, 0°, 45°, 0° \text{ or } 180°)$, $(\theta_1, \phi_1, \gamma_1, \eta_1) = (45°, 45°, 45°, 0° \text{ or } 180°)$, $(\theta_1, \phi_1, \gamma_1, \eta_1) = (45°, 45°, 45°, 0° \text{ or } 180°)$.

5. CRB OF DOA AND POLARIZATION ESTIMATION IN THE PRESENCE OF MUTUAL COUPLING

With a total of $N_s > 1$ snapshots taken at the distinct times, the correlation matrix is

$$\mathbf{R}(\mathbf{Q}, \mathbf{Y}) = \mathbf{b}R_s\mathbf{b}^H + \sigma^2\mathbf{I}$$

where $\mathbf{b} = \mathbf{PG}$, $\mathbf{Q} = [\theta, \phi, \gamma, \eta]$, $\mathbf{Y} = [A_1, B_1, C_1, D_1, \ldots, A_{N-1}, B_{N-1}, C_{N-1}, D_{N-1}]$.

Let $N_s = 1$. If $N_s \neq 1$, every element of Fisher matrix $\mathbf{F}$ is multiplied by N_s. $\mathbf{F}$ is a $4N \times 4N$ symmetric matrix, and the CRLB of any unbiased estimator is $[\mathbf{F}]_{ii}^{-1}$.

$$\mathbf{F} = \begin{bmatrix} tr\left[\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Q}}\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Q}}\right] & tr\left[\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Q}}\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Y}}\right] \\ tr\left[\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Y}}\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Q}}\right] & tr\left[\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Y}}\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\mathbf{Y}}\right] \end{bmatrix}$$

$$\frac{\partial\mathbf{R}}{\partial\mathbf{Q}} = \left[\frac{\partial\mathbf{R}}{\partial\theta}, \frac{\partial\mathbf{R}}{\partial\phi}, \frac{\partial\mathbf{R}}{\partial\gamma}, \frac{\partial\mathbf{R}}{\partial\eta}\right], \quad \frac{\partial\mathbf{R}}{\partial\theta} = \dot{\mathbf{P}}_\theta\,\mathbf{G}R_s\mathbf{b}^H + \mathbf{b}R_s\mathbf{G}^H\,\dot{\mathbf{P}}_\theta^H$$

$\frac{\partial\mathbf{R}}{\partial\phi}$, $\frac{\partial\mathbf{R}}{\partial\gamma}$ and $\frac{\partial\mathbf{R}}{\partial\eta}$ have the same structure as $\frac{\partial\mathbf{R}}{\partial\theta}$.

$$\dot{\mathbf{P}}_\theta = \frac{\partial\mathbf{P}}{\partial\theta} = \begin{bmatrix} \frac{\partial\mathbf{C}}{\partial\theta}q_1 + \mathbf{C}\frac{\partial q_1}{\partial\theta} & & \\ & \ddots & \\ & & \frac{\partial\mathbf{C}}{\partial\theta}q_{N-1} + \mathbf{C}\frac{\partial q_{N-1}}{\partial\theta} \end{bmatrix}$$

$$\dot{\mathbf{P}}_\gamma = \frac{\partial \mathbf{P}}{\partial \gamma} = \begin{bmatrix} \frac{\partial \mathbf{C}}{\partial \gamma} q_1 & & \\ & \ddots & \\ & & \frac{\partial \mathbf{C}}{\partial \gamma} q_{N-1} \end{bmatrix}$$

The form of $\dot{\mathbf{P}}_\phi$ is similar to $\dot{\mathbf{P}}_\theta$, and $\dot{\mathbf{P}}_\eta$ to $\dot{\mathbf{P}}_\gamma$.

$$\frac{\partial \mathbf{R}}{\partial \mathbf{Y}} = \left[\frac{\partial \mathbf{R}}{\partial A_1}, \frac{\partial \mathbf{R}}{\partial C_1}, \frac{\partial \mathbf{R}}{\partial B_1}, \frac{\partial \mathbf{R}}{\partial D_1}, \ldots, \frac{\partial \mathbf{R}}{\partial A_{N-1}}, \frac{\partial \mathbf{R}}{\partial C_{N-1}}, \frac{\partial \mathbf{R}}{\partial B_{N-1}}, \frac{\partial \mathbf{R}}{\partial D_{N-1}}\right]$$

where $\frac{\partial \mathbf{R}}{\partial C_l}$, $\frac{\partial \mathbf{R}}{\partial B_l}$ and $\frac{\partial \mathbf{R}}{\partial D_l}$ have the same structure as $\frac{\partial \mathbf{R}}{\partial A_l} = \mathbf{P}\,\dot{\mathbf{G}}_{A_l} R_s \mathbf{b}^H + \mathbf{b} R_s \dot{\mathbf{G}}_{A_l}^H \mathbf{P}^H$, with $\dot{\mathbf{G}}_{A_l} = e_{4l+1}$, $\dot{\mathbf{G}}_{C_l} = e_{4l+2}$, $\dot{\mathbf{G}}_{B_l} = e_{4l+3}$, $\dot{\mathbf{G}}_{D_l} = e_{4l+4}$ and with e_i denotes a $4N \times 1$ vector whose i-th ($i = 4l+1, 4l+2, 4l+3, 4l+4$) element is 1 and the other elements are zero.

6. SIMULATIONS

In this section, some simulations are conducted to verify the efficacy of the proposed mutual coupling calibration algorithm for six-component electromagnetic vector array.

[Experiment 1] we assume that the reference signal source impinge upon a 4-element irregularly spaced 3-D array with elements at the Cartesian coordinates $\left(\frac{\lambda}{2}\right) \times \{(0,0,0), (1,0,0), (0,2.7,0), (0,0,1)\}$, the reference signal source has the following parameter values: $(\theta, \phi, \gamma, \eta) = (45°, 50°, 30°, 90°)$. Suppose that the mutual coupling variable A_l obey a uniform distribution over the interval $[0.8i, 1.2i]$, and variables B_l, C_l, D_l are uniformly distributed in the interval $[-0.2i, 0.2i]$. For each specific SNR, 500 snapshots are used and 200 Monte Carlo trials are carried out.

The curves with circular points in Figs. 2 and 3, respectively, plot the elevation angle and azimuth angle estimated by the proposed method, at different signal-to-noise ration (SNR) levels. In the SNR range (namely, at or above 0 dB) the biases of DOA are less than 0.5° in elevation angle and 0.2° in azimuth angle. The curves with little triangles in Figs. 2 and 3, respectively, plot the elevation angle and azimuth angle estimated by the method without taking mutual coupling into account. The biases are larger than 1° and they both do not decrease with increasing SNR decibel values.

Figures 4, 5, and 6, respectively plot elevation angle, azimuth angle and mutual coupling estimation standard deviation versus SNR. Comparing standard deviation of the proposed method to that of CRLB, it shows that the difference between them becomes less and

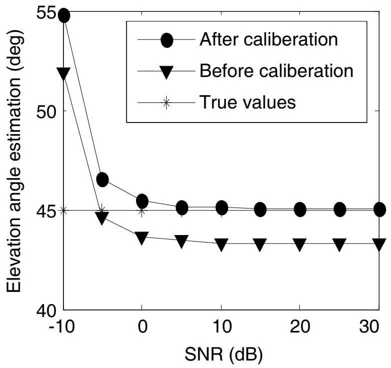

Figure 2. Elevation estimation versus SNR.

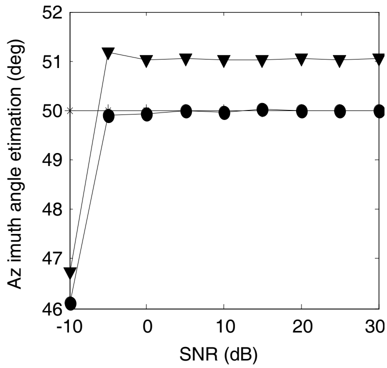

Figure 3. Azimuth estimation versus SNR.

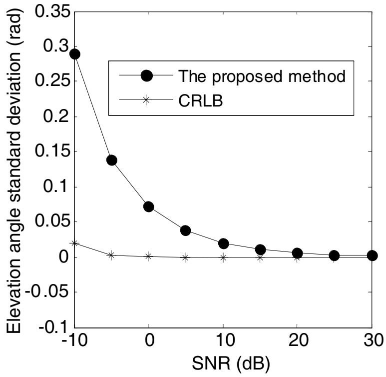

Figure 4. Elevation angle standard deviation versus SNR.

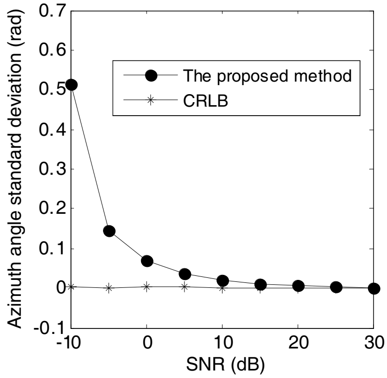

Figure 5. Azimuth angle standard deviation versus SNR.

less with the increasing SNR decibel values. It verifies the efficacy of the proposed mutual coupling calibration algorithm for six-component electromagnetic vector array.

Figure 7 plots the estimation of DOA before and after calibration at SNR $= 15\,\text{dB}$. It shows that before calibration the mean of DOA estimation is $(43°, 51°)$ which deviates from the true value $(45°, 50°)$, while after calibration it is nearly the same as the true value.

[Experiment 2] MUSIC spatial spectrum versus azimuth angle and elevation angle are given to confirm the performance of the mutual coupling calibration method.

The simulation settings are listed as follows: $(\theta_1, \gamma_1, \eta_1) = (\theta_2, \gamma_2, \eta_2) = (40°, 40°, 70°)$, $\phi_1 = 10°$, $\phi_2 = 60°$ in Figure 8; $(\phi_1, \gamma_1, \eta_1) = (\phi_2, \gamma_2, \eta_2) = (40°, 40°, 70°)$, $\theta_1 = 10°$, $\theta_2 = 60°$ in

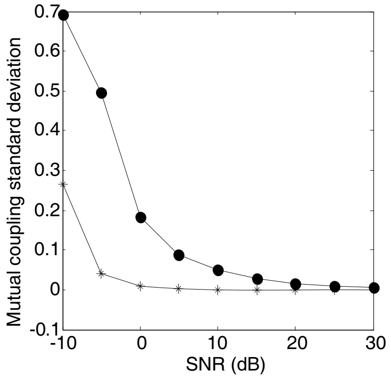

Figure 6. Mutual coupling standard deviation versus SNR.

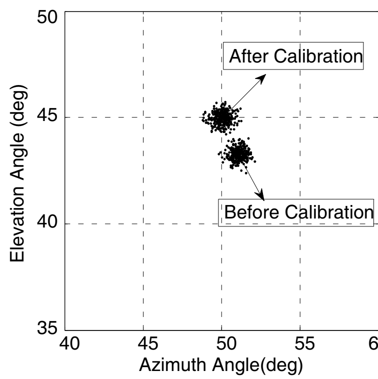

Figure 7. Estimation of DOA equation before and after calibration.

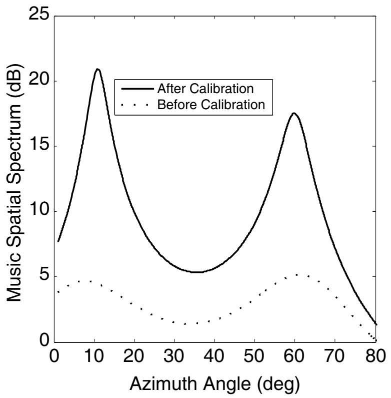

Figure 8. Music spatial spectrum versus azimuth angle.

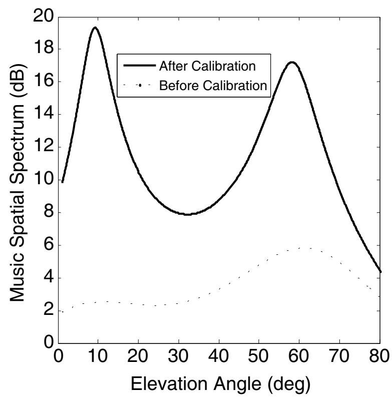

Figure 9. Spatial spectrum versus elevation angle.

Figure 9. For $SNR = 0\,\mathrm{dB}$, 500 snapshots are used and 200 Monte Carlo trials are carried out. From these estimates, it is evident that before calibration MUSIC spatial spectrum algorithm fails to resolve the two signals. The proposed mutual coupling calibration method, however, achieved a consistent resolution performance, more selective peaks that coincide with the true source's DOAs.

7. CONCLUSION

This paper introduces a novel mutual coupling calibration and correction algorithm offering closed-form solutions of mutual coupling to arbitrarily spaced arrays of electromagnetic vector-sensors. To calibrate the mutual coupling across the six collocated antennas of electromagnetic vector sensor, a mathematic model is established. The CRLB for mutual coupling are derived and compared to the proposed method. This innovational approach uses an auxiliary element and a reference signal source to estimate the mutual coupling of the whole array simultaneously. The proposed algorithm requires no a priori information of the reference signal source, and does not need to search and match operation that would decrease considerably to the computational load.

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