

MAGNETO-KINEMATICAL AND ELECTRO-KINEMATICAL FIELDS

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Abstract—The problem of the motion of a magnetic field due to the motion of a permanent magnet has been subject of scientific controversy for many decades. However, the similar question, pertaining to the motion of an electric field due to the motion of a permanent charge, has been neglected by the scientific community, tacitly admitting this specific purely kinematical phenomenon. Such an evidently skew position is under theoretical consideration on an experimental ground. It is shown a profound symmetry between electro-kinematics on the one hand and magneto-kinematics on the other, and also the radical dissimilarity of both from electrodynamics.

1. INTRODUCTION

In previous works [1, 2] we presented the experimental results proving an existence in Nature of the “dragging” of a magnetic field \mathbf{B} by moving at a speed \mathbf{v} permanent magnet (magneto-kinematical Zajev-Dokuchajev effect), which induces in the laboratory electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$ (in the SI units). It would be reasonable to expect natural existence of a symmetric electro-kinematical phenomenon, conjugal to the Z-D effect. J. C. Maxwell dreamed and hoped to ‘*at least verify our supposition that a moving electrified body is equivalent to an electric current*’ [3, p. 370, article 770]. Moreover, he suggested the real scheme of a relevant experimentation, but... ‘*The unified view of electricity and magnetism which was then emerging from Maxwell’s work suggested that any moving charge ought to cause a magnetic field, but experimental proof was hard to come by*’ [4, p. 241].

Shortly after these opinions have been published an American physicist Henry Rowland, working ‘*in the laboratory of the Berlin*

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University through the kindness of Professor Helmholtz' [4, p. 242], first qualitatively showed an electro-kinematical effect in his experiments with a charged disk (vulcanite plate 21.1 cm in diameter gilt on both side), revolving with a angle velocity of 61 rev/sec. His article titled 'On the Magnetic Effect of Electric Convection' was published in [5]. The scheme of the Rowland's apparatus is given in [4, p. 242]. The field subject to measurement was a million times less than the earth's magnetic field in the laboratory. In the chapter 9 entitled '*Maxwell's equations for moving media*' Panofsky and Phillips write: '*Hence the moving polarized dielectric will produce a magnetic field which is indistinguishable from that of a magnetized material. This effect has been demonstrated experimentally by Roentgen, Eichenwald, and others*' [6, p. 165]. At the beginning of the 20-th century A. A. Eichenwald (a Russian physicist of the German origin) fulfilled a series of painstakingly organised experiments [7]. He worked with the "Rowland's current" and the "Roentgen's current" and quantitatively corroborated the existence of the magnetic field $\mathbf{B} = \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$ (too small in value because of the square light speed in the denominator), induced in the laboratory by moving at a speed \mathbf{v} charged (or polarized) rigid bodies, so called convection currents. It would be fair to name this physical phenomenon the Rowland-Eichenwald effect (R-E effect).

Both fields (appearing in the Z-D effect and in the R-E effect) are kinematically sourced and so its properties likely differ in comparison to those of ordinarily providing magnetostatic and electrostatic fields. Inasmuch as the laws determining appearance of these fields are known from quantitative laboratorial experiments, the following step into researching may be done in a pure theoretical manner. In this paper we proceed from easy achieved physical sources having the well known analytically described fields. All the velocities \mathbf{v} dealing with are supposed to be big enough in the common sense, but much less than the light speed: $v \ll c$.

Let (x', y', z') be the proper frame of reference, where a source of the static field \mathbf{F}' is at rest. It may be either a stable permanent magnet with a magnetisation \mathbf{M} , providing flux density \mathbf{B} as shown in Figure 1(a), or a conducting surface with a charge Q , providing electric strength \mathbf{E} , Figure 1(b). This field is space variable and may be described by a function $\mathbf{F}(\mathbf{r}')$, where $\mathbf{r}' = (x', y', z')$ is a radius-vector with coordinates x', y', z' . In the laboratory reference frame with coordinates x, y, z there is a stable closed circuit and the magnet moves through it at a constant velocity $\mathbf{v} = (v_x, v_y, v_z)$. In every point $\mathbf{r} = (x, y, z)$ of the laboratory system we have the time-variable field

$$\begin{aligned} \mathbf{F}(\mathbf{r}) &= \mathbf{F}(x, y, z) \\ &= \mathbf{F}'(ax + by + cz + d - v_x t, ex + fy + gz + h - v_y t, ix + jy + kz + l - v_z t) \end{aligned}$$

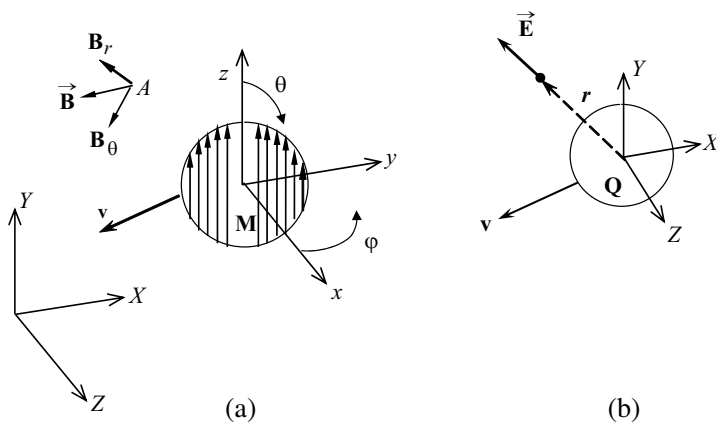


Figure 1.

$$= \mathbf{F}'(x'y'z').$$

Here the values $a, b, c, d, e, f, g, h, i, j, k, l$ are constant coefficients. The function $\mathbf{r}'(\mathbf{r})$ is $\mathbf{r}' = \mathbf{r} - \mathbf{v}t$, where the moment $t = 0$ corresponds to the position of the origin of the coordinate system (x, y, z) in the point $\mathbf{r}' = (d, h, l)$. The partial derivation of the composite function over time gives the well known expression:

$$\begin{aligned} \frac{\partial \mathbf{F}}{\partial t} &= \frac{\partial \mathbf{F}'}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \mathbf{F}'}{\partial y'} \frac{\partial y'}{\partial t} + \frac{\partial \mathbf{F}'}{\partial z'} \frac{\partial z'}{\partial t} \\ &= \frac{\partial \mathbf{F}'}{\partial x'} (-v_x) + \frac{\partial \mathbf{F}'}{\partial y'} (-v_y) + \frac{\partial \mathbf{F}'}{\partial z'} (-v_z) = -(\mathbf{v} \cdot \nabla) \mathbf{F}' \end{aligned} \quad (1)$$

for the “convective” time derivative through the space dependence (symbol “ \cdot ” denotes the scalar product). The equality (1) presents an implicit dependence of the field vector on time. This dependence originating from a mechanical transfer is in contrast with electro-dynamical processes, in which case the dependence on time is explicit. It will be shown what a deep chasm lays between.

2. FIELDS OF KINEMATICAL ORIGIN INDUCED BY A MOVING MAGNETIZED SPHERE

A sphere of diameter d , with a uniform magnetization \mathbf{M} of magnitude M and parallel to the z axis, is embedded in vacuum (Figure 1). Let (r, θ, φ) be the system of spherical coordinates with polar axis directed along the vector \mathbf{M} and with origin in the centre of the sphere. Magnetic field in the external space may be expressed through simple

elementary functions [8, p. 183]. In this case the components of the magnetic flux density (SI-units) are

$$B_r = p \frac{\cos \theta}{r^3}, \quad B_\theta = \frac{p \sin \theta}{2 r^3}, \quad B_\varphi = 0,$$

where p is a constant numerical coefficient depending on using system of units. The projections of the vector \mathbf{B} on axes of Cartesian coordinates are

$$B_x = 3p \frac{\sin \theta \cos \theta \cos \varphi}{r^3}, \quad B_y = 3p \frac{\sin \theta \cos \theta \sin \varphi}{r^3}, \quad B_z = p (2 - 3 \sin^2 \varphi).$$

The length of a radius-vector into an arbitrary point A is $r = \sqrt{x^2 + y^2 + z^2}$ and appeared trigonometry functions may be expressed as following:

$$\sin \theta = \frac{\sqrt{x^2 + y^2}}{r}, \quad \cos \theta = \frac{z}{r}, \quad \sin \varphi = \frac{y}{\sqrt{x^2 + y^2}}, \quad \cos \varphi = \frac{x}{\sqrt{x^2 + y^2}}.$$

Substituting these expressions into Cartesian projections of the vector \mathbf{B} we have

$$B_x = 3p \frac{xz}{r^5}, \quad B_y = 3p \frac{yz}{r^5}, \quad B_z = p \frac{2z^2 - x^2 - y^2}{r^5}.$$

Let the own frame of reference, where a magnetised sphere is at rest, has the axes (x, y, z) in parallel with the axes X, Y, Z of the laboratory reference frame as shown in Figure 2. The magnetised sphere provides magnetic field with space-variable flux density \mathbf{B} . Let the sphere is moving relative to the laboratory reference frame along the X axis at a constant velocity $\mathbf{v} = (v, 0, 0)$. There is a time-dependence between coordinates of both systems:

$$x = X - vt, \quad y = Y, \quad z = Z,$$

where the origins of both systems coincide at the moment in time $t_0 = 0$. So magnetic vector $\mathbf{B}(\mathbf{r}')$ in any point $\mathbf{r}' = (X, Y, Z)$ of the laboratory system is equal to the magnetic vector $\mathbf{B}(\mathbf{r})$ in the point $\mathbf{r} = (X - vt, Y, Z)$ of the moving one. With accuracy to the constant factor p we have

$$B_X = 3 \frac{(X - vt)Z}{r^5}, \quad B_Y = 3 \frac{YZ}{r^5}, \quad B_Z = \frac{2Z^2 - (X - vt)^2 - Y^2}{r^5}, \quad (2)$$

where $\mathbf{r} = (X - vt, Y, Z)$. The corresponding electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$ of the kinematical origin has the following components:

$$E_X = 0, \quad E_Y = v \frac{2Z^2 - (X - vt)^2 - Y^2}{r^5}, \quad E_Z = -3v \frac{YZ}{r^5}, \quad (3)$$

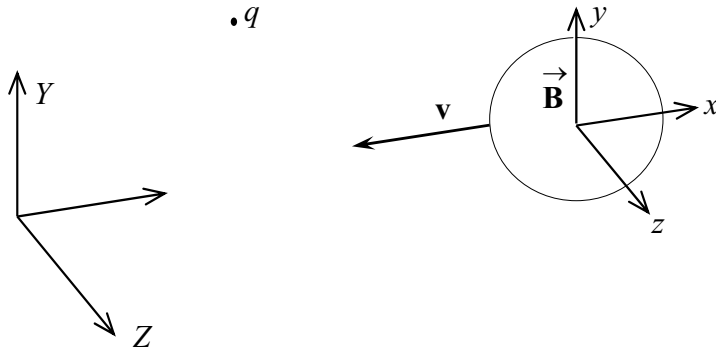


Figure 2.

and the inverse Lorentz force $q\mathbf{B} \times \mathbf{v}$ exerts on a charge q fixed in a point A of the laboratory coordinates (X, Y, Z) . This force is variable in time although the magnetic field is stationary at its source. So we have an implicit dependence originating from a mechanical transfer similar to (1). The question is what a difference would be with electromagnetic waves, in which case the dependence on time is explicit.

The partial derivatives of the magnetic components (2) in the laboratory system are

$$\begin{aligned} \frac{\partial B_X}{\partial X} &= 3Z \frac{Y^2 + Z^2 - 4(X-vt)^2}{r^7}, & \frac{\partial B_Y}{\partial Y} &= 3Z \frac{(X-vt)^2 + Z^2 - 4Y^2}{r^7}, \\ \frac{\partial B_Z}{\partial Z} &= 3Z \frac{3(X-vt)^2 + Y^2 - Z^2}{r^7}; \\ \frac{\partial B_X}{\partial Y} &= -15 \frac{YZ(X-vt)}{r^7}, & \frac{\partial B_X}{\partial Z} &= 3(X-vt) \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7}; \\ \frac{\partial B_Y}{\partial X} &= -15 \frac{YZ(X-vt)}{r^7}, & \frac{\partial B_Y}{\partial Z} &= 3Y \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7}; \\ \frac{\partial B_Z}{\partial X} &= 3(X-vt) \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7}, & \frac{\partial B_Z}{\partial Y} &= 3Y \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7}; \\ \frac{\partial B_X}{\partial t} &= -3vZ \frac{Y^2 + Z^2 - 4(X-vt)^2}{r^7}, & \frac{\partial B_Y}{\partial t} &= 15v \frac{(X-vt)YZ}{r^7}, \\ \frac{\partial B_Z}{\partial t} &= 3v(X-vt) \frac{4Z^2 - (X-vt)^2 - Y^2}{r^7}. \end{aligned}$$

From these formulae we have

$$\begin{aligned}\operatorname{div} \mathbf{B} &= \frac{\partial B_X}{\partial X} + \frac{\partial B_Y}{\partial Y} + \frac{\partial B_Z}{\partial Z} = 0; \\ \operatorname{curl} \mathbf{B} &= \left(\frac{\partial B_Z}{\partial Y} - \frac{\partial B_Y}{\partial Z}, \frac{\partial B_X}{\partial Z} - \frac{\partial B_Z}{\partial X}, \frac{\partial B_Y}{\partial X} - \frac{\partial B_X}{\partial Y} \right) = (0, 0, 0).\end{aligned}$$

The partial derivatives of the induced electric components (3) are

$$\begin{aligned}\frac{\partial E_X}{\partial X} &= \frac{\partial E_X}{\partial Y} = \frac{\partial E_X}{\partial Z} \equiv 0, \quad \frac{\partial E_Y}{\partial Y} = 3vY \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7}, \\ \frac{\partial E_Z}{\partial Z} &= -3vY \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7}; \\ \frac{\partial E_Y}{\partial X} &= -3vX \frac{4Z^2 - (X-vt)^2 - Y^2}{r^7}, \quad \frac{\partial E_Y}{\partial Z} = 3vZ \frac{3(X-vt)^2 + 3Y^2 - 2Z^2}{r^7}; \\ \frac{\partial E_Z}{\partial X} &= 15v \frac{(X-vt)YZ}{r^7}, \quad \frac{\partial E_Z}{\partial Y} = -3vZ \frac{(X-vt)^2 - 4Y^2 + Z^2}{r^7}; \\ \frac{\partial E_X}{\partial t} &= 0, \quad \frac{\partial E_Y}{\partial t} = -3v^2(X-vt) \frac{4Z^2 - (X-vt)^2 - Y^2}{r^7}, \\ \frac{\partial E_Z}{\partial t} &= -15v^2 \frac{(X-vt)YZ}{r^7}.\end{aligned}$$

From these formulae we have

$$\begin{aligned}\operatorname{div} \mathbf{E} &= \frac{\partial E_X}{\partial X} + \frac{\partial E_Y}{\partial Y} + \frac{\partial E_Z}{\partial Z} = 0; \\ \operatorname{curl} \mathbf{E} &= \left(\frac{\partial E_Z}{\partial Y} - \frac{\partial E_Y}{\partial Z}, -\frac{\partial E_Z}{\partial X}, \frac{\partial E_Y}{\partial X} \right) \\ &= \left(-3vZ \frac{4(X-vt)^2 - Y^2 - Z^2}{r^7}, -15v \frac{(X-vt)YZ}{r^7}, \right. \\ &\quad \left. -3v(X-vt) \frac{4Z^2 - (X-vt)^2 - Y^2}{r^7} \right).\end{aligned}$$

From comparison we find that $\operatorname{curl} \mathbf{E} = -\partial \mathbf{B} / \partial t$ is in accordance with Faraday's law. But then $\operatorname{curl} \mathbf{B} \neq \frac{1}{c^2} \partial \mathbf{E} / \partial t$ is in contradiction with the Ampere-Maxwell law. This specific circumstance hints at the necessity to check the wave equation. The second partial derivatives of the magnetic X -component are

$$\begin{aligned}\frac{\partial^2 B_X}{\partial X^2} &= \frac{\partial}{\partial X} \left(3Z \frac{Y^2 + Z^2 - 4(X-vt)^2}{r^7} \right) \\ &= 15(X-vt)Z \frac{4(X-vt)^2 - 3Y^2 - 3Z^2}{r^9};\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 B_X}{\partial Y^2} &= \frac{\partial}{\partial Y} \left(-15 \frac{YZ(X-vt)}{r^7} \right) = 15(X-vt)Z \frac{6Y^2 - (X-vt)^2 - Z^2}{r^9}; \\ \frac{\partial^2 B_X}{\partial Z^2} &= \frac{\partial}{\partial Z} \left(3(X-vt) \frac{(X-vt)^2 + Y^2 - 4Z^2}{r^7} \right) \\ &= 15(X-vt)Z \frac{4Z^2 - 3(X-vt)^2 - 3Y^2}{r^9},\end{aligned}$$

and Laplacian operator applying to the magnetic component B_X would result in zero:

$$\nabla^2 B_X = \frac{\partial^2 B_X}{\partial X^2} + \frac{\partial^2 B_X}{\partial Y^2} + \frac{\partial^2 B_X}{\partial Z^2} = 0.$$

In a similar manner we have

$$\begin{aligned}\frac{\partial^2 B_Y}{\partial X^2} &= 15YZ \frac{6(X-vt)^2 - Y^2 - Z^2}{r^9}; \\ \frac{\partial^2 B_Y}{\partial Y^2} &= 15YZ \frac{4Y^2 - 3(X-vt)^2 - 3Z^2}{r^9}; \\ \frac{\partial^2 B_Y}{\partial Z^2} &= 15YZ \frac{4Z^2 - 3(X-vt)^2 - 3Y^2}{r^9}; \\ \nabla^2 B_Y &= \frac{\partial^2 B_Y}{\partial X^2} + \frac{\partial^2 B_Y}{\partial Y^2} + \frac{\partial^2 B_Y}{\partial Z^2} = 0. \\ \frac{\partial^2 B_Z}{\partial X^2} &= 3 \frac{Y^4 - 4(X-vt)^4 - 4Z^4 - 3(X-vt)^2 Y^2 + 27(X-vt)^2 Z^2 - 3Y^2 Z^2}{r^9}; \\ \frac{\partial^2 B_Z}{\partial Y^2} &= 3 \frac{(X-vt)^4 - 4Y^4 - 4Z^4 - 3(X-vt)^2 Y^2 - 3(X-vt)^2 Z^2 + 27Y^2 Z^2}{r^9}; \\ \frac{\partial^2 B_Z}{\partial Z^2} &= 3 \frac{3(X-vt)^4 + 3Y^4 + 8Z^4 + 6(X-vt)^2 Y^2 - 24(X-vt)^2 Z^2 - 24Y^2 Z^2}{r^9}; \\ \nabla^2 B_Z &= \frac{\partial^2 B_Z}{\partial X^2} + \frac{\partial^2 B_Z}{\partial Y^2} + \frac{\partial^2 B_Z}{\partial Z^2} = 0.\end{aligned}$$

Hence, the vector equality is valid: $\nabla^2 \mathbf{B} \equiv \mathbf{0}$.

The second partial derivatives of the magnetic vector over time are

$$\begin{aligned}\frac{\partial^2 B_X}{\partial t^2} &= \frac{\partial}{\partial t} \left(-3vZ \frac{Y^2 + Z^2 - 4(X-vt)^2}{r^7} \right) \\ &= 15v^2(X-vt)Z \frac{4(X-vt)^2 - 3Y^2 - 3Z^2}{r^9}; \\ \frac{\partial^2 B_Y}{\partial t^2} &= \frac{\partial}{\partial t} \left(15v \frac{(X-vt)YZ}{r^7} \right) = 15v^2YZ \frac{6(X-vt)^2 - Y^2 - Z^2}{r^9};\end{aligned}$$

$$\frac{\partial^2 B_Z}{\partial t^2} = \frac{\partial}{\partial t} \left(3v \frac{(X - vt) [4Z^2 - (X - vt)^2 - Y^2]}{r^7} \right) \neq 0.$$

(The last formula implies the *identically* satisfied inequality.) It is obvious that the vector wave equation cannot be satisfied by magnetic vector \mathbf{B} of a moving magnetized sphere:

$$\nabla^2 \mathbf{B} - \frac{1}{v^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} \neq \mathbf{0}. \quad (4)$$

Firstly, this result have been adduced in [9] without a full proof.

3. FIELDS OF KINEMATICAL ORIGIN INDUCED BY A MOVING CHARGED SPHERE

A sphere made of a conductive material is charged with a constant superficial density so that the full charge q is uniformly spread over all the surface of the sphere. In this case the electrostatic field in air outside the sphere coincides with the field of a point charge q focused in the centre of the sphere:

$$\mathbf{E} = \frac{q\mathbf{r}}{4\pi\epsilon_0 r^3}.$$

In any point A electrical vector \mathbf{E} is collinear with the radius-vector of this point (Figure 3).

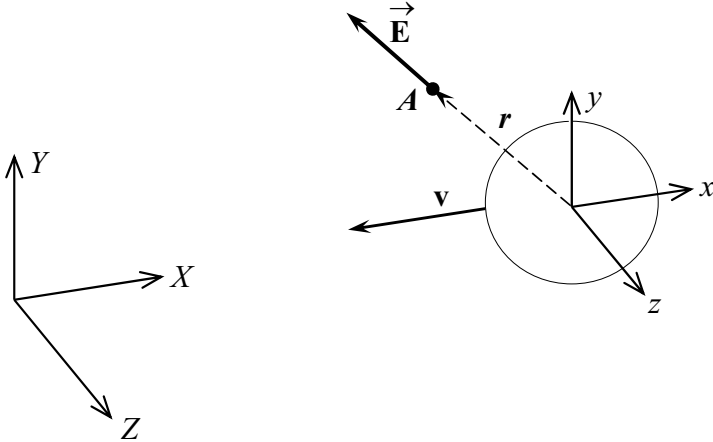


Figure 3.

In the Cartesian system with an accuracy to the constant factor $q/4\pi\epsilon_0$, electric vector $\mathbf{E}_{x,y,z}$ has the components

$$E_x = \frac{x}{r^3}, \quad E_y = \frac{y}{r^3}, \quad E_z = \frac{z}{r^3}.$$

The charged sphere is moving relative to the laboratory reference frame with coordinates X, Y, Z along the X axis at a constant velocity $\mathbf{v} = (v, 0, 0)$, and there is a dependency in time between coordinates of both systems:

$$x = X - vt, \quad y = Y, \quad z = Z.$$

At any point of the laboratory space at a given moment t there is a variable electric vector $\mathbf{E}_{X,Y,Z}$ with components

$$E_X = \frac{X - vt}{r^3}, \quad E_Y = \frac{Y}{r^3}, \quad E_Z = \frac{Z}{r^3}; \quad r = \sqrt{(X - vt)^2 + Y^2 + Z^2}. \quad (5)$$

The corresponding magnetic field $\mathbf{B} = \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$ of the electrokinematical origin (R-E effect) has the following components with accuracy to the factor $q/4\pi\epsilon_0 c^2$:

$$B_X = 0, \quad B_Y = -v \frac{Z}{r^3}, \quad B_Z = v \frac{Y}{r^3}. \quad (6)$$

The partial derivatives of the electric components (5) are

$$\begin{aligned} \frac{\partial E_X}{\partial X} &= \frac{Y^2 + Z^2 - 2(X - vt)^2}{r^5}, & \frac{\partial E_Y}{\partial Y} &= 3vY \frac{(X - vt)^2 + Z^2 - 2Y^2}{r^5}, \\ \frac{\partial E_Z}{\partial Z} &= -3vY \frac{(X - vt)^2 + Y^2 - 2Z^2}{r^5}; \\ \frac{\partial E_X}{\partial Y} &= -3 \frac{(X - vt)Y}{r^5}, & \frac{\partial E_X}{\partial Z} &= -3 \frac{(X - vt)Z}{r^5}, & \frac{\partial E_Y}{\partial X} &= -3 \frac{(X - vt)Y}{r^5}; \\ \frac{\partial E_Y}{\partial Z} &= -3 \frac{YZ}{r^5}, & \frac{\partial E_Z}{\partial X} &= -3 \frac{(X - vt)Z}{r^5}, & \frac{\partial E_Z}{\partial Y} &= -3 \frac{YZ}{r^5}; \\ \frac{\partial E_X}{\partial t} &= v \frac{2(X - vt)^2 - Y^2 - Z^2}{r^5}, & \frac{\partial E_Y}{\partial t} &= 3v \frac{(X - vt)Y}{r^5}, \\ \frac{\partial E_Z}{\partial t} &= 3v \frac{(X - vt)Z}{r^5}. \end{aligned}$$

Using above expressions we have

$$\begin{aligned} \operatorname{div} \mathbf{E} &= \frac{\partial E_X}{\partial X} + \frac{\partial E_Y}{\partial Y} + \frac{\partial E_Z}{\partial Z} = 0; \\ \operatorname{curl} \mathbf{E} &= \left(\frac{\partial E_Z}{\partial Y} - \frac{\partial E_Y}{\partial Z}, \frac{\partial E_X}{\partial Z} - \frac{\partial E_Z}{\partial X}, \frac{\partial E_Y}{\partial X} - \frac{\partial E_X}{\partial Y} \right) = (0, 0, 0). \end{aligned}$$

The partial derivatives of the induced magnetic components (6) are

$$\begin{aligned}\frac{\partial B_X}{\partial X} &= \frac{\partial B_X}{\partial Y} = \frac{\partial B_X}{\partial Z} = \frac{\partial B_X}{\partial t} = 0; & \frac{\partial B_Y}{\partial X} &= 3v \frac{(X - vt)Z}{r^5}, \\ \frac{\partial B_Y}{\partial Y} &= 3v \frac{YZ}{r^5}, & \frac{\partial B_Y}{\partial Z} &= v \frac{2Z^2 - (X - vt)^2 - Y^2}{r^5}; \\ \frac{\partial B_Z}{\partial X} &= -3v \frac{(X - vt)Y}{r^5}, & \frac{\partial B_Z}{\partial Y} &= v \frac{(X - vt)^2 + Z^2 - 2Y^2}{r^5}, & \frac{\partial B_Z}{\partial Z} &= -3v \frac{YZ}{r^5}; \\ \frac{\partial B_Y}{\partial t} &= -3v^2 \frac{(X - vt)Z}{r^5}, & \frac{\partial B_Z}{\partial t} &= 3v^2 \frac{(X - vt)Y}{r^5}.\end{aligned}$$

So we are able to calculate

$$\operatorname{div} \mathbf{B} = 0 + 3v \frac{YZ}{r^5} - 3v \frac{YZ}{r^5} = 0$$

and

$$\operatorname{curl} \mathbf{B} = \left(v \frac{2(X - vt)^2 - Z^2 - Y^2}{r^5}, \quad 3v \frac{(X - vt)Y}{r^5}, \quad 3v \frac{(X - vt)Z}{r^5} \right).$$

The comparison values of the respective components with one another gives us the following vector equations:

$$\operatorname{curl} \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{curl} \mathbf{E} \neq - \frac{\partial \mathbf{B}}{\partial t}.$$

In this case, on the contrary, Ampere-Maxwell law is carried out, but Faraday's law isn't. As above, we could have calculated the second derivatives and check the induced electric vector $\mathbf{E}_{X,Y,Z}$ to meet wave equation similar to (4), but it is needless to do because in the next section a mathematical proof will be done.

It should be noted that R-E effect is valid for any rigid carrier of charge. In particular, it is undoubtedly present in every device with a moving charged part. For example, an electrostatic motor — of cheap construction, not involving a wire-winding and heavy core — makes use of the Coulomb repulsion-attraction between the stationary and rotating plates. Let us make estimation for a disk of 1 m in diameter rotating at 6000 rev/min. The speed at its rim would be $v \approx 300$ m/sec and with an electric strength $E = 300$ kV/m the so called “convection current” provides magnetic field $B = 10^{-9}$ T = 10^{-5} Gauss. Thus, four the orders of value less than the natural Earth magnetic field enable us to recognise the scientific feat of Rowland and Eichenwald who managed to register this hardly discernible trace of developing magnetism.

It would be much more reliable to observe R-E effect on an accelerator of electrons. The current $I = 1$ Amp in the beam of 5 MeV electrons (a rather moderate energy) must afford $B = 2$ Gauss at a distance of 1 cm. That is ten times bigger than the magnetic field existing around a wire with the same current.

4. KINEMATICS FOR MOVING FIELDS

Consider the electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$ of the magneto-kinematical origin, appearing in a laboratory when a permanent magnet is moving with a constant velocity $\mathbf{v} = (v, 0, 0)$ as it is shown in the Figure 2. The components of \mathbf{E} are

$$E_X = 0, \quad E_Y = vB_Z = vB_Z 0, \quad E_Z = -vB_Y = -vB_Y;$$

the components of $\text{curl } \mathbf{E}$ are

$$(\text{curl } \mathbf{E})_X = -v \frac{\partial B_Y}{\partial Y} - v \frac{\partial B_Z}{\partial Z}, \quad (\text{curl } \mathbf{E})_Y = v \frac{\partial B_Y}{\partial X}, \quad (\text{curl } \mathbf{E})_Z = v \frac{\partial B_Z}{\partial X};$$

and the components of $\partial \mathbf{B} / \partial t$ are

$$\begin{aligned} \frac{\partial B_X}{\partial t} &= \frac{\partial B_X}{\partial(X-vt)} \frac{\partial(X-vt)}{\partial t} = -v \frac{\partial B_X}{\partial X}, \\ \frac{\partial B_Y}{\partial t} &= \frac{\partial B_Y}{\partial(X-vt)} \frac{\partial(X-vt)}{\partial t} = -v \frac{\partial B_Y}{\partial X}, \\ \frac{\partial B_Z}{\partial t} &= \frac{\partial B_Z}{\partial(X-vt)} \frac{\partial(X-vt)}{\partial t} = -v \frac{\partial B_Z}{\partial X}. \end{aligned}$$

In vacuum the divergence $\text{div } \mathbf{B} = 0$, so we have

$$\frac{\partial B_X}{\partial X} = -\frac{\partial B_Y}{\partial Y} - \frac{\partial B_Z}{\partial Z},$$

where from

$$\frac{\partial B_X}{\partial t} = v \frac{\partial B_Y}{\partial Y} + v \frac{\partial B_Z}{\partial Z}.$$

The comparison of corresponding components for $\text{curl } \mathbf{E}$ and $\partial \mathbf{B} / \partial t$ brings us to Faraday's law

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Starting from a magnetic field $\mathbf{B} = \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$ of an electro-kinematical origin we have the components

$$\begin{aligned} B_X &= 0, \quad B_Y = -\frac{v}{c^2} E_Z, \quad B_Z = \frac{v}{c^2} E_Y; \\ (\text{curl } \mathbf{B})_X &= \frac{v}{c^2} \frac{\partial E_Y}{\partial Y} + \frac{v}{c^2} \frac{\partial E_Z}{\partial Z}, \quad (\text{curl } \mathbf{B})_Y = -\frac{v}{c^2} \frac{\partial E_Y}{\partial X}, \\ (\text{curl } \mathbf{B})_Z &= -\frac{v}{c^2} \frac{\partial E_Z}{\partial X}; \\ \frac{\partial E_X}{\partial t} &= \frac{\partial E_X}{\partial(X-vt)} \frac{\partial(X-vt)}{\partial t} = -v \frac{\partial E_X}{\partial X} \\ \frac{\partial E_Y}{\partial t} &= \frac{\partial E_Y}{\partial(X-vt)} \frac{\partial(X-vt)}{\partial t} = -v \frac{\partial E_Y}{\partial X}, \end{aligned}$$

$$\frac{\partial E_Z}{\partial t} = \frac{\partial E_Z}{\partial(X-vt)} \frac{\partial(X-vt)}{\partial t} = -v \frac{\partial E_Z}{\partial X}.$$

By comparing corresponding components for $\text{curl } \mathbf{B}$ and $\partial \mathbf{E} / \partial t$ and taking into account the equality $\text{div } \mathbf{E} = 0$ (for free space), we come to the Ampere-Maxwell law

$$\text{curl } \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}.$$

The Ampere-Maxwell law alone or Faraday's law alone are not sufficient for a vector field to satisfy the wave equation; to be so it is necessary for both laws take place [10, pp. 122–123].

Suppose, that relative to a moving frame of reference with the coordinates (x, y, z) there exist a stationary vector field $\mathbf{F}(x, y, z)$. Let x axis, y axis and z axis be parallel to the X axis, Y axis and Z axis of the laboratory coordinate system respectively (Figure 3), the constant velocity $\mathbf{v} = (v_X, v_Y, v_Z)$, and the origins of both coordinate systems are coincident with one another at the moment of time $t_0 = 0$. Then the field in laboratory system may be written as follows: $\mathbf{F}_{XYZ}(X, Y, Z) = \mathbf{F}_{xyz}(X - tv_X, Y - tv_Y, Z - tv_Z)$. A partial derivative of the component $F_X(X, Y, Z)$ over variable X , for example, should be obtained through differentiation of the composite function:

$$\frac{\partial F_X}{\partial X} = \frac{\partial F_x}{\partial x} \cdot \frac{\partial x}{\partial X} = \frac{\partial F_x}{\partial x} \cdot 1 = \frac{\partial F_x}{\partial x}.$$

A similar procedure is valid for differentiation over any space variable and, hence, either partial derivative of the field in the laboratory frame coincides with the corresponding partial derivative in the moving frame: such a *derivative similarity* takes place.

There are several mathematical identities involving fields and differential operators which hold for any smoothly-varying fields. A well known one involving the Laplasian operator $\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}$ is

$$\text{curl curl } \mathbf{F} \equiv \text{grad div } \mathbf{F} - \nabla^2 \mathbf{F}.$$

Let us use this equation for an external magnetic vector of a moving permanent magnet. In its own coordinate system $\text{div } \mathbf{B}_{xyz} = 0$, $\text{curl } \mathbf{B}_{xyz} = 0$, so owing to the derivative similarity we have $\text{div } \mathbf{B}_{XYZ} = 0$, $\text{curl } \mathbf{B}_{XYZ} = 0$. As a result, $\text{curl curl } \mathbf{B}_{XYZ} \equiv 0$, $\text{grad div } \mathbf{B}_{XYZ} \equiv 0$, and it is a must that $\nabla^2 \mathbf{B}_{XYZ} \equiv 0$. Inasmuch as the second time derivative $\frac{\partial^2 \mathbf{B}_{XYZ}}{\partial t^2} \neq 0$ for a finite source, while $\mathbf{v} \neq \mathbf{0}$, the magnetic vector \mathbf{B}_{XYZ} cannot satisfy the wave equation. Just the same inference is valid for the electric field \mathbf{E}_{XYZ} of a moving charged surface, and consequently we have an analogous to (4) inequality

$$\nabla^2 \mathbf{E} - \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \neq 0. \quad (7)$$

These *kinematical* fields are transferring in space not like a ripple on the river, but like the water itself in the stream (it is worth recalling that specific stars named magnetars have a magnetic field with the density of water). That is a pure electric kinematics or magnetic kinematics, not an electromagnetic dynamics. The wave equation plays role of the main watershed between two different realms of electromagnetism.

It is common knowledge that the special relativity theory takes into account the retardation of a signal in the Minkowski space. O. D. Jefimenko conceived a fruitful idea of retardation in the Galilean coordinates. Hence, a new type of electromagnetic interaction has been hypothetically discovered: *'The electric field created by time-variable currents is very different from all other fields encountered in electromagnetic phenomena. Therefore a special name should be given to it. Taking into account that the cause of this field is a motion of electric charges (current), we may call it the **electrokinetic field**, and we may call the force which this field exerts on an electric charge the **electrokinetic force**'* [11, pp. 28–29]. The question arises as to what impact on the kinematical fields might appear if we take into account a signal's retardation? Needless to say, this makes all the formulae much more complicated, but it would be a waste effort because the main implication of the above analysis remains untouched.

Indeed, the properly accounted retardation may redeem a non-zero value for the first term in the inequalities (4) and (7), but this possible addition would be apparently not more than a (v/c) order of magnitude. (In the above adduced example with a rotating disk the ratio of velocities is equal to 10^{-6} .) The second term in (4) and (7) has an essentially non-zero value, depending on the position of an electric (or magnetic) source only. This causes the inequalities (4) and (7) to persist in some range of the velocity values: $v < c$. Thereby, neither special relativity nor retardation effect in the sense of [11] can disprove the established fact — the existence in nature of two fundamentally different types of induction: dynamical and kinematical.

It is clear from the above text that the derivative similarity plays a key role in the demonstration, but it pertains only to a rectilinear movement. If a rigid source of electric (magnetic) field arbitrary moves, both coordinate systems — proper and the laboratory's — are related by an orthogonal transformation which provides curl and divergence to remain invariant. This results also with the integral form of definition applied to them. But it should be kindly remembered that in the case of an arbitrary moving charge an electro-dynamic field is present together with the kinematical one.

5. CONCLUSION

The main implication of the above analysis is that there exists a deep symmetry between electricity and magnetism not only in the dynamics but also in the kinematics. The latter being an example of moving permanent fields. The Biot-Savart force of electro-kinematical origin is the adjacent counterpart for the magneto-kinematical Ampere force. Unlike dynamical fields, which are amenable to both Faraday's law and Ampere-Maxwell law, kinematical fields obey either Faraday's law or Ampere-Maxwell law only. For dynamical fields is impossible to state a cause-effect relation: electric and magnetic vectors in the electromagnetic wave are equiphase. For kinematical fields, on the contrary, the Ampere-Maxwell law, involved in the electro-kinematical case, determinates magnetic vector to be an effect of the electric cause, and vice versa, the Faraday's law, involved in the magneto-kinematical case, imposes magnetic vector to be the cause for electric one. Consequently, there are no electro-kinematic or magneto-kinematic waves in Nature.

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