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ANALYSIS OF TRANSMISSION PROPERTIES IN A PHOTONIC QUANTUM WELL CONTAINING SUPERCONDUCTING MATERIALS

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Abstract—Properties of wave transmission in a photonic quantum well (PQW) structure containing superconducting materials are theoretically investigated. We consider two possible PQW structures, $(AB)^P(CD)^Q(AB)^P$-asymmetric and $(AB)^P(CD)^Q(BA)^P$-symmetric, where the host photonic crystal (PC) $(AB)^P$ is made of dielectrics, A = $SrTiO_3$, B = Al_2O_3, and the PQW $(CD)^Q$ contains C = A and superconducting layer D = $YBa_2Cu_3O_{7-x}$, a typical high-temperature superconducting thin film. Multiple transmission peaks can be seen within the photonic band gap (PBG) of $(AB)^P$ and the number of peaks is directly determined by the stack number of PQW, i.e., it equals Q-1. Additionally, the results show that symmetric PQW structure is preferable to the design of a multichannel transmission filter. The effect of stack number of photonic barrier is also illustrated. Such a filter operating at terahertz with feature of multiple channels is of technical use in superconducting optoelectronic applications.

1. INTRODUCTION

The use of photonic crystals (PCs) to design a narrowband transmission filter has been of interest to the community. In general, the filter channel, which is represented by the transmission peak

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in the transmission spectrum, is designed to be located within the photonic band gap (PBG) of a PC. This filter can be simply achieved by introducing a defect layer into a PC to break the translational symmetry [1]. The filtering device can also be achieved by the photonic crystal waveguide structure [2]. Theoretical methods on the calculation of photonic band structures can be seen in an excellent review article [3].

In order to enhance the spectral efficiency in the PBG, a multichannel filter is of technical need in practical applications. To engineer PBG with multiple channels, photonic quantum well (PQW) structure can meet the goal [4]. Multiple channels within the PBG are produced due to the photonic confinement effect. These channels are nearly discrete (quantized) in the transmission spectrum. This effect is similar to the electronic quantum well in semiconductor. Electron energy is quantized in quantum well systems, including semiconductor quantum well, metal quantum wells, and quantum dots [5–7]. The photonic quantized states are not only seen in one-dimensional (1D) PC [4] but also in two-dimensional and three-dimensional PC structures [8, 9]. These photonic discrete states, which have a high transmittance because of the resonant tunneling, can thus be used to design an optical filter with multiple channels [10–13].

For a 1D PQW, it has a structure of $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{AB})^P$, where the host PC $(\mathrm{AB})^P$ plays the role of Bragg mirror, whereas the sandwiched PC $(\mathrm{CD})^Q$ is the PQW. It has been shown that the number of multiple channels is just equal to Q, the number of periods of the PQW when all the constituent materials, A, B, C, and D, are simple dielectrics [4]. This result also holds when the host PC has a constituent which is a negative-index material (NIM) [14]. In addition, the use of NIM causes the multiple channels to be weakly dependent on angle of incidence and polarization.

In this work, we consider the PQW which contains the superconductor as one of constituents. PCs containing superconductors and dielectric are generally called the superconductor-dielectric photonic crystals (SDPCs) which have attracted much attention in the past decade [15–20]. More recently, the use of superconducting thin film as a defect layer in PC has shown that it can be a tuning agent in a transmission filter [21–25]. That is, the position of transmission peak can be changed by the variation of thickness in superconducting thin film. Motivated by the SDPC, we would like to study the wave properties in a dielectric PC which has a superconducting PQW as a defect. We shall consider two possible PQW structures. The first one is $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{AB})^P$, which is referred to as the asymmetric structure. The other called the symmetric structure is $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{BA})^P$.

Here, the host PC $(AB)^P$ or $(BA)^P$ is made of two dielectrics, i.e., A = $SrTiO_3$ (STO), B = Al_2O_3 (AlO) and the PQW $(CD)^Q$ contains C = A and D is a superconducting layer taken to be the typical high-temperature superconducting system, $YBa_2Cu_3O_{7-x}$ (YBCO). It is worth mentioning that we have selected STO and AlO because these two dielectrics are commonly used in fabrication of YBCO [22, 23]. We shall investigate the filtering properties based on the transmittance spectra. The dependence of stack number of superconducting PQW, Q, will be systematically analyzed. The proposed photonic structure will be of technical use for signal processing in superconductor photonic applications.

2. BASIC EQUATIONS

Let us first consider asymmetric PQW structure $(AB)^P(CD)^Q(AB)^P$, where the host PC $(AB)^P$ are made of A = STO, B = AlO, and the PQW $(CD)^Q$ consists of C = STO, and D = YBCO, respectively. The whole structure immersed in air is diagramed in the upper panel of Figure 1. The wave is assumed to be normally launched at the left boundary and consequently the transmission is calculated at the right boundary. To explore properties of wave transmission, transfer matrix method (TMM) will be employed [26]. This method is elegant and widely used in the study of one-dimensional photonic layered structures [27–35]. In using TMM, it is necessary to have the refractive index of each material. For STO and AlO, they are $n_A = 2.437$ and $n_B = 1.767$, respectively [21–23]. As for the YBCO, the refractive index $n_D = \sqrt{\varepsilon_D}$, where the relative permittivity is described by two-fluid model along with London local electrodynamics. If the temperature is well below the critical temperature, the superconductor can be regarded as a lossless medium and the permittivity takes the form [16, 19]

$$\varepsilon_D = 1 - \frac{1}{\omega^2 \mu_0 \varepsilon_0 \lambda_L^2}, \tag{1}$$

where the temperature-dependent London penetration length λ_L is expressed as

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1-(T/T_c)^p}}, \tag{2}$$

where $\lambda_L(0)$ is the London penetration length at $T = 0\,\mathrm{K}$, $p = 4$, and T_c is the superconducting critical temperature.

According to TMM, the total transfer matrix for asymmetric

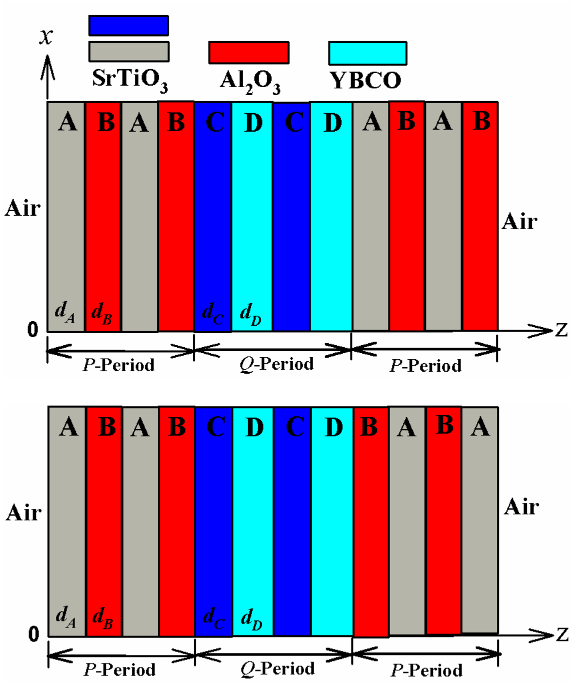


Figure 1. Schematic diagram for two superconducting PQW structures of $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{AB})^P$ (top, asymmetric) and $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{BA})^P$ (bottom, symmetric), where A = C = $SrTiO_3$, B = Al_2O_3, and D = YBCO, respectively. Here, the stack numbers of the host PC and PQW are P and Q respectively. In addition, the corresponding thicknesses are denoted by d_A, d_B, d_C, and d_D, respectively.

PQW structure is expressed as [26]

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = D_0^{-1}\left(M_A M_B\right)^P \left(M_C M_D\right)^Q \left(M_A M_B\right)^P D_0, \qquad (3)$$

where $M_j = D_j P_j D_j^{-1}$, $j = A$, B, C and D is the transfer matrix for layer j. The dynamical matrix D_j and translational matrix P_j can be found in Ref. [26]. Similarly, a modification in Eq. (3) leads to the result for symmetric PQW structure. Then, the transmittance Γ is determined by the following relation,

$$\Gamma = \left|\frac{1}{M_{11}}\right|^2. \qquad (4)$$

3. NUMERICAL RESULTS AND DISCUSSION

Let us examine the photonic band structure for the PCs $(\mathrm{AB})^{20}$ and $(\mathrm{CD})^{20}$. In Figure 2, we have plotted transmittance spectra for these two PCs. In PC $(\mathrm{AB})^{20}$, the thicknesses of A and B are $d_A = 0.4a$ and $d_B = 0.6a$, where $a = 1\,\mu\mathrm{m}$ is taken. In PC $(\mathrm{CD})^{20}$, the thicknesses of C and D are $d_C = 0.48a$ and $d_D = 0.39a$, Here, the temperature is fixed at $T = 4.2\,\mathrm{K}$ which is well below $T_c = 92\,\mathrm{K}$ for YBCO. It can be seen that the first pass band of $(\mathrm{CD})^{20}$ is completely overlapping with the first PBG of $(\mathrm{AB})^{20}$. It should be noted that the overlapping is strongly dependent on the values of d_A, d_B, d_C, and d_D. This is because both photonic band structures of PCs $(\mathrm{AB})^{20}$ and $(\mathrm{CD})^{20}$ will be respectively affected by the variations of d_A, d_B and d_C, d_D. In order to achieve this overlapping, we can first fix the values of d_A and d_B for the photonic band gap shown in Figure 2. Next, by carefully varying d_C and d_D, it can lead to the goal of overlapping. The process can be done vice versa. That is, we first fix d_C and d_D for the assigned pass band, and then vary d_A and d_B so that the overlapping can be obtained. Thus, in the PQW structures of $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{AB})^P$ and $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{BA})^P$, $(\mathrm{AB})^P$ or $(\mathrm{BA})^P$ acts as a photonic barrier and $(\mathrm{CD})^Q$ plays as a role of photonic quantum well. The overlapping frequency region is at $\omega a/2\pi c = 0.2207$–0.2726, which is equal to $f = \omega/2\pi = 66.21$–$81.78\,\mathrm{THz}$. In the analysis that follows we would like to engineer this overlapping region for the purpose of multiple-filtering phenomenon in the PQW structure.

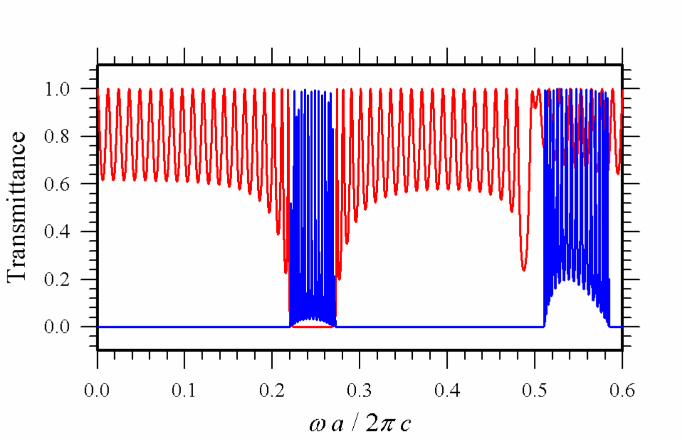
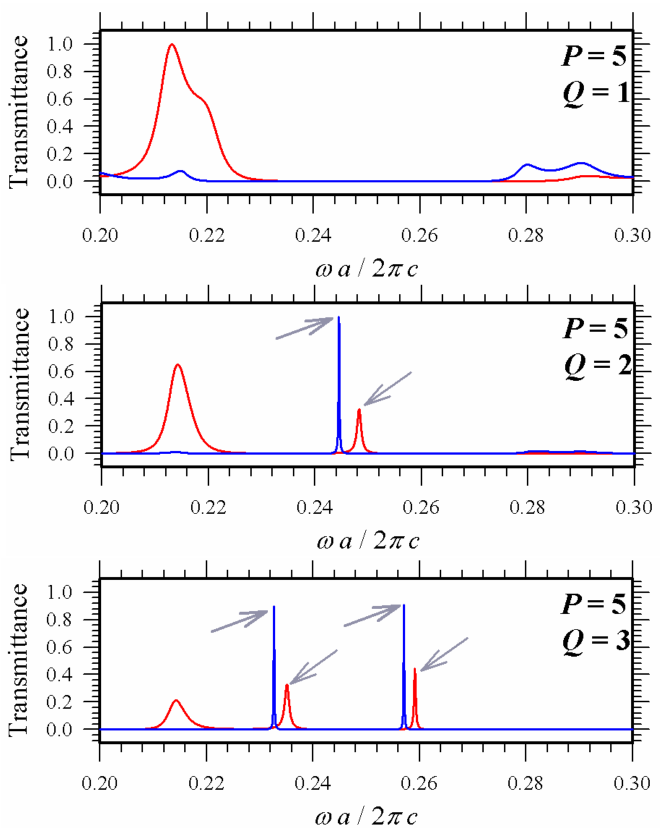


Figure 2. The calculated transmittance spectra for $(\mathrm{AB})^{20}$ (red) and $(\mathrm{CD})^{20}$ (blue) at $T = 4.2\,\mathrm{K}$. A photonic pass band of $(\mathrm{CD})^{20}$ completely overlaps with a photonic band gap of $(\mathrm{AB})^{20}$.

With the same parameters in Figure 2, we combine two PCs to form the PQW structure. As mentioned previously, the asymmetric PQW structure is $(AB)^P(CD)^Q(AB)^P$ whereas $(AB)^P(CD)^Q(BA)^P$ is symmetric. In Figure 3, we plot transmittance spectra for asymmetric (red curve) and symmetric (blue curve) PQW structures near the above-mentioned overlapping region. Here, we take $P = 5$ and change value of Q as $Q = 1, 2, 3, 4,$ and 5, respectively. Some features are of note. First, it can be seen that there are Q-1 peaks for a given value of Q. All peaks are indicated by the gray arrows. Thus, multiple filtering channels can be achieved by increasing Q, the stack number of the center PQW. By the way, at $P = 5$ and $Q = 1$, there exhibits the broad features appearing at the side bands. This can be explained as follows. Since at $Q = 1$, the resonance has not yet happened, in this case, the wave in a single superconducting layer sandwiched by two PCs of $(AB)^5$ is evanescent because the permittivity of superconductor



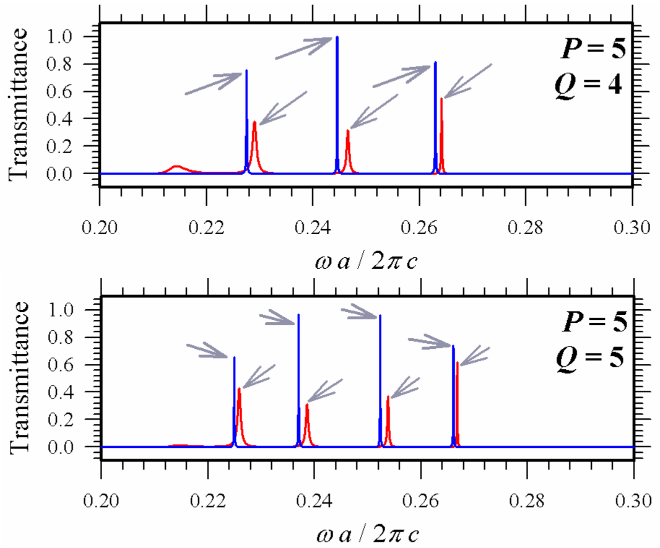
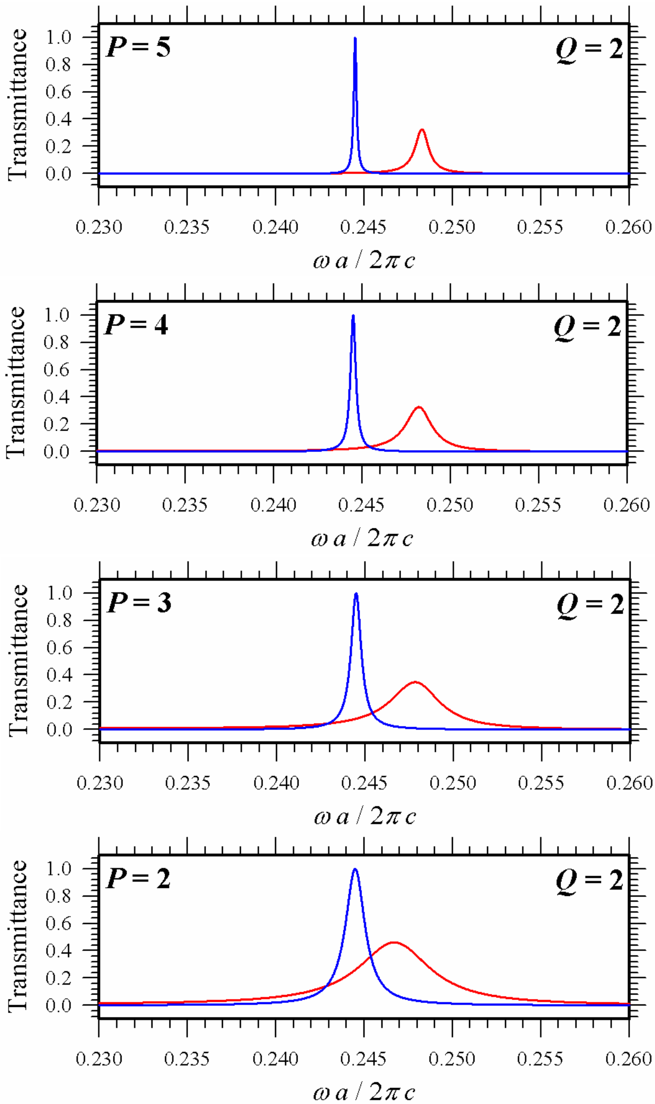


Figure 3. Calculated transmittance spectra for the asymmetric (red) and symmetric (blue) PQW structures at $P = 5$ for different $Q = 1$, 2, 3, 4, and 5, respectively. It is seen that there are Q-1 peaks at a given Q.

is negative. Thus, the superconducting layer acts as a coupling center between two pass bands. Second, the peak is sharper in the symmetric structure and the peak height is also relatively larger than that of the asymmetric one. The distribution of the peaks in the symmetric PQW structure is apparently discrete. Thus, as far as the filter design is concerned, it is preferable to use the symmetric PQW structure to design a multichannel transmission filter. This result is in sharp contrast to that of the all-dielectric PQW structure [4]. In Ref. [4], the multichannel filtering feature is seen in the asymmetric structure and there are Q peaks for a given value of Q when all constituents A, B, C, and D are dielectric materials. Third, for each transmission peak, the peak frequency in the asymmetric structure is higher than that of the symmetric. Fourth, at $Q = 4$, the peak height of the central peak is higher than the other two peaks. Similar behavior is also seen at $Q = 5$, where the central two peaks have a higher peak height compared to peaks at the left and right sides. Finally, for the asymmetric PQW structure, the peak at the highest frequency also has the highest peak height.

We have investigated the role played by the Q-number in the transmission properties. We now turn our attention to the effect of P-

number, the stack number of the left and right photonic Bragg mirrors. The calculated results are shown in Figure 4, in which we have fixed $Q = 2$ (for one transmission peak) at different values of $P = 5$, 4, 3, 2, and 1, respectively. For the symmetric PQW structure, the blue curves illustrate that the peak frequency as well as the peak height remain nearly unchanged as P changes. The decreasing in P-value will cause the peak to be broadened which, in turn, leads to the decrease in the



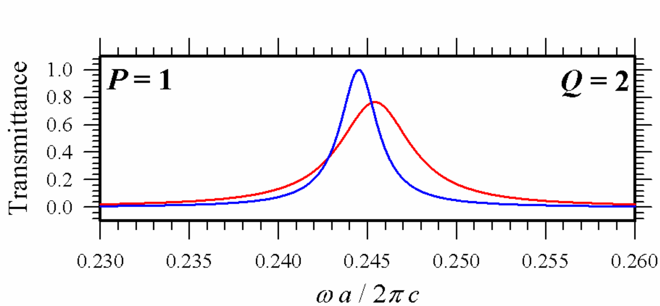
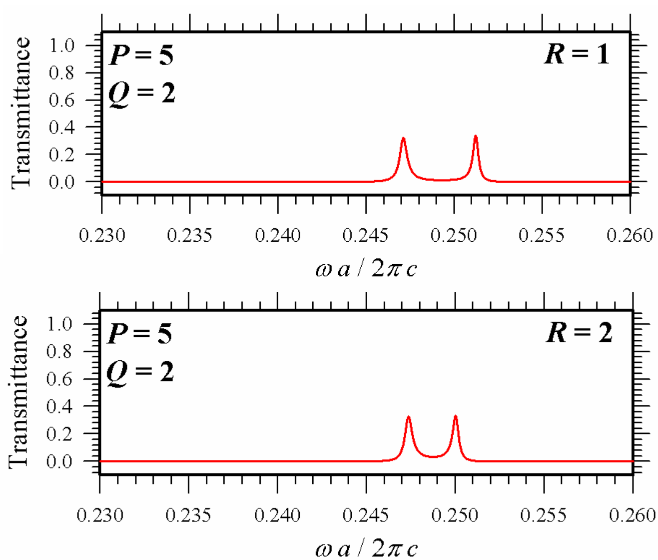


Figure 4. Calculated transmittance spectra for the asymmetric (red) and symmetric (blue) PQW structures at $Q = 2$ for different $P = 5$, 4, 3, 2, and 1, respectively.

quality factor for the resonant peak.

On the other hand, the effect of P-value is more salient in asymmetric PQW structure (red curves). In this case, like in symmetric one, the peak shape will be broadened as P decreases. However, the peak height is significantly enhanced at $P = 1$. In addition, the peak frequency is red-shifted as P decreases. Moreover, the peak frequency at $P = 1$ is close to that of symmetric structure. Thus, in asymmetric PQW structure, the number of stack of photonic barrier can be used to shift the peak frequency. As a result, as far as



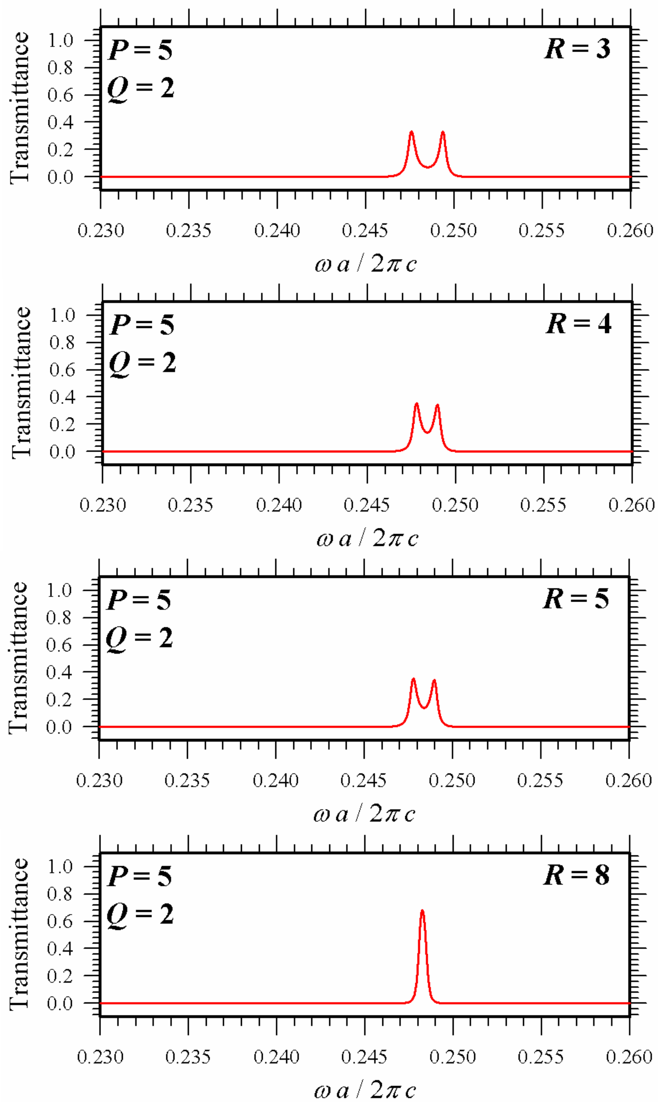


Figure 5. Calculated transmittance spectra for the DPQW structure at $P = 5$ and $Q = 2$ for different $R = 1$, 2, 3, 4, 5, and 8, respectively.

the tuning is concerned, the use of asymmetric structure is preferable to the design of a tunable filter.

Before we go to the conclusion, let us extend the above PQW structure to the so-called double photonic quantum well (DPQW) structure. The considered DPQW is $(\mathrm{AB})^P(\mathrm{CD})^Q(\mathrm{AB})^R(\mathrm{CD})^Q(\mathrm{AB})^P$, where the central $(\mathrm{AB})^R$ is regarded as the coupling center between the two adjacent PQWs. In Figure 5, we show the calculated spectra for different values in $R = 1$, 2, 3, 4, 5 and 8, respectively. It is seen that, at $R = 1$, the original single peak (See the red curve in the first panel of Figure 4) is split into two peaks due to the interaction between the two PQWs. The separation between two peaks is then decreases as R increases. When R is increased such that $R = 8$, the two peaks finally merge as a single peak. In this case, the interaction between the two PQWs is absent due to the large size in $(\mathrm{AB})^R$. The results in Figure 5 illustrate that more channels can be obtained by the use of the DPQW structure. However, the stack number of the central $(\mathrm{AB})^R$ is limited to a small number in order to increase the channel number. In addition, the role played by the central $(\mathrm{AB})^R$ is analogous to the well width in the doped double-heterojunction structure reported by Simserides and Triberis [36].

4. CONCLUSION

In summary, transmission properties for the PQW structures containing superconductor have been investigated in this work. Based on the above analyses, we can draw the following conclusions. First, the number of transmission peaks is equal to the stack number of PQW minus one, which is sharp contrast to the all-dielectric PQW structure. Second, the symmetric PQW structure has a sharp filtering feature compared to the asymmetric PQW. Third, for both structures, the peaks are broadened as the stack number of the photonic barriers decreases. In addition, the stack number of photonic barriers can be used to play as a tunable agent in order to obtain a tunable filter.

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