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RADIATION FROM AXIAL SLOT ON AN INFINITELY LONG CONDUCTING CIRCULAR CYLINDER ECCENTRICALLY COATED WITH DIELECTRIC AND PARTLY EMBEDDED IN A GROUND PLANE

Hassan A. Ragheb* **and Essam E. Hassan**

Electrical Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

Abstract—An analytical formulation is developed for an axial slot on an infinitely long conducting circular cylinder eccentrically coated with dielectric and partially embedded in a ground plane. A computer program has been developed based on the resulting formulation. The problem is highly significant since the ground plane could be the body of an aircraft, a ship, or any other mobile system. The effect of the ground plane can be used to enhance the radiation characteristics in some cases. It is also a support for the antenna instead of using other mechanical mounting supporting system. Numerical results for different antenna configurations are illustrated. The results show that how one may shape the pattern by offsetting the slotted cylinder from the center of the dielectric coating.

1. INTRODUCTION

Radiation from axial slot on a circular conducing cylinder has been the subject of extensive investigations. For instance, the residue series, geometrical optics representation [1], Green's function formulation [2] and Fourier integral representation [3] have been employed for analytical treatment of different slots on a circular conducting cylinder. Also, the dielectric coated conducting slotted circular cylinder has been examined [4, 5]. Various methods similar to those used for the circular conducting slotted antenna were employed in the two-dimensional (2-D) and three dimensional cases to obtain different radiation characteristics of the dielectric-coated slotted cylindrical antenna [6–8]. The concentric case of a dielectric coated slotted

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* Corresponding author: Hassan Ali Ragheb (hragheb@kfupm.edu.sa).

conducting cylinder in a ground plane has been also tackled [9]. Also Radiation from a slot on an elliptic conducting cylinder coated by a dielectric has been extensively investigated [10–15].

In all of the previous work, the effect of mounting the antenna on any communication system has been ignored. The present work is generalizing the problem by considering the metallic slotted cylinder eccentric with respect to the dielectric coating it. This arrangement can be used to enhance the antenna radiation characteristics and to optimize its radiation pattern. In addition, the eccentricity is used for shaping the radiation beam in a specific direction. The ground plane can be used to support the slotted dielectric-coated conducting circular cylindrical antenna. The ground plane could be the body of an air craft, a ship, or any other mobile system.

2. FORMULATION OF THE TM CASE

Figure 1 illustrates a cross section of the geometry of the problem. The conducting circular cylinder has a radius "a" while the dielectric coating has an outer radius "b", permittivity ε and permeability μ. The center of the conducting cylinder is located at $(d,\ \alpha)$ with respect to the global coordinated $(x,\ y)$ at the center of the dielectric coating. Another local coordinates $(x_1,\ y_1)$ at the center of the conducting cylinder are defined. The GP is assumed to be perfectly conducting and the axial slot is centered at ϕ_o and has an angular angle equals to 2θ in terms of local coordinates. We divide the space surrounding the conducting cylinder into two regions. The first region lies outside the dielectric coating while the second is inside it.

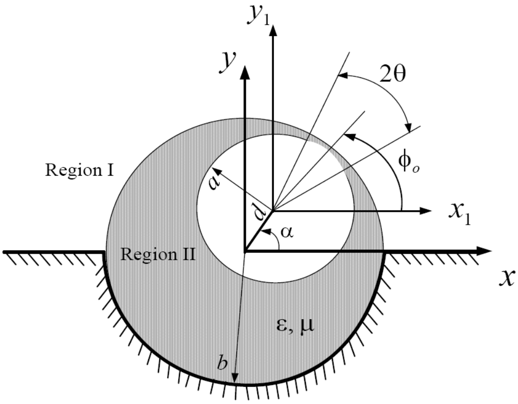

Figure 1. Geometry of the problem.

The electric field in region I must have zero value at ϕ equal to 0 and π. Therefore, the solution of the Helmholtz wave equation results in a z-component of the electric field in region I in terms of the global coordinates in the form:

$$E_z^{(I)} = E_o \sum_{n=1}^{\infty} A_n H_n^{(2)}(k_o r) \sin n\phi \tag{1}$$

The time dependence $e^{j\omega t}$ is assumed and omitted throughout. k_o is the free space wavenumber $(2\pi/\lambda)$ where λ is the wavelength. Similarly, the z-component of the electric field in region II in terms of the local coordinates (x_1, y_1) can be written as:

$$E_z^{(II)} = E_o \sum_{n=-\infty}^{\infty} \{B_n J_n(kr_1) + C_n Y_n(kr_1)\}\ e^{jn\phi_1} \tag{2}$$

where $k = k_o\sqrt{\mu_r \varepsilon_r}$, $J_n(x)$ and $Y_n(x)$ are Bessel functions of the first and second kind, respectively, of order n and argument x. The field component $H_z = 0$ and H_ϕ is equal to $(1/j\omega\mu)(\partial E_z/\partial r)$. The boundary condition of the vanishing tangential component of the electric field on the surface of conducting cylinder, except on the axial slot, is applied to obtain

$$E_o \sum_{n=-\infty}^{\infty} \{B_n J_n(ka) + C_n Y_n(ka)\}\, e^{jn\phi_1} = \begin{cases} E_z(\phi_1) & \phi_o - \theta \le \phi_1 \le \phi_o + \theta \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $E_z(\phi_1)$ is the electric field on the axial slot aperture [10], which is given as:

$$E_z(\phi_1) = E_o \cos\left(\frac{\pi(\phi_1 - \phi_o)}{2\theta}\right) \tag{4}$$

Multiplying both sides of (3) by $e^{-jm\phi_1}$ and integrating over ϕ_1 from 0 to 2π, i.e.,

$$\int_0^{2\pi} e^{j(n-m)\phi_1} d\phi_1 = \begin{cases} 0 & n \neq m \\ 2\pi & n = m \end{cases}$$

$$2\pi \{B_m J_m(ka) + C_m Y_m(ka)\} = \int_{\phi_o-\theta}^{\phi_o+\theta} e^{-jm\phi_1} \cos\left(\frac{\pi(\phi_1 - \phi_o)}{2\theta}\right) d\phi_1$$

$$B_m J_m(ka) + C_m Y_m(ka) = \frac{2\theta \cos m\theta}{\pi^2 - 4\theta^2 m^2} e^{-jm\phi_o}$$

Let

$$X_m = \frac{2\theta \cos m\theta}{\pi^2 - 4\theta^2 m^2}$$

Then, one obtains

$$B_m = \frac{1}{J_m(ka)} \left\{ X_m^{\mathrm{TM}} e^{-jm\phi_o} - C_m Y_m(ka) \right\} \tag{5}$$

The boundary condition of the zero tangential electric field at $r = b$ and $\pi < \phi < 2\pi$ and continuous fields (i.e., E_z and H_ϕ) across the aperture $0 < \phi < \pi$ must be enforced. In order to enforce such boundary conditions the electric field of Equation (3) must be expressed in terms of the global coordinates. This can be done using the addition theorem of Bessel functions [16], namely.

$$B_n(kr_1)e^{jn\phi_1} = \sum_{m=-\infty}^{\infty} J_m(kd)B_{n+m}(kr)\ e^{j(n+m)\phi}e^{-jm\alpha} \tag{6}$$

where $B_n(x)$ could be $J_n(x)$ or $Y_n(x)$. Now employing (6) into (3), one can get:

$$E_z^{(\mathrm{II})} = E_o \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-jm\alpha} J_m(kd) \left\{ B_n J_{n+m}(kr) + C_n Y_{n+m}(kr) \right\} e^{j(n+m)\phi} \tag{7}$$

Substituting for B_n from (5) into (7), with some mathematical manipulations, one obtains:

$$E_z^{(\mathrm{II})} = E_o \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-jm\alpha} \frac{J_m(kd)}{J_n(ka)} \Big\{ X_n e^{-jn\phi_o} J_{n+m}(kr) - C_n \left[Y_n(ka)J_{n+m}(kr) - J_n(ka)Y_{n+m}(kr) \right] \Big\} e^{j(n+m)\phi} \tag{8}$$

Now the boundary conditions of continuous electric fields (i.e., E_z) across the aperture at $r = b$ and $0 < \phi < \pi$ is applied to obtain the following equation:

$$\sum_{n=1}^{\infty} A_n H_n^{(2)}(k_o b) \sin n\phi = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-jm\alpha} \frac{J_m(kd)}{J_n(ka)} \left\{ X_n e^{-jn\phi_o} J_{n+m}(kb) + C_n \psi_{n,n+m} \right\}\ e^{j(n+m)\phi} \tag{9}$$

where $\psi_{n,n+m} = [J_n(ka)Y_{n+m}(kb) - Y_n(ka)J_{n+m}(kb)]$.

The summation over n can be expanded while replacing m by $-m$ in the second term the following equation results in:

$$\sum_{n=1}^{\infty} A_n H_n^{(2)}(k_o b) \sin n\phi = \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)} \left\{ X_n e^{-j(m\alpha+n\phi_o)} J_{n+m}(kb) + C_n e^{-jm\alpha} \psi_{n,n+m} \right\} e^{j(n+m)\phi}$$

$$+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}\left\{X_n e^{j(m\alpha+n\phi_o)}J_{n+m}(kb)+D_n e^{jm\alpha}\psi_{n,n+m}\right\}$$
$$e^{-j(n+m)\phi} \tag{10}$$

where $D_n = (-1)^n C_{-n}$, Equation (10) can be mathematically manipulated to obtain:

$$\sum_{n=1}^{\infty}A_n H_n^{(2)}(k_o b)\sin n\phi =$$
$$\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_o(ka)}X_0 J_m(kb)e^{-jm\alpha}\{\cos m\phi+j\sin m\varphi\}$$
$$+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\frac{2J_m(kd)}{J_n(ka)}\ X_n J_{n+m}(kb)\ \cos(m\alpha+n\phi_o)\ \cos(n+m)\phi$$
$$+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\frac{2J_m(kd)}{J_n(ka)}\ X_n J_{n+m}(kb)\ \sin(m\alpha+n\phi_o)\ \sin(n+m)\phi$$
$$+C_0\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_0(ka)}\ \psi_{0,m}e^{-jm\alpha}\ \{\cos m\phi+j\sin m\varphi\}$$
$$+\sum_{n=1}^{\infty}C_n\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}\ \psi_{n,n+m}e^{-jm\alpha}\ \{\cos(n+m)\phi+j\sin(n+m)\varphi\}$$
$$+\sum_{n=1}^{\infty}D_n\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}\ \psi_{n,n+m}e^{jm\alpha}\{\cos(n+m)\phi-j\sin(n+m)\varphi\} \tag{11}$$

Multiplying both sides of the above equation by $\sin p\phi$ ($p = 1, 2, 3, \ldots$) and integrating both sides over ϕ from 0 to π. The following integrals results in:

$$\frac{\pi}{2}A_p H_p^{(2)}(k_o b) = f_p + \sum_{n=0}^{\infty} C_n g_{p,n} + \sum_{n=1}^{\infty} D_n q_{p,n} \tag{12}$$

where

$$f_p = \sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_0(ka)}\ \left\{X_0 e^{-jm\alpha}J_m(kb)\right\}\left\{\begin{array}{ll}\pm j\pi/2 & m=\pm p\\ N_{p,0+m} & m\neq\pm p\end{array}\right\}$$
$$+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}2\frac{J_m(kd)}{J_n(ka)}\ \ X_n J_{n+m}(kb)\cos(m\alpha+n\phi_o)$$
$$\left[\left\{\begin{array}{ll}0 & n+m=\pm p\\ N_{p,n+m} & n+m\neq\pm p\end{array}\right\}\right]$$

$$+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}2\frac{J_m(kd)}{J_n(ka)}\; X_nJ_{n+m}(kb)\sin(m\alpha+n\phi_o)$$
$$\left[\begin{Bmatrix} 0 & n+m\neq\pm p \\ \pm\pi/2 & n+m=\pm p \end{Bmatrix}\right] \tag{13}$$

$$g_{p,n} = \sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}\; e^{-jm\alpha}\psi_{n,n+m}\begin{Bmatrix} \pm j\pi/2 & n+m=\pm p \\ N_{p,n+m} & n+m\neq\pm p \end{Bmatrix} \tag{14}$$

$$q_{p,n} = \sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}\; e^{jm\alpha}\psi_{n,n+m}\begin{Bmatrix} \mp j\pi/2 & n+m=\pm p \\ N_{p,n+m} & n+m\neq\pm p \end{Bmatrix} \tag{15}$$

$$N_{p,n+m} = \frac{p[1-(-1)^{(n+m)+p}]}{p^2-(n+m)^2} \tag{16}$$

The second boundary condition of zero tangential electric field at $r = b$ and $\pi < \phi < 2\pi$ leads to:

$$E_z^{(\mathrm{II})} = E_o\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}e^{-jm\alpha}\frac{J_m(kd)}{J_n(ka)}$$
$$\left\{X_ne^{-jn\phi_o}J_{n+m}(kb)+C_n\psi_{n,n+m}\right\}\; e^{j(n+m)\phi}=0 \tag{17}$$

Equation (17) can be expanded and manipulated with the result:

$$\begin{aligned}
0 = &\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_0(ka)}\left\{X_0e^{-jm\alpha}J_m(kb)\right\}\{\cos m\phi+j\sin m\phi\} \\
&+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}2\frac{J_m(kd)}{J_n(ka)}X_nJ_{n+m}(kb)\cos(m\alpha+n\phi_o)\cos(n+m)\phi \\
&+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}2\frac{J_m(kd)}{J_n(ka)}X_nJ_{n+m}(kb)\sin(m\alpha+n\phi_o)\sin(n+m)\phi \\
&+\sum_{n=0}^{\infty}C_n\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}e^{-jm\alpha}\psi_{n,n+m}\{\cos(n+m)\phi+j\sin(n+m)\phi\} \\
&+\sum_{n=1}^{\infty}D_n\sum_{m=-\infty}^{\infty}\frac{J_m(kd)}{J_n(ka)}e^{jm\alpha}\psi_{n,n+m}\{\cos(n+m)\phi-j\sin(n+m)\phi\}
\end{aligned} \tag{18}$$

Multiplying the above equation by $\sin p\phi$ and integrating over ϕ from π to 2π.

$$R_p = \sum_{n=0}^{\infty}C_nS_{p,n}+\sum_{n=1}^{\infty}D_nT_{p,n} \tag{19}$$

$$R_p = \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_0(ka)} \left\{X_0 e^{-jm\alpha} J_m(kb)\right\} \left\{ \begin{array}{ll} \mp j\pi/2 & m = \pm p \\ N^{\rm TM}_{p,0+m} & m \neq \pm p \end{array} \right\}$$
$$+\sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\frac{J_m(kd)}{J_n(ka)} \ X_n J_{n+m}(kb)\cos(m\alpha + n\phi_o)$$
$$\left[\left\{ \begin{array}{ll} 0 & n+m = \pm p \\ N^{\rm TM}_{p,n+m} & n+m \neq \pm p \end{array} \right\}\right]$$
$$-\sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\frac{J_m(kd)}{J_n(ka)} \ X_n J_{n+m}(kb)\sin(m\alpha + n\phi_o)$$
$$\left[\left\{ \begin{array}{ll} 0 & n+m \neq \pm p \\ \pm\pi/2 & n+m = \pm p \end{array} \right\}\right] \tag{20}$$

$$S_{p,n} = -\sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)} \ e^{-jm\alpha} \psi_{n,n+m} \left\{ \begin{array}{ll} \mp j\pi/2 & n+m = \pm p \\ N^{\rm TM}_{p,n+m} & n+m \neq \pm p \end{array} \right\} \tag{21}$$

$$T_{p,n} = -\sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)} \ e^{jm\alpha} \psi_{n,n+m} \left\{ \begin{array}{ll} \pm j\pi/2 & n+m = \pm p \\ N^{\rm TM}_{p,n+m} & n+m \neq \pm p \end{array} \right\} \tag{22}$$

The magnetic field $H_\phi = (1/j\omega\mu)(\partial E_z/\partial r)$ must be continuous on the boundary at $r = b$ and $0 \leq \phi \leq \pi$. This results in:

$$\frac{k_o}{j\omega\mu} \sum_{n=1}^{\infty} A_n H_n^{(2)\prime}(k_o b) \sin n\phi = \frac{k}{j\omega\mu} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-jm\alpha} \frac{J_m(kd)}{J_n(ka)}$$
$$\left\{X_n e^{-jn\phi_o} J'_{n+m}(kb) - C_n \ \psi_{n,n+m}\right\} e^{j(n+m)\phi} \tag{23}$$

where the Wronskian

$$\psi'^{\rm TM}_{n,n+m} = J_n(ka) Y'_{n+m}(kb) - Y_n(ka) J'_{n+m}(kb) \tag{24}$$

Equation (23) can be expanded as:

$$\frac{1}{\sqrt{\varepsilon_r}} \sum_{n=1}^{\infty} A^{\rm TM}_n H_n^{(2)\prime}(k_o b) \sin n\phi =$$
$$\sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_0(ka)} \ X^{\rm TM}_0 e^{-jm\alpha} J'_m(kb) \ \{\cos m\phi + j \sin m\phi\}$$
$$+\sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\frac{J_m(kd)}{J_n(ka)} \ X^{\rm TM}_n J'_{n+m}(kb) \cos(m\alpha + n\phi_o) \cos(n+m)\phi$$
$$+\sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\frac{J_m(kd)}{J_n(ka)} \ X^{\rm TM}_n J'_{n+m}(kb) \sin(m\alpha + n\phi_o) \sin(n+m)\phi$$

$$+\sum_{n=0}^{\infty} C_n^{\rm TM} \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)}\, e^{-jm\alpha}\psi_{n,n+m}^{\prime\rm TM}\{\cos(n+m)\phi+j\sin(n+m)\phi\}$$

$$+\sum_{n=1}^{\infty} D_n^{\rm TM} \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)} e^{jm\alpha}\psi_{n,n+m}^{\prime\rm TM}\{\cos(n+m)\phi-j\sin(n+m)\phi\} \quad (25)$$

Multiplying both sides of the above equation by $\sin p\phi$ ($p = 1, 2, 3, \ldots$) and integrating both sides over ϕ from 0 to π. The following integrals results in:

$$\frac{\pi}{2\sqrt{\varepsilon_r}} A_p^{\rm TM} H_p^{(2)\prime}(k_o b) = f_p' + \sum_{n=0}^{\infty} C_n^{\rm TM} g_{p,n}' + \sum_{n=1}^{\infty} D_n^{\rm TM} q_{p,n}' \quad (26)$$

$$f_p' = \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_0(ka)} \left\{X_0^{\rm TM} e^{-jm\alpha} J_m'(kb)\right\} \left[\begin{Bmatrix} j\pi/2 & m = \pm p \\ N_{p,n+m}^{\rm TM} & m \neq \pm p \end{Bmatrix}\right]$$

$$+\sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\frac{J_m(kd)}{J_n(ka)}\ X_n^{\rm TM} J_{n+m}'(kb)\cos(m\alpha + n\phi_o)$$

$$\begin{Bmatrix} 0 & n+m = \pm p \\ N_{p,n+m}^{\rm TM} & n+m \neq \pm p \end{Bmatrix}$$

$$+\sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\frac{J_m(kd)}{J_n(ka)}\ X_n^{\rm TM} J_{n+m}'(kb)\sin(m\alpha + n\phi_o)$$

$$\begin{Bmatrix} 0 & n+m \neq \pm p \\ \pi/2 & n+m = \pm p \end{Bmatrix} \quad (27)$$

$$g_{p,n}' = \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)}\ e^{-jm\alpha}\psi_{n,n+m}^{\prime\rm TM} \begin{Bmatrix} \pm j\pi/2 & n+m = \pm p \\ N_{p,n+m}^{\rm TM} & n+m \neq \pm p \end{Bmatrix} \quad (28)$$

$$q_{p,n}' = \sum_{m=-\infty}^{\infty} \frac{J_m(kd)}{J_n(ka)}\ e^{jm\alpha}\psi_{n,n+m}^{\prime\rm TM} \begin{Bmatrix} \mp j\pi/2 & n+m = \pm p \\ N_{p,n+m}^{\rm TM} & n+m \neq \pm p \end{Bmatrix} \quad (29)$$

Multiplying Equation (12) by $\frac{1}{\sqrt{\varepsilon_r}} H_p^{(2)\prime}(k_o b)$ and Equation (26) by $H_p^{(2)}(k_o b)$ and then subtract, one obtains

$$V_p = \sum_{n=0}^{\infty} U_{p,n} C_n + \sum_{n=1}^{\infty} W_{p,n} D_n \quad (30)$$

$$V_p = H_p^{(2)}(k_o b) f_p' - \frac{1}{\sqrt{\varepsilon_r}} H_p^{(2)\prime}(k_o b) f_p \quad (31)$$

$$U_{p,n} = \frac{1}{\sqrt{\varepsilon_r}} H_p^{(2)\prime}(k_o b) g_{p,n} - H_p^{(2)}(k_o b) g_{p,n}' \quad (32)$$

$$W_{p,n} = \frac{1}{\sqrt{\varepsilon_r}} H_p^{(2)'}(k_o b) q_{p,n} - H_p^{(2)}(k_o b) q'_{p,n} \tag{33}$$

Equations (19) and (30) can be re-written in a matrix from as:

$$\lfloor U_{p,n} \rfloor \, [C_n] + \lfloor W_{p,n} \rfloor \, [D_n] = \lfloor V_p \rfloor \tag{34}$$

$$\lfloor S_{p,n} \rfloor \, [C_n] + \lfloor T_{p,n} \rfloor \, [D_n] = \lfloor R_p \rfloor \tag{35}$$

The above matrix equation can be solved as

$$\left\{ [U_{p,n}]^{-1} [W_{p,n}] - [S_{p,n}]^{-1} [T_{p,n}] \right\} \, [D_n] = [U_{p,n}]^{-1} [V_p] - [S_{p,n}]^{-1} [R_p] \tag{36}$$

Once $[D_n]$ is obtained then $[C_n]$ can also be obtained from (30) and A_n from:

$$\frac{\pi}{2} A_p H_p^{(2)}(k_o b) = f_p + \sum_{n=0}^{\infty} C_n g_{p,n} + \sum_{n=1}^{\infty} D_n q_{p,n} \tag{37}$$

The primed Bessel functions in the above expressions denote their derivatives with respect to the argument. The infinite dimension of the above matrix has to be truncated to a finite value. It must be done while ensuring the convergence of the infinite series.

2.1. Radiation Pattern and Slot Conductance

The far-zone radiation pattern for the electric field of the TM case can be calculated using the asymptotic expression of the Hankel function, i.e.,

$$E_z^{(\mathrm{I})} = \sqrt{\frac{2}{\pi k r}} e^{-jkr} e^{j\pi/4} P(\phi) \tag{38}$$

where

$$P(\phi) = \sum_{n=0}^{\infty} j^n A_n \sin n\phi \tag{39}$$

2.2. Results and Discussion

The first example presented here is for a geometry with the parameters, $a = 0.51\lambda$, $b = 0.7\lambda$, $\varepsilon_r = 3.1$, $\alpha = 90^\circ$, $\phi_o = 90^\circ$ and d varies as shown in Fig. 2. The solid curve corresponding to $d = 0$ is the same as the example reported in [9]. The comparison between this curve and the corresponding one in [9] showed an excellent agreement. As one can see the increase in d has affected the radiation pattern in which a main lobe beam width increases with d on the expense of two side lobes.

Figure 3 shows the far field radiation pattern for the parameters, $a = 0.5\lambda$, $b = 0.7\lambda$, $\varepsilon_r = 3.1$, $\alpha = -90^\circ$, $\phi_o = 90^\circ$ and different values

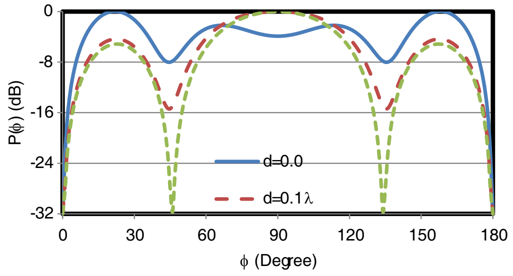

Figure 2. Far field radiation pattern for different offset distances.

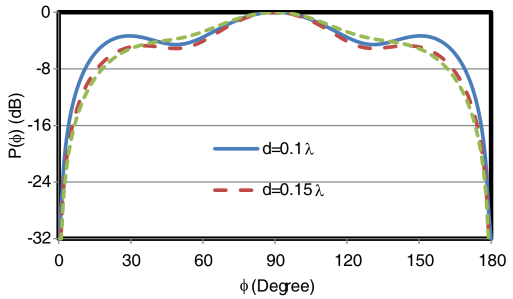

Figure 3. Far field radiation pattern for different offset distances.

of d. In this example d is displaced such that the conducting slotted cylinder is shifted towards the conducting ground plane. As one can see from Fig. 3, the effect of varying d in this direction has a minimal effect on the radiation pattern.

The next example corresponds to a slot positioned at $\phi_o = 0°$ for deferent eccentric distance d. The other parameters are considered as, $a = 0.5\lambda$, $b = 0.7\lambda$, $\varepsilon_r = 3.1$, and $\alpha = 0°$. Radiation patterns corresponding to four different values of d are illustrated in Fig. 4. As one can see as the slotted cylinder is displaced off center, the maximum of the radiation pattern is displaced at 20° while the back lob starts to decrease.

The next example corresponds to a slot positioned at $\phi_o = 270°$ for deferent eccentric distance d. The other parameters are considered as, $a = 0.5\lambda$, $b = 0.7\lambda$, $\varepsilon_r = 3.1$, and $\alpha = 90°$. Radiation patterns corresponding to four different values of d are illustrated in Fig. 5. As one can see as the slotted cylinder displaced off center the radiation pattern which has three lobs starts to combine into one lobe. Moreover, the higher the eccentricity the higher the beam width.

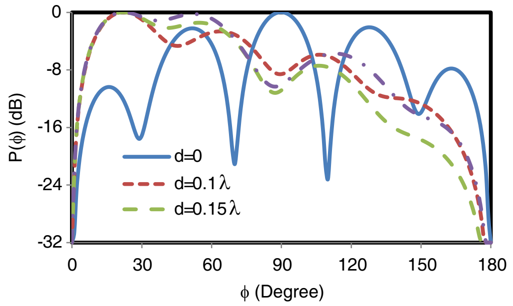

Figure 4. Far field radiation pattern for different offset distances.

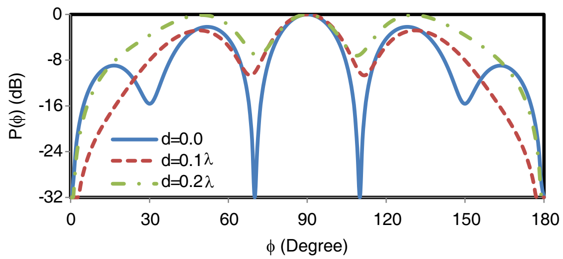

Figure 5. Far field radiation pattern for different offset distances.

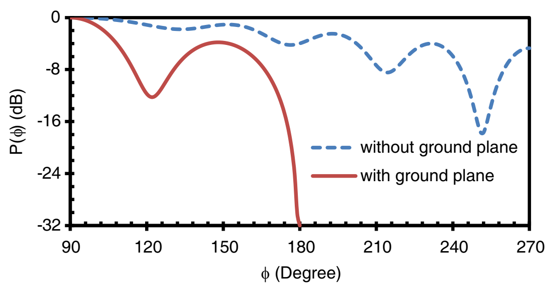

Figure 6. Far field radiation pattern for slotted cylinder coated with dielectric With and without ground plane.

In the last example the slot is positioned at $\phi_o = 90^\circ$ for $d = 0$. The other parameters are considered as, $a = 0.5\lambda$, $b = 0.7\lambda$, $\varepsilon_r = 3.1$. Radiation patterns corresponding to coated slotted cylinder with and without ground plane are shown in Fig. 6. As one can see the ground plane produces more directive pattern in the upper half space.

3. CONCLUSION

The axial slot on a conducting circular cylinder coated with eccentric dielectric and partly embedded in a ground plane has been analyzed. Results corresponding to different geometrical parameters are presented. It is also a support for the antenna instead of using mechanical mounting supporting system. The results show how one can shape the pattern by offsetting the slotted cylinder from the center of the dielectric coating.

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