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A NOVEL SYNTHESIS PROCEDURE FOR ULTRA WIDEBAND (UWB) BANDPASS FILTERS

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Abstract—In this paper, a novel synthesis procedure is presented to achieve optimum solution for UWB filter parameters. It is found that the narrowband approximation is not valid for any arbitrary powered rational type filtering function. For wider bandwidths, the frequency dependent terms have significant effects on the frequency response. Hence, extracted filtering function cannot be mapped to generalize Chebyshev polynomials. This paper will provide an exact synthesis procedure for step impedance resonators (SIR's) type UWB bandpass filters. To validate the synthesis procedure prototypes are designed and fabricated. Simulated and measured results show good agreement with proposed theory.

1. INTRODUCTION

Ultra wideband (UWB) technology plays a vital role in the development of short range high-data-rate communication systems and wireless personal area networks. Since the release of unlicensed UWB communication spectrum from 3.1 to 10.6 GHz, many filter topologies have been proposed to achieve UWB spectrum mask [1–9]. Over the years, theory of classical narrowband microwave bandpass filter has been well established [10, 11]. However, these classical theories are not valid for filters having fractional bandwidth greater than 50%. One such example is reported in [12] and [13], where based on synthesis a new formula is presented to design relatively wider passband using

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parallel-coupled transmission-line resonator, but due to the frequency dependent terms involved in even-odd mode impedance equations, the maximum fractional bandwidth achieved is 50%. Since the filtering function obtained from parallel coupled lines has frequency dependent term in denominator so the approximation $\theta = \pi/2$ is valid only in the vicinity of the center frequency. Hence, for ultra wide bandwidth this frequency dependent term will distort the frequency response. To overcome these constraints, synthesis method is proposed in [9] using isolated cascaded high-pass and low-pass sections. The developed synthesis method is based on an iterative algorithm for Butterworth and Chebyshev type response. However, there are some limitations like equal ripple response for Chebyshev case is not achieved, the fractional bandwidth of 109.4% for UWB spectrum remained unattainable and the return loss level is not flexible. Moreover, the fabricated prototype size is increased due to cascaded network and hence not suitable for hand handle UWB devices. In [5], a direct synthesis procedure is reported for UWB filters using composite series and shunt stubs. Generalize Chebyshev type filtering function is used to synthesize the parameter values. However, this type of filtering function is not valid for any arbitrary powered rational filtering function. Moreover, the filtering function follows a specific periodic pattern which cannot be mapped on all types of filtering functions. Hence, there is a need of a synthesis procedure that is free of these restrictions. Now in order to synthesize the filtering function, following goals have to be considered for Chebyshev type frequency response.

(i) For a given topology, obtain the maximum number of transmission poles in the passband for optimum selectivity.
(ii) Exhibit the desired passband bandwidth.
(iii) Attain equal ripple behavior at the passband for optimum loss performance.
(iv) Obtain the desired return loss determined by a prescribed ripple level factor ϵ.

This paper is organized in the following manner. Initially, even-odd mode analysis to find the overall transfer function has been presented followed by the extraction of the quasi-generalize filtering function using the transfer function. Afterwards, our proposed synthesis procedure is discussed that has been used on extracted filtering function to get exact solutions for the filter parameters. This section also discusses the proposed method in the light of above mentioned goals. Later, the fabricated prototypes are presented that validates our proposed theory. Finally the last section concludes the paper. The actual scope of this work is to produce a novel synthesis procedure in order to give a better understanding of the designed filter theory.

2. PROPOSED SYNTHESIS PROCEDURE FOR SIR'S TYPE UWB FILTERS

Shown in Fig. 1 is a quasi-generalize configuration of UWB filter with its equivalent circuit model [1]. The UWB filter consists of low impedance line $k\theta_o$ attached to high impedance quarter wavelength θ_o parallel coupled lines. Where, k is an integer and is related to filter order by $N = k + 3$. Here, z_o is the characteristic impedance of the middle line whereas, z_{even} and z_{odd} are the even and odd-mode characteristic impedances of parallel coupled line section. z_{ue} is the impedance of unit element. The parallel coupled line is realized by two θ_o open circuited stubs separated by unit element.

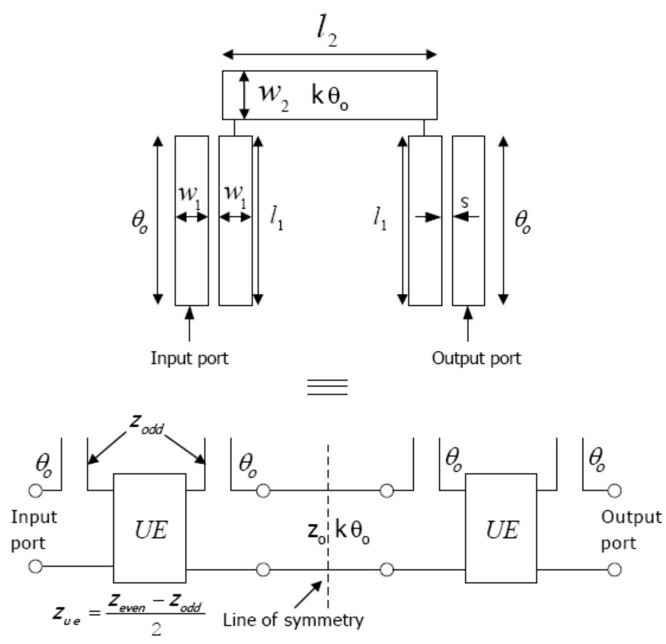

Figure 1. A SIR's with parallel coupled line UWB filter and its equivalent circuit.

Due to the symmetry of design, even-odd mode analysis has been adopted to find the transfer function S_{21}. Now the aim is to extract filtering function $F(\theta)$ from transfer function using Equation (1).

$$|S_{21}(\theta)|^2 = \frac{1}{1+F^2(\theta)} \tag{1}$$

and due to unitary condition, the reflection coefficient $S_{11}(\theta)$ is related by the equation

$$|S_{11}(\theta)|^2 = 1 - |S_{21}(\theta)|^2. \tag{2}$$

Quasi-generalize filtering function is given in Appendix A. Integer value k will produce the required filter order. The filtering function is of the form:

$$F(\theta) = \frac{Numinator(\theta)}{Denominator(\theta)} \tag{3}$$

The analysis of filtering function shows that it is no more a standard first order Chebyshev polynomial due to a frequency dependent term in the denominator, hence cannot be solved using Chebyshev polynomial of first kind. For narrowband filters, this frequency dependent term can be neglected because the bandwidth is in close vicinity of center frequency and hence can not distort the filtering response. Whereas, for fractional bandwidth wider then 20%, this frequency dependence will cause significant effect on filtering response. In the forthcoming sections extracted filtering function will be synthesized to achieve ideal parameter values for $k = 1$ and 2.

2.1. Synthesis of Fourth Order UWB Filter

For forth order UWB filter $k = 1$, the filtering function in (A1) leads to Equation (4). Here the unknown coefficients are redefined for the given case.

$$F(\theta) = \frac{A\cos^4(\theta) + B\cos^2(\theta) + C}{\sin(\theta)}, \tag{4}$$

where,

$$A = \frac{\left[(z_{ue}+z_{\text{odd}})^2-1\right](z_o+z_{ue}+z_{\text{odd}})^2}{2\,{z_{ue}}^2 z_o}, \tag{5}$$

$$B = \left\{\left[2-(z_{cl}+z_{\text{odd}})^2\right]{z_o}^2 + \left(1-2{z_{ue}}^2\right)(z_{ue}+z_{\text{odd}})^2 + 2\left(1-{z_{ue}}^2\right)(z_{ue}+z_{\text{odd}})z_o\right\} \times \left\{2\,{z_{ue}}^2 z_o\right\}^{-1}, \tag{6}$$

$$C = \frac{{z_{ue}}^4 - {z_o}^2}{2{z_{ue}}^2 z_o}. \tag{7}$$

It is required that $z_o, z_{ue}, z_{\text{odd}} > 0$. The filtering function has frequency dependent term in the denominator which will distort the equal ripple characteristic and bandwidth of filter. This effect can be nullified by restructuring the filtering function as shown in the following section.

2.1.1. Restructure the Filtering Function

Filtering function is redefine to achieve mentioned goals, the ripple factor is extracted by normalizing the filtering function in the following form:

$$F(\theta) \equiv \epsilon\, \frac{\tilde{F}(\theta)}{\tilde{F}(\theta_N)}, \tag{8}$$

where θ_N is the normalizing electrical length, and the normalization factor is taken as

$$\tilde{F}(\theta) = \frac{\cos^4(\theta) + \alpha\, \cos^2(\theta) + \zeta}{\sin(\theta)}, \tag{9}$$

where

$$A = \frac{\epsilon}{\tilde{F}(\theta_N)}, \tag{10}$$

$$\alpha = \frac{B}{A}, \tag{11}$$

$$\zeta = \frac{C}{A}. \tag{12}$$

The ripple factor ϵ is related to the return loss by

$$\epsilon = \sqrt{10^{L_R/10} - 1}. \tag{13}$$

From (9), the values of the parameters A, B and C can be obtained by requiring the transfer function

$$|S_{21}(\theta)|^2 = \frac{1}{1 + \epsilon^2 \left| \dfrac{\tilde{F}(\theta)}{\tilde{F}(\theta_N)} \right|^2}, \tag{14}$$

This is related to using the first derivatives to find the relative maxima and minima of the transfer function. In order to generate the filter coefficients for equal ripple response, let the denominator of (2) equal to Q,

$$Q = 1 + F^2(\theta). \tag{15}$$

By Chain rule and using (8), the derivative of reflection coefficient (2) is

$$\frac{d}{d\theta}|S_{11}(\theta)|^2 = \frac{1}{Q^2} \times \frac{2\epsilon^2 \tilde{F}(\theta)}{\tilde{F}^2(\theta_N)} \times \frac{d\tilde{T}(\theta)}{d\theta}. \tag{16}$$

The vanishing of Q^{-1} corresponds to the reflection poles at $\theta = 0$ and π, and the vanishing of $\tilde{T}(\theta)$ corresponds to the reflection zeros which are located at

$$\theta_{zero}^{(\pm,\pm)} = \arccos\left(\pm\frac{1}{2}\sqrt{-2\alpha \pm 2\sqrt{\alpha^2 - 4\zeta}}\right), \tag{17}$$

where the $\pm$ sign notation stated above is understood. In order to ensure the existence of four zeros in the transfer function, it is required that

$$\alpha < 0 \quad \text{and} \quad \zeta > 0, \tag{18}$$

and satisfy the inequalities

$$\alpha^2 - 4\zeta \;>\; 0, \tag{19}$$

$$\alpha + \zeta + 1 \;>\; 0. \tag{20}$$

Since $\theta \in (0, \pi)$, we identify the location of the reflection zeros, in the order from left to right: $\theta^{(+,+)}$, $\theta^{(+,-)}$, $\theta^{(-,-)}$, $\theta^{(-,+)}$. The ripple peak frequencies are found by solving

$$\frac{d\tilde{F}(\theta)}{d\theta} = \frac{[3\cos^4\theta + (\alpha - 4)\cos^2\theta - (2\alpha + \zeta)]\cos\theta}{\sin^2\theta}. \tag{21}$$

Equation (21) has a simple root which is located at $\theta_{\text{peak}}^{(c)} = \frac{\pi}{2}$ (corresponding to center peak). By the standard quadratic formula,

$$\theta_{\text{peak}}^{(\pm,\pm)} = \arccos\left(\pm\sqrt{\frac{4-\alpha}{6} \pm \sqrt{\left(\frac{4-\alpha}{6}\right)^2 + \frac{2\alpha+\zeta}{3}}}\right). \tag{22}$$

It can be shown that $\cos(\theta_{\text{peak}}^{(\pm,+)}) > 1$ for all α and ζ in the domain defined by (18)–(20). Thus, the ripple peaks corresponding to these solutions do not exist and are omitted. The remaining roots, $\theta_{\text{peak}}^{(+,-)}$ and $\theta_{\text{peak}}^{(-,-)}$ correspond to the first and third ripple peak in the passband, respectively.

The equal-ripple can be forced in place by matching the filtering function at cut-off frequency with the first and second peak, i.e.,

$$\tilde{F}(\theta_L) = -\tilde{F}\left(\theta_{\text{peak}}^{(+,-)}\right) = \tilde{F}\left(\frac{\pi}{2}\right). \tag{23}$$

where θ_L is the angular of the lower cut-off frequency. From these simultaneous Equation (23), we obtain

$$\alpha = \frac{3}{4}\left[\cos\left(\frac{\Delta_{BW}}{2}\right) + \frac{1}{3}\right]^2 - \frac{4}{3}, \tag{24}$$

$$\zeta = \frac{1}{4}\sin^2\left(\frac{\Delta_{BW}}{2}\right)\left[1 - \cos\left(\frac{\Delta_{BW}}{2}\right)\right], \tag{25}$$

where Δ_{BW} is the bandwidth in degrees which is related to filter frequency by $\theta = \frac{\pi}{2} \cdot \frac{f}{f_c}$, here f_c is the center frequency. α and ζ can be found using (24) and (25), which are used to find unknown coefficients given in Equations (27)–(28). Finally, characteristic Impedance can be evaluated using (5)–(7).

Figure 2 shows the ideal frequency response of 7.5 GHz bandwidth using the synthesized parameter values given in Table 1 with return loss of 15 dB.

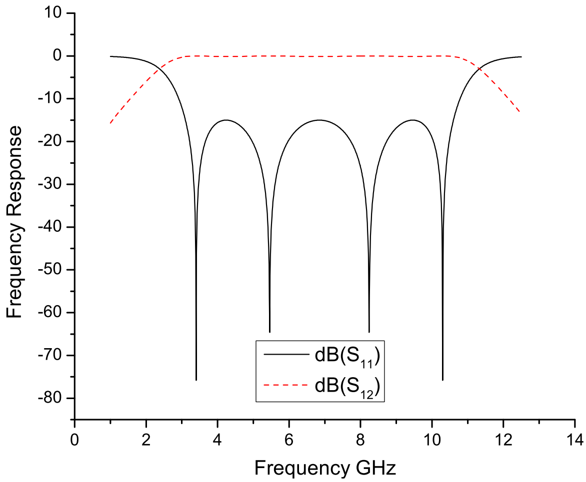

Figure 2. Ideal frequency response using synthesized parameter values.

2.2. Synthesis of Fifth Order UWB Filter

In order to achieve better selectivity, more number of transmission poles will be needed in the given passband. For fifth order $k = 2$, filtering function (A1) reduce to (26). Parameter values will be redefined again for fifth order as shown in Equation (26).

$$F(\theta) = \frac{A\cos^5(\theta) + B\cos^3(\theta) + C\cos(\theta)}{\sin(\theta)}, \tag{26}$$

where $A \neq 0$ and the filter coefficients A, B and C are given by

$$A = \frac{\left(q^2 - 1\right)\left(q + z_o\right)^2}{z_o k^2}, \tag{27}$$

$$B = \frac{z_o^2\left(2 - q^2\right) + z_o q\left(3 - 2k^2 - q^2\right) + q^2\left(1 - 2k^2\right)}{z_o k^2}, \tag{28}$$

$$C = \frac{k^4 - z_o^2 + z_o q \left(k^2 - 1\right)}{z_o k^2}, \tag{29}$$

where $z_o > 0$ is the normalized characteristic impedance of middle-section transmission line, $q > 0$ and $k > 0$ are the coupling coefficients. Due to the fact that

$$q = \frac{z_{\text{even}} + z_{\text{odd}}}{2} \quad \text{and} \quad k = \frac{z_{\text{even}} - z_{\text{odd}}}{2}, \tag{30}$$

where $z_{\text{even}} > z_{\text{odd}} > 0$ are the normalized even and odd-mode characteristic impedance of coupled-line pair, so the coupling coefficients are bounded by $q > k$.

For achieving Chebyshev equiripple response filtering function is normalized using ripple factor ϵ such as.

$$A = \frac{\epsilon}{\tilde{F}(\bar{\theta})}, \tag{31}$$

Then the filtering function reads

$$\tilde{F}(\theta) = \frac{\cos^5(\theta) + \beta \cos^3(\theta) + \gamma \cos(\theta)}{\sin(\theta)}, \tag{32}$$

where

$$\beta = \frac{B}{A}, \tag{33}$$

$$\gamma = \frac{C}{A}. \tag{34}$$

Now the same procedure is adopted as described in previous section to find transmission poles in passband. It is obvious from (32) that one of the zeros of the filtering function is located at $\theta_{zero}^{(3)} = \pi/2$ and the others are

$$\theta_{zero\pm}^{(1)} = \arccos\left(\pm\sqrt{-\frac{1}{2}\beta + \sqrt{\frac{1}{4}\beta^2 - \gamma}}\right), \tag{35}$$

$$\theta_{zero\pm}^{(2)} = \arccos\left(\pm\sqrt{-\frac{1}{2}\beta - \sqrt{\frac{1}{4}\beta^2 - \gamma}}\right). \tag{36}$$

In order to obtain maximal number of reflection zeros, all zeros must be real and different. Therefore, β is required to be negative, whereas γ is positive and satisfies the inequalities

$$\beta^2 - 4\gamma > 0 \tag{37}$$

and

$$0 < -\beta \pm \sqrt{\beta^2 - 4\gamma} < 2. \tag{38}$$

This implies that

$$\beta + \gamma + 1 > 0. \tag{39}$$

In order to find ripple peaks, the first derivative of filtering function is set to zero. Thus the passband peak frequencies correspond to the vanishing of

$$\frac{d\tilde{F}(\theta)}{d\theta} = \frac{4\cos^6\theta + (2\beta - 5)\cos^4\theta - 3\beta\cos^2\theta - \gamma}{\sin^2\theta}, \tag{40}$$

and this leads to solving the equation

$$\cos^6\theta + \left(\frac{1}{2}\beta - \frac{5}{4}\right)\cos^4\theta - \frac{3}{4}\beta\cos^2\theta - \frac{1}{4}\gamma = 0. \tag{41}$$

The solutions of (41) are given by the standard cubic formula

$$\cos\theta^{(1)}_{peak\pm} = \pm\sqrt{\left(\frac{5}{12} - \frac{1}{6}\beta\right) - \frac{1}{2}(\tau_+ + \tau_-) - i\frac{\sqrt{3}}{2}(\tau_+ - \tau_-)}, \tag{42}$$

$$\cos\theta^{(2)}_{peak\pm} = \pm\sqrt{\left(\frac{5}{12} - \frac{1}{6}\beta\right) - \frac{1}{2}(\tau_+ + \tau_-) + i\frac{\sqrt{3}}{2}(\tau_+ - \tau_-)}, \tag{43}$$

$$\cos\theta^{(3)}_{peak\pm} = \pm\sqrt{\left(\frac{5}{12} - \frac{1}{6}\beta\right) + \tau_+ + \tau_-}, \tag{44}$$

where

$$\tau_\pm = \left(-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3}\right)^{1/3}, \tag{45}$$

where

$$a = -\frac{1}{12}(\beta + 2)^2 - \frac{3}{16}, \tag{46}$$

$$b = \frac{1}{108}(\beta + 2)^3 - \frac{1}{4}(\beta + \gamma) - \frac{7}{32}. \tag{47}$$

From the given domain in Equations (37)–(39), it is found that the real values of $\theta^{(3)}_{peak+}$ and $\theta^{(3)}_{peak-}$ do not exist, hence can be neglected. Now, let $(\tilde{\beta})$ and $(\tilde{\gamma})$ be the point for which the peak's level is equalized inside the passband. Due to the symmetry of the response, it is sufficient to consider the first and second peaks where the reflection coefficients have the same value,

$$\left|S_{11}\left(\theta^{(1)}_{peak+}\right)\right|^2 = \left|S_{11}\left(\theta^{(2)}_{peak+}\right)\right|^2. \tag{48}$$

Since the filtering function oscillates with frequency, (48) reduced to the filtering functions which are related by

$$\tilde{F}\left(\theta^{(1)}_{peak+}\right) = -\tilde{F}\left(\theta^{(2)}_{peak+}\right). \tag{49}$$

This equation contains two unknown parameters, for the determination of which we need another simultaneous equation which can be obtained by equating the response at low cut-off frequency, θ_L, and the value at peak frequency, i.e.,

$$\tilde{F}(\theta_L) = -\tilde{F}\left(\theta^{(1)}_{peak+}\right). \tag{50}$$

Unlike the fourth order, solution of these equations is extremely difficult to obtain by using substitution method. So in order to solve these tedious Equations (49) and (50) Newton-Raphson's method [14] is used, which involves of simultaneous zeroing

$$F_1(\beta,\gamma) = \tilde{F}\left(\theta^{(1)}_{peak+}\right) + \tilde{F}\left(\theta^{(2)}_{peak+}\right), \tag{51}$$

$$F_2(\beta,\gamma) = \tilde{F}\left(\theta^{(1)}_{peak+}\right) + \tilde{F}(\theta_L). \tag{52}$$

We compute the sequence defined as

$$\Gamma_{n+1} = \Gamma_n - \mathrm{J}^{-1}\Phi, \tag{53}$$

where

$$\Phi = \begin{pmatrix} F_1(\beta_n,\gamma_n) \\ F_2(\beta_n,\gamma_n) \end{pmatrix}, \tag{54}$$

$$\Gamma_n = \begin{pmatrix} \beta_n \\ \gamma_n \end{pmatrix}, \tag{55}$$

and J is the Jacobian matrix

$$\mathrm{J} = \begin{pmatrix} \frac{\partial F_1}{\partial \beta} & \frac{\partial F_1}{\partial \gamma} \\ \frac{\partial F_2}{\partial \beta} & \frac{\partial F_2}{\partial \gamma} \end{pmatrix}, \tag{56}$$

evaluated at β_n and γ_n. To finding the elements of J matrix, we take the partial derivative of Equation (40) with respect to β and γ.

$$\frac{\partial \cos\theta^{(3)}_{peak\pm}}{\partial \beta} = \frac{\left(3 - 2\cos^2\theta^{(3)}_{peak\pm}\right)\cos\theta^{(3)}_{peak\pm}}{24\cos^4\theta^{(3)}_{peak\pm} + 4(2\beta-5)\cos^2\theta^{(3)}_{peak\pm} - 6\beta}, \tag{57}$$

$$\frac{\partial \cos\theta^{(3)}_{peak\pm}}{\partial \gamma} = \left\{\left[24\cos^4\theta^{(3)}_{peak\pm} + 4(2\beta-5)\cos^2\theta^{(3)}_{peak\pm} - 6\beta\right]\cos\theta^{(3)}_{peak\pm}\right\}^{-1}, \tag{58}$$

Using (40), (57) and (58) with the first and second frequency peaks in place, the elements of matrix J are given by

$$\frac{\partial F_1}{\partial \beta} = \sum_{i=1}^{2} \left\{ -\frac{1}{\sin\theta_{peak+}^{(i)}} \frac{\partial \tilde{F}\left(\theta_{peak+}^{(i)}\right)}{\partial \theta_{peak+}^{(i)}} \cdot \frac{\partial \cos\theta_{peak+}^{(i)}}{\partial \beta} + \frac{\cos^3\theta_{peak+}^{(i)}}{\sin\theta_{peak+}^{(i)}} \right\}, \quad (59)$$

$$\frac{\partial F_1}{\partial \gamma} = \sum_{i=1}^{2} \left\{ -\frac{1}{\sin\theta_{peak+}^{(i)}} \frac{\partial \tilde{F}\left(\theta_{peak+}^{(i)}\right)}{\partial \theta_{peak+}^{(i)}} \cdot \frac{\partial \cos\theta_{peak+}^{(i)}}{\partial \gamma} + \frac{\cos\theta_{peak+}^{(i)}}{\sin\theta_{peak+}^{(i)}} \right\}, \quad (60)$$

$$\frac{\partial F_2}{\partial \beta} = -\frac{1}{\sin\theta_{peak+}^{(1)}} \frac{\partial \tilde{F}\left(\theta_{peak+}^{(1)}\right)}{\partial \theta_{peak+}^{(1)}} \cdot \frac{\partial \cos\theta_{peak+}^{(1)}}{\partial \beta} \quad (61)$$

$$\frac{\partial F_2}{\partial \gamma} = -\frac{1}{\sin\theta_{peak+}^{(1)}} \frac{\partial \tilde{F}\left(\theta_{peak+}^{(1)}\right)}{\partial \theta_{peak+}^{(1)}} \cdot \frac{\partial \cos\theta_{peak+}^{(1)}}{\partial \gamma}. \quad (62)$$

By using appropriate initial guess for β_n and γ_n, the solution of Equation (52) is achieved. The normalizing factor $\tilde{F}(\overline{\theta})$ can be evaluated straightforwardly using (32) and $\theta = \theta_L$. Now by choosing desired ripple level ϵ, the filtering coefficients can be evaluated by using (31), (33) and (34). The characteristic impedances can then be evaluated using (27)–(29). Presented synthesis procedure gave full control of ripple level and bandwidth. Fig. 3 shows the synthesized

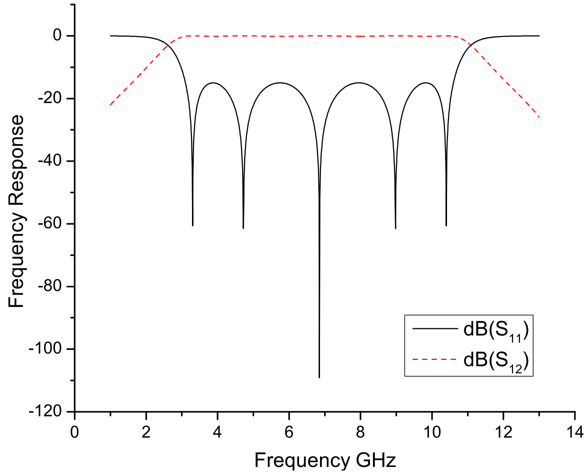

Figure 3. Ideal frequency response using synthesized parameter values.

Table 1. Synthesized values of filter parameters shown in Fig. 1 for various bandwidths and ripple levels ($f_c = 6.85\,\text{GHz}$).

Bandwidth (GHz)	Ripple Level (dB)	Order N	z_{even} (Ω)	z_{odd} (Ω)	z_0 (Ω)
7.5	10	4	174.75	31.292	74.186
5.0	10	4	215.63	79.176	67.125
2.5	10	4	349.66	223.19	57.663
7.5	20	4	124.24	13.293	55.668
5.0	20	4	143.28	38.99	49.186
2.5	20	4	214.92	120.47	40.344
7.5	30	4	108.24	5.4943	51.137
5.0	30	4	116.85	20.206	45.245
2.5	30	4	158.27	73.078	35.16
7.5	10	5	171.93	37.589	53.073
5.0	10	5	212.8	88.024	33.441
2.5	10	5	351.71	233.73	15.729
7.5	20	5	122.55	17.95	42.79
5.0	20	5	141.43	46.891	26.187
2.5	20	5	217.91	131.02	11.775
7.5	30	5	107.11	8.7357	43.08
5.0	30	5	115	27.859	26.497
2.5	30	5	161.53	84.605	11.235
3.7	15	4	201.58	91.516	50.604
4.3	15	5	182.48	80.237	23.718

frequency response of 7.5 GHz bandwidth with return loss of 15 dB using parameter values given in Table 1.

This synthesis procedure can be used for all types of filtering functions extracted from the planer filter topology. Here it is worth mentioning that the most critical part is to find the solutions after setting the goals. For small N, these equations are solved using standard substitution method. Whereas, for high N where the complexity of equations is increased, approximation methods are used with appropriate initial guess.

As for designers starting from a given specifications, it is often a long journey to reach final design of a typical filter. Therefore, the synthesis procedure is of critical importance in filter design because this approach provides a number of benefits, e.g., a remarkable reduction in computational time and flexibility to design high-order filter structures with many unknown dimensions. The exact dimensions obtained from synthesis will be a good initial guess for more accurate EM-based simulation, tuning and optimization. Moreover, the synthesis approach of filter design provides a deep physical insight into the operating principle for the developed microwave circuits, which is of core importance for understanding which ultimately results in even finer designs and ideas. Table 1 shows some synthesized filter parameter values for forth and fifth order UWB filter.

3. FABRICATED PROTOTYPES

In order to validate the proposed synthesis, full-wave EM simulation is done on Advanced Design System (ADS) [15] using microstrip line with RT/duroid 5880 ($\varepsilon_r = 2.2$, $\tan\delta = 0.0009$ and height $h = 787\,\mu\text{m}$). Tapered line is used at I/O ports to match with 50 Ohm SMA connectors and also to ease constraint on even and odd mode impedance of parallel coupled lines which are used to achieve tight coupling for wider frequency response. It is worth mentioning here that for fourth order, simulator tuning and optimization have not been used. The filter parameters are achieved using impedance values in Table 1. Fabricated prototypes are shown in Fig. 4 for fourth and fifth order UWB bandpass filters. The resulting simulated frequency response shown in Fig. 5 is no more equalized due to the effect of the parasitic elements, substrate dielectric and conductive losses, which are not considered in the theory. Moreover, the investigated band is extremely wide, it is typical to anticipate that the effect of the

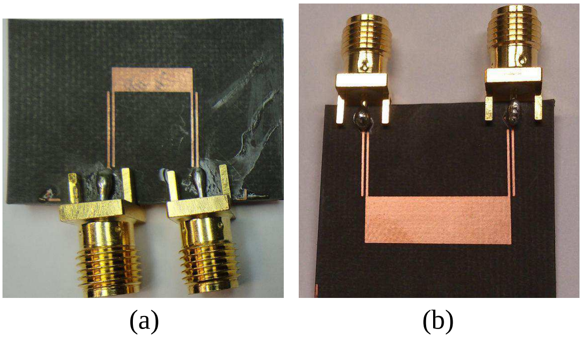

Figure 4. Fabricated prototypes. (a) 4th order UWB filter $(0.28 \times 0.36)\lambda_g$. (b) 5th order UWB filter $(0.56 \times 0.46)\lambda_g$.

non-ideal elements used in the simulation would be significant. For fourth order, 3.7 GHz of bandwidth has been selected for fabrication. Using synthesized impedance values given in Table 1, parameter values labeled in Fig. 1 are determined as: $l_1 = 8532.87\,\mu\text{m}$, $w_1 = 212.47\,\mu\text{m}$, $l_2 = 7959.24\,\mu\text{m}$, $w_2 = 2364.04\,\mu\text{m}$ and $s = 296.738\,\mu\text{m}$. Similarly, for fifth order, 4.3 GHz of bandwidth is selected for prototype fabrication. The parameter values labeled in Fig. 1 are determined as l_1 =

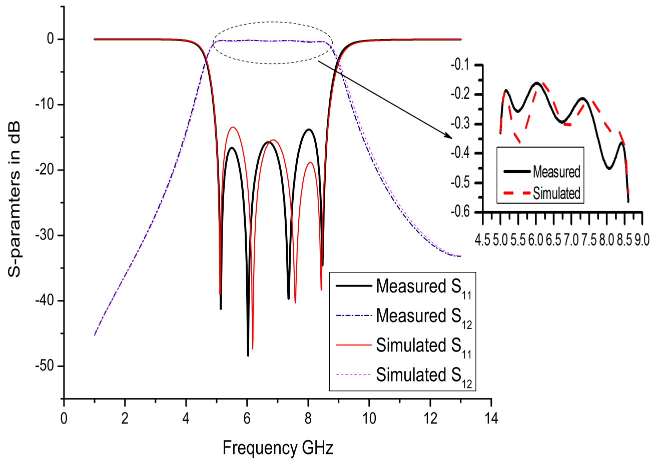

Figure 5. Simulated and measured frequency response for fourth order UWB filter.

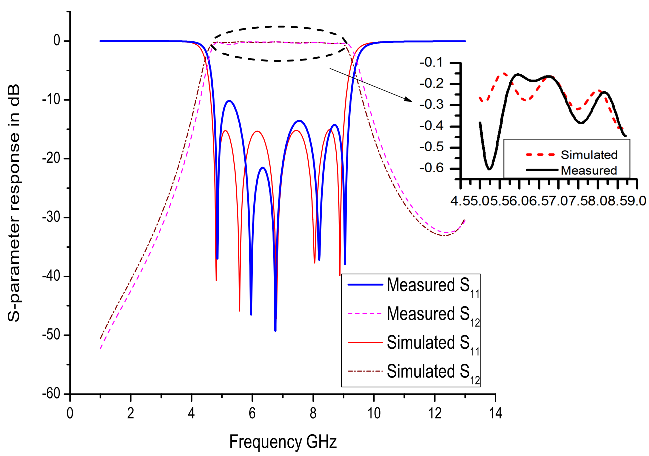

Figure 6. Simulated and measured frequency response for fifth order UWB filter.

7918.35 μm, $w_1 = 333.775$ μm, $l_2 = 16080.5$ μm, $w_2 = 5675.3$ μm and $s = 219.186$ μm. Simulated and measured results are shown in Figs. 5 and 6 for the fourth and fifth order UWB filters, respectively. Post fabrication tuning is done to get better results. Measured frequency response is well correlated with theory with some discrepancies in ripple level. Possible reasons for this are fabrication errors and isolation problem (due to the close vicinity of I/O ports). In addition, due to fabrication limitations fractional bandwidth for fourth and fifth order is reduced to 54.01% and 62.773% respectively. To achieve fractional bandwidth of 109.489%, coupled line spacing should be very small in order to get tight coupling between parallel coupled lines.

4. CONCLUSION

An optimum design procedure has been presented in this paper using SIR's. The proposed synthesis procedure is implemented on filtering function extracted from the equivalent circuit of quasi-generalized UWB filter prototype. Maximum transmission poles in the passband have been shown by setting nominator of filtering function equals to zero. Ripple peaks are realized by taking derivative of the filtering function. Ideal values for filter parameters are extracted for both the fourth and fifth orders. This proposed synthesis procedure can be used for all types of filtering functions. In order to validate the design procedure, UWB bandpass filter is fabricated with four and five transmission poles. Simulated and measured results are well correlated.

APPENDIX A. QUASI-GENERALIZE FILTERING FUNCTION

$$\begin{aligned} F(\theta) \;=\; & \Big\{\Big[\big(A\cos(\theta)^8 + B\cos(\theta)^6 + C\cos(\theta)^4 + D\cos(\theta)^2 + E\big) \\ & \big(\cos(k\theta)^2\big) + \big(F\cos(\theta)^4 + G\cos(\theta)^2 + H\big)\sin(\theta)\sin(k\theta) \\ & \big(I\cos(\theta)^2 + J\big)\cos(\theta)\cos(k\theta) + \big(F'\cos(\theta)^4 + G'\cos(\theta)^2 \\ & + H'\big)^2\Big]^{\frac{1}{2}}\Big\} \times \Big\{\sin(\theta)^2\Big\}^{-1} \end{aligned} \qquad \text{(A1)}$$

where A, B, C, D, E, F, G, H, I, J, F', G' and H' are the filtering coefficients given as:

$$\begin{aligned} A \;=\; & \Big\{-\big({z_{ue}}^4 + 6\left[{z_o}^2 + {z_{\rm odd}}^2\right]{z_{ue}}^2 + \left[12\,{z_o}^2 z_{\rm odd} + 4\,{z_{\rm odd}}^3\right] z_{ue} + {z_o}^4 \\ & + 4{z_{ue}}^3 z_{\rm odd} + {z_{\rm odd}}^4 + 6\,{z_o}^2 {z_{\rm odd}}^2\big)\big(\left[z_{ue} + z_{\rm odd}\right]^2 - 1\big)^2\Big\} \end{aligned}$$

$$
\begin{aligned}
&\times\Big\{4\, {z_{ue}}^4 {z_o}^2\Big\}^{-1}, \\
B \;=\;& \Big\{2\left(2\, {z_{ue}}^6 + 8\, z_{\rm odd}\, {z_{ue}}^5 + \left(12\, {z_{\rm odd}}^2 - 1 + 9{z_o}^2\right) {z_{ue}}^4 + (-4\, z_{\rm odd}\right. \\
&+8\, {z_{\rm odd}}^3 + 24\, {z_o}^2 z_{\rm odd}\big)\, {z_{ue}}^3 + \left(2\, {z_{\rm odd}}^4 + \left(24\, {z_o}^2 - 6\right) {z_{\rm odd}}^2 + {z_o}^4\right. \\
&\left.-9{z_o}^2\right) {z_{ue}}^2 + 2\left(\left(6z_o^2 - 2\right) z_{\rm odd}^2 + z_o^4 - 9z_o^2\right) z_{\rm odd} z_{ue} + \left(3{z_o}^2 - 1\right) \\
&z_{\rm odd}^4 + \left(z_o^4 - 9z_o^2\right) z_{\rm odd}^2 - 2z_o^4\big) \left(\left(z_{ue} + z_{\rm odd}\right)^2 - 1\right) \Big\} \\
&\times\Big\{4\, {z_{ue}}^4 {z_o}^2\Big\}^{-1}, \\
C \;=\;& \Big\{- 6{z_{ue}}^8 - 24{z_{ue}}^7 z_{\rm odd} - \left(18{z_o}^2 + 36{z_{\rm odd}}^2 - 6\right) {z_{ue}}^6 - \left(24{z_{\rm odd}}^3\right. \\
&\left.-20z_{\rm odd} + 60\, {z_o}^2 z_{\rm odd}\right) {z_{ue}}^5 - \left(6\, {z_{\rm odd}}^4 - \left[26 - 78\, {z_o}^2\right] {z_{\rm odd}}^2 + {z_o}^4\right. \\
&\left.+1 - 38\, {z_o}^2\right) {z_{ue}}^4 - 4\left(\left[12\, {z_o}^2 - 4\right] {z_{\rm odd}}^2 + {z_o}^4 + 1 - 24\, {z_o}^2\right) {z_{ue}}^3 \\
&z_{\rm odd} + \left(\left[-12\, {z_o}^2 + 4\right] {z_{\rm odd}}^4 + \left[-6\, {z_o}^4 + 96\, {z_o}^2 - 6\right] {z_{\rm odd}}^2 + 6{z_o}^4\right. \\
&\left.-18{z_o}^2\right) {z_{ue}}^2 - 4z_{\rm odd}\left(\left[1 - 12{z_o}^2 + {z_o}^4\right] {z_{\rm odd}}^2 - 3{z_o}^4 + 9{z_o}^2\right) z_{ue} \\
&+ \left(-1 + 12\, {z_o}^2 - {z_o}^4\right) {z_{\rm odd}}^4 + \left(6{z_o}^4 - 18\, {z_o}^2\right) {z_{\rm odd}}^2 - 6\, {z_o}^4\Big\} \\
&\times\Big\{4\, {z_{ue}}^4 {z_o}^2\Big\}^{-1}, \\
D \;=\;& \Big\{4\, {z_{ue}}^8 + 8\, {z_{ue}}^7 z_{\rm odd} + \left(6{z_o}^2 - 2 + 4\, {z_{\rm odd}}^2\right) {z_{ue}}^6 + \left(12{z_o}^2 - 4\right) \\
&z_{\rm odd} {z_{ue}}^5 + \left(\left[6\, {z_o}^2 - 2\right] {z_{\rm odd}}^2 - 16\, {z_o}^2\right) {z_{ue}}^4 - 24{z_{ue}}^3 {z_o}^2 z_{\rm odd} \\
&-2{z_o}^2\left({z_o}^2 + 6\, {z_{\rm odd}}^2 - 3\right) {z_{ue}}^2 - 4{z_o}^2 z_{\rm odd}\left({z_o}^2 - 3\right) z_{ue} \\
&+ \left(-2{z_o}^4 + 6\, {z_o}^2\right) {z_{\rm odd}}^2 + 4{z_o}^4\Big\} \times \Big\{4\, {z_{ue}}^4 {z_o}^2\Big\}^{-1}, \\
E \;=\;& \Big\{ - {z_{ue}}^8 - {z_o}^4 + 2\, {z_{ue}}^4 {z_o}^2\Big\} \times \Big\{4\, {z_{ue}}^4 {z_o}^2\Big\}^{-1}, \\
F \;=\;& \Big\{\left({z_{ue}}^2 + 2z_{ue} z_{\rm odd} + {z_{\rm odd}}^2 + {z_o}^2\right) \left(z_{ue} + z_{\rm odd}\right) \left(\left[z_{ue} + z_{\rm odd}\right]^2 - 1\right) \Big\} \\
&\times\Big\{4\, {z_{ue}}^4 {z_o}^2\Big\}^{-1}, \\
G \;=\;& \Big\{ \left(z_{ue} + z_{\rm odd}\right) \left(-2\, {z_{ue}}^4 - 4{z_{ue}}^3 z_{\rm odd} + \left[1 - {z_o}^2 - 2{z_{\rm odd}}^2\right] {z_{ue}}^2\right. \\
&\left.+ \left[2 - 2{z_o}^2\right] z_{\rm odd}\, z_{ue} + \left[1 - {z_o}^2\right] {z_{\rm odd}}^2 + 2{z_o}^2\right) \Big\} \times \Big\{{z_{ue}}^4 z_o\Big\}^{-1}, \\
H \;=\;& \Big\{ \left({z_{ue}}^4 - {z_o}^2\right) \left(z_{ue} + z_{\rm odd}\right) \Big\} \times \Big\{{z_{ue}}^4 z_o\Big\}^{-1},
\end{aligned}
$$

$$
\begin{aligned}
I &= \left\{ \left([z_{ue} + z_{\text{odd}}]^2 - 1 \right) (z_{ue} + z_{\text{odd}}) \right\} \times \left\{ {z_{ue}}^4 z_o \right\}^{-1}, \\
J &= \left\{ \left(1 - {z_{ue}}^2 \right) (z_{ue} + z_{\text{odd}}) \right\} \times \left\{ {z_{ue}}^4 z_o \right\}^{-1}, \\
F' &= \left\{ 4\, z_o\, (z_{ue} + z_{\text{odd}})\, F \right\}, \\
G' &= \left\{ 4\, z_o\, (z_{ue} + z_{\text{odd}})\, G \right\}, \\
H' &= \left\{ 4\, z_o\, (z_{ue} + z_{\text{odd}})\, H \right\},
\end{aligned}
$$

Here $z_o, z_{ue}, z_{\text{odd}} > 0$.

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