

## HIGH-DYNAMIC DOA ESTIMATION BASED ON WEIGHTED $L_1$ MINIMIZATION

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**Abstract**—In high dynamic environment, due to the rapid relative movement between receiver and transmitter, the DOA (Direction of Arrival) of signals will change even between two consecutive snapshots. Thus, covariance-based DOA estimation algorithms are ineffective. Compressive sensing algorithms, as a kind of novel DOA estimation algorithms, are still effective with only one snapshot. At the same time, it is noted that the DOA changing is limited by relative moving speed and distance between receiver and transmitter. In this paper, a DOA tracking algorithm based on weighted  $L_1$  minimization is proposed which utilizing the DOA changing scope between two consecutive snapshots as a prior to improve the tracking performance. Different from other multiple snapshots compressive sensing algorithms which assumed fixed DOA among multiple consecutive snapshots, the proposed algorithm takes into account the DOA changing among different snapshots. The simulation results demonstrate the advantages of the proposed algorithm.

### 1. INTRODUCTION

DOA (Direction of Arrival) estimation problem is a major problem in many engineering applications, such as smart antennas, mobile communications, radio astronomy, sonar, navigation. Many DOA estimation algorithms have been proposed, see [1] and references therein. These algorithms can be roughly classified into two categories: single snapshot algorithms and multiple snapshots algorithms. The most representative single snapshot algorithm is DAS (Delay-and-Sum) algorithm [1]. Many other algorithms are included in multiple

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*Received 14 June 2013, Accepted 10 August 2013, Scheduled 20 August 2013*

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snapshots algorithms, e.g., Capon, MUSIC, ESPRIT [2, 3]. Of course, there also are some algorithms which can be adopted with single snapshot and multiple snapshots, e.g., ML (Maximum Likelihood) algorithm [4]. Generally, the performances of multiple snapshots algorithms are better than that of single snapshot algorithms. However, the multiple snapshots algorithms assume that the DOAs are fixed in multiple consecutive snapshots. In high dynamic environment, due to the rapid relative movement between receiver and transmitter, the DOAs will rapidly change. Thus, the assumption of fixed DOAs is irrational. In this case, single snapshot algorithms are a better choice. However, as stated before, performances of single snapshot algorithms are generally worse than that of multiple snapshots algorithms, such as DOA resolution and estimation error. Therefore, it is interesting to propose a new single snapshot algorithm to improve DOA estimation performance in high dynamic environment.

In recent years, compressive sensing algorithms have been recognized as a kind of novel high resolution DOA estimation algorithms [5]. The sparse property of spatial spectrum is utilized if there is only a limited number of point sources. Different with conventional DOA estimation algorithms, compressive sensing algorithms have high DOA resolution based on same number of snapshot. Thus it is an alternative choice in high dynamic environment where it is difficult, if not impossible, to obtain enough number of snapshots for the fixed DOAs. The representative compressive sensing algorithms include convex relaxation algorithms [6, 7] and greedy algorithms [8, 9]. It has been proved that the compressive sensing algorithms can accurately recover the sparse signal under certain conditions.

In compressive sensing framework, many DOA estimation algorithms have been proposed by utilizing one or multiple snapshots [5]. In the multiple snapshots case, the DOAs are generally assumed as fixed. However, in high dynamic environment, the DOAs will change even between two consecutive snapshots. Thus, almost all multiple snapshots compressive sensing algorithms are ineffective. At this point, a simple choice is to estimate DOAs based on only one snapshot. At the same time, it is also noted that the DOAs changing between two consecutive snapshots is limited by the relative moving speed and distance between receiver and transmitter. Thus the DOAs changing is constrained in a known scope, which can be utilized as a prior information, to improve the performance of the single snapshot compressive sensing algorithms.

There are also some DOA estimation algorithms in compressive sensing framework to deal with DOA dynamic changing [10, 11].

One algorithm is to incorporate Kalman signal evolution model to compressive sensing framework [10]. By utilizing the known signal evolution model as a prior, the algorithm improves the performance of DOA estimation. However, in many practical applications, it is difficult to obtain this model. Another algorithm is compressive sensing algorithm with partially known support information [11]. The DOA estimation performance is also improved by utilizing the partially known support information as a prior information. When there is no known support information, the algorithm degrades to the conventional compressive sensing algorithm. However, in high dynamic environment, all DOAs will change among multiple consecutive snapshots. Thus the assumption about the partially known support information is also irrational in high dynamic environment.

In this paper, a DOA tracking algorithm based on weighted  $L_1$  minimization algorithm [12] is proposed. Different from other algorithms, the signal evolution model is not assumed known and all DOAs will change between two consecutive snapshots. The proposed algorithm is also different with reweighted  $L_1$  algorithm where the weights are based on the estimated signal vector in the last iteration. Then the performance is improved by including multiple weighted  $L_1$  optimization problems for one snapshot which leads to a higher complexity. The proposed algorithm includes only one weighted  $L_1$  minimization problem for one snapshot which utilizing the DOA changing limitation as a prior to improve the performance.

The reminder of the paper is organized as follows. The problem formulation and compressive sensing algorithm are presented in Section 2. The Section 3 is devoted to the proposed DOA tracking algorithm based on weighted  $L_1$  minimization. The performance evaluation of proposed algorithm is included in Section 4. Section 5 concludes the paper.

## 2. PROBLEM FORMULATION AND COMPRESSIVE SENSING ALGORITHM

### 2.1. Problem Formulation

Consider a uniform linear array (ULA) consisting of  $M$  identical and omnidirectional elements. The inter-element spacing is the half of wavelength of the signals as  $d = \lambda/2$ . Assume  $K$  far field narrow band sources are impinging upon the array from different angles. Thus the resulting steering vector can be expressed as follows

$$\mathbf{a}(\phi_k) = \left[ 1, e^{-j2\pi \sin(\phi_k)}, \dots, e^{-j2\pi(M-1) \sin(\phi_k)} \right]^T \quad (1)$$

where  $(*)^T$  denotes matrix transpose, and  $\phi_k \in [-90^\circ, 90^\circ]$  is the DOA of the  $k$ th incident signal. And the steering matrix at the  $i$ th snapshot is as follows

$$\mathbf{A}'_i = [\mathbf{a}(\phi_1^i), \dots, \mathbf{a}(\phi_K^i)] \quad (2)$$

where  $\phi_k^i$  is the DOA of the  $k$ th signal for the  $i$ th snapshot.

Then the array output snapshot can be given as follows

$$\mathbf{y}_i = \mathbf{A}'_i \mathbf{s}'_i + \mathbf{n}_i \quad (3)$$

where  $\mathbf{y}_i$  is the  $i$ th received snapshot,  $\mathbf{s}'_i = [\mathbf{s}'_{i1}, \dots, \mathbf{s}'_{iK}]^T$  the corresponding signal vector, and  $\mathbf{n}_i$  the zero-mean Gaussian measurements noise with covariance matrix  $\delta \mathbf{I}$ .

The DOA estimation is to obtain  $\phi_k^i$  by utilizing one snapshot or multiple snapshots. As stated before, for multiple snapshots algorithms, it is generally assumed that all DOAs of signals are fixed among multiple consecutive snapshots. However, in high dynamic environment, the DOAs will change even between two consecutive snapshots. Thus many multiple snapshots algorithms are ineffective.

Compressive sensing algorithms, due to the high resolution capability, have been adopted for a new kind of DOA estimation algorithms. Best of all, it is even effective with single snapshot. Therefore, compressive sensing algorithms maybe a good choice for DOA tracking in high dynamic environment.

## 2.2. Compressive Sensing Algorithms

Compressive sensing algorithms are a kind of approximately convex optimization algorithms to find a sparse solution by utilizing the sparsity as a prior information. In order to utilize compressive sensing algorithms to solve the DOA estimation problem, the received signal model has to be transformed into a sparse representation which fitting the compressive sensing model. The key idea is to divide the total DOA space into a potential DOA set with a given grid spacing, e.g.,  $\Theta = \{-90^\circ, -89^\circ, \dots, 88^\circ, 89^\circ\}$  with grid spacing  $1^\circ$ . The gridding spacing  $1^\circ$  will be used in the following parts if not stated otherwise. Thus the steering matrix  $\mathbf{A}'_i$  can be transformed to an overcomplete dictionary  $\mathbf{A}$

$$\mathbf{A} = [\mathbf{a}(-90^\circ), \dots, \mathbf{a}(89^\circ)] \quad (4)$$

where the number of columns is equal to the size of the potential DOA set  $\Theta$ , e.g.,  $N = 180$ . As other many references, it is assumed that the DOAs of signals are included in the potential DOA set, e.g.,  $\phi_k^i \subseteq \Theta$  for all  $k$ . If the potential DOA set does not include the DOAs of signals, an effective way is to decrease the grid spacing and enlarge the size of the potential DOA set.

Then the upper Equation (3) can be reformulated as

$$\mathbf{y}_i = \mathbf{A}\mathbf{s}_i + \mathbf{n}_i \quad (5)$$

The vector  $\mathbf{s}_i$  denotes a new signal vector which relates to the signal vector  $\mathbf{s}'_i$  as follows

$$\mathbf{s}_{in} = \begin{cases} \mathbf{s}'_{ik}, & \theta_n = \phi_k^i \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where  $\mathbf{s}_{in}$  ( $1 \leq n \leq N$ ) and  $\theta_n$  are the  $n$ th element of the sparse vector  $\mathbf{s}_i$  and the potential DOA set  $\Theta$ , respectively.

As the number of signals is far less than the size of the potential DOA set ( $N \gg K$ ), the most elements of the signal vector  $\mathbf{s}_i$  are zero. Thus the signal vector  $\mathbf{s}_i$  is a sparse vector. Then the DOA estimation problem has been transformed into a standard compressive sensing problem where  $\mathbf{A}$  and  $\mathbf{s}_i$  are measurement matrix and sparse vector, respectively. Therefore, all compressive sensing algorithms can be utilized to obtain DOA estimation.

The most direct way to incorporate sparsity prior is to formulate following optimization problem

$$\min_{\mathbf{s}_i} \|\mathbf{s}_i\|_0 \quad (7)$$

$$\text{subject to} \quad \|\mathbf{y}_i - \mathbf{A}\mathbf{s}_i\|_2 \leq \epsilon \quad (8)$$

where  $\|\mathbf{s}_i\|_0$  denotes the number of non-zero elements in the vector  $\mathbf{s}_i$ , and  $\epsilon$  is a parameter related to the noise vector  $\mathbf{n}_i$ .

However, the upper optimization problem is an NP-hard problem. In order to overcome this problem, a well-known method is to approximate it by following convex optimization problem, e.g., BPDN (Basis Pursuit Denoise) [7]

$$\min_{\mathbf{s}_i} \|\mathbf{s}_i\|_1 \quad (9)$$

$$\text{subject to} \quad \|\mathbf{y}_i - \mathbf{A}\mathbf{s}_i\|_2 \leq \epsilon \quad (10)$$

where  $\|\mathbf{s}_i\|_1$  is the  $L_1$  norm of vector  $\mathbf{s}_i$ :  $\|\mathbf{s}_i\|_1 = \sum_n |\mathbf{s}_{in}|$ . Though there are also compressive sensing algorithms which utilizing multiple snapshots to improve performance, they are ineffective in high dynamic environment as the rapidly changing of DOAs.

### 3. PROPOSED DOA TRACKING ALGORITHM

In order to improve the performance of BPDN algorithm, iteratively reweighted basis pursuit algorithm [12] has been proposed. Different with the BPDN algorithm, it improves performance by giving different weights to different elements in  $L_1$  norm. In every iteration, the

present weights are set based on the estimated signal vector in the last iteration.

In a mathematical formulation, it iteratively solves a sequence of weighted subproblems

$$\min_{\mathbf{s}_i} \quad \|\mathbf{W}\mathbf{s}_i\|_1 \quad (11)$$

$$\text{subject to} \quad \|\mathbf{y}_i - \mathbf{A}\mathbf{s}_i\|_2 \leq \epsilon \quad (12)$$

where  $\mathbf{W}$  is a diagonal matrix with  $w_n$  along the main diagonal and zeros elsewhere.

The weight  $w_n$  in the  $l$ th iteration is updated according to

$$w_n = \left( \hat{\mathbf{s}}_{in}^{l-1} + \gamma \right)^{-1} \quad (13)$$

where  $\hat{\mathbf{s}}_{in}^{l-1}$  is the  $n$ th element of the estimated vector in the  $(l-1)$ th iteration, and  $\gamma$  is introduced to prevent undefined weight when  $\hat{\mathbf{s}}_{in}^{l-1} = 0$ . Theoretical results have proved that, by iteratively utilizing the prior information obtained in the last iteration, the iteratively reweighted basis pursuit algorithm improves the performance than that of BPDN algorithm. Note that the little weights will have their corresponding coefficients increased and the large weights will have their corresponding coefficients decreased. Thus the object function can be further decreased in current iteration and the elements corresponding to larger coefficients is more possible to be included in the estimated support set.

It should be noted that the iteratively reweighted basis pursuit algorithm estimates the signal vector based on the same one snapshot. Thus the DOAs are unchanged in all iterations. In other words, compared with the BPDN algorithm, it improves the performance at the cost of multiple weighted  $L_1$  optimization subproblems. The complexity of the iteratively reweighted basis pursuit algorithm will be improved with increasing of the number of iterations. Therefore, the complexity is more higher than the BPDN algorithm which may prohibit the iteratively reweighted basis pursuit algorithm from being utilized in high dynamic environment.

Though the DOAs will change between two consecutive snapshots in high dynamic environment, the changing is limited by the relative moving speed and relative distance between receiver and transmitter. Once the relatively maximum moving speed and relatively minimum distance are identified, it is easy to obtain the DOA changing scope between two consecutive snapshots. Even if the relative maximum moving speed and minimum distance are not known accurately, it is also possible to estimate the approximate DOA changing scope based

on the concrete application. Thus, as a prior information, it is possible to be utilized to improve the performance of DOA estimation.

Once the DOA changing scope between two consecutive snapshots is known, the scope of the DOAs in present snapshot is easily to be obtained based on the estimated DOAs in the last snapshot. Then a possible way to incorporate the prior information is to set the weights in the present snapshot based on the DOA scope and the estimated signal vector in the last snapshot. If the weights are still set as the Equation (13) by ignoring the prior information, the weights according to the estimated DOAs in the last snapshot will be set less than that of the neighbors. Then the estimated DOAs are more possible unchanged which is contrary with the fact of DOA changing.

For every estimated DOA in the last snapshot, a better way is to give the same weights to the DOAs included in the corresponding DOA scope. It means that all DOAs in the DOA scope have same probability to be estimated as a real DOA, which is contrary to the iteratively reweighted basis pursuit algorithm. In the sense of possibility theory, it assumed that all DOAs in one DOA scope have the same a priori possibility.

Based on the discussions before, the weight in the  $i$ th snapshot can be set as following

$$w_{in} = \begin{cases} (\hat{\mathbf{s}}_{(i-1)k} + \gamma)^{-1}, & \theta_i \in (\hat{\phi}_{(i-1)k} - \Delta, \hat{\phi}_{(i-1)k} + \Delta) \\ \gamma^{-1}, & \text{otherwise} \end{cases} \quad (14)$$

where  $w_{in}$  is the  $n$ th main diagonal element of the weight matrix in the  $i$ th snapshot,  $\hat{\mathbf{s}}_{(i-1)k}$  the estimated coefficient according to the  $k$ th estimated DOA  $\hat{\phi}_k$  in the  $(i-1)$ th snapshot, and  $\Delta$  the possibly maximum DOA changing scope between two consecutive snapshots.

Thus the estimated signal vector in the present snapshot is obtained by solving the following weighted  $L_1$  optimization algorithm

$$\min_{\mathbf{s}_i} \|\mathbf{W}_i \mathbf{s}_i\|_1 \quad (15)$$

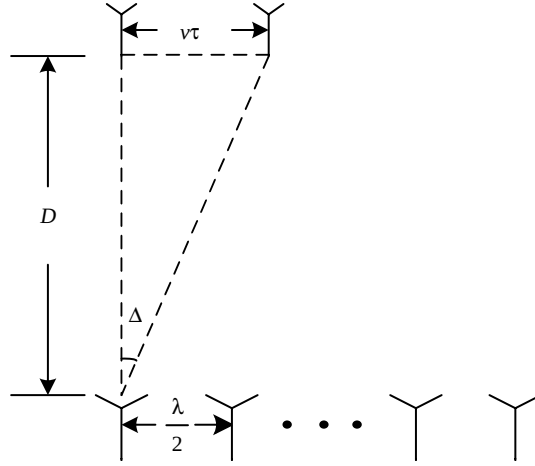
$$\text{subject to } \|\mathbf{y}_i - \mathbf{A} \mathbf{s}_i\|_2 \leq \epsilon \quad (16)$$

where  $\mathbf{W}_i = \text{diag}\{w_{i1}, \dots, w_{iN}\}$ . Once the signal vector is estimated, the estimated DOAs can be obtained by finding the peaks of the estimated signal vector.

The remaining problem is how to obtain the DOA changing scope  $\Delta$ . If the relative maximum speed and minimum distance between receiver and transmitter are known, the  $\Delta$  can be set as following

$$\Delta = \arctan \frac{v\tau}{D} \quad (17)$$

where  $v$  is the relative maximum speed,  $\tau$  the time interval between two consecutive snapshots, and  $D$  the relative minimum distance. The geometric relationship between receiver and transmitter is shown in the Figure 1.



**Figure 1.** Geometric relationship between the transmitter and the receiver.

In some applications, parameters  $v$  and  $D$  can be easily obtained. However, in some cases, it is difficult to accurately obtain these parameters. Then,  $\Delta$  can be set slightly larger than that in the Equation (17) which ensures real DOAs are included in the DOA scope. Otherwise, the real DOAs maybe excluded from the DOA scope which leads to the wrong estimated DOAs.

It is clear that the performance will degrade with increasing of the DOA scope. If there is only one signal,  $\Delta$  is set so large that the DOA scope includes all possible DOAs, e.g.,  $\Delta = \Theta$ . Then the proposed algorithm degrades to the standard BPDN algorithm. However, the simulations show that a slightly larger DOA scope only leads to a slight performance degradation. The accurate  $\Delta$ , of course, provides more prior information which will improve the performance of the proposed algorithm.

When two real DOAs are close to each other, the corresponding DOA scopes maybe coincide to each other. Then the weights in the overlap scope can be set as the less one. The simulations show that the proposed algorithm is robust for the overlap scope. When the estimated DOAs are not accurate in the last snapshot, the proposed is still robust as long as the real DOAs are included in the DOA scopes.



So far, the proposed algorithm includes only one weighted  $L_1$  optimization problem for one snapshot. However, it is easily to extend to include multiple weighted  $L_1$  optimization iterations. It means that in present snapshot, the weights in the first iteration are set based on the estimated signal vector in the last snapshot. In the other iterations, as the reweighted basis pursuit algorithm, the weights are set based on the estimated signal vector in the last iteration. In this case, the cost of improved performance is the increasing of the complexity.

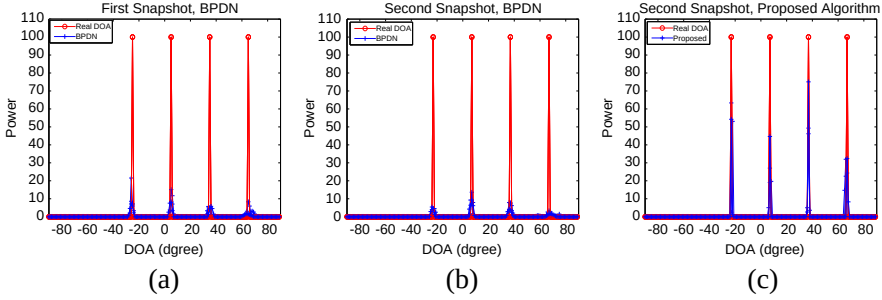
According to the complexity, the rationality of proposed algorithm can be described in two ways. On one hand, the  $L_1$  optimization problem, as a specific convex programming problem, can be solved by linear programming [13]. At the same time, there have been some more efficient algorithms which are proposed for weighted  $L_1$  optimization problem [14] and streaming signals [15]. On the other hand, for the offline DOA tracking problem, the complexity of proposed algorithm is totally acceptable.

It is also worth to point out that the number of sources is not necessary to be known for the proposed algorithm. In addition, as only one snapshot is utilized to obtain the estimated DOAs, the signal coherence also has no impact on the performance of the proposed algorithm.

#### 4. SIMULATIONS

In this section, several numerical simulations will demonstrate the performance of the proposed algorithm. Unless otherwise stated, a linear array is employed which includes  $M = 12$  elements with half-wavelength spacing. Equal modulus are assumed for all sources and the phases are selected randomly for all sources. The additive noise is assumed as complex white Gaussian noise. As only one snapshot is utilized to obtain the estimated DOAs, the conventional DOA estimation algorithms which are based on the signal statistical property will not be included in most simulations. Compared with the BPDN algorithm, the proposed algorithm improves performance by utilizing the prior information of DOA changing scope. Therefore, the proposed algorithm is compared with the BPDN algorithm between two consecutive snapshots.

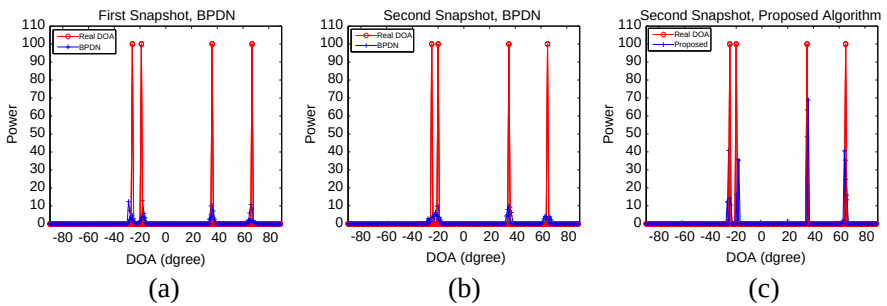
In Figure 2, the proposed algorithm is compared with the BPDN algorithm where enough DOA separation among all DOAs is assumed. Thus the BPDN algorithm and the proposed algorithm can resolve all DOAs. In the first snapshot, only the BPDN algorithm is provided as there is no prior information. In the second snapshot, BPDN algorithm and the proposed algorithm are evaluated separately. In order to keep



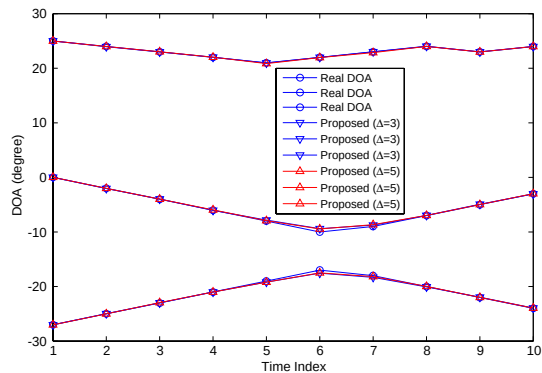
**Figure 2.** Performance comparison in the case with enough DOA separation. (a) The BPDN algorithm for the first snapshot. (b) The BPDN algorithm for the second snapshot. (c) The proposed algorithm for the second snapshot.

the figure clear, only three trials are included in the Figure. The power of all sources is assumed as 20 dB and the power of noise is 0 dB, i.e.,  $SNR_1 = \dots = SNR_4 = 20$  dB. The DOA changing scope is set as  $\Delta = 4^\circ$  which is an estimated DOA changing scope. In the first snapshot, all DOAs are set as  $\{-25^\circ, -5^\circ, 35^\circ, 65^\circ\}$ . In the second snapshot, all DOAs are set as  $\{-23^\circ, -3^\circ, 37^\circ, 67^\circ\}$  which means a  $2^\circ$  DOA changing for all DOAs between two consecutive snapshots. The figure shows that, as stated before, the BPDN algorithm and the proposed algorithm can accurately estimate all DOAs in the second snapshot. However, the Figure 2 shows that the estimated signal vector of the proposed algorithm is much sparser than that of the BPDN algorithm.

In Figure 3, the proposed algorithm is compared with the BPDN algorithm where the BPDN algorithm cannot resolve the little DOA separation. The power of sources and noise is same as that of in Figure 2. The DOAs are set as  $\{-26^\circ, -19^\circ, 36^\circ, 67^\circ\}$  in the first snapshot. In the second snapshot, the DOAs are set as  $\{-25^\circ, -20^\circ, 35^\circ, 65^\circ\}$ . Therefore, the DOA changes of all DOAs are different as  $\{1^\circ, -1^\circ, -1^\circ, -2^\circ\}$ . As that in Figure 2, the DOA changing scope is also set as  $\Delta = 4^\circ$ . In the first snapshot, the BPDN algorithm can resolve all DOA where the minimum DOA separation is  $7^\circ$ . In the second snapshot, the BPDN algorithm cannot resolve the two DOAs whose separation is  $5^\circ$ . However, the proposed algorithm can still resolve the two DOAs with  $5^\circ$  DOA separation. Therefore, the proposed algorithm has higher resolution than the BPDN algorithm for DOA tracking in high dynamic environment. This result is consistent with that of reweighted basis pursuit algorithm which has higher resolution than the BPDN algorithm.



**Figure 3.** Performance comparison in the case with enough DOA separation. (a) The BPDN algorithm for the first snapshot. (b) The BPDN algorithm for the second snapshot. (c) The proposed algorithm for the second snapshot.



**Figure 4.** The DOA tracking based on the proposed algorithms with different DOA changing scope.

In order to simulate a DOA tracking process, multiple consecutive snapshots are included in the Figure 4. In the first snapshot, three DOAs are assumed as  $\{-26^\circ, 1^\circ, 26^\circ\}$ . For every DOA, the DOA changing in later consecutive snapshots is different. Specially, in order to test the stability of the proposed algorithm, the different DOA changing scopes are also considered. If the DOA changing scope is not accurately obtained, two different DOA changing scope are set as  $\Delta = 3^\circ$  and  $\Delta = 5^\circ$  which is unchanged in the tracking process. The simulation shows that the proposed algorithm can accurately track all DOAs in all snapshots. Another conclusion is that the performance of the proposed algorithm will slightly degrade with a slight larger DOA changing scope. Therefore, the proposed algorithm is robust for an

estimated DOA changing scope. On the other hand, even if we have no a priori information about the DOA changing scopes, the proposed algorithm will degrade to the conventional BPDN algorithm where the weight matrix is set as a unit matrix.

## 5. CONCLUSION

In high dynamic environment, as the rapid changing of signal DOAs, conventional DOA estimation algorithms are ineffective as there is no enough snapshots to obtain the statistical property of the signal. However, compressed sensing algorithms, e.g., the BPDN algorithm, are still effective as the estimated DOA can be obtained based on only one snapshot. Compared with the BPDN algorithm, the reweighted basis pursuit algorithm improves the performance by iteratively solving multiple weighted  $L_1$  optimization subproblems. However, the cost is a higher complexity. In this paper, a weighted  $L_1$  minimization algorithm is proposed to improve the performance by utilizing the DOA changing scope as a prior information. The weights in present snapshot are set based on the prior information and the estimated signal vector in the last snapshot. Different with the reweighted basis pursuit algorithm, only one weighted  $L_1$  optimization problem has to be solved for one snapshot. The simulations demonstrate the effectiveness of the proposed algorithm.

## ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Nos. 61179064, 61172112, 61271404), Fundamental Research Funds for Central Universities (Nos. ZXH2010D012, ZXH2009A003, ZXH2012M006), and National Science Foundation of Tianjin Municipality (No. 10ZCKFGX04000).

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