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A COMPRESSIVE SENSING SIGNAL DETECTION FOR UWB RADAR

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Abstract—A major challenge in UWB signal processing is the requirement for very high sampling rate under Shannon-Nyquist sampling theorem which exceeds the current ADC capacity. Radar signal is essentially a delayed and scaled version of the transmitted pulse, determined by sparse parameters such as time delays and amplitudes. A system for sampling UWB radar signal at an ultra-low sampling rate based on the Finite Rate of Innovation (FRI) and the estimation of time delays and amplitudes to detect UWB radar signal is presented in the paper. This sampling scheme which acquires the Fourier series coefficients often results in sparse parameter extraction for UWB radar signal detection. The parameters such as time-delays and amplitudes are estimated using the total variation norm minimization. With this system, the UWB radar signal can be accurately reconstructed and detected with overwhelming probability at the rate much lower than Nyquist rate. The simulation results show that the proposed approach offers very good recovery performances for noisy UWB radar signal using very small number of samples, which is effective for sampling and detecting UWB radar signal.

1. INTRODUCTION

Because of the advantage of the ability of high spatial resolution and object-distinction capability, ultra-wideband (UWB) technology has been widely utilized in radar system. UWB radar transmits the pulse of very short period, characterized by its very high bandwidth, up to several gigahertzes. To digitize such UWB radar signal, a very high sampling rate is required according to Shannon-Nyquist sampling theorem [1], namely, the received signals must be sampled at twice

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their baseband bandwidth. However, the very high sampling rate brings a great challenge of acquiring and detecting the UWB radar signal at the receiver side, especially for a lower rate analog-to-digital converter (ADC), which requires less computational power. To address this issue, some special sampling systems (e.g., parallel ADCs) are developed. The U.S. Army Research Laboratory (ARL) has been developed low-frequency UWB radar systems [2] with inexpensive ADC to sample UWB radar signal, in which a much higher equivalent sampling rate can be achieved via the synchronous time equivalence sampling technique where the same radar signal waveform of interest is acquired with many observations with different phase offsets and the resulting under-sampled data records are then interleaved to yield the equivalent over-sampled data record. The idea of this technique is same to the multicoset sampling scheme [3]. However, except that this technique has a drawback due to the longer time required to complete a data acquisition cycle, it is suggested that this technique introduces an inherent bandwidth limitation which distorts the samples due to a preceding low-pass filter in practice [3]. In addition, the emerging compressed sensing (CS) theory [4, 5] indicates that sparse signals may be reconstructed with high probability by relatively fewer non-adaptive random measurements beyond Nyquist limit with optimization techniques, if the signal has a sparse representation in some bases or frames. Based on the CS theory, Tropp et al. [6] presented an efficient sampling of sparse bandlimited signal, named as Random Demodulator. The input signal is mixed by multiplying it with a high-rate pseudorandom sign sequence and the mixed output is then integrated and dumped at sub-Nyquist rate. In the recovery stage, CS algorithms are performed to recover the original signal from these samples. Random Demodulator is restricted to measure the on-grid tones and performs poorly for the off-grid tone model.

Some continuous-time signal classes, such as streams of Dirac, piecewise polynomials, are called the Finite Rate of Innovation (FRI) which are not bandlimited but still have a finite number of degrees of freedom per unit time [7]. Many classes of FRI signal can be recovered from samples taken at the rate of innovation, which is far below the Nyquist rate [7, 8]. Vetterli et al. [7] formulated the relationship between the signal's Fourier series coefficients and its unknown parameters in the form of a spectral analysis problem. The latter may be solved using existing techniques, given a subset of consecutive Fourier series coefficients, with a minimal cardinality of $2L$. Lie et al. [9] proposed UWB Direction of Arrival estimation using digital channelization receiver architecture which split the UWB array output into multiple frequency channels and then down-convert each channel into much lower frequency, hence allowing the low sampling rate ADC to be used. Xampling framework [10] based on FRI is also developed to capture the information carried by an analog signal, by filtering with an appropriate sampling kernel and then sampling it at the sub-Nyquist rate. Xampling aims to successfully acquire the FRI signal at the minimal sampling rate. The two sampling schemes based on single processing channel and multiple processing channels respectively were developed to extract a small subset of the detected signal's frequency samples [11, 12].

Generally, the radar signals are formed by scattering of a transmitted pulse from multiple reflectors, may be modeled by a relatively small number of pulses, all replicas of some known transmitted pulse shape. Given the number of reflected pulses by L, the radar signals can be defined by $2L$ degrees of freedom corresponding to the replicas unknown time delays and amplitudes, which are described within the FRI framework. The UWB radar signal have a wide band but a small non-zero information rate. The sampling scheme based on FRI can capture the information of UWB radar signal using a small number of samples, even a minimal number $2L$, which is less than the required number of samples when employing Random Demodulator sampling scheme, since it still acquires some samples with zero information. Thus in the paper, we follow the FRI sampling scheme which is described in the papers [10, 12] to acquire sub-Nyquist samples, which allows us to detect UWB radar signals using very small number of samples.

To estimate the underlying unknown parameters such as time delays and amplitudes, many mature techniques, such as spectral analysis frameworks [13], can be used, however, these techniques require prior knowledge of the model order L and are not highly stable in the presence of noise. In the real case, the model order L is unknown. Another alternative approach is CS algorithm based on l_1 norm minimization [14], which need quantize the analog time axis with a small-size discrete grid, but some conditions, such as restricted isometry property [15] and discrete uncertainty principle [16] that are required in CS to guarantee perfect signal recovery by ℓ_1 sparsity constraint, are not well satisfied due to the off-grid problem [17], which seriously degrade the recovery performance of l_1 norm minimization. The convex program based on total variation minimization which can be interpreted as being the continuous analog to the ℓ_1 norm for discrete signals can overcome these conditions [17]. In the paper, we use the convex program based on total variation minimization to estimate the unknown parameters form a small subset of Fourier series coefficients. Simulation results showed that convex program based on total variation minimization yields better performances than recovery algorithms based on ℓ_1 norm minimization to detect the UWB radar with both on-grid and off-grid models using very small number of samples.

The remainder of the paper is organized as follows. The next section establishes the mathematical foundation of our radar signal model. Section 3 introduces the sampling scheme to extract Fourier series coefficients at sub-Nyquist rate for UWB radar signal within FRI framework. In Section 4, we estimate the unknown parameters form a small subset of Fourier series coefficients using CS based on the total-variation minimization. The results for the simulated and experimental radar signals are presented in Section 5.

2. UWB RADAR SIGNAL MODEL

The basic principle of UWB radar is to transmit electromagnetic (EM) radar pulse to image the subsurface. When the transmitted EM wave reflects from a scatter or a boundary with different dielectric constants in a scene, the receiving antenna records the reflected return signal. A model which is a simple linear superposition of targets is obtained as

$$x(t; r_0) = \int \frac{\sigma(r) g(t - \tau(r; r_0))}{A(r; r_0)} dr \tag{1}$$

where $x(t; r_0)$ is the measured space-time data at the antenna position r_0, $\tau(r; r_0)$ the total travel time from the transmitting antenna to the target space point r and back onto the receiver, $\sigma(r)$ the reflection coefficient at the target space point r, and $A(r; r_0)$ accounts for spreading and losses during prorogation.

Under the common point-target model, according to Born approximation, the received signal $x(t)$ of (1) can be written as

$$x(t) = \sum_{i=1}^{L} \sigma_i g(t - \tau_i), \quad t \in [0, \tau) \tag{2}$$

where L is the number of scatters in the target space, τ_i the total trip delay from the transmitting antenna to the scatter i and back to receiving antenna, σ_i the complex amplitude proportional to the target's radar cross section (RCS), dispersion attenuation and spreading losses through propagation, and τ the total receiving period. Thus, the received signal is essentially a delayed and scaled version of the transmitted pulse, which is defined by the following parameters: transmitted pulse $g(t)$, the number of scatters reflected L, time delays $\{\tau_i\}_{i=1}^{L}$, and complex amplitudes $\{\sigma_i\}_{i=1}^{L}$. We make the following assumptions on UWB radar signal:

- UWB transmitted pulse $g(t)$ is known;
- Each time delay is in the total signal period: $\{\tau_i\}_{i=1}^L \subset [0, \tau)$; and
- $g(t-\tau_i) = 0, t \notin (0, \tau)$, so that $x(t) = 0, t \notin (0, \tau)$.

3. ULTRA-LOW SAMPLING RATE SCHEME FOR UWB RADAR SIGNAL

Since the received signal $x(t)$ is confined to the interval $[0, \tau)$, we can extend $x(t)$ in a Fourier series as

$$x(t) = \sum_{k \in \mathcal{Z}} X[k] e^{j\frac{2\pi}{\tau}kt}, \quad t \in [0, \tau), \tag{3}$$

where

$$X[k] = \frac{1}{\tau} \int_0^\tau x(t) e^{-j\frac{2\pi}{\tau}kt} dt \tag{4}$$

Substituting (2) into (4), we obtain

$$\begin{aligned} X[k] &= \frac{1}{\tau} \sum_{i=1}^{L} \sigma_i \int_0^\tau g(t-\tau_i) e^{-j\frac{2\pi}{\tau}kt} dt \\ &= \frac{1}{\tau} \sum_{i=1}^{L} \sigma_i e^{-j\frac{2\pi}{\tau}k\tau_i} \int_{-\infty}^{\infty} g(t) e^{-j\frac{2\pi}{\tau}kt} dt = \frac{1}{\tau} G\left(\frac{2\pi}{\tau}k\right) \sum_{i=1}^{L} \sigma_i e^{-j\frac{2\pi}{\tau}k\tau_i} \end{aligned} \tag{5}$$

where $G(w)$ denotes the continuous time Fourier transformation of $G(t)$.

Choose a set $\mathcal{K}$ of K indices for which $G(\frac{2\pi}{\tau}k) \neq 0, \forall k \in \mathcal{K}$. Such a integer subset exists for UWB radar transmitted pulse due to its very large relative bandwidth. (5) can be rewritten as

$$Y[k] = \frac{X[k]}{\frac{1}{\tau} G\left(\frac{2\pi}{\tau}k\right)} = \sum_{i=1}^{L} \sigma_i e^{-j\frac{2\pi}{\tau}k\tau_i} \tag{6}$$

Denote $V(\mathbf{t})$ as the $K \times L$ Vandermonde matrix with klth element given by $e^{-j\frac{2\pi}{\tau}k\tau_l}$, where $\mathbf{t} = \{\tau_1, \ldots, \tau_L\}$ is the vector of the unknown time delays. In addition, let $\alpha = (\sigma_1, \ldots, \sigma_L)^T$, and $y = (Y[1], \ldots, Y[K])^T$, we may now write (6) in the matrix form as

$$y = V(\mathbf{t})\alpha \tag{7}$$

Given vector y, the problem of retrieving α and $\{\tau_1, \tau_2, \ldots, \tau_L\}$ in (7) is a standard sum-of-exponentials problem. As long as $K \geq 2L$ and the time delays are distinct, these time delays $\{\tau_i\}_{i=1}^L$ can be estimated using methods such as annihilating filter [7] or ESPRIT [18]. Once the time-delays are known, the linear set of Equations (7) may be solved via least-squares for the unknown amplitudes $\{\alpha_i\}_{i=1}^{L}$. Due to the Vandermonde form of $V(t)$, it is left invertible as long as $K \geq L$, so that $\alpha = V^{\dagger}(t)y$, where $V^{\dagger}$ is the pseudoinverse of V.

Given the certain vector of $K \geq 2L$ Fourier series coefficients X, we can determine the parameters $\{\tau_i, \sigma_i\}_{i=1}^{L}$ to detect the target. To acquire Fourier series coefficients, we employed a Xampling scheme described in the papers [10, 12], which allows extraction of its necessary samples of Fourier series coefficients at sub-Nyquist rate (see Figure 1). The multichannel sampling scheme is used to directly extract the required Fourier series coefficients from the signal, where the analog input signal $x(t)$ in each channel is firstly mixed with the harmonic signal $e^{-j\frac{2\pi}{\tau}kt}$, integrated over the whole time period, and then sampled. An alternative Xampling method uses the Sum of Sincs filter, following by uniform sampling of the output with sub-Nyquist sampling rate as described in [11].

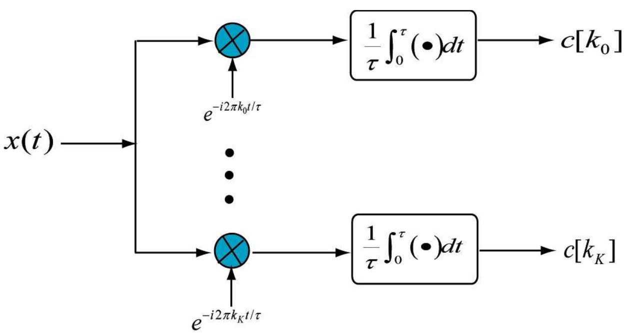


Figure 1. Sampling scheme of Fourier series coefficients.

As we have seen in the above analysis, the minimal sample rate of the signal $x(t)$ is $\frac{2L}{\tau}$, which is usually much lower than Nyquist sampling rate. Let $k_0 = \lfloor f_c * \tau \rfloor$, where f_c is the central frequency of the transmitted pulse, and $\lfloor \alpha \rfloor$ represents the largest integer less than α. We can choose the integers $\mathcal{K} = \{k + k_0,\ k = -m, \ldots, m\}$, where $m = \frac{K}{2}$ (assume K is a even number.), such that $G(\frac{2\pi}{\tau}k) \neq 0, k \in \mathcal{K}$. The number of Fourier series coefficients is a parameter which controls the tradeoff between sampling rate and recovery performances. In our numerical experiments, we will investigate the effect of the number of Fourier series coefficients.

4. SIGNAL RECOVERY USING COMPRESSING SENSE

The total-variation (TV) norm is the continuous version which can be interpreted as finding the shortest linear combination — in an l_1 sense — of elements taken from a continuous and infinite dictionary [17] is used to estimate the unknown parameters of the UWB radar signal. The TV norm minimization problem of (7) is expressed as:

$$\begin{array}{ll} \text{minimize}_{\alpha} & ||\alpha||_{TV} \\ \text{subject to} & ||V\alpha - y||_2 \leq \delta \end{array} \tag{8}$$

where δ is the noise level. The solution of (8) can be obtained by using semidefinite program (SDP) as the following [17]: the convex problem dual to (8) is

$$\begin{array}{ll} \text{maximize}_{c} & \text{Re} < y, c > -\delta||c||_2 \\ \text{subject to} & ||V^* c||_\infty \leq 1 \end{array} \tag{9}$$

where V^* is conjugate transpose of V.

For a consecutive subset of the Fourier series coefficients (i.e., choose the consecutive indices $\mathcal{K} = \{k + k_0,\ k = -m, \ldots, m\}$), the dual problem can be recast as SDP:

$$\begin{array}{ll} \text{maximize}_{c,Q} & \text{Re} < y, c > -\delta||c||_2 \\ \text{subject to} & \begin{bmatrix} Q & c \\ c^* & 1 \end{bmatrix} \succeq 0 \\ & \sum_{i=1}^{m-j} Q_{i,i+j} = \begin{cases} 1, & j = 0, \\ 0, & j = 1, 2, \ldots, m-1 \end{cases} \end{array} \tag{10}$$

where Q is an $m \times m$ Hermitian matrix. SDP can be solved using software CVX. Once a dual solution c is found, the solution of (8) can be estimated in the following way: the trigonometric polynomial

$$p(t) = \sum_{k \in \mathcal{K}} c_k e^{i2\pi kt} \tag{11}$$

satisfies the following properties

$$\begin{cases} |p(t_i)| = 1, & t_i \in T \\ |p(t_i)| < 1, & t_i \notin T \end{cases}$$

where T is the support of time delays. Consider the function

$$q(t) = 1 - |p(t)|^2 = 1 - \sum_{k=-(m-1)}^{m-1} u_k e^{i2\pi kt}, \quad u_k = \sum_j c_j \bar{c}_{j-k} \tag{12}$$

Let $z = e^{i2\pi kt}$, $z^{m-1}q(z)$ is a real polynomial of degree of $2(m-1)$, which is either equal to zero everywhere or has at most $m - 1$ roots on the unit circle. We can obtain the support T of time delays by locating the roots of $z^{m-1}q(z)$ on the unit circle. Having obtained the estimation for the time delays, the amplitudes α of the signal can be reconstructed by solving the system of Equation (7).

Consecutive Fourier series coefficients can be easily obtained using a simple low-pass filter [10], but it is suggested to use a non-consecutive set of Fourier series coefficients randomly selected in a distributed manner from wide frequency aperture, which greatly increases the resolution of the underlying signal, i.e., an exponential improvement [19]. Tang et al. [19] proposed an atomic norm minimization approach, similar to the TV norm minimization, to recover the missing Fourier series coefficients. Assume that a subset of entries $\mathcal{K}$ selected at random form a set $\{y_k, k \in \mathcal{K}\}$ of consecutive Fourier coefficients are observed, as prescribed in the paper [19], a natural algorithm for estimating the missing samples of a sparse sum of complex exponentials is the atomic norm minimization problem

$$\begin{array}{ll} \text{minimize}_{\widetilde{y}} & ||\widetilde{y}||_{\mathcal{A}} \\ \text{subject to} & |\widetilde{y}_j - y_j| < \delta, \quad j \in \mathcal{K} \end{array} \tag{13}$$

where $||y||_{\mathcal{A}}$ is the atomic norm of $\mathcal{A}$ associated with conv($\mathcal{A}$) (the convex hull of $\mathcal{A}$), defined by

$$||\widetilde{y}||_{\mathcal{A}} = \inf\{t > 0 | \widetilde{y} \in t\text{conv}(\mathcal{A})\}$$

(13) is equivalent to the following SDF:

$$\begin{array}{ll} \text{minimize}_{u,y,t} & \text{trace}(\text{Toep}(u)) + t \\ \text{subject to} & \begin{bmatrix} \text{Toep}(u) & y \\ y^* & t \end{bmatrix} \succeq 0 \\ & |\widetilde{y}_j - y_j| < \delta, \quad j \in \mathcal{K} \end{array} \tag{14}$$

where Toep(u) denotes the Toeplitz matrix whose first column is equal to u. By solving (14), the whole set $\{y_k, k \in \mathcal{K}\}$ of consecutive Fourier coefficients can be obtained, then following the above procedure, unknown signal parameters such as time delays and amplitudes can be estimated. Tang et al. [19] had showed that the resolution of the signal from $\frac{4}{s \log s \log n}$ of the consecutive samples to $\frac{4}{n}$ of random samples, where $n = |\mathcal{K}|$ and s is the number of time delays.

5. NUMERICAL AND EXPERIMENTAL SIMULATION

This section investigates the performances of signal reconstruction and echo detection in a UWB radar system. Assume that transmitted signal $g(t)$ is Gaussian monocycle pulse with the central frequency $f_c = 1.5$ GHz, showed in the Figure 2. To achieve accurate reconstruction

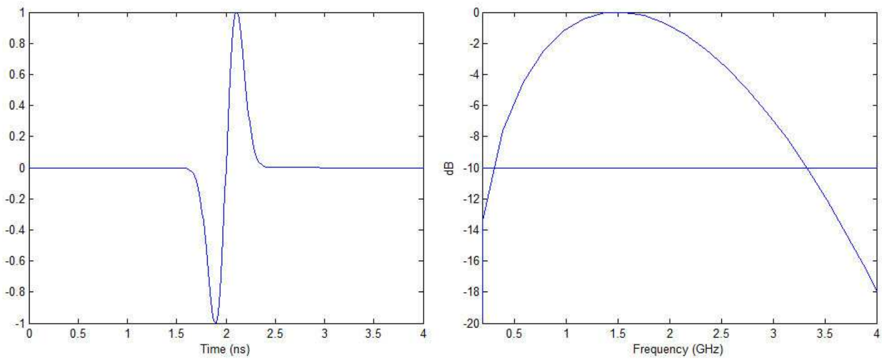


Figure 2. Gaussian monocycle pulse with the central frequency $f_c = 1.5\,\mathrm{GHz}$.

of the signal, based on Shannon-Nyquist sampling theorem, we set the "raw" sampling rate and the corresponding sampling interval of the testing echo signal be $f_s = 3B = 9\,\mathrm{GHz}$ and $T_s = 1/f_s$ respectively, which results to 1024 samples for the testing signal. Assume that the echo signal includes five target echoes with time delays $[50T_s, 100T_s, 120T_s, 400T_s, 800T_s]$ and the amplitudes [0.8 1.0 0.6 0.9 0.5], respectively. The signals were corrupted by zero-mean white Gaussian noise, with variance σ^2 determined such that the SNR, defined with respect to the weakest target as

$$\mathrm{SNR} = \frac{1}{\sigma^2} \min_{l=1,\dots,L} |a_l|^2 \tag{15}$$

To measure the recovery performances, we define successful detection ζ as

$$\zeta = \begin{cases} 1\#\{\widehat{\tau_i}\} == L \quad \text{and} \quad |\widehat{\tau_i} - \tau_i| \le T_s, & i = 1, \dots, L \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

and Root Mean Square Error (RMSE) metric of the time delays and amplitudes as:

$$\alpha = \frac{\sqrt{\frac{1}{L}\sum_i^L (\widehat{\tau_i} - \tau_i)^2}}{T_s} \qquad \beta = \sqrt{\frac{1}{L}\sum_i^L (\widehat{a_i} - a_i)^2} \tag{17}$$

where $\{\widehat{\tau}_i, \widehat{a}_i\}_{i=1}^{L}$ are the estimated time delays and amplitudes, and $\{\tau_i, a_i\}_{i=1}^{L}$ are the actual time delays and amplitudes. In order to statistically analyze the stable results, we also measure the probability of recovery of the experiment as

$$P = \frac{\#\{\zeta == 1\}}{\#\{\text{experiments repeated}\}} \tag{18}$$

and each experiment is repeated 100 times.

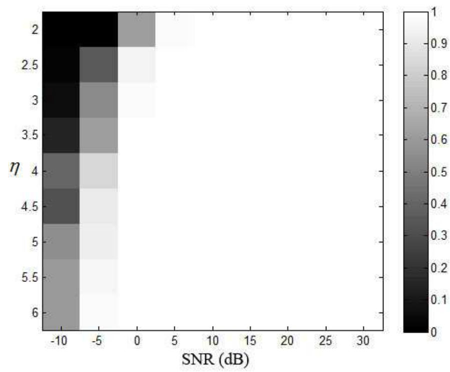


Figure 3. Probability of recovery vs. oversampling factor η and SNR.

We choose a subset of entries $\mathcal{K}$ at random such that $G(\frac{2\pi k}{\tau}), k \in \mathcal{K}$ is above -3 dB. Define the number of chosen Fourier series coefficients as $K = 2\lfloor \eta L \rfloor + 1$, where $\eta > 1$ is the desired factor. Figure 3 showed the probability of recovery for multiple combinations of SNR and the factor η, where $\eta = 2 : 0.5 : 6$ corresponding to sampling rates from 180 MHz to 540 MHz which are far below Nyquist rate. From the Figure 3, the target echoes can be successfully detected at very low sampling rate and noisy Fourier series coefficients. It is observed that increasing the number of Fourier series coefficients allows the method to perform at lower SNR values. In particular, even with small number of Fourier series coefficients, all the target echoes can successfully be detected as long as their amplitudes are above the variance of the noise, i.e., SNR ≥ 0. When SNR < 0, we need more samples to detect the weak target with high successful recovery rate. Figure 4 illustrates the RMSE for the estimated time-delays and amplitudes of the targets, which shows that we can robustly estimate the parameters of radar signals with a very low sampling rate.

We also compare the performances of TV-norm minimization method with ℓ_1 norm minimization solved by Orthogonal Matching

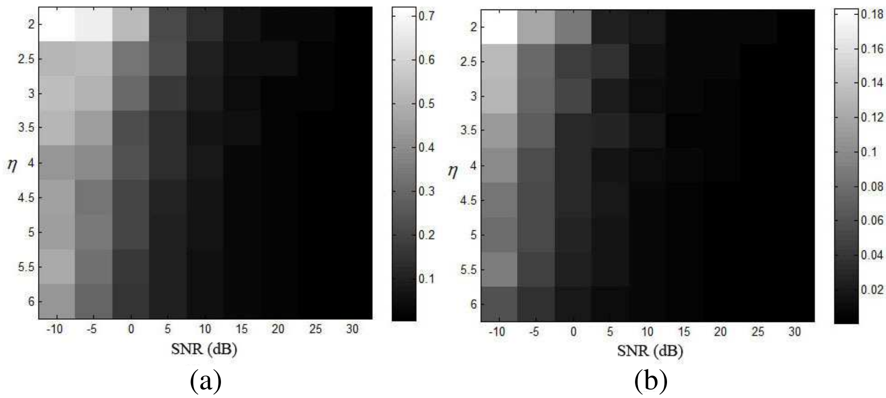
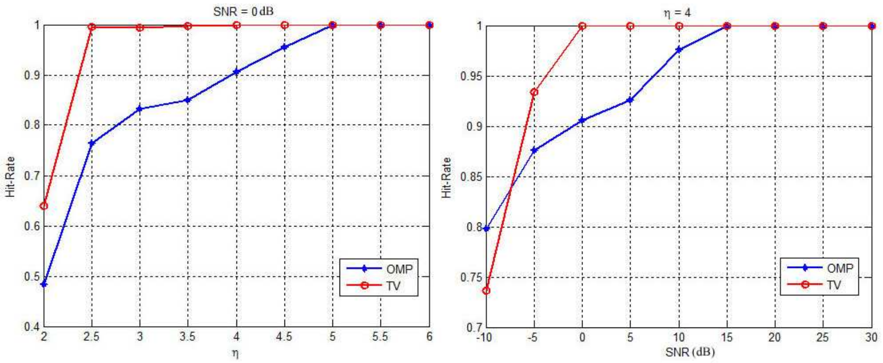


Figure 4. RMSE for the estimated (a) time-delays and (b) amplitudes of the targets.

Pursuit (OMP) [14]. We define the Hit-Rate in the following manner:

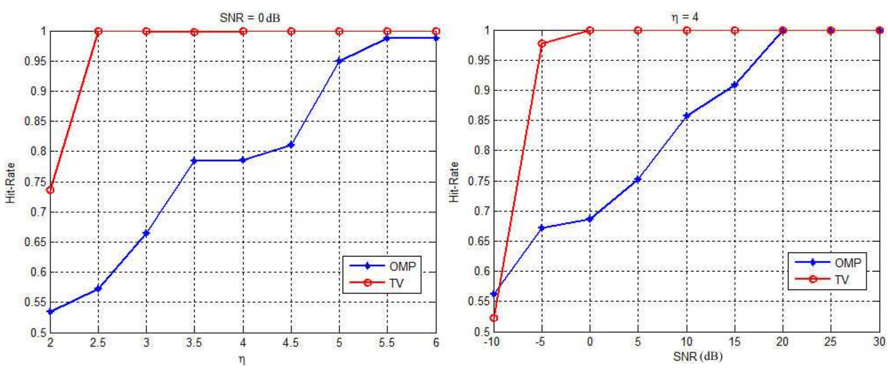
$$\text{Hit-Rate} = \frac{1}{L} \left|\left\{ \widehat{\tau_i} \,\middle|\, |\widehat{\tau_i} - \tau_i| \leq \epsilon_{th}, \quad i = 1, \ldots, L \right\}\right| \tag{19}$$

where we choose $\epsilon_{th} = \frac{1}{2B}$. Figure 5 illustrates the results of two methods using the above signal as a function of the factor η or a function of SNR, where for OMP method, the time axis is quantized with a T_s quantization step, i.e., all the time delays are on the grid. For a given noisy signal (SNR $= 0$ dB) or a fixed Fourier series coefficients ($\eta = 4$), the method of TV-norm minimization yields much better performances than OMP method, especially using lower number of Fourier series coefficients, and for lower noisy signal. In order to investigate the off-grid problem, we set five target echoes with time delays $[50.5T_s, 100.2T_s, 120.4T_s, 400.7T_s, 800.3T_s]$ off the grid. Figure 6 shows the results of two methods using the off-grid signal. Examining the results, it is not surprising that the Hit-Rate for the off-grid signal is lower than that for the on-grid signal using OMP method, while TV-norm minimization yields the similar performances. Therefore, we can conclude that TV-norm minimization method outperformed the ℓ_1 norm minimization, since the former does not quantize the time axis and does not degrade the performances due to the off-grid problem. In addition, we also examine the same situation expect that the transmitted signal is the Ricker pulse. We obtain the similar performances, showed in the Figure 7. These simulations demonstrate the detection performances of our propose method are barely affected by the choice of the transmitted pulse, as long as it has enough large bandwidth to choose the appropriate Fourier series coefficients.



(a) (b)

Figure 5. A comparison of on-grid signal model for a given (a) SNR $= 0\,\mathrm{dB}$ and (b) $\eta = 4$.



(a) (b)

Figure 6. A comparison of off-grid signal model for a given (a) SNR $= 0\,\mathrm{dB}$ and (b) $\eta = 4$.

Next we examine the results obtained by the proposed method to raw through-wall radar signal acquired by GSSI radar equipment. The raw signal was collected with frequency range 1–3 GHz in the time period 25 ns with 1024 time points. Figure 8 depicts the transmitted pulse and raw signal collected using this transmitted pulse. The signals are reconstructed using 20 or 30 samples of Fourier series coefficients respectively, which are far less than the sampling rate of raw signal (downsampling rate $\frac{1024}{20}$ and $\frac{1024}{30}$), as also showed in the Figure 8. Figure 9 showed reconstructed signal SNR values using the different numbers of samples of Fourier series coefficients, calculated

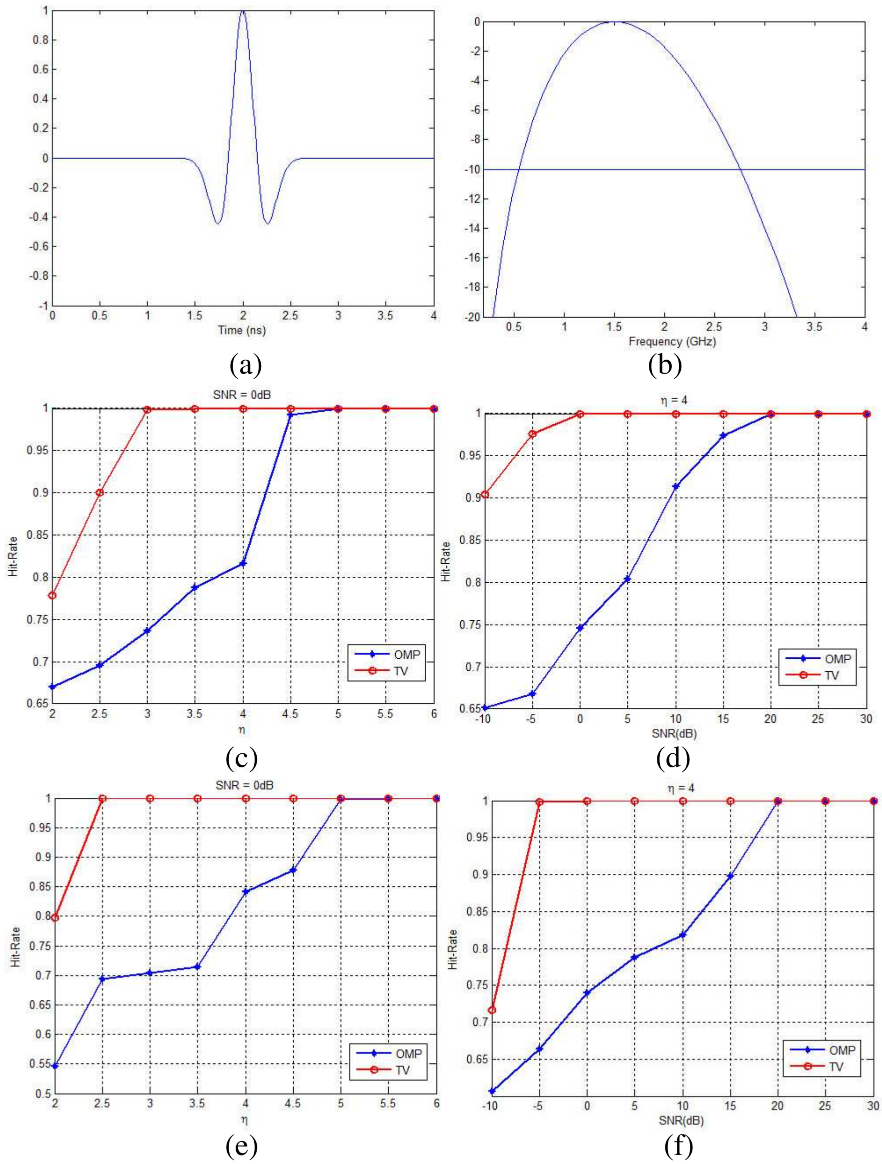


Figure 7. The transmitted ricker pulse in (a) time domain and (b) frequency domain; A comparison of on-grid signal model for a given (c) SNR = 0 dB and (d) $\eta = 4$; A comparison of off-grid signal model for a given (e) SNR = 0 dB and (f) $\eta = 4$.

for the reconstructed signal, after envelope detection with the Hilbert transform as:

$$\text{sSNR} = 10\log_{10}\frac{\displaystyle\int_0^{\tau} \left|H\left(\widehat{x}(t)\right)\right|^2 dt}{\displaystyle\int_0^{\tau} \left|H\left(x(t)-\widehat{x}(t)\right)\right|^2} \tag{20}$$

where $H(\cdot)$ denote the Hilbert transform, and $\widehat{x}(t)), x(t)$ are the reconstructed and actual signal. From these results, we can find that the reconstructed signals perfectly match the raw signal, and the target echoes are successfully detected using very small number of samples.

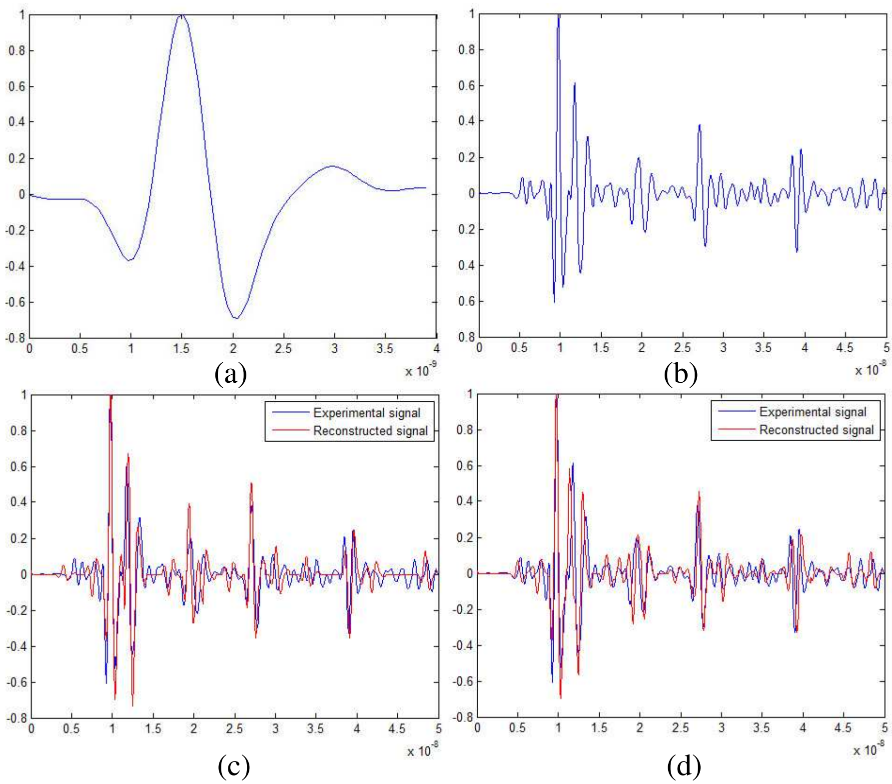


Figure 8. (a) Transmitted pulse used in GSSI. (b) Though-wall radar signal collected using GSSI. (c) Reconstructed signal using 20 Fourier series coefficients. (d) Reconstructed signal using 30 Fourier series coefficients.

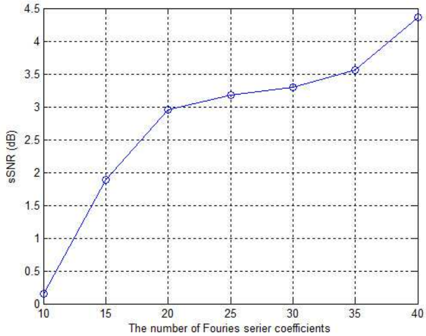


Figure 9. Reconstructed signal SNR values using the different numbers of samples of Fourier series coefficients.

6. CONCLUSION

We have presented in this paper an effective approach to detect UWB radar echo signals at sub-Nyquist rate based on CS. We made use of FRI framework to acquire samples at an ultra-low sampling rate and estimated the radar parameters such as time-delays and amplitudes based on TV-norm minimization. The simulation results demonstrated that the UWB radar signal can be detected at an ultra-low sampling rate. Compared to ℓ_1 norm minimization, TV-norm minimization has seen its clear advantage in the simulation. This study suggests that the proposed approach offers very good recovery performances for noisy UWB radar signal using very small number of samples, which is effective for realistic UWB radar scenarios. Future work will involve testing UWB radar signal under the complex environment, particularly for very week target echo detection, and validating the results on the realistic UWB radar scenarios.

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REFERENCES

1. Hanson, K. M., "Communication in the presence of noise," *Proceedings of the IRE*, Vol. 37, 10–21, 1949.

2. Nguyen, L. H., "Signal and image processing algorithms for the army research lab ultra-wideband synchronous impulse reconstruction (UWB sire) radar," Tech. Rep. ARL-TR-4784, ARL, 2009.
3. Mishali, M. and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals," *IEEE Journal of Selected Topics in Signal Process*, Vol. 4, No. 2, 375–391, 2010.
4. Candes, E. J. and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, Vol. 25, No. 2, 21–30, 2008.
5. Duarte, M. F. and Y. C. Eldar, "Structured compressed sensing: From theory to applications," *IEEE Transactions on Signal Processing*, Vol. 59, No. 9, 4053–4085, 2011.
6. Tropp, J. A., J. N. Laska, M. F. Duarte, J. K. Romberg, and R. Baraniuk, "Beyond Nyquist: Efficient sampling of sparse bandlimited signals," *IEEE Transactions on Information Theory*, Vol. 56, No. 1, 520–544, 2010.
7. Vetterli, M., P. Marziliano, and T. Blu, "Sampling signals with finite rate of innovation," *IEEE Transactions on Signal Processing*, Vol. 5, No. 6, 1417–1428, 2002.
8. Uriguen, J., Y. C. Eldar, P. L. Dragotti, and Z. Ben-Haim, "Sampling at the rate of innovation: Theory and applications," *Compressed Sensing: Theory and Applications*, Y. C. Eldar and G. Kutyniok (eds.), Cambridge University Press, 2012.
9. Lie, J. P., B. P. Ng, and C. M. See, "Multiple UWB emitters DOA estimation employing time hopping spread spectrum," *Progress In Electromagnetics Research*, Vol. 78, 83–101, 2008.
10. Michaeli, T. and Y. C. Eldar, "Xampling at the rate of innovation," *IEEE Transactions on Signal Processing*, Vol. 60, No. 3, 1121–1133, 2012.
11. Tur, R., Y. C. Eldar, and Z. Friedman, "Innovation rate sampling of pulse streams with application to ultrasound imaging," *IEEE Transactions on Signal Processing*, Vol. 59, No. 4, 1827–1142, 2011.
12. Gedalyahu, K., R. Tur, and Y. C. Eldar, "Multichannel sampling of pulse streams at the rate of innovation," *IEEE Transactions on Signal Processing*, Vol. 59, No. 4, 1491–1504, 2011.
13. Stoica, P. and R. Moses, *Introduction to Spectral Analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1997.
14. Wagner, N., Y. C. Eldar, and Z. Friedman, "Compressed

beamforming in ultrasound imaging," *IEEE Transactions on Signal Processing*, Vol. 60, No. 9, 4643–4657, 2012.

15. Candes, E. J., J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on Information Theory*, Vol. 52, No. 2, 489–509, 2006.
16. Donoho, D. L. and P. B. Stark, "Uncertainty principles and signal recovery," *SIAM Journal on Applied Mathematics*, Vol. 49, No. 3, 906–931, 1989.
17. Candes, E. J. and C. Fernandez-Granda, "Towards a mathematical theory of super-resolution," *Communications on Pure and Applied Mathematics*, 2013, doi: 10.1002/cpa.21455.
18. Roy, R. and T. Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. 37, No. 7, 984–995, 1989.
19. Tang, G., B. N. Bhaskar, P. Shah, and B. Recht, "Compressed sensing of the grid," *50th Annual Allerton Conference on Communication, Control, and Computing*, 778–785, Allerton, Monticello, USA, 2012.