

## **AN ALTERNATIVE METHOD FOR DIFFERENCE PATTERN FORMATION IN MONOPULSE ANTENNA**

**Jafar R. Mohammed\***

Communication Engineering Department, College of Electronic Engineering, University of Mosul, Mosul 41001, Iraq

**Abstract**—Difference patterns are vital for the successful function of tracking radar employing monopulse techniques to estimate target direction. Traditional monopulse antenna pattern synthesis methods require the use of two independent distributions, e.g., Taylor and Bayliss distributions, for formation of sum and difference patterns for one antenna. Hence, these approaches require a feed network of considerable complexity. In this letter, a method for forming difference pattern in linear arrays using a very simple beamforming network and two additional elements is described. The sum pattern is determined by adding signals received by original radiating elements of the array whereas the difference pattern is determined by subtracting the output of the sum pattern from signals received from two external edge elements. The proposed method used to generate these two patterns offers significant hardware and software savings over current methods.

### **1. INTRODUCTION**

Monopulse angle estimation methods require the formation of sum and difference patterns. The beam peak of the sum pattern is used to detect a target, whereas the boresight null of the difference pattern is used to determine accurately a target's angular location. These two patterns are usually synthesized by controlling the excitation coefficients of the array elements. The excitation coefficients (distributions) of optimum sum and difference patterns are different, e.g., Taylor distribution [1] for sum pattern formation and Bayliss distribution [2] for difference pattern formation. In other words, it is required two independent distributions to achieve good features in both patterns. Thus, these

---

*Received 29 June 2013, Accepted 13 August 2013, Scheduled 21 August 2013*

\* Corresponding author: Jafar Ramadhan Mohammed (jafarram@yahoo.com).

methods often require the design and realization of beamforming networks of considerable complexity [3–6]. This complexity can be reduced by utilizing subarray configuration [7,8]. In these cases, the excitations for the sum pattern are fixed, whereas the difference mode excitations depend on the weighting coefficients of each subarray [5, 9, 10].

Recently, the simplification of the feed network has also been investigated by sharing some excitations for the sum and difference channels, e.g., see [11,12]. These methods rely on the use of an optimization procedure which gives the optimal solutions. However, the computational cost of such optimization algorithms rapidly increases with the antenna size [13, 14].

In [15], a technique for obtaining simultaneously wide-angular nulling in the sum and difference patterns of a monopulse antenna that utilizes two additional elements, one at each end of the original array, was investigated. In this letter, we exploit the use of two additional elements to produce a difference pattern. The sum pattern is determined by adding signals received by original radiating elements of the array. Then the amplitude and phase of those two additional elements are properly adjusted so that the main beam of the two additional elements array pattern is aligned to that of the sum pattern at the target direction. The difference pattern is determined by subtracting the output of the sum pattern from signals received from two additional radiating elements.

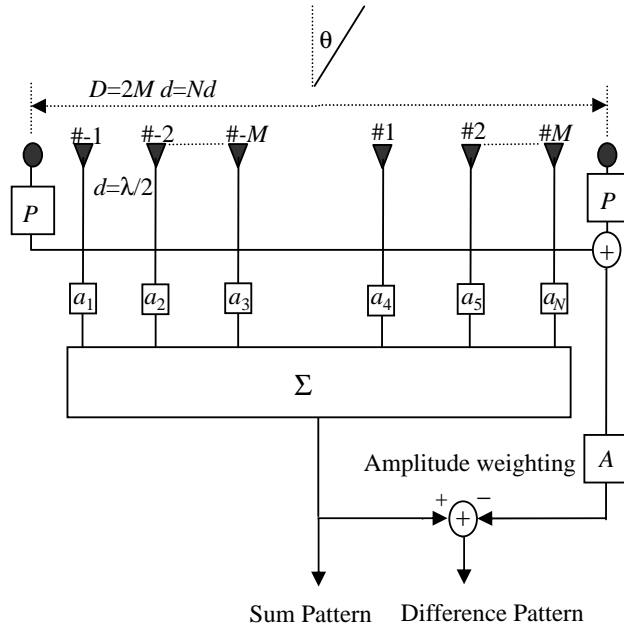
## 2. DESCRIPTION OF THE PROPOSED METHOD

The structure of the proposed method is shown in Fig. 1, which consists of original array elements to form a sum pattern and augmented antenna (i.e., original array elements with two external edge elements) to form a difference pattern. The sum pattern has a narrow mainlobe for high angular accuracy, whereas the difference pattern has a null in the direction of the desired signal. The detail of each part is detailed in the next sections.

### 2.1. Sum Pattern

An array of an even number of isotropic elements  $N = 2M$  (where  $M$  is an integer) is positioned symmetrically along the  $x$ -axis, as shown in Fig. 1. The separation between the elements is  $d = \lambda/2$  and  $M$  elements are placed on each side of the origin.

Assuming that the amplitude excitation is symmetrical about the origin, the array factor for nonuniform amplitude distribution can be



**Figure 1.** Block diagram of the proposed method for sum and difference patterns.

written as

$$AF_{Sum}(u) = a_1 e^{+j \frac{1}{2} k d u(\theta, \emptyset)} + a_3 e^{+j \frac{3}{2} k d u(\theta, \emptyset)} + \dots + a_M e^{+j \frac{(2M-1)}{2} k d u(\theta, \emptyset)} + a_{-1} e^{-j \frac{1}{2} k d u(\theta, \emptyset)} + a_{-3} e^{-j \frac{3}{2} k d u(\theta, \emptyset)} + \dots + a_{-M} e^{-j \frac{(2M-1)}{2} k d u(\theta, \emptyset)} \quad (1)$$

which in normalized form reduces to

$$AF_{Sum}(u) = \sum_{m=1}^M a_m \cos \left[ \frac{(2m-1)}{2} k d u(\theta, \emptyset) \right] \quad (2)$$

where  $u(\theta, \emptyset) = \sin \theta \cos \emptyset - \sin \theta_{peak}$  (note that the radiation patterns are plotted for  $-90^\circ \leq \theta \leq 90^\circ$  and  $\emptyset = 0^\circ$ ),  $k$  is the wave number which is equal to  $k = \frac{2\pi}{\lambda}$ ,  $\theta$  the angular position of the field point, and  $\theta_{peak}$  the angular position of the beam peak. And  $a_m$  are the excitation coefficients of the original array elements. Here, we choose these excitation coefficients  $a_m$  according to cosine distribution. Then, the corresponding symmetric array factor due to such distribution will be [15]

$$AF_{Sum}(\theta) = 2\pi N d \frac{\cos \left[ \frac{1}{2} k N d (\sin \theta - \sin \theta_{peak}) \right]}{\pi^2 - (k N d (\sin \theta - \sin \theta_{peak}))^2} \quad (3)$$

## 2.2. Difference Pattern

The augmented array consists of the aforementioned original array and an adding two-element array. These two elements are separated by a distance  $D$  and symmetrically positioned with respect to the centre of the array as shown in Fig. 1. Let the amplitude of their excitation be  $A$  and the phase of their excitation be  $P$ . Let a subscript “+” denote the element at position  $\frac{+1}{2}D$  and let a subscript “-” denote the element at position  $\frac{-1}{2}D$ . The total field radiated by the augmented array can be written [15],

$$AF_{Diff}(\theta) = 2\pi Nd \frac{\cos \left[ \frac{1}{2}kNd(\sin \theta - \sin \theta_{peak}) \right]}{\pi^2 - (kNd(\sin \theta - \sin \theta_{peak}))^2} + A_+ e^{jP_+} e^{j\frac{1}{2}kD \sin \theta} + A_- e^{jP_-} e^{-j\frac{1}{2}kD \sin \theta} \quad (4)$$

We wish to set the values of  $A$  and  $P$  for each added element so as to achieve the objective of producing difference pattern that has a null centred at a target direction ( $\theta_{peak}$ ). We proceed as follows.

First, since the first term of (4) is real, the second and third terms will have to be complex conjugates. Thus,  $P_+ = -P_- = P$  and  $A_+ = A_- = A$ . Equation (4) then becomes,

$$AF_{Diff}(\theta) = \underbrace{2\pi Nd \frac{\cos \left[ \frac{1}{2}kNd(\sin \theta - \sin \theta_{peak}) \right]}{\pi^2 - (kNd(\sin \theta - \sin \theta_{peak}))^2}}_{\text{original array}} + \underbrace{2A \cos \left[ \frac{1}{2}kD \sin \theta - P \right]}_{\text{two added elements}} \quad (5)$$

Next, we choose  $D = dN$  and set the phase of the two added elements to be equal to  $P = -\pi + \frac{1}{2}kD \sin \theta_{peak}$  so that the main lobe of the two-element array pattern is aligned to the main lobe of the sum pattern at the target direction and then to set the amplitude of the two added elements,  $A$ , to place a null in the direction of the desired signal (i.e.,  $AF_{Diff}(\theta)|_{\theta=\theta_{peak}} = 0$ ). Substituting the value of phase,  $P$ , into (5), the result is,

$$AF_{Diff}(\theta) = 2\pi Nd \frac{\cos \left[ \frac{1}{2}kNd(\sin \theta - \sin \theta_{peak}) \right]}{\pi^2 - (kNd(\sin \theta - \sin \theta_{peak}))^2} - 2A \cos \left[ \frac{1}{2}kNd(\sin \theta - \sin \theta_{peak}) \right] \quad (6)$$

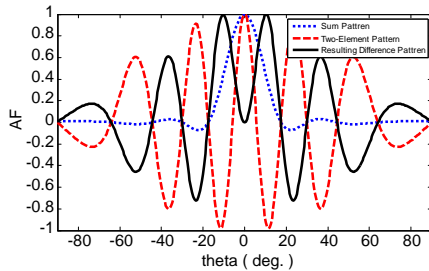
From (6), it is clear that the difference pattern can be obtained by properly adjusting the value of  $A$ ; that is, setting

$$A = \pi N d \frac{1}{\pi^2 - (k N d \sin \theta_{peak})^2} \quad (7)$$

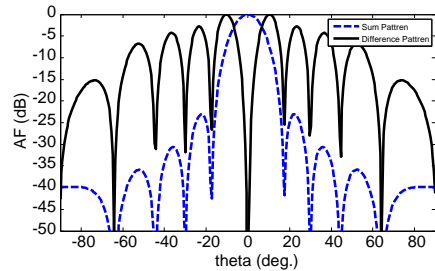
results in a difference pattern that has a null at target direction, i.e., at  $\theta = \theta_{peak}$ ,

$$AF_{Diff}(\theta) = 2\pi N d \frac{\cos \left[ \frac{1}{2} k N d (\sin \theta - \sin \theta_{peak}) \right]}{\pi^2 - (k N d (\sin \theta - \sin \theta_{peak}))^2} - 2\pi N d \frac{\cos \left[ \frac{1}{2} k N d (\sin \theta - \sin \theta_{peak}) \right]}{\pi^2 - (k N d \sin \theta_{peak})^2} \quad (8)$$

To verify that the difference pattern has a null at target direction  $\theta_{peak} = 0^\circ$ , we plot the resulting radiation patterns according to (3) and (8), of the original  $N$ -element array, two-element array, and augmented array for  $D = dN = 0.5\lambda * 10 = 4.5\lambda$  and  $\theta_{peak} = 0^\circ$ . The results are shown in Fig. 2. As can be seen from this figure that the main lobes of the two-element array and original  $N$ -element array patterns are exactly aligned at target direction  $0^\circ$ . As a result of subtracting between these two antenna patterns, the produced null in the resulting difference pattern is deep and sharp enough. Note that the sum pattern has low sidelobes since we have cosine distribution, whereas the resulting difference pattern has relatively high side lobes.



**Figure 2.** Coincidence between main lobes of the two-element pattern and original sum pattern at the target direction  $\theta_{peak} = 0^\circ$ .



**Figure 3.** Resulting difference pattern and original sum pattern for cosine distribution,  $N = 10$ ,  $d = \lambda/2$ ,  $A = 1$ , and  $\theta_{peak} = 0^\circ$ .

### 3. COMPARISON OF THE METHODS

The methods described in [11,12] and the proposed method aim to generate both sum and difference patterns with maximum number of

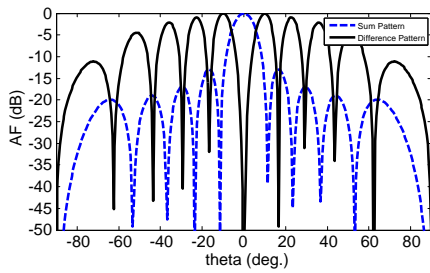
the common amplitude excitations, thus reducing the complexity of the required feeding network. Comparing the procedure of the common excitation by the methods described in [11, 12] (its array architecture was shown in [11, Fig. 1]) with that by the proposed method (its array architecture was shown in Fig. 1), it is found that using the proposed method is easier and faster (because it requires few simple mathematical operations to generate difference pattern whereas the methods described in [11, 12] require at least 50% of the overall number of array elements to be readjusted iteratively via an optimization procedure so that the radiation pattern may be switched from a sum to difference pattern). Moreover, the proposed method is suitable for real-time implementation because it can be used to reconfigure the radiation pattern from a sum to a difference mode by the addition of two-element array and perform few simple mathematical operations rather than recalculating the excitation coefficients of the original antenna elements (i.e., the coefficients which are not common).

The only advantage of the previous methods over the proposed one is that the generated difference pattern has lower sidelobes. More details can be found in the following section.

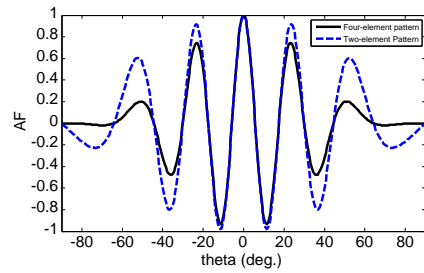
#### 4. NUMERICAL SIMULATIONS

In order to point out the effectiveness of the proposed method, a number of numerical experiments have been performed. In the following, we assume a uniform linear array with 10 elements and half-wavelength element spacing. The two added elements are then spaced one-quarter wavelength from the end elements. We want to generate a difference pattern without changing the relative amplitudes and/or phases of the excitations of the original array elements which are responsible for generating sum pattern. As a first test case, the proposed method is applied to the cosine distribution, the resulting sum and difference patterns of this case are shown in Fig. 3. In this example, the coincidence between nulls of both sum and difference patterns is evident. The first sidelobe of the resulting difference pattern goes down to  $-3$  dB whereas other sidelobes have much lower values when  $\theta$  moves away from main lobe.

In the second test example, the proposed method is applied to the uniform distribution. The results are shown in Fig. 4. Here, the resulting difference pattern is exactly orthogonal to the sum pattern where sidelobe peaks' in the difference pattern are located at the nulls in the sum pattern. Also, note that the resulting difference pattern has high sidelobe level. Generally, high sidelobe level in the resulting difference pattern is due to the fact that the two-element pattern



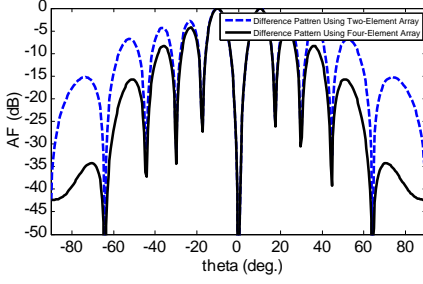
**Figure 4.** Resulting difference pattern and original sum pattern for uniform distribution,  $N = 10$ ,  $d = \lambda/2$ ,  $A = 1$ , and  $\theta_{peak} = 0^\circ$ .



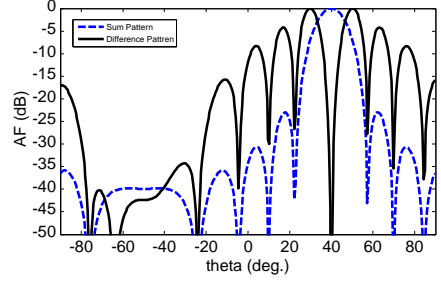
**Figure 5.** Sidelobe levels of the two-element array pattern and four-element array pattern. The element pattern  $\cos \theta$  is included for calculation of these aforementioned patterns.

has many grating lobes instead of sidelobes. Thus, when subtracting the sum pattern which has low sidelobes from the two-element array pattern which has high grating lobes results in a new pattern with high sidelobes.

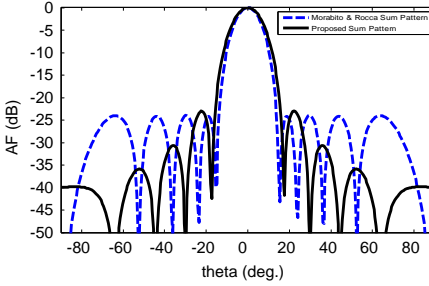
In order to reduce the level of sidelobes in the resulting difference pattern, we suggest to consider the two existing edge elements of the original array which are separated by a distance equal to  $d(N-1) = 4\lambda$  along with those two additional elements which are separated by a distance equal to  $dN = 4.5\lambda$ . The radiation pattern of these boundary four elements array is shown in Fig. 5 together with the pattern of the two additional elements. It can be seen that the four-element pattern is decaying more rapidly than two-element pattern. This helps to reduce the sidelobe levels of the resulting difference pattern as shown in Fig. 6. It can be seen from this figure that, by using four-element array (two additional elements plus two existing edge elements) instead of only two additional elements, a better reduction in the level of the most sidelobes can be obtained. It is worthy to mention that in [16], the authors used an additional 4-element auxiliary array to reduce the sidelobe level in linear array. However, there is a fundamental difference between the technique presented in [16] and the proposed technique. Another way to reduce sidelobes in the resulting difference pattern is by taking into account the element pattern whose response is cosine raised to a specified power  $n$  (i.e.,  $\cos^n(\theta)$  where the exponent  $n$  is real number greater than or equal to 1). Raising the response pattern to powers greater than one concentrates the response at target direction and reduces its sidelobes.



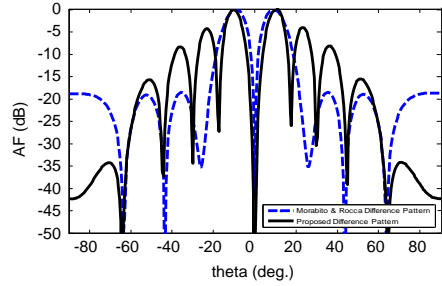
**Figure 6.** Resulting difference pattern with four-element array compared to difference pattern with two-element array for cosine distribution,  $N = 10$ ,  $d = \lambda/2$ ,  $A = 1$ , and  $\theta_{peak} = 0^\circ$ .



**Figure 7.** Resulting difference pattern and original sum pattern for scanned main lobe.  $\theta_{peak} = 40^\circ$ ,  $N = 10$ ,  $d = \lambda/2$ , and  $A = 1$ .



(a)



(b)

**Figure 8.** (a) Sum patterns of the proposed and Morabito & Rocca methods. (b) Difference patterns of the proposed and Morabito & Rocca methods. In both cases,  $N = 10$ ,  $d = \lambda/2$ ,  $A = 1$ , and  $\theta_{peak} = 0^\circ$ .

Next, consider an array with the main lobe steered to  $40^\circ$  from broadside and apply four-element pattern as before. The result is shown in Fig. 7. It should be noted here that to achieve steered difference pattern, a proper phase shift should be applied equally to both the original and two additional elements.

Finally, the resulting sum and difference patterns are to be compared with the method proposed by Morabito and Rocca [11]. As in [11], six elements among 10 are shared between the two patterns, while the other parameters that are used to obtain optimal sum-difference synthesis are the same as given in [11]. Fig. 8(a) shows the sum patterns, while Fig. 8(b) shows the difference patterns of the



aforementioned methods. It can be seen that the sum patterns of both methods have low sidelobes. Since we used cosine distribution with the proposed method, the general side lobe structure is mainly lower than that of the Morabito & Rocca sum pattern. On the other hand, the sidelobe structure of the Morabito & Rocca difference pattern is lower than that of the proposed method. Nevertheless, the resulting difference pattern has narrower null-to-null beamwidth (34.4 deg.) than that of Morabito & Rocca difference pattern (52 deg.). Narrow main lobes are desirable feature to increase angular accuracy.

## 5. CONCLUSION

It is clear from present study that the radiation pattern of the two external edge elements can be specifically designed to align the location of its main lobe with that of the original sum pattern. Then subtracting these two patterns allows formation of difference pattern. The resultant difference pattern has relatively high side lobes and narrow beamwidth. The sidelobe level may be improved by using four edge elements instead of only two edge elements. Unlike the traditional sum and difference pattern formation approaches which require the use of  $2N$  excitation coefficients for  $N$  elements array, the proposed method uses only  $N + 2$  excitation coefficients to generate both the sum and difference patterns.

## REFERENCES

1. Taylor, T. T., "Design of circular apertures for narrow beamwidth and low sidelobes," *IEEE Trans. Antennas Propag.*, Vol. 8, No. 1, 17–22, Jan. 1960.
2. Milligan, T. A., "Bayliss line-source distribution," *Modern Antenna Design*, Vol. 7, Section 4, 158–161, Hoboken, NJ, 2005.
3. Monzingo, R. A., R. L. Haupt, and T. W. Miller, *Introduction to Adaptive Arrays*, 2nd edition, SciTech Publishing, 2011.
4. Morabito, A. F., A. Massa, P. Rocca, and T. Isernia, "An effective approach to the synthesis of phase-only reconfigurable linear arrays," *IEEE Trans. Antennas Propag.*, Vol. 60, No. 8, 3622–3631, Aug. 2012.
5. Alvarez-Folgueiras, M., J. Rodriguez-Gonzales, and F. Ares-Pena, "Optimal compromise among sum and difference patterns in monopulse antennas: Use of subarrays and distributions with common aperture tail," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 17–18, 2301–2311, 2009.

6. Hosseini, S. A. and Z. Atlasbaf, "Optimization of side lobe level and fixing quasi-nulls in both of the sum and difference patterns by using continuous ant colony optimization (ACO) method," *Progress In Electromagnetics Research*, Vol. 79, 321–337, 2008.
7. Oliveri, G. and L. Poli, "Synthesis of monopulse sub-arrayed linear and planar array antennas with optimized sidelobes," *Progress In Electromagnetics Research*, Vol. 99, 109–129, 2009.
8. Manica, L., P. Rocca, A. Martini, and A. Massa, "An innovative approach based on a tree-searching algorithm for the optimal matching of independently optimum sum and difference excitations," *IEEE Trans. Antennas Propag.*, Vol. 56, No. 1, 58–66, Jan. 2008.
9. McNamara, D. A., "Synthesis of sub-arrayed monopulse linear arrays through matching of independently optimum sum and difference excitations," *IEE Proc.*, Vol. 135, No. 5, 293–296, 1988.
10. Manica, L., P. Rocca, M. Benedetti, and A. Massa, "A fast graph-searching algorithm enabling the efficient synthesis of sub-arrayed planar monopulse antennas," *IEEE Trans. Antennas Propag.*, Vol. 57, No. 3, 652–663, Mar. 2009.
11. Morabito, A. F. and P. Rocca, "Optimal synthesis of sum and difference patterns with arbitrary sidelobes subject to common excitations constraints," *IEEE Antennas and Wireless Propagation Letters*, Vol. 10, 623–626, Jul. 2010.
12. Alvarez-Folgueiras, M., J. Rodriguez-Gonzales, and F. Ares-Pena, "Synthesizing Taylor and Bayliss linear distributions with common aperture tail," *Electron. Lett.*, Vol. 45, No. 11, 18–19, 2009.
13. Qu, Y., G. Liao, S.-Q. Zhu, and X.-Y. Liu, "Pattern synthesis of planar antenna array via convex optimization for airborne forward looking radar," *Progress In Electromagnetics Research*, Vol. 84, 1–10, 2008.
14. Li, W.-T., X.-W. Shi, L. Xu, and Y.-Q. Hei, "Improved GA and PSO culled hybrid algorithm for antenna array pattern synthesis," *Progress In Electromagnetics Research*, Vol. 80, 461–476, 2008.
15. Mohammed, J. R. and K. H. Sayidmarie, "A new technique for obtaining wide-angular nulling in the sum and difference patterns of monopulse antenna," *IEEE Antennas and Wireless Propagation Letters*, Vol. 11, 1242–1245, 2012.
16. Aboud, A. H. and K. H. Sayidmarie, "Reduction of sidelobe structure in slotted waveguide phased arrays by a 4-element auxiliary antenna," *Proc. of 3rd IEEE Int. Conf. on Systems, Signals & Devices*, Tunisia, March 21–24, 2005.