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OPTIMAL POLARIZATION DESIGN FOR DIRECTION FINDING USING MIMO ELECTROMAGNETIC VECTOR-SENSOR ARRAY

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Abstract—In this paper, transmitter polarization optimization is firstly proposed to improve the accuracy of azimuth-elevation arrival angles estimation within MIMO electromagnetic vector-sensor array (EMVA). Minimizing of Cramér-Rao bound is used as cost function for the optimal design of transmitting signal polarization. Computer simulation results verify that the optimal polarization design provides increased estimation accuracy of direction finding in MIMO-EMVA, compared with that of using fixed polarization of transmitting signal. Moreover, the optimal polarization design retains all advantages of using fixed polarization of transmitting signal for MIMO-EMVA direction finding.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar [1], which uses multiple antennas to simultaneously transmit diverse waveforms and uses multiple antennas to receive the reflected signals, has many potential advantages over conventional phased-array radar (CPAR). For example, MIMO radar has more flexible spatial transmitting beam pattern design [2] and higher resolution spatial spectral estimates [3] than that of CPAR, etc..

The electromagnetic vectorsensor with six component-antennas lying in a point-like geometry was introduced into array signal processing by Nehorai and Paldi [4] in 1994. It generally uses three orthogonal- electric-dipoles and magnetic-loops to measure the three electric-field- and three magnetic-field-parameters of the

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incident signals, respectively. The electromagnetic vector-sensor array (EMVA, note that an array composed of two orthogonal electric-dipoles vector-sensors is called as polarization sensitive array) has been extensively investigated during the past two decades owing to numerical advantages over conventional scalar-sensor array [5], especially in direction-of-arrival (DOA) estimation field [6, 7]

Polarimetric MIMO radar [8–10] and MIMOEMVA [11] combines waveform diversity offered by MIMO radar with polarization diversity offered by polarization sensitive vector-sensor to further improve direction finding accuracy. Especially for MIMOEMVA [11], it also provides automatic pairing between azimuth and elevation estimation, and require no restrictions on the localization of transmitting antennas. We then can avoid the pairing processing between azimuth and elevation estimation and calibration for the positions of transmitting antennas. Despite their outstanding contributions, all of the aforementioned methods work in fixed polarization of transmitting signal. However, it is well known that controlling the polarization information of transmitting waveform enables improving the performance of target estimation, detection, and tracking [12].

In this paper, transmitter polarization optimization based on minimizing of Cramér-Rao bound (CRB) is firstly proposed to estimate two-dimensional DOA for MIMO-EMVA. The optimal polarization design provides increased estimation accuracy of direction finding in MIMO-EMVA, compared with that of using fixed polarization of transmitting signal [11]. Moreover, the optimal polarization design retains all advantages of using fixed polarization of transmitting signal for MIMO-EMVA direction finding.

2. PROBLEM FORMULATION

Exploiting the same MIMO-EMVA configuration as that of Ref. [11], the returns due to the mth transmitted waveform can be recovered by a matched filter for the nth received antenna, which can be expressed as [11, Eq. (8)]

$$\mathbf{x}_{m,n}(p) = \sum_{k=1}^{K} r_k(p)\mathbf{Q}(\theta_k, \phi_k)\mathbf{S}_k\xi\exp\Big[-j2\pi/\lambda(\mathbf{p}_{t,m} + \mathbf{p}_{r,n})^T\mathbf{r}_k\Big] \in \mathbb{C}^{6\times1}, \quad (1)$$

where $r_k(p)$ denotes the complex reflection coefficient of kth target at the pth pulse assumed to the Swerling II model. $\mathbf{Q}(\theta_k, \phi_k) \in \mathbb{R}^{6\times2}$ denotes the response of an electromagnetic vector-sensor (see [11, Eq. (1)]). $\theta_k \in [0, \pi]$ and $\phi_k \in [0, 2\pi)$ denote kth target's elevation and azimuth angles. $\mathbf{S}_k \in \mathbb{C}^{2\times 2}$ represents the scattering matrix of kth target. $\xi = [\sin\gamma \exp(j\eta),\, \cos\gamma]^T$ denotes the transmitted waveform polarization, where $\gamma \in [0,\, \frac{\pi}{2}]$ is the auxiliary polarization angle and $\eta \in [0, \pi]$ is the polarization phase difference. λ is the signal wavelength. $\mathbf{p}_{t,m} = [x_{t,m},\, y_{t,m},\, z_{t,m}]^T$ and $\mathbf{p}_{r,n} = [x_{r,n},\, y_{r,n},\, z_{r,n}]^T$ are the localizations of mth transmitted and nth received electromagnetic vector-sensor, respectively, which can be arbitrary. $\mathbf{r}_k = [u_k,\, v_k,\, w_k]^T = [\sin\theta_k \cos\phi_k,\, \sin\theta_k \sin\phi_k,\, \cos\theta_k]^T$ denotes the normalized Poynting vector. $(\cdot)^T$ denotes the transpose. K is the number of targets. Assume that total transmitting vector-sensors is M and total receiving vector-sensor is N. In noise case, we stack all the matching filtered data into a column vector:

$$\mathbf{x}(p) = \mathbf{A}\mathbf{r}(p) + \mathbf{n}(p), \tag{2}$$

where $\mathbf{A} = [\mathbf{a}_1,\, \mathbf{a}_2,\, \ldots,\, \mathbf{a}_K]$ is an $6MN \times K$ matrix, the columns of which are K steering vectors: $\mathbf{a}_k = \mathbf{Q}(\theta_k, \phi_k)\mathbf{S}_k\xi \otimes \mathbf{b}_{r,k} \otimes \mathbf{b}_{t,k} \in \mathbb{C}^{6MN\times 1}$, $k = 1,\, \ldots,\, K$. $\mathbf{b}_{t,k} = [\exp(-j2\pi/\lambda \mathbf{p}_{t,1}^T \mathbf{r}_k),\, \ldots,\, \exp(-j2\pi/\lambda\, \mathbf{p}_{t,M}^T \mathbf{r}_k)]^T \in \mathbb{C}^{M\times 1}$ and $\mathbf{b}_{r,k} = [\exp(-j2\pi/\lambda \mathbf{p}_{r,1}^T \mathbf{r}_k),\, \ldots,\, \exp(-j2\pi/\lambda\, \mathbf{p}_{r,N}^T \mathbf{r}_k)]^T \in \mathbb{C}^{N\times 1}$ denote the transmitted steering vector and the received steering vector, respectively. $\otimes$ denotes the Kronecker product. $\mathbf{r}(p) = [r_1(p),\, \ldots,\, r_K(p)]^T$. $\mathbf{n}(p)$ represents an $6MN \times 1$ complex Gaussian white noise vector with zeros mean and covariance matrix $\sigma_n^2 \mathbf{I}_{6MN}$. Therefore, the goal of this paper is to select the transmitting polarization state $(\gamma,\, \eta)$ to obtain the best accuracy of azimuth-elevation $(\phi,\, \theta)$ arrival angles estimation from Equation (2).

3. OPTIMAL POLARIZATION DESIGN BASED ON MINIMIZING OF CRB

First, we must define a cost function to select the optimal transmitting polarization. It is well known that the CRB is a universal lower bound on the variance for all unbiased estimators. Thus, we will consider the optimal polarization design subject to minimizing the CRB cost-function. The CRB equals the inverse of Fisher information matrix (FIM). Given that the system output $\mathbf{x}(p)$ is a zero-mean Gaussian random process, then the (i, j)th entry of the FIM for P snapshots equals

$$[\mathrm{FIM}(\boldsymbol{\psi})]_{i,j} = P \cdot Tr\left\{\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \psi_i}\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \psi_j}\right\}, \tag{3}$$

where $\boldsymbol{\psi} = [\theta,\, \phi]^T$ is the unknown parameter to be estimated. Note that the covariance matrix equals $\mathbf{R} = E\{\mathbf{x}(p)\mathbf{x}^H(p)\} = \sigma_s^2 \sum_{k=1}^{K} \mathbf{a}_k \mathbf{a}_k^H +$

$\sigma_n^2\mathbf{I}_{6MN}$, where σ_s^2 denotes signal power. We then have: $\frac{\partial\mathbf{R}}{\partial\psi_k} = \frac{\sigma_s^2\partial\mathbf{a}_k\mathbf{a}_k^H}{\partial\psi_k} = \sigma_s^2\frac{\partial\mathbf{a}_k}{\partial\psi_k}\mathbf{a}_k^H + \sigma_s^2\mathbf{a}_k\frac{\partial\mathbf{a}_k^H}{\partial\psi_k}$, where $\psi_k = \theta_k,\ \phi_k$. When ψ_k denotes the elevation angle θ_k:

$$\begin{aligned}\frac{\partial\mathbf{a}_k}{\partial\theta_k} &= \mathbf{U}_k\mathbf{S}_k\xi\otimes\mathbf{b}_{r,k}\otimes\mathbf{b}_{t,k} + \mathbf{Q}(\theta_k,\phi_k)\mathbf{S}_k\xi\otimes\{[\mathbf{b}_{r,k}\otimes\mathbf{b}_{t,k}]\\ &\quad\odot\left\{[c_{r,k,1},\ldots,c_{r,k,N}]^T\otimes[c_{t,k,1},\ldots,c_{t,k,M}]^T\right\}\right\},\end{aligned}\tag{4}$$

where

$$\begin{aligned}c_{r,k,n} &= -j\frac{2\pi}{\lambda}(x_{r,n}\cos\theta_k\cos\phi_k + y_{r,n}\cos\theta_k\sin\phi_k - z_{r,n}\sin\theta_k),\\ &\qquad n=1,\ldots,N,\\ c_{t,k,m} &= -j\frac{2\pi}{\lambda}(x_{t,m}\cos\theta_k\cos\phi_k + y_{t,m}\cos\theta_k\sin\phi_k - z_{t,m}\sin\theta_k),\\ &\qquad m=1,\ldots,M,\end{aligned}$$

and

$$\mathbf{U}_k = \begin{bmatrix}-u_k, & -v_k, & -w_k, & 0, & 0, & 0\\ 0, & 0, & 0, & u_k, & v_k, & w_k\end{bmatrix}^T.$$

In Equation (4), $\odot$ denotes the Hadamard product. When ψ_k denotes the azimuth angle ϕ_k:

$$\begin{aligned}\frac{\partial\mathbf{a}_k}{\partial\phi_k} &= \mathbf{V}_k\mathbf{S}_k\xi\otimes\mathbf{b}_{r,k}\otimes\mathbf{b}_{t,k} + \mathbf{Q}(\theta_k,\phi_k)\mathbf{S}_k\xi\otimes\{[\mathbf{b}_{r,k}\otimes\mathbf{b}_{t,k}]\\ &\quad\odot\left\{[d_{r,k,1},\ldots,d_{r,k,N}]^T\otimes[d_{t,k,1},\ldots,d_{t,k,M}]^T\right\}\right\},\end{aligned}\tag{5}$$

where

$$\begin{aligned}&\mathbf{V}_k\\ &= \begin{bmatrix}-\cos\theta_k\sin\phi_k, & \cos\theta_k\cos\phi_k, & 0, & -\cos\phi_k, & -\sin\phi_k, & 0\\ -\cos\phi_k, & -\sin\phi_k, & 0, & \cos\theta_k\sin\phi_k, & -\cos\theta_k\cos\phi_k, & 0\end{bmatrix}^T,\end{aligned}$$

and

$$\begin{aligned}d_{r,k,n} &= -j\frac{2\pi}{\lambda}(-x_{r,n}v_k + y_{r,n}u_k),\\ d_{t,k,m} &= -j\frac{2\pi}{\lambda}(-x_{t,m}v_k + y_{t,m}u_k).\end{aligned}$$

Until now, the FIM of azimuth-elevation ($\theta_k,\ \phi_k,\ k=1,\ \ldots,\ K$) can be easily evaluated, then the CRB can be expressed as:

$$\mathrm{CRB}(\theta_k) = \left[\mathrm{FIM}^{-1}\right]_{k,k}\mathrm{CRB}(\phi_k) = \left[\mathrm{FIM}^{-1}\right]_{K+k,K+k}\tag{6}$$

Finally, searching the two dimensional polarization of transmitting signal: $\{\gamma\in[0,\ \frac{\pi}{2}],\ \eta\in[0,\pi]\}$, we pick up the polarization $(\gamma_{\mathrm{opt}},\ \eta_{\mathrm{opt}})$

corresponding to minimum sum of CRB for $(\theta_k,\, \phi_k,\, k = 1,\, \ldots,\, K)$ as the optimal result:

$$(\gamma_{\text{opt}}, \eta_{\text{opt}}) = \arg\min_{\gamma,\eta} \left\{ \sum_{k=1}^{K} [\text{CRB}(\theta_k) + \text{CRB}(\phi_k)] \right\} \tag{7}$$

To evaluate the CRB, we must know the value of the targets' parameters. Those parameters can be offered by a tracking filter of the system according to the previous parameters estimation values [12]. The targets' parameters can also be offered by the estimation results using the algorithm in [11].

4. SIMULATION RESULTS

In this section, the ESPRIT-based algorithm in [11] is used to verify the effectiveness of our CRB-based polarization design. The root mean squared error (RMSE) of proposed method is computed by the ESPRIT-based algorithm. We perform a grid search over the possible waveform polarizations to obtain the minimum RMSE. It is called as the best polarization method. The angles estimation algorithm with non-optimal for polarization in [11] is called as fixed polarization method. In the following simulations, we compare proposed method with the best polarization method and fixed polarization method.

The MIMO-EMVA with $M = 2$ and $N = 2$ is considered. The first transmitted electromagnetic vector-sensor locates at the origin. The position of the second electromagnetic vector-sensor can be arbitrary Here we set $\mathbf{p}_{t,2} = [5\lambda,\, 6\lambda,\, 7\lambda]^T$. And we set $\mathbf{p}_{r,1} = [1\lambda,\, 3\lambda,\, 2\lambda]^T$ and $\mathbf{p}_{r,2} = [5\lambda,\, 3\lambda,\, 4\lambda]^T$. We assume that there are $K = 2$ closely spaced targets, which are located at the angles $(\theta_1,\, \theta_2) = (40°,\, 42°)$, $(\phi_1,\, \phi_2) = (25°,\, 29°)$. The two scattering matrices are $\mathbf{S}_1 = \begin{bmatrix} 2j, & 0.5 \\ 0.5, & -j \end{bmatrix}$ and $\mathbf{S}_2 = \begin{bmatrix} j, & 1-j \\ 1-j, & 0.5 \end{bmatrix}$. The number of snapshots is $P = 200$. The RMSE is defined as $\text{RMSE} = \sqrt{\frac{1}{K}\sum\limits_{k=1}^{K} E[(\hat{\theta}_k - \theta_k)^2 + (\hat{\phi}_k - \phi_k)^2]}$. The number of Monte Carlo trials is 1000. Fixed polarization method uses the random polarization state Here we set $(\gamma, \eta) = (50°, -90°)$

Figure 1 shows the results of 50 Monte Carlo tests with $SNR =$ 5 dB. The crosses denote the true location of the targets, and the asterisks denote the results estimated by the proposed method and fixed polarization method. It is shown that the two-dimensional directions are automatically paired and all targets are well localized.

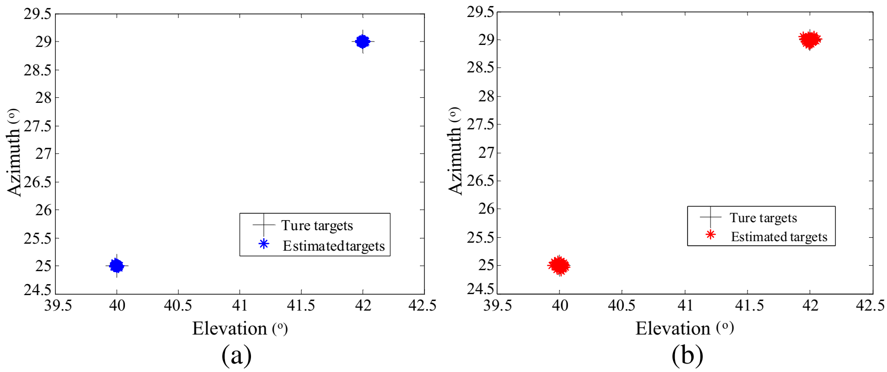

Figure 1. Paired results of two targets. (a) Proposed method. (b) Fixed polarization method.

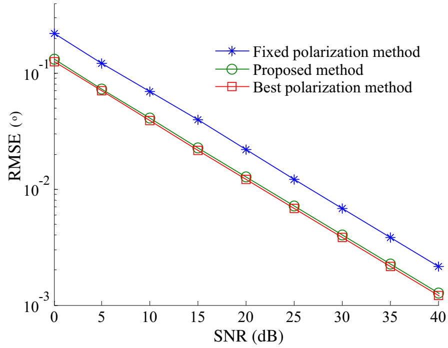

Figure 2. RMSE of angle estimation versus SNR.

It implies that the proposed method retains all advantages of using fixed polarization method for MIMO-EMVA direction finding.

Figure 2 shows the RMSE of estimation versus SNR. It can be seen from the figure that the RMSE of the proposed algorithm is about 2.4 dB lower than that of the fixed polarization method, and it is very close to that of the best polarization method. It verifies that the proposed polarization optimization method is resultful.

5. CONCLUSION

In this paper, we present a CRB-based transmitting polarization design algorithm for two-dimensional angles estimation in MIMO-EMVA. The proposed method provides better estimation performance than that of fixed polarization method and obtains almost the best optimized result. In addition, the proposed method keeps all advantages of fixed polarization method because it only changes the transmitting polarization.

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