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MODIFIED BAYESIAN BEAMFORMER FOR BINNING ERROR ELIMINATION

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Abstract—Constrained Least Mean Square (CLMS) algorithm is used to adapt the antenna array weights. CLMS in its simple form fails to capture the Signal of Interest (SOI) if there is an error in the Direction of Arrival (DOA) estimation. Moreover, it will consider the SOI as an interferer and create null in the desired DOA. The large gain will be towards the detected wrong direction. Derivative constraints and Bayesian beamformer are two techniques used to overcome such a problem. Derivative constraints destroy a lot of Degrees of Freedom (DOF). Bayesian beamformer destroys only one DOF but vulnerable to binning error. The proposed algorithm overcomes the problem of binning error in the Bayesian beamformer with only one extra DOF.

1. INTRODUCTION

Adaptive Antenna (Smart Antenna) is one of the challenging techniques used to improve the antenna performance. It depends on sorting multiple antenna elements into an array to receive multiple delayed copies of the signal [1]. For a given array geometry, we can control what is received, amplified or discarded by adapting weights after the antenna elements. The main problem of the adaptive antenna is the weight system adaptive algorithm and its performance.

The Constrained LMS (CLMS) is one of the most popular techniques used in the array weight adaptation [2]. It depends on minimizing the output power while satisfying some constraints (e.g., directional constraints). The CLMS adaptation algorithm is mainly

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designed for given constraints. When these constraints are directional constraints (the common case), the error in Direction of Arrival (DOA) detection reduces the algorithm performance. This leads to the need to some modifications to get robustness in the beamformer towards directional errors.

One of the oldest techniques to create robust beamformer with respect to direction errors is adding derivative constraints to the CLMS beamformer. Derivative constraints force the beam pattern derivatives to be equal zero in the direction of the SOI adding robustness to the beamformer [3]. The derivative constraints continuously cover the desired interval of uncertainty but destroy a lot of Degrees of Freedom (DOF) to cover a large interval.

Direct mathematical modeling is used in [4–7] to keep acceptable gain in the worst point in the interval of uncertainty, this leads to too complex mathematical problem, numerical solutions are involved.

Another technique of the efficient robust beamformers is in [8, 9], it depends on dividing the interval of uncertainty into small bins. The Robust beamformer is the sum of the beamformers of the mid-points of the bins weighted by some weights. In [8] fuzzy logic based weights are used, in [9] the Angle of Arrival (AOA)'s probability density function (pdf) based weights are used to form Bayesian beamformer. Bayesian beamformer can efficiently track the SOI with high directivity regardless of the error. Although Bayesian beamformer is an efficient robust beamformer, it suffers from the problem of binning errors due to its discrete nature.

In this letter we use only first order derivative constraint with the Bayesian beamformer to fix the binning error. The new technique solves the problem of the binning error by using first order derivative constraint. At the same time it solves the problem of destroying a lot of DOF because it uses the Bayesian beamformer in its main structure. This letter is organized as follows, in Section 2 the system model is described, and in Section 3 Constrained Least Mean Square is reviewed. Two types of constraints, directional and derivative constraints are also reviewed. Section 4 reviews the Bayesian beamformer, Section 5 introduces the proposed algorithm. Section 6 shows the simulation results and Section 7 concludes the letter.

2. SYSTEM MODEL

Consider a linear uniform adaptive array with M isotropic antennas separated by a distance d between neighbors, $k+1$ independent transmitted signals impinge the array from directions $\theta_0, \theta_1, \theta_2, \ldots, \theta_k$, with the broadside direction. θ_0 is the direction of the SOI. θ_1, $\theta_2, \ldots, \theta_k$ are the directions of the interferers. Each direction θ_i, is associated with a vector a_i namely the steering vector of this direction, $i = 0, 1, 2, \ldots, k$. In the matrix form the following equations describe the system:

$$
\begin{aligned}
\bar{x} &= A\bar{s} + \bar{n} \\
\bar{s} &= \begin{bmatrix} s_0 & s_1 & \ldots & s_K \end{bmatrix}^T \\
A &= \begin{bmatrix} \bar{a}_0 & \bar{a}_1 & \ldots & \bar{a}_K \end{bmatrix} \\
\bar{a}_k &= \begin{bmatrix} 1 \\ e^{j\varphi_k} \\ \vdots \\ e^{jm\varphi_k} \\ \vdots \\ e^{j(M-1)\varphi_k} \end{bmatrix}, \\
\varphi_k &= 2\pi \frac{d}{\lambda} \sin(\theta_k), \quad y = \bar{w}^H \bar{x}, \quad R_{xx} = E\left(\bar{x}\bar{x}^H\right)
\end{aligned}
\tag{1}
$$

x is the received signal at the array input, A an $M \times (k+1)$ matrix that contains the steering vectors as its columns, n the thermal noise vector, w the array weight vector, and y the array output. One of the signals s_i impinges the array from the direction θ_i, which is the Signal of Interest (SOI). The vector $\bar{a}_i$ is the associated steering vector. The other signals are interferers. R_{xx} is the covariance matrix of the input calculated by the input samples.

To measure the antenna array performance for a given weight we use two parameters, the antenna pattern and the Signal to Interferers and Noise Ratio (SINR). The antenna pattern (beampattern) is an indicator to the response of the antenna to a given direction. For a given direction, the antenna response is $|w^H a|$, a is the steering vector of the angle of concern. The Signal to Interferers and Noise Ratio (SINR) is the ratio between the signal power to the interferers and the noise power passed to the output through the weight of interest. It is calculated using the following relation;

$$\mathrm{SINR} = \frac{\bar{w}^H R_{ss} \bar{w}}{\bar{w}^H R_{ni} \bar{w}} \tag{2}$$

R_{ss}: The covariance matrix of the input signal assuming no noise and interferers.

R_{ni}: The covariance matrix of the input signal withthe SOI removed.

3. CONSTRAINED LEAST MEAN SQUARE

In general, the main goal of the communication system is keeping the receiver output always equal to the transmitted signal from the transmitter of interest. In the adaptive antenna mathematical model the number of the constraints, which keeps this state up depends on the available knowledge about the SOI and the transmission medium. More information usually means fewer DOF destroyed for a given performance level.

CLMS efficiently manages the available DOF (the number of antennas in the array) in the system, it can assign a given number of the DOF to given constraints and use the remaining DOF to satisfy the unknown mandatory constraints. CLMS minimizes the output power (satisfying the unknown constraints during this process) and at the same time keeps some known constraints up. Mathematically, it can be modeled as

$$\text{Minimize } \bar{w}^H R_{ni}\bar{w} \ s.t. \ C^H\bar{w} = \bar{f} \tag{3}$$

C and f are respectively the constraint matrix and the response vector, which specify the known constraints and the required responses that must be satisfied by the solution. To solve this problem using Lagrange multiplier the following cost function has to be minimized with respect to w with the constraints satisfied [2]

$$J = \left(\bar{w}^H R_{ni}\bar{w}\right) + \left(C^H\bar{w} - \bar{f}\right) \tag{4}$$

It has been solved in [2], approximating R_{ni} to R_{xx} for practical considerations

$$\bar{w}_{\text{CLMS}} = R_{xx}^{-1}C\left(C^H R_{xx}^{-1}C\right)^{-1}\bar{f} \tag{5}$$

Based on Equation (4), the cost function can be divided into two parts (demonstrated in parentheses). The first part is quadratic and is required to be minimized, closed to zero as possible as it can be. The second part is also required to be zero. Given that the known required constraints are usually less than the available DOF then the second part is a simple underdetermined linear system. To solve this linear system, DOF equal to the number of constraints are destroyed. The DOF destroyed by this part can be called 'pre-set' constraints. The remaining DOF can be used to minimize the first part and can be called "float" constraints. More available DOF tends to better minimization (closer to zero), in antenna language, deeper nulls. Modeling a physical task in fewer DOF leads to more DOF for the minimization process. In the following, we will show two types of the linear constraints can be satisfied by CLMS.

3.1. Directional Constraints

Directional constraints mean constraining the gain in a given direction to be a given value, e.g., unity gain in the direction of the SOI or zero gain in the directions of the known interferers. Directional constraints are represented in the constraint matrix columns by the steering vectors of the directions, the equivalent locations in the response vector are filled by the desired gains. In the simple case, only the direction of the SOI (θ_0) is known. In this case let $C = \bar{a}_0$, $\bar{a}_0$ is defined in Equation (1) and $\bar{f} = 1$. Equation (5) is simplified to Minimum Variance Distortion less (MVDR) beamformer

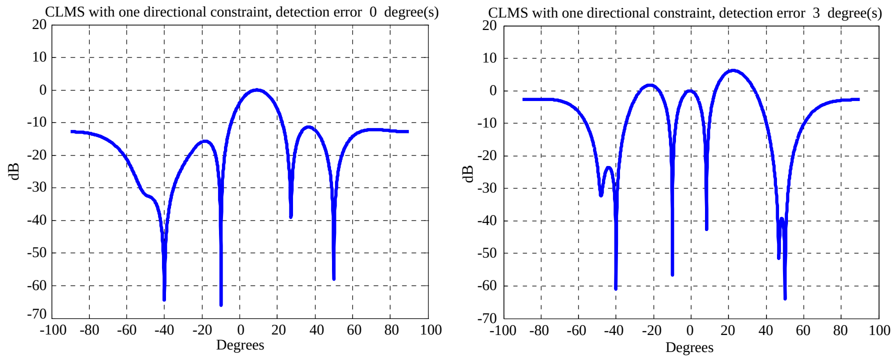
$$\bar{w}_{\mathrm{MVDR}} = \frac{R_{xx}^{-1}\bar{a}_0}{\bar{a}_0^H R_{xx}^{-1}\bar{a}_0} \tag{6}$$

3.2. Derivative Constraints

Derivative constraint(s), introduced in [3], is a type of linear constraints used in the adaptive array. It means constraining the derivative(s) of the beam-pattern to be equal to zero at a given direction, the detected DOA. The DOA of arrival may be vulnerable to errors due to several practical considerations. Using the derivative constraints in conjunction with the directional constraints keeps the high gain for an interval around the detected angle not for a point giving robustness to the beamformer with respect to the direction estimation errors. Higher order zero derivatives lead to wider band of high gain which means more robustness. The derivative constraint(s) is represented in the constraint matrix C and the response vector f by adding the following column to the constraint matrix, steered to the looking direction, with zero in the corresponding element in the response vector

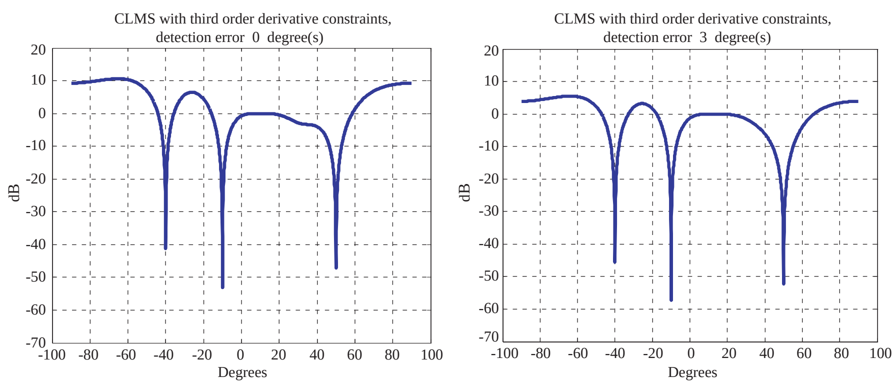
$$\begin{bmatrix} 1^n \\ 2^n \\ \vdots \\ M^n \end{bmatrix}, \quad n \text{ is the derivative order} \tag{7}$$

The above explanation can be shown in Figure 1–Figure 3. They are plotted for 7 elements array (7 DOF) with four signals impinge the array. The signal from 10 degrees is the SOI. Interferers are coming from -40, -10 and 50 degrees. Figure 1(a) shows the beampattern of the conventional case (unity gain directional constraint towards the SOI with no detection error). The SOI is successfully captured. The interferers are discarded by the nulls in their directions. Figure 1(b) shows the same case with 3 degrees detection error. In the case of detection error CLMS assumes the true direction as an interferer and creates null in its direction. It is clear in Figure 1(b) that null has been created at 10 degrees (the SOI). Derivative constraints are used to overcome the problem of detection error. Figure 2 and Figure 3 show the third and the fourth derivative constraints respectively. In Figure 2 the beam is forced due to the derivative constraints to keep high gain



(a) (b)

Figure 1. CLMS with 7 elements, one directional constraints (MVDR), SOI: 10 deg, interferers: -40, -10, 50. (a) One directional constraint. No detection error. (b) One directional constraint. 3 degrees detection error.



(a) (b)

Figure 2. CLMS with 7 elements, 1 directional constraints and 3rd order derivative constraints, SOI: 10 deg, interferers: -40, -10, 50. (a) 3rd order derivative constraints. No detection error. (b) 3rd order derivative constraints. 3 degrees detection error.

around the detected DOA, thus the SOI is correctly captured regardless of any error. Also the nulls have been created in the corrected locations. In Figure 3 the fourth order derivative constraints work well around the DOA of the SOI regardless there is an error or not. Although fourth order derivative constraints succeed to keep the robustness around the SOI, it fails to create nulls in the direction of the interferers. This can be considered due to consuming (5 of 7) DOF (one directional constraints and four derivatives) in the pre-set constraints. There are only two remaining while we have three interferers (need at least one DOF for each interferer). Also we can see the effect of the number of the DOF comparing the nulls in Figure 1 and Figure 2. The nulls are deeper in Figure 1. This can be considered due to the fact that more DOF were destroyed in the derivative constraints, thus the minimization process works using fewer DOF.

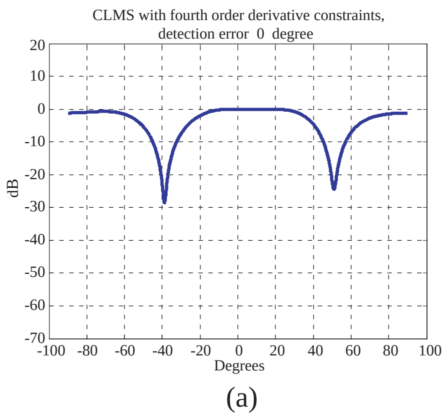


Figure 3. CLMS with 7 elements, 1 directional constraints and 4th order derivative constraints, SOI: 10 deg, interferers: -40, -10, 50. (a) 4th order derivative constraints. No detection error. (b) 4th order derivative constraints. 3 degrees detection error.

4. BAYESIAN BEAMFORMER

Bayesian beamformer [9] is one of the efficient robust beamformers. It assumes the DOA as a random variable. The priori probability density function of the DOA is assumed known, usually constant. The a posteriori pdf of the true DOA given the array input $p(\theta/x)$ is

$$p\left(\theta|\bar{x}\right) = \frac{p\left(\theta\right)\, p\left(\bar{x}|\theta\right)}{\sum_{j=1}^{L} p\left(\theta\right)\, p\left(\bar{x}|\theta\right)} \tag{8}$$

It has been approximated in [9] to

$$p(\theta|\bar{x}) = cpe^{K\gamma \frac{1}{\bar{a}_i^H R_K^{-1} \bar{a}_i}} \tag{9}$$

K is the number of samples, p the a priori probability, c a constant to hold the sum of the probabilities to 1, γ: a constant controls the behavior of the algorithm, and R_K the sample covariance matrix equal $\frac{1}{K}\sum_{i=1}^{K} \bar{x}_i \bar{x}_i^H$.

Bayesian beamformer [9] is the sum of the MVDR beamformers (Equation (6)) along the interval of uncertainty weighted by the a posteriori pdf. Practically, to be able to calculate the Bayesian beamformer the interval of uncertainty is divided into L bins. The Bayesian beamformer is the sum of the MVDR beamformers of the mid-points of the small L bins weighted by their a posteriori pdf.

$$\bar{w}_{Bay} = \sum_{i=1}^{L} \bar{w}_i p(\theta_i|\bar{x}) \tag{10}$$

$\bar{w}_i$ is the MVDR weight at angle θ_i, θ_i is the mid-point of the ith bin, then

$$\bar{w}_{Bay} = \sum_{i=1}^{L} \frac{R_k^{-1}\bar{a}_i}{\bar{a}_i^H R_K^{-1} \bar{a}_i} cpe^{K\gamma \frac{1}{\bar{a}_i^H R_K^{-1} \bar{a}_i}} \tag{11}$$

Giving a closer look to Equations (5), (7) and (10) one can show that the derivative constraints destroy DOF equals to the derivative order while the Bayesian beamformer uses only one degree of freedom in each calculated beamformer. This reduction in the number of destroyed DOF is in the expense of higher computational complexity (calculating L beamformers) and binning error (The desired DOA doesn't match exactly one of the calculated beamformers). Binning error is recovered in the proposed algorithm using first order derivative constraint.

5. FIRST DERIVATIVE CONSTRAINT BASED BAYESIAN BEAMFORMER

It has been shown in the previous section that Bayesian beamformer is vulnerable to binning error. This means that the Bayesian beamformer does not cover the whole interval of uncertainty. It only covers given points along the interval and suffers from performance degrading if the desired DOA falls between the mentioned points. In opposition to Bayesian beamformer, the derivative constraints continuously cover the interval of uncertainty in the expense of destroying more DOF.

In our proposed technique we only used the first order derivative constraint (destroying only one more degree of freedom) to maintain the discontinuity coverage in the interval of uncertainty.

As shown above, to construct a beamformer with unity gain and first order derivative constraint at $\theta = 0$, $C\&f$ are of the form

$$C = \begin{bmatrix} 1 & 1^n \\ 1 & 2^n \\ \vdots & \vdots \\ 1 & M^n \end{bmatrix}, \quad f = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{12}$$

To add the first order derivative constraint to each beamformer the constraint matrix C_i is the steered version of the constraint matrix of (12) and f is constant for all beamformers. Consequently, the constraint matrix C_i and the response vector f are

$$C_i = \begin{bmatrix} 1 & & & & & 0 \\ & e^{j\varphi_i} & & & & \\ & & \ddots & & & \\ & & & e^{jm\varphi_i} & & \\ & & & & \ddots & \\ 0 & & & & & e^{j(M-1)\varphi_i} \end{bmatrix} \begin{bmatrix} 1 & 1^n \\ 1 & 2^n \\ \vdots & \vdots \\ 1 & M^n \end{bmatrix}, \quad \bar{f} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{13}$$

In this case

$$\bar{w}_i = R_k^{-1} C_i \left(C_i^H R_k^{-1} C_i \right)^{-1} \bar{f} \tag{14}$$

$$\bar{w}_{Bay} = \sum_{i=1}^{L} \bar{w}_i p\left(\theta_i | \bar{x}\right) \tag{15}$$

$\bar{w}_i$ is the ith weight calculated from (14), $p(\theta_i|\bar{x})$ is the a posteriori pdf calculated from (9), substituting (9) and (14) in (15) we get

$$\bar{w}_{Bay} = \sum_{i=1}^{L} R_k^{-1} C_i (C_i^H R_k^{-1} C_i)^{-1} \bar{f} cpe^{K\gamma \frac{1}{\bar{a}_i^H R_K^{-1} \bar{a}_i}} \tag{16}$$

then

$$\bar{w}_{Bay} = \sum_{i=1}^{L} R_k^{-1} C_i \left(C_i^H R_k^{-1} C_i \right)^{-1} \begin{bmatrix} cpe^{K\gamma \frac{1}{\bar{a}_i^H R_K^{-1} \bar{a}_i}} \\ 0 \end{bmatrix} \tag{17}$$

This beam covers each bin separately by the first order derivative constraint. Given that it covers each bin separately using different beamformers, it destroys only two DOF in each one. It can cover the whole interval of the Bayesian beamformer and at the same time can take the advantage of the derivative constraints, no binning error, in the expense of only one extra degree of freedom. It is shown in the simulation section the differences between the algorithms in the ability of interferers' cancellation due to the available DOF.

6. SIMULATION RESULTS

Assume a uniform linear 12 elements sensor array spaced by d, $d/\lambda = 0.5$. Four signals impinge the array from angles -75, -55, 0, 45 degrees with the broadside. The signal coming from 0 degree is the SOI. The first, third order derivative constraints, conventional Bayesian and the proposed algorithm have been simulated. For all cases the uncertainty range is assumed $[-6 : +6]$ degrees. Two division scenarios have been studied in each case. The first scenario has three bins with centers $[-6,\ 0,\ 6]$, the second has five bins with centers $[-6,\ -3,\ 0,\ 3,\ 6]$. The results were averaged over 10 trails.

Figure 4 shows the beampattern of the conventional Bayesian and the proposed algorithm at 3 degrees error. We can notice that both algorithms work properly because 3 degrees error exactly matches one of the bins centers. Figure 5 shows the beampattern of both algorithms at 2 degrees error which does not match any bin center. The conventional Bayesian beamformer created null towards the DOA of the SOI while the proposed algorithm succeeds to maintain high gain in the desired direction.

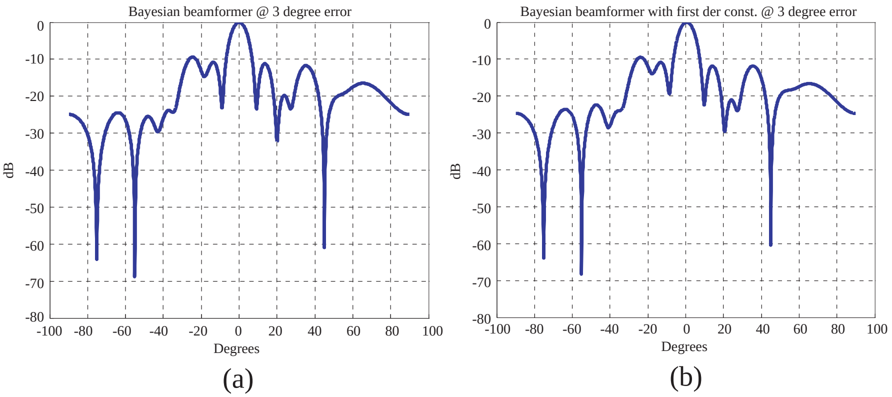


Figure 4. The case of 3 degree error, matches one of the bins. SOI: 0 deg. Interferers: -75, -55, 45 degrees. (a) Conventional Bayesian beamformer. No detection error. (b) Bayesian with first derivative.

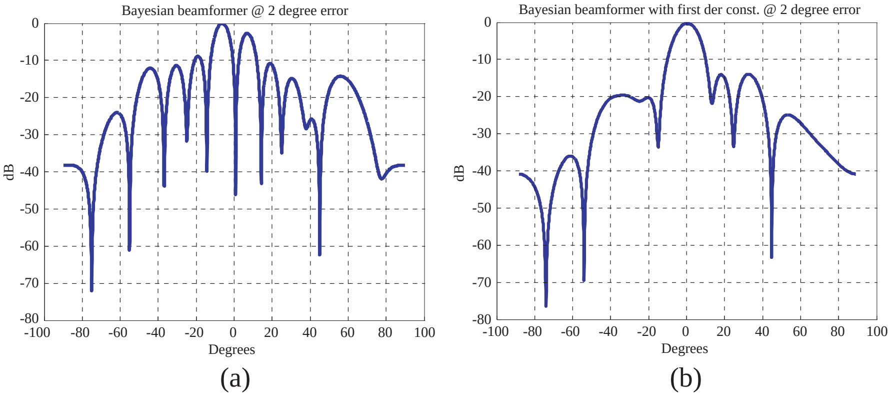


Figure 5. The case of 2 degree error, DOES NOT match one of the bins. SOI: 0 deg. Interferers: −75, −55, 45 degrees. (a) Conventional Bayesian Beamformer. (b) Bayesian with first derivative.

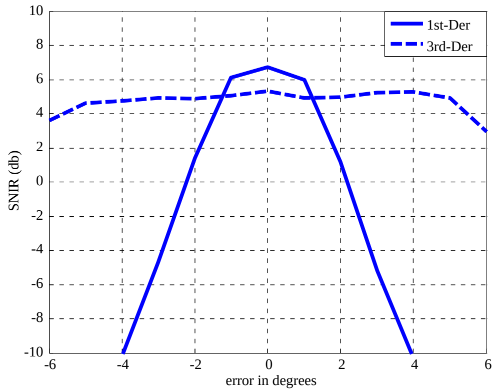


Figure 6. SINR of the first and third order derivative constraints.

Figure 6 shows the Signal to Interferers and Noise Ratio (SINR) along the interval of uncertainty for the first and third order derivative constraints, the graph is same for both cases of bins scenarios because the derivative constraints behavior is independent of the number of bins. It is clear that the third order derivative can continuously cover larger interval with SINR less than the peak of the first order. This is because the first order has more DOF for minimization. Figure 7 shows the SINR along the interval of uncertainty to the cases of three and five bins. From Figure 7 we can see that the conventional Bayesian beamformer responds with low SINR if the desired DOA does not

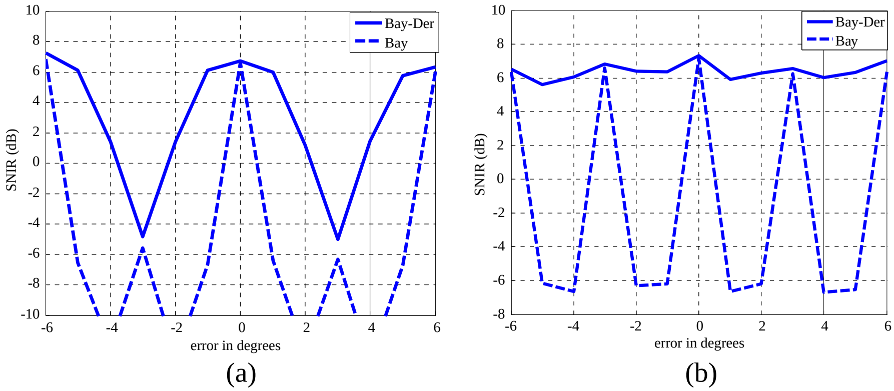


Figure 7. SINR of the conventional Bayesian beamformer and the proposed beamformer. (a) The case of 3 bins with centers $[-6, 0, 6]$. (b) The case of 5 bins with centers $[-6, -3, 0, 3, 6]$.

exactly match one of the bins. The proposed algorithm maintains high SINR around the bins centers. If the number of bins is large enough the interval of uncertainty will be covered continuously. Comparing Figures 7(a) and (b) we can see the effect of the number of bins. In the case of three bins, Figure 7(a), the first order derivative could not cover the whole bin around the center, thus more bins are required. In Figure 7(b), five bins with first order derivative constraint can continuously cover the interval of uncertainty even if it matches one of the bins centers or not. Now we have two algorithms which can cover the interval of uncertainty with accepted SINR, third order derivative constraints and the proposed algorithm,comparing them in Figure 6 and Figure 7(b) we can see that the proposed algorithm responds with higher SINR, this is because the third order derivative constraints destroys more DOF while the proposed algorithm uses more DOF for the minimization process.

7. CONCLUSIONS

In this letter, after reviewing the needed tasks we proposed a new beamformer algorithm for robustness beamforming. Our proposed algorithm succeeds to overcome the problem of binning error in the Bayesian beamformer using only one additional degree of freedom. With only one additional degree of freedom destroyed, the minimization process has no notable effect. In the conventional Bayesian beamformer the binning error degrades the performance in several small intervals inside the interval of uncertainty. Overcoming the binning error in the proposed algorithm developed the performance in the previously degraded intervals. The simulation verified the expected results.

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