ANALYTICAL CALCULATION OF THE INDUCTANCE OF PLANAR ZIG-ZAG SPIRAL INDUCTORS

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Abstract—An analytical procedure for the calculation of the inductance of planar zig-zag spiral inductors is proposed. The procedure is based on the partial inductance concept and models the inductor as a series of a number of parts. The self-inductance of each individual part, which has the shape of a parallelogram, and the mutual inductance between any two parts of the inductor are determined. The inductance of a planar zig-zag spiral inductor can thus be obtained for any width, length and angle of the saw-tooth configuration. The procedure is validated with experimental measurements; the agreement between estimated and measured inductances is very good.

1. INTRODUCTION

The calculation of inductance, historically important in power engineering applications, has recently grown new interest due to the development of contactless power transfer systems [1]. In particular, planar inductors are used as intermediate resonators between the transmitting and receiving coils to improve the efficiency of the wireless power transfer, channeling the magnetic field in resonance condition. Inductors and resonators are also essential components in radio frequency and microwave integrated circuits for making low noise amplifiers, oscillators, impedance matching networks and filters. In several of these applications, these components are shaped in planar spiral geometries etched or milled on a printed circuit board (PCB) and the accurate prediction of the inductance allows the determination of the resonant frequency of the structure. A number of papers have been developed to model planar rectangular spiral inductors. Their relative

easy layout requires extensive computation if modelled numerically using full-wave electromagnetic methods [2]; the solution may be indeed accurate, however run times may be long and the analysis may be limited to inductors of just a few turns only. The inductance of planar rectangular spiral inductors can be more efficiently calculated with analytical procedures, as in [3, 4]. For more complex geometries of inductors, such as square, hexagonal, octagonal and circular shapes, approximate expressions are given [5].

In this paper, an analytical procedure for the calculation of the inductance of a planar spiral inductor with saw-toothed shaped sides based on the partial inductance concept is presented. This geometry is derived from the spiral antenna with zig-zag arms [6] and its structure is shown in Fig. 1. This type of inductor can be of interest in several applications where there is a need to increase the total length of the inductor and its inductance without changing its external dimensions.
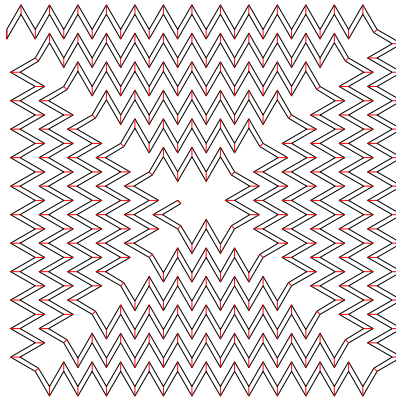
Figure 1. Planar zig-zag spiral inductor subdivided in N parts.

The paper is organized as follows. In Section 2, the procedure to calculate the inductance of a planar zig-zag spiral inductor is first outlined; then, the closed-form expressions for the partial self- and mutual inductances of thin parallelograms are given. Section 3 presents the application of the proposed approach to planar zig-zag spiral structures and the comparison with measurement results.

2. INDUCTANCE CALCULATION

The inductance of a planar zig-zag spiral inductor can be conveniently calculated by means of a method initially outlined by Grover [7] and significantly expanded by Ruehli [8], who further developed the

concepts in [7] under a new comprehensive theory of inductance known as the theory of partial inductance. The method is based on the calculation of the partial inductance of straight loop elements, as it is demonstrated that an inductance contribution can be uniquely associated to each element of a closed loop. The total inductance of a loop is then equal to the sum of the partial self-inductances of each straight element plus all the partial mutual inductances between the elements. There is no unique choice of the elements into which a circuit is divided. In [8], it is demonstrated that unique inductances can be obtained also for incomplete loops, such as planar zig-zag spiral inductors: the open loop inductance is the closed loop inductance with the partial inductances of the closing path removed.

The partial self- and mutual-inductances of straight conductors of rectangular cross-section are given in approximate and exact forms in [7–12], respectively. In this paper, the calculation of the inductance of a planar zig-zag spiral inductor is carried out by neglecting the thickness of the conductor. The planar zig-zag spiral inductor is partitioned into N parts or elements, as shown in Fig. 1. Each part has the shape of a thin parallelogram, with the exception of the two adjacent parts concurrent at the points where the horizontal sides meet the vertical sides. For layout requirements, in fact, these parts need to be slightly different from a parallelogram. The total inductance L of the inductor of Fig. 1 is then the sum of all partial self- and mutual inductances of the N parts:

$$L = \sum_{i=1}^{N} \sum_{j=1}^{N} M_{pij} \qquad (1)$$

where M_{pij} $(j \neq i)$ is the partial mutual inductance between any two parallelograms i and j of the spiral inductor, and M_{pij} $(j = i)$ is the partial self-inductance of the ith parallelogram.

2.1. Analytical Calculation of the Partial Self-inductance of a Thin Parallelogram

The zig-zag spiral inductor has a constant cross section of width w and negligible thickness along its length, as shown in Fig. 2. The angle of the spiral is θ, with $0 \leq \theta < \pi/2$, and the length of each spiral element is l. The inductor may then be thought as a bundle of parallel filaments, each of width dx and carrying a current density constant along the length of each filament. The current density is then assumed uniform throughout the zig-zag spiral inductor. With reference to Fig. 3 the partial self-inductance of a thin parallelogram can be evaluated as a four-fold integral from the definition of the partial
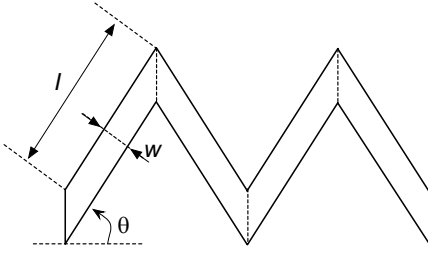
Figure 2. Particular of a single side with its geometrical dimensions.
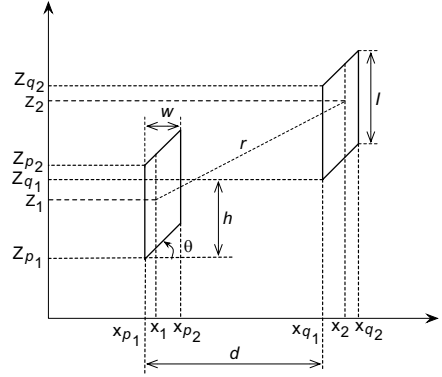
Figure 3. Two parallel thin parallelograms.

mutual inductance between two parallel thin parallelograms p and q each with a constant current density

$$M_{pq} = \frac{\mu_0}{4\pi} \frac{1}{w_p} \frac{1}{w_q}$$
$$\times \int_0^{w_p} \int_0^{w_q} \int_0^{l_p} \int_0^{l_q} \frac{dx_1 dx_2 dz_1 dz_2}{\sqrt{(x_2 - x_1 + d)^2 + [z_2 - z_1 + (x_2 - x_1)\tan\theta + h]^2}} \quad (2)$$

where μ_0 is the magnetic permeability of free space, w_p, l_p and w_q, l_q are the width and the length of the parallelograms p and q, respectively, $d = x_{q_1} - x_{p_1}$ and $h = z_{q_1} - z_{p_1}$. As the current density is assumed constant along the length of each filament, the partial mutual inductance between two filaments of the two parallelograms is given by Neumann's formula

$$M_f = \frac{\mu_0}{4\pi} \int_0^{l_1} \int_0^{l_2} \frac{dz_1 dz_2}{r} \quad (3)$$

where r is the distance between two elements of length dz_1 and dz_2 of the two filaments of length l_1 and l_2, respectively.

The partial self-inductance of a thin parallelogram can be found from (2) by performing the integration over the same area. The two parallelograms p and q are then the same, and thus letting $w_p = w_q = w$, $l_p = l_q = l$, $d = 0$ and $h = 0$ in (2) the expression for the partial self-inductance $L_p = M_{pp}$ of a thin parallelogram is obtained as

$$L_p = \frac{\mu_0}{4\pi} \frac{1}{w^2} \int_0^w \int_0^w \int_0^l \int_0^l \frac{dx_1 dx_2 dz_1 dz_2}{\sqrt{(x_2 - x_1)^2 + [z_2 - z_1 + (x_2 - x_1)\tan\theta]^2}}. \quad (4)$$

In general, the solution of the four-fold integration is obtained by introducing new variables $u = x_2 - x_1$ and $v = z_2 - z_1$ which yields the expression

$$L_p = \frac{\mu_0}{4\pi} \frac{1}{w^2} \int_0^w dx_1 \int_{-x_1}^{w-x_1} du \int_0^l dz_1 \int_{-z_1}^{l-z_1} \frac{dv}{\sqrt{u^2 + (v + u\tan\theta)^2}}. \quad (5)$$

The partial self-inductance of a thin parallelogram of width w and length l is then

$$L_p = \frac{\mu_0}{4\pi} \frac{1}{6w^2} \left[4t \left(w^3 + \frac{l^3}{F_5^3} \right) \log(F_5 - t) + 4w^3 F_5 + \frac{4l^3}{F_5^2} \right.$$
$$-2(F_1 + F_2) \left(w^2 + \frac{l^2}{F_5^2} \right) + 2w^2(3l + wt) \log \left(\frac{wt + F_1 + l}{w} \right)$$
$$+2w^2(3l - wt) \log \left(\frac{-wt + F_2 + l}{w} \right) + \frac{6wl^2}{F_5} \log \left(\frac{F_1 F_5 + F_3}{l} \right)$$
$$\left. +\frac{6wl^2}{F_5} \log \left(\frac{F_2 F_5 + F_4}{l} \right) + \frac{2l^3 t}{F_5^3} \log \left(\frac{F_1 F_5 + F_3}{F_2 F_5 + F_4} \right) \right] \quad (6)$$

where

$$F_1 = \sqrt{w\left(wt^2 + w + 2lt\right) + l^2},$$
$$F_2 = \sqrt{w\left(wt^2 + w - 2lt\right) + l^2},$$
$$F_3 = wt^2 + w + lt,$$
$$F_4 = wt^2 + w - lt,$$
$$F_5 = \sqrt{t^2 + 1},$$

and $t = \tan\theta$. It can be verified that (6) reduces to the expression of the self-inductance of a thin rectangle [8, 11–13] for $t = 0$.
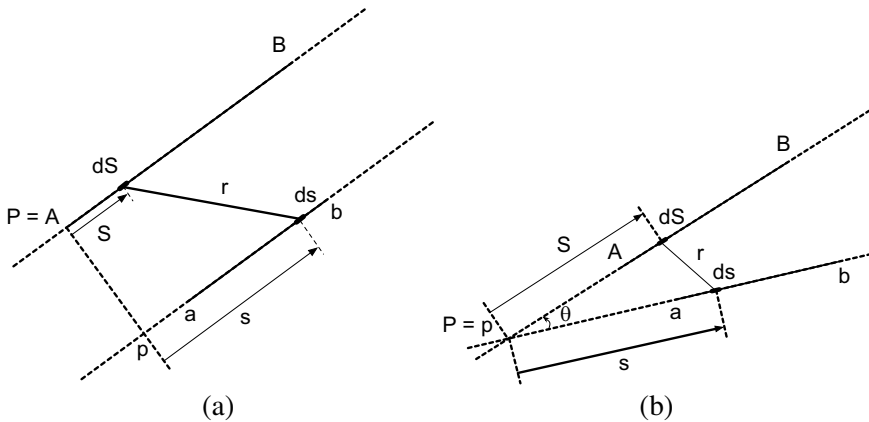
2.2. Partial Mutual Inductance between Zig-zag Spiral Parts

It is not straightforward to find a closed form expression for the partial mutual inductance between any two parallel thin parallelograms p and q of the planar spiral inductor. An approximation can then be adopted for the parallelograms, which are represented as straight filaments. The segments forming the axis of the planar zig-zag spiral inductor may be chosen as the filaments representing the parallelograms, as Fig. 4 shows. The partial mutual inductance between two parallelograms can then be calculated as the partial mutual inductance between two filaments, which is given by the Neumann's formula (3). Campbell [14] first proposed a general solution of the Neumann integral for any two

straight wires with negligible thickness in any relative location in space. Here the solutions in a form more suitable for implementation in a computer code are proposed. There are three possible configurations for any two filaments of a planar zig-zag spiral inductor: i) the filaments are parallel, ii) the filaments are incident at a point forming an angle to each other, iii) the filaments are perpendicular. The last case is the simplest as the partial mutual inductance between any two perpendicular filaments is always zero. The possible filament configurations are shown in Figs. 5(a) and (b).

Figure 4. Planar zig-zag spiral inductor of square shape (side dimension l_0) with its axis.

(a)   (b)

Figure 5. Configurations of any two filaments of a planar zig-zag spiral inductor. (a) Parallel straight filaments. (b) Incident straight filaments forming an angle θ to each other.

2.2.1. Partial Mutual Inductance between Parallel Straight Filaments

The partial mutual inductance M_{pf} between any two parallel filaments AB, ab in any relative position in space can be found from Neumann's formula (3), where $r^2 = Pp^2 + (S - s)^2$. With reference to Fig. 5(a), $P \equiv A$, Pp is the common perpendicular of the two filaments, S and s are the distances from the common perpendicular Pp of the two elements dS and ds in the positive directions along AB and ab, respectively. From [14], it can be found that

$$
\begin{aligned}
M_{pf} = \frac{\mu_0}{4\pi} \Big[&\sqrt{(pa - PA)^2 + Pp^2} - \sqrt{(pb - PA)^2 + Pp^2} \\
&-\sqrt{(pa - PB)^2 + Pp^2} + \sqrt{(pb - PB)^2 + Pp^2} \\
&+(PA - pa) \log \left(\sqrt{(pa - PA)^2 + Pp^2} + pa - PA \right) \\
&-(PA - pb) \log \left(\sqrt{(pb - PA)^2 + Pp^2} + pb - PA \right) \\
&-(PB - pa) \log \left(\sqrt{(pa - PB)^2 + Pp^2} + pa - PB \right) \\
&+(PB - pb) \log \left(\sqrt{(pb - PB)^2 + Pp^2} + pb - PB \right) \Big] \quad (7)
\end{aligned}
$$

that being $PA = 0$ can be simplified as

$$
\begin{aligned}
M_{pf} = \frac{\mu_0}{4\pi} \Big[&\sqrt{pa^2 + Pp^2} - \sqrt{pb^2 + Pp^2} \\
&-\sqrt{(pa - PB)^2 + Pp^2} + \sqrt{(pb - PB)^2 + Pp^2} \\
&-pa \log \left(\sqrt{pa^2 + Pp^2} + pa \right) \\
&+pb \log \left(\sqrt{pb^2 + Pp^2} + pb \right) \\
&-(PB - pa) \log \left(\sqrt{(pa - PB)^2 + Pp^2} + pa - PB \right) \\
&+(PB - pb) \log \left(\sqrt{(pb - PB)^2 + Pp^2} + pb - PB \right) \Big]. \quad (8)
\end{aligned}
$$

A particular case of this configuration occurs when the two filaments are aligned and offset. In this case, $P \equiv A \equiv p$ and thus $Pp = PA = 0$ so that (8) reduces to

$$
\begin{aligned}
M_{pf} = \frac{\mu_0}{4\pi} \big[&-pa \log(2pa) - (PB - pa) \log 2(pa - PB) \\
&+pb \log(2pb) + (PB - pb) \log 2(pb - PB) \big]. \quad (9)
\end{aligned}
$$

2.2.2. Partial Mutual Inductance between Straight Filaments Incident at an Angle

The general case of the partial mutual inductance M_{if} between any two coplanar filaments AB, ab incident at an angle θ that do not share a common point can be derived from Neumann's formula (3), where $r^2 = S^2 - 2Ss\cos\theta + s^2$. In fact, with reference to Fig. 5(b), $P \equiv p$ and then $Pp = 0$; S and s are the distances from the point $P \equiv p$ of the two elements dS and ds in the positive directions along AB and ab, respectively. From [14], it can be found that

$$
\begin{aligned}
M_{if} = \frac{\mu_0}{4\pi} \cos\theta \\
\Big[pa \log \left(\sqrt{-2paPA\cos\theta + pa^2 + PA^2} - pa\cos\theta + PA \right) \\
+ PA \log \left(\sqrt{-2paPA\cos\theta + pa^2 + PA^2} - PA\cos\theta + pa \right) \\
- pa \log \left(\sqrt{-2paPB\cos\theta + pa^2 + PB^2} - pa\cos\theta + PB \right) \\
- PB \log \left(\sqrt{-2paPB\cos\theta + pa^2 + PB^2} - PB\cos\theta + pa \right) \\
- PA \log \left(\sqrt{-2PApb\cos\theta + PA^2 + pb^2} - PA\cos\theta + pb \right) \\
- pb \log \left(\sqrt{-2PApb\cos\theta + PA^2 + pb^2} - pb\cos\theta + PA \right) \\
+ pb \log \left(\sqrt{-2pbPB\cos\theta + pb^2 + PB^2} - pb\cos\theta + PB \right) \\
+ PB \log \left(\sqrt{-2pbPB\cos\theta + pb^2 + PB^2} - PB\cos\theta + pb \right) \Big]. \quad (10)
\end{aligned}
$$

The case of two filaments starting from a common point is readily available from (10) letting $P \equiv p \equiv A \equiv a$ and thus $PA = pa = 0$:

$$
\begin{aligned}
M_{if} = \frac{\mu_0}{4\pi} \cos\theta \left[-PB \log \left(PB(1 - \cos\theta) \right) - pb \log \left(pb(1 - \cos\theta) \right) \right. \\
+ pb \log \left(\sqrt{-2pbPB\cos\theta + pb^2 + PB^2} - pb\cos\theta + PB \right) \\
\left. + PB \log \left(\sqrt{-2pbPB\cos\theta + pb^2 + PB^2} - PB\cos\theta + pb \right) \right]. \quad (11)
\end{aligned}
$$

3. RESULTS

The total inductance of a planar zig-zag spiral inductor is then calculated with (1) using (6) for the partial self-inductance of the individual parts of the inductor and (8), (10) for the partial mutual inductance between any two parts of the inductor. The procedure

was implemented in a Matlab$^{\text{TM}}$ computer code. In the calculation it is possible to proceed through the elements of the spiral inductor in one of the two possible directions. For example, choosing the clockwise direction, the procedure starts from the part in the top left corner of the planar zig-zag spiral inductor (see Fig. 4). The planar zig-zag spiral inductor of square shape is defined through the overall dimension of the outer side, l_0, the width w and spacing s_l of the lands, the angle of the spiral, θ, and the number of turns, n. According to the geometrical dimensions, the outer and inner contours of the spiral are built in a cartesian coordinate system, from which the axis is obtained. The axis is subdivided into segments, each one corresponding to an individual parallelogram of the zig-zag spiral inductor; the coordinates of the start and end points of each segment, as well as the angle the segment connecting the two points makes with the positive x axis, are collected in an array. A $N \times N$ matrix of the partial self- and mutual inductances of the N spiral parts is then built. For each individual parallelogram of the spiral inductor, the partial self-inductance is calculated with (6). The partial mutual inductance between the straight filaments representing any two parallelograms of the spiral is calculated as follows. In case the filaments are parallel, the code calculates the projection of the point $P \equiv A$ of the segment AB on the segment ab (see Fig. 5(a)) and calculates the partial mutual inductance with (8). If the two segments AB and ab belong to the same line, then the code calculates the partial mutual inductance with (9). For filaments incident at an angle different than $\pi/2$, the intersection between the lines connecting A and B and a and b is found (point $P \equiv p$, see Fig. 5(b)), and the partial mutual inductance is given by (10). If the filaments AB and ab are incident and share a common point (viz., $P \equiv p \equiv A \equiv a$), the partial mutual inductance is calculated with (11). Differently, the partial mutual inductance is equal to zero if the filaments are perpendicular. The partial self- and mutual inductances of the planar zig-zag spiral parts are then collected in a $N \times N$ symmetric matrix $\mathbf{L}_p$

$$\mathbf{L}_p = \begin{bmatrix} L_{p1} & M_{p12} & \ldots & M_{p1i} & \ldots & M_{p1j} & \ldots & M_{p1N} \\ M_{p12} & L_{p2} & \ldots & M_{p2i} & \ldots & M_{p2j} & \ldots & M_{p2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ M_{p1i} & M_{p2i} & \ldots & L_{pi} & \ldots & M_{pij} & \ldots & M_{piN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ M_{p1j} & M_{p2j} & \ldots & M_{pij} & \ldots & L_{pj} & \ldots & M_{pjN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ M_{p1N} & M_{p2N} & \ldots & M_{piN} & \ldots & M_{pjN} & \ldots & L_{pN} \end{bmatrix} \tag{12}$$

where $L_{p_i} = M_{p_{ii}}$ is the partial self-inductance of the element i, and $M_{p_{ij}}$ is the partial mutual inductance between the elements i and j. The sum of the elements of $\mathbf{L}_p$ along any ith row or column yields the partial inductance of the ith part of the planar zig-zag spiral inductor; the sum of all the elements of $\mathbf{L}_p$ yields the total inductance (1) of the planar zig-zag spiral inductor

$$L_{ZSI} = \sum_{i=1}^{N} \sum_{j=1}^{N} M_{pij} \tag{13}$$

The analytical calculations are validated by means of measurements carried out with an Agilent 4263B LCR impedance analyzer, at
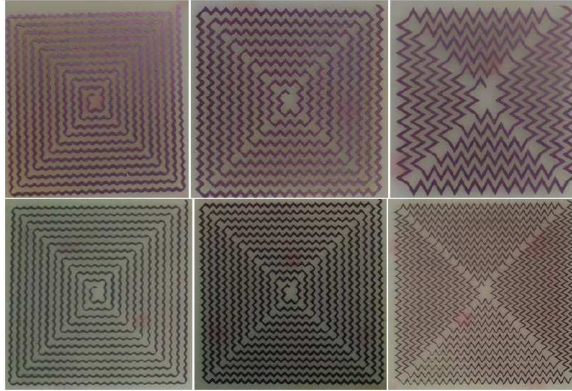
Figure 6. Samples of planar zig-zag spiral inductors of Table 1. From left to right, top row: samples I–III; bottom row: samples IV–VI.

Table 1. Comparison between the total inductance values obtained with (13), L_{ZSI}, with FastHenry, $L_{ZSI,FH}$, and with experimental measurements, $L_{ZSI,m}$.

Sample	n	N	l_0 [mm]	θ	w [mm]	s_l [mm]	L_{ZSI} [µH]	$L_{ZSI,FH}$ [µH]	$L_{ZSI,m}$ [µH]
I	11	1056	60	$\pi/6$	1.0	1.2	3.34	3.37	3.5
II	9	720	60	$\pi/4$	1.0	1.2	2.51	2.48	2.6
III	6	336	60	$\pi/3$	1.0	1.2	1.54	1.39	1.5
IV	12	1248	80	$\pi/6$	1.0	1.7	5.35	5.41	5.7
V	12	1248	80	$\pi/4$	1.0	1.2	5.54	5.61	5.6
VI	12	1248	80	$\pi/3$	0.7	0.9	6.88	6.29	6.4
VII	14	1680	130	$\pi/3$	0.9	1.1	14.13	12.99	13.5

the operating frequency of 100 Hz, and with calculations carried out with FastHenry, a multipole-accelerated 3D inductance extraction program based on magnetoquasistatic analysis [15]. Seven planar zig-zag spiral inductor samples were fabricated with the photochemical etching technique. The characteristics of the samples are summarized in Table 1. Samples I, II and III have the same overall dimension of the outer side, same land width and spacing between lands of adjacent turns; their angles are $\pi/6$, $\pi/4$ and $\pi/3$, respectively. With these geometrical parameters, the three samples have different number of turns (11, 9 and 6, respectively), and different number of parts (1056, 720 and 336, respectively). Samples IV, V and VI have the same overall dimension of the outer side and are designed in order to have the same number of turns and parts being their angles $\pi/6$, $\pi/4$ and $\pi/3$, respectively. To obtain this, they present different land width and spacing between lands of adjacent turns. Sample VII was built with a larger outer dimension than all other samples and an angle of $\pi/3$. Samples I to VI are shown in Fig. 6. The calculations and measurements of the inductance of the planar zig-zag spiral inductors are collected in Table 1; as it can be noticed, the comparison shows a very good agreement. As regards the calculations with FastHenry, it has to be noticed that each parallelogram of the planar zig-zag spiral inductor is represented as a thin rectangular parallelepiped of width w and length l. Table 2 shows the comparison between the partial self-inductance values of each part of the considered planar zig-zag spiral inductors obtained with the exact expression (6), L_{exact}, and approximated with the self-inductance of a rectangle, $L_{\mathrm{rectangle}}$. It can be noticed that the

Table 2. Comparison between the partial self-inductance values of each part of the planar zig-zag spiral inductor obtained with the exact expression (6), L_{exact}, and approximated with the partial self-inductance of a rectangle, $L_{\mathrm{rectangle}}$.

Sample	l_0 [mm]	θ	w [mm]	l [mm]	L_{exact} [nH]	$L_{\mathrm{rectangle}}$ [nH]
I	60	$\pi/6$	1.0	1.47	0.516	0.523
II	60	$\pi/4$	1.0	2.2	0.911	0.935
III	60	$\pi/3$	1.0	4.4	2.35	2.42
IV	80	$\pi/6$	1.0	1.8	0.695	0.703
V	80	$\pi/4$	1.0	2.2	0.911	0.935
VI	80	$\pi/3$	0.7	3.2	1.73	1.78
VII	130	$\pi/3$	0.9	4.35	2.32	2.39

Table 3. Partial self-inductance [nH] of parallelograms of different dimensions calculated with (6) as a function of the angle θ.

θ	$w = 1, l = 10$ [mm]	$w = 2, l = 10$ [mm]	$w = 2, l = 20$ [mm]
$\arctan(0.0)$	7.06	5.74	14.11
$\arctan(0.5)$	7.05	5.72	14.1
$\arctan(1.0)$	7.03	5.68	14.06
$\arctan(1.5)$	7.00	5.62	13.99
$\arctan(2.0)$	6.95	5.54	13.91
$\arctan(2.5)$	6.91	5.45	13.81
$\arctan(3.0)$	6.85	5.36	13.71
$\arctan(3.5)$	6.80	5.25	13.59
$\arctan(4.0)$	6.73	5.15	13.47

use of the exact expression (6) allows a minor error in the calculation of the total inductance of the planar zig-zag spiral inductor. As it can be expected, the difference between the partial self-inductance of a parallelogram calculated with (6) and that of a rectangle increases with the angle of the parallelogram, as Table 3 shows.

4. CONCLUSION

An analytical procedure for the determination of the inductance of planar zig-zag spiral inductors is presented in this paper. The inductor is partitioned into a number of parts, each with the shape of a parallelogram. The procedure is based on the partial inductance concept and consists in determining the partial self-inductance of each part and the partial mutual inductance between any two parts of the inductor. The partial self-inductance expression of a thin parallelogram is obtained in closed-form; as regards the partial mutual inductance calculations, the thin parallelograms are represented by segments (e.g., their axes), and the partial mutual inductances between filaments in any relative position are calculated. The comparison between analytical predictions and measurement shows a very good agreement. The exact analytical expression for the partial self-inductance of a parallelogram, presented in this paper, allows the error in the calculation of the inductance of the planar zig-zag spiral inductor to be reduced.

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