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THREE-HOP AF DISTRIBUTED RELAY SYSTEMS UNDER POWER CONSTRAINTS

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Abstract—This paper proposes an amplify-and-forward (AF) distributed relay network consisting of a one-source-one-destination pair and two-level N relays. Optimal relay amplifying matrices (or vectors) at the relays in the first and second levels are determined based on the minimum mean square error (MMSE) criterion. Power is globally, locally, and aggregately constrained at the relays in the first and second levels, independently or separately. With the derived optimal relay amplifying matrices, bit error rate (BER), mean square error (MSE) behavior, and the achievable rate are investigated. It is also proven that minimizing the MSE is equal to maximizing the signal-to-noise ratio (SNR) in a three-hop AF wireless relay network.

1. INTRODUCTION

Relay communication is well known for increasing the gain of diversity order, extending the propagation range, and improving communication reliability [1–5]. In addition, due to the lowest complexity and shortest delay at the relays compared to some protocols, such as decode-and-forward and compute-and-forward [2, 3], an amplify-and-forward (AF) relay protocol has become more attractive in wireless relay networks [4, 5]. Hence, over the past years, the two-hop AF wireless nondistributed and nondistributed relay systems have been investigated in most of the literature [6–8].

However, the three-hop AF wireless relay network has not been studied much in the literature. In particular, the three-hop AF wireless distributed relay network can be applicable in a 5G candidate system, e.g., the cloud radio access network if the distributed relays are connected through optical fibers to a central service station in the

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cloud radio access network. The three-hop AF wireless distributed relay system is of more considerable practical importance compared to the two-hop AF one. Hence, recently, Gomadam and Jafar in [9] made a large contribution in the three-hop AF wireless distributed relay system. Their effort in determining the optimal relay amplification matrices (or vectors) for the three-hop wireless relay network is valuable. An iterative algorithm was proposed for solving the relay optimization problem of the three-hop AF distributed relay network under a total relay power constraint based on the signal-to-noise ratio (SNR) at the destination [9].

Additionally, the authors of this paper have proposed the three-hop AF wireless distributed relay system in designing optimal relay amplifying matrices under various power constraints during data transmission [10]. Here, the relay amplifying matrix is an $N \times N$ matrix in order to minimize the mean square error (MSE) between the equalized signal at the destination and the originally transmitted signal from the source, where N is the number of relays at each hop.

Furthermore, according to the roles of relay, the relay amplifying matrix can be either diagonal or nondiagonal. For example, the relay amplifying matrix in [10] is nondiagonal, assuming that the relays can exchange their channel coefficients with each other. In other words, the diagonal relay amplifying matrix was not investigated [10]. In particular, the diagonal relay amplifying matrix in the three-hop AF wireless distributed relay system consisting of all nodes with only a single antenna is more practical than the nondiagonal one. The local power constraint (LPC) in the first and second levels was not considered [9, 10]. Hence, the authors of this paper were motivated to determine the optimal diagonal relay amplifying matrix in the three-hop AF wireless distributed relay network based on minimizing the MSE. Moreover, power will be globally, locally, and aggregately constrained at the relays during data transmission. The definition of the global power constraint (GPC) means that total power usage at the relays in the first and second levels is limited to p_1 and p_2, respectively, i.e., $p_1 = \sum_{i=1}^{N} p_{1_i}$ and $p_2 = \sum_{i=1}^{N} p_{2_i}$, while the definition of LPC means that the individual relay transmission power at the i-th relay in the first and second levels is limited to p_{1_i} and p_{2_i}, respectively. Additionally, the definition of aggregate power constraint (APC) means that the sum of transmit power at the relays in both the first and second levels is limited to $p_1 + p_2 = p_{\mathrm{T}}$ [11, 12]. Depending on the locations of power constraint, the different overall system performance can be yielded. Moreover, if either the first or the third stage power in Fig. 1 is very high, then the three-hop AF wireless distributed relay network can be reduced to the two-hop one. In

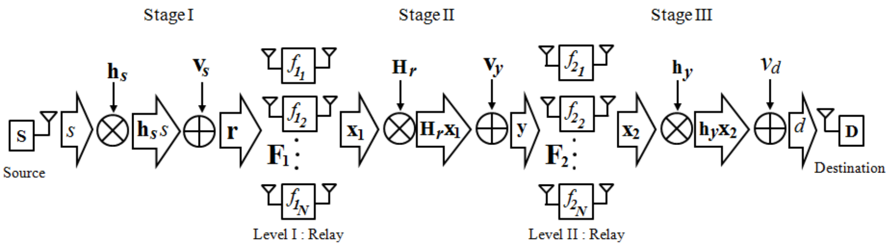

Figure 1. Three-hop AF wireless distributed relay network with one-source-one-destination pair and two-level N relay nodes.

particular, if both the first and third stage powers are very high, then it can be reduced to the point-to-point multiple-input multiple-output wireless network. This is one of characteristics of the three-hop AF wireless distributed relay network proposed in this paper. Finally, it is proven that minimizing the MSE is identical to maximizing the SNR in a three-hop AF wireless relay network.

2. SYSTEM MODEL AND DATA TRANSMISSION

An AF wireless distributed relay network with $2N$-relay nodes between a source and a destination is shown in Fig. 1. A three-stage protocol is employed for data transmission. In Stage I, a source transmits a signal symbol $s \in \mathbf{C}^{1\times 1}$ to the relays in the first level. In Stage II, relays in the first level multiplied by a relay amplifying matrix $\mathbf{F}_1$ retransmit their received signals to the relays in the second level; then they retransmit their received signals to a destination multiplied by another relay amplifying matrix $\mathbf{F}_2$ in Stage III. Here, the uppercase boldface (e.g., $\mathbf{F}_1$ and $\mathbf{F}_2$) and italic characters (e.g., s) are denoted, respectively, by a matrix and a scalar.

The complex received signal column vector $\mathbf{r} \in \mathbf{C}^{N\times 1}$ at the relays in the first level is written as

$$\mathbf{r} = \mathbf{h}_s s + \mathbf{v}_s \tag{1}$$

where $\mathbf{v}_s \in \mathbf{C}^{N\times 1}$ is a zero mean complex thermal additive white Gaussian noise (AWGN) column vector with covariance matrix $\sigma^2_{v_s}\mathbf{I}_N$, and $\mathbf{h}_s \in \mathbf{C}^{N\times 1}$ is the channel coefficient complex column vector from the source to the relays in the first level as

$$\mathbf{h}_s = [h_{s,1}, h_{s,2}, \ldots, h_{s,N}]^T. \tag{2}$$

Here, the lowercase boldface character (e.g., $\mathbf{r}$, $\mathbf{v}_s$, $\mathbf{v}_s$) is denoted by a vector. The amplified signal complex column vector $\mathbf{x}_1 \in \mathbf{C}^{N\times 1}$ at the relays in the first level with the transmitted power $\mathbf{x}_1^H \mathbf{x}_1 = p_1$ is given by

$$\mathbf{x}_1 = \mathbf{F}_1 \mathbf{r} \tag{3}$$

where $\mathbf{F}_1 \in \mathbf{C}^{N\times N}$ is a relay amplifying matrix for the linear processing operation at the relays in the first level. Here, $(\cdot)^H$ stands for the Hermitian of $(\cdot)$. The complex received signal column vector $\mathbf{y} \in \mathbf{C}^{N\times 1}$ at the relays in the second level can be represented as

$$\mathbf{y} = \mathbf{H}_r \mathbf{x}_1 + \mathbf{v}_y \tag{4}$$

where $\mathbf{v}_y \in \mathbf{C}^{N\times 1}$ is also a zero-mean, complex, thermal AWGN column vector with covariance matrix $\sigma_{v_y}^2 \mathbf{I}_N$, and $\mathbf{H}_r \in \mathbf{C}^{N\times N}$ is the channel coefficient complex matrix from relays in the first level to relays in the second level as

$$\mathbf{H}_r = [\mathbf{h}_{r,1}, \mathbf{h}_{r,2}, \ldots, \mathbf{h}_{r,N}]^T. \tag{5}$$

Here, $\mathbf{h}_{r,n} = \left[h_{r,n,1}, \ldots, h_{r,n,N}\right]$, $n = 1, \ldots, N$, is a row vector, representing the channel coefficients from all relays in the first level to the n-th relay in the second level. Additionally, here, $(\cdot)^T$ denotes the transpose of $(\cdot)$. The amplified signal complex column vector $\mathbf{x}_2 \in \mathbf{C}^{N\times 1}$ at the relays in the second level with the transmitted power $\mathbf{x}_2^H \mathbf{x}_2 = p_2$ is given by

$$\mathbf{x}_2 = \mathbf{F}_2 \mathbf{y} \tag{6}$$

where $\mathbf{F}_2 \in \mathbf{C}^{N\times N}$ is also a relay amplifying matrix for the linear processing operation at the relays in the second level. Finally, the received complex signal $d \in \mathbf{C}^{1\times 1}$ at the destination can be written as

$$d = \mathbf{h}_y \mathbf{x}_2 + v_d \tag{7}$$

where $v_d \in \mathbf{C}^{1\times 1}$ is a zero-mean, complex, thermal AWGN variable with variance $\sigma_{v_d}^2$, and $\mathbf{h}_y \in \mathbf{C}^{1\times N}$ is the channel coefficient complex row vector from the i-th relay in the second level to the destination as

$$\mathbf{h}_y = [h_{y,1},\, h_{y,2},\, \ldots,\, h_{y,N}]. \tag{8}$$

Substituting (1), (3), (4), and (6) into (7), the received complex signal $d \in \mathbf{C}^{1\times 1}$ at the destination can be rewritten as

$$d = \mathbf{h}_y \mathbf{F}_2 \mathbf{H}_r \mathbf{F}_1 \mathbf{h}_s s + \mathbf{h}_y \mathbf{F}_2 \mathbf{H}_r \mathbf{F}_1 \mathbf{v}_s + \mathbf{h}_y \mathbf{F}_2 \mathbf{v}_y + v_d. \tag{9}$$

Each channel coefficient $h_{s,i}$, $h_{r,n,i}$, and $h_{y,i}$, $i = 1, \ldots, N$, is assumed to be independent identically distributed (i.i.d.) with a zero-mean, circular, complex Gaussian of unit variance and quasi-static Rayleigh fading so that they stay fixed during data transmission. Obtaining knowledge of channels at the relays and destination can be performed through pilot symbols. In addition, the signal d in (9) is equalized by a scalar gain factor α^{-1} to produce the estimated $\hat{s}$, i.e., $\hat{s} = \alpha^{-1}d$. Using (9), the cost function $J(\mathbf{F}_1, \mathbf{F}_2) \triangleq E\left[|\hat{s} - s|^2\right]$ at the destination can be written, using the MSE [13], as

$$\begin{aligned} J(\mathbf{F}_1, \mathbf{F}_2) \;=\; & \alpha^{-2}\sigma_s^2|\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1\mathbf{h}_s|^2 - 2\alpha^{-1}\sigma_s^2\mathrm{Re}[\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1\mathbf{h}_s] \\ & + \alpha^{-2}\sigma_{v_s}^2||\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1||^2 + \alpha^{-2}\sigma_{v_y}^2||\mathbf{h}_y\mathbf{F}_2||^2 \\ & + \alpha^{-2}\sigma_{v_d}^2 + \sigma_s^2. \end{aligned} \tag{10}$$

Here, $E[\cdot]$ and $\mathrm{Re}[\mathbf{A}] = (\mathbf{A} + \mathbf{A}^*)/2$ represent the expectation and real operators, respectively. Additionally, the notations $|\cdot|$ and $||\cdot||$ denote the absolute value of any scalar and 2-norm of any vector, respectively. In (10), the scalar gain factor α^{-1} is in fact the Wiener filter [14], which is given as

$$\alpha^{-1} = \sigma_s^2 p_d^{-1}\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1\mathbf{h}_s \tag{11}$$

where $p_d = \sigma_s^2|\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1\mathbf{h}_s|^2 + \sigma_{v_s}^2||\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1||^2 + \sigma_{v_y}^2||\mathbf{h}_y\mathbf{F}_2||^2 + \sigma_{v_d}^2$. Substituting (11) into (10), $J(\mathbf{F}_1, \mathbf{F}_2)$ in (10) can be rewritten as

$$J(\mathbf{F}_1, \mathbf{F}_2) = \sigma_s^2(\mathrm{SNR} + 1)^{-1} \tag{12}$$

where

$$\mathrm{SNR} = \frac{\sigma_s^2|\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1\mathbf{h}_s|^2}{\sigma_{v_s}^2||\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r\mathbf{F}_1||^2 + \sigma_{v_y}^2||\mathbf{h}_y\mathbf{F}_2||^2 + \sigma_{v_d}^2} \tag{13a}$$

$$= \mathbf{h}_y\mathbf{H}_t\mathbf{H}_w^{-1}\mathbf{H}_t^H\mathbf{h}_y \tag{13b}$$

where $\mathbf{H}_w = \sigma_{v_y}^2(\mathbf{H}_y \odot \mathbf{I}_N) + \sigma_{v_d}^2 p_2^{-1}(\mathbf{H} \odot \mathbf{I}_N)$, $\mathbf{H}_y = \mathbf{h}_y^H\mathbf{h}_y$, $\mathbf{H} = \sigma_s^2\mathbf{H}_r\mathbf{F}_1\mathbf{h}_s\mathbf{h}_s^H\mathbf{F}_1^H\mathbf{H}_r^H + \sigma_{v_s}^2\mathbf{H}_r\mathbf{F}_1\mathbf{F}_1^H\mathbf{H}_r^H + \sigma_{v_y}^2\mathbf{I}_N$, $\mathbf{H}_t = \mathrm{diag}(\mathbf{h}_t)$, $\mathbf{h}_t = \mathbf{H}_r\mathbf{F}_1\mathbf{h}_s$, $E[v_d] = 0$, $E[|s|^2] = \sigma_s^2$, and $E[\mathbf{v}_s] = E[\mathbf{v}_y] = \mathbf{0}_N$. Here, $\mathbf{H}_w$ is a symmetric positive definitive Hermitian matrix with rank N. In addition, an $N \times N$ identity matrix, a diagonal matrix, and an $N \times 1$ zero vector consisting of all zero entries are denoted, respectively, by $\mathbf{I}_N$, $\mathrm{diag}(\cdot)$, and $\mathbf{0}_N$. The notations $\odot$ and $(\cdot)^{-1}$ are denoted, respectively, by the Hadamard (elementwise) product operator and the inverse of $(\cdot)$. From (12), it can be seen that minimizing the MSE is equivalent to maximizing the SNR. However, it is difficult to directly solve the SNR maximization problem because the power constraints are interdependent, as shown in (3) and (6). Hence, this paper focuses on the MSE minimization problem using the Karush-Kuhn-Tucker conditions [15], unlike [9].

Additionally, as stated earlier, the relay amplifying matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ will be diagonal, i.e., $\mathbf{F}_1 = \mathrm{diag}(\mathbf{f}_1) = \mathrm{diag}(f_{1_1}, \ldots, f_{1_N})$ and $\mathbf{F}_2 = \mathrm{diag}(\mathbf{f}_2) = \mathrm{diag}(f_{2_1}, \ldots, f_{2_N})$, similar to [9]. Let the $N \times 1$

vector $\mathbf{a} = \mathrm{diag}(\mathbf{A})$ denote the diagonal elements of $\mathbf{A}$. Hence, using this notation, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed by $\mathbf{f}_1 = \mathrm{diag}(\mathbf{F}_1)$ and $\mathbf{f}_2 = \mathrm{diag}(\mathbf{F}_2)$, respectively.

3. MMSE RELAY SCHEMES

3.1. Global Power Constraint (GPC)

As stated in the previous section, in order to determine optimum relay amplifying matrices $\mathbf{F}_1$ and $\mathbf{F}_2$, the minimum mean square error (MMSE) criterion between the equalized signal $\hat{s}$ at the destination and the originally transmitted signal s from the source will be applied by constraining the total power at the relays in the first and second levels, independently and separately. Hence, the desired MMSE formulation to derive optimum relay amplifying matrices under relay power-constraint conditions in the first and second levels can be written as

$$\left(\mathbf{F}_1^{\dagger}, \mathbf{F}_2^{\dagger}\right) = \arg\min_{\mathbf{F}_1, \mathbf{F}_2} J(\mathbf{F}_1, \mathbf{F}_2) \tag{14a}$$

$$\text{s.t.}\quad E\left[||\mathbf{x}_1||^2\right] = p_1 \text{ and } E\left[||\mathbf{x}_2||^2\right] = p_2 \tag{14b}$$

where the superscript † denotes the optimum. Using vector forms $\mathbf{f}_1$, $\mathbf{f}_2$, and the Hadamard product operator, the cost function $J(\mathbf{F}_1, \mathbf{F}_2)$ in (14a) can be written as

$$J(\mathbf{F}_1, \mathbf{F}_2) = \alpha^{-2}\mathbf{f}_1^H\left(\mathbf{H}_a^T \odot \mathbf{H}_c\right)\mathbf{f}_1 - 2\alpha^{-1}\sigma_s^2\mathrm{Re}\left[\mathbf{f}_1^H\mathbf{h}_b\right] + \sigma_s^2 + \alpha^{-2}\sigma_{v_y}^2\mathbf{f}_2^H\left(\mathbf{H}_y \odot \mathbf{I}_N\right)\mathbf{f}_2 + \alpha^{-2}\sigma_{v_d}^2 \tag{15a}$$

$$= \alpha^{-2}\mathbf{f}_2^H\left(\mathbf{H}^T \odot \mathbf{H}_y\right)\mathbf{f}_2 - 2\alpha^{-1}\sigma_s^2\mathrm{Re}\left[\mathbf{f}_2^H\mathbf{h}_a\right] + \alpha^{-2}\sigma_{v_d}^2 + \sigma_s^2 \tag{15b}$$

where $\mathbf{H}_a = \sigma_s^2\mathbf{h}_s\mathbf{h}_s^H + \sigma_{v_s}^2\mathbf{I}_N$, $\mathbf{H}_c = \mathbf{H}_r^H\mathbf{F}_2^H\mathbf{h}_y^H\mathbf{h}_y\mathbf{F}_2\mathbf{H}_r$, $\mathbf{h}_b = \mathrm{diag}(\mathbf{H}_r^H\mathbf{F}_2^H\mathbf{h}_y^H\mathbf{h}_s^H)$, and $\mathbf{h}_a = \mathrm{diag}(\mathbf{h}_y^H\mathbf{h}_s^H\mathbf{F}_1^H\mathbf{H}_r^H)$. Here, (15a) and (15b) will be used for determining $\mathbf{f}_1$ and $\mathbf{f}_2$, respectively. The total power constraints at the relays in the first and second levels can be represented from (3) and (6), respectively, as

$$p_1 = \mathbf{f}_1^H(\mathbf{H}_a \odot \mathbf{I}_N)\mathbf{f}_1 \tag{16}$$

$$p_2 = \mathbf{f}_1^H(\mathbf{H}_a^T \odot \mathbf{H}_b)\mathbf{f}_1 + \sigma_{v_y}^2\mathbf{f}_2^H\mathbf{f}_2 \tag{17a}$$

$$= \mathbf{f}_2^H(\mathbf{H} \odot \mathbf{I}_N)\mathbf{f}_2 \tag{17b}$$

where $\mathbf{H}_b = \mathbf{H}_r^H\mathbf{F}_2^H\mathbf{F}_2\mathbf{H}_r$. Here, (17a) and (17b) will be used for determining $\mathbf{f}_1$ and $\mathbf{f}_2$, respectively. Since the total power at the relays in the first and second levels is constrained, respectively, to p_1 and p_2, the constrained Lagrangian optimization [15] can be formulated as

$$L(\mathbf{F}_1, \mathbf{F}_2, \alpha, \lambda_1, \lambda_2) = J(\mathbf{F}_1, \mathbf{F}_2) + \lambda_1\big(E\big[||\mathbf{x}_1||^2\big] - p_1\big) + \lambda_2\big(E\big[||\mathbf{x}_2||^2\big] - p_2\big) \quad (18)$$

where λ_1 and λ_2 are Lagrangian multipliers. For expositional convenience, $L(\mathbf{F}_1, \mathbf{F}_2, \alpha, \lambda_1, \lambda_2)$ in (18) is henceforth simply stated as $L(\mathbf{F}, \lambda)$. Taking the partial derivatives of (18) with regard to $\{\mathbf{f}_1^H, \mathbf{f}_2^H, \alpha, \lambda_1, \text{and } \lambda_2\}$, respectively, and using the linear and nonlinear properties of the complex vector derivative [16], and equating the derivatives to zero result in

$$\frac{\partial \mathcal{L}(\mathbf{F}, \lambda)}{\partial \mathbf{f}_1^H} = \alpha^{-2}\Big(\mathbf{H}_a^T \odot \mathbf{H}_c\Big)\mathbf{f}_1 + \lambda_2\Big(\mathbf{H}_a^T \odot \mathbf{H}_b\Big)\mathbf{f}_1 - \alpha^{-1}\sigma_s^2\mathbf{h}_b + \lambda_1\Big(\mathbf{H}_a \odot \mathbf{I}_N\Big)\mathbf{f}_1 = \mathbf{0}_N \quad (19)$$

$$\frac{\partial \mathcal{L}(\mathbf{F}, \lambda)}{\partial \mathbf{f}_2^H} = \alpha^{-2}\Big(\mathbf{H}^T \odot \mathbf{H}_y\Big)\mathbf{f}_2 + \lambda_2\Big(\mathbf{H} \odot \mathbf{I}_N\Big)\mathbf{f}_2 - \alpha^{-1}\sigma_s^2\mathbf{h}_a = \mathbf{0}_N \quad (20)$$

$$\frac{\partial \mathcal{L}(\mathbf{F}, \lambda)}{\partial \alpha} = -2\alpha^{-3}\mathbf{f}_2^H\Big(\mathbf{H}^T \odot \mathbf{H}_y\Big)\mathbf{f}_2 + 2\alpha^{-2}\sigma_s^2\mathrm{Re}\Big[\mathbf{f}_2^H\mathbf{h}_a\Big] - 2\alpha^{-3}\sigma_{v_d}^2 = 0 \quad (21)$$

$$\frac{\partial \mathcal{L}(\mathbf{F}, \lambda)}{\partial \lambda_1} = \mathbf{f}_1^H\Big(\mathbf{H}_a \odot \mathbf{I}_N\Big)\mathbf{f}_1 - p_1 = 0 \quad (22)$$

$$\frac{\partial \mathcal{L}(\mathbf{F}, \lambda)}{\partial \lambda_2} = \mathbf{f}_1^H\Big(\mathbf{H}_a^T \odot \mathbf{H}_b\Big)\mathbf{f}_1 + \sigma_{v_y}^2\mathbf{f}_2^H\mathbf{f}_2 - p_2 = 0 \quad (23a)$$

$$\Big(\text{or } \mathbf{f}_2^H\Big(\mathbf{H} \odot \mathbf{I}_N\Big)\mathbf{f}_2 - p_2 = 0\Big). \quad (23b)$$

Here, (23a) and (23b) will be used for determining $\mathbf{f}_1$ and $\mathbf{f}_2$, respectively.

Using the trace property, i.e., $tr(\mathrm{Re}\{\mathbf{A}\}) = tr(\mathbf{A})$ if $tr(\mathbf{A}) = tr(\mathbf{A}^H)$ (here, $tr(\cdot)$ is trace of $(\cdot)$), the optimal solutions $\mathbf{f}_1^\dagger$, $\mathbf{f}_2^\dagger$, $\alpha^\dagger$, $\lambda_1^\dagger$, and $\lambda_2^\dagger$ for the GPC at the relays in the first and second levels can be written, respectively, as

$$\mathbf{f}_1^\dagger = \frac{\mathbf{H}_z^{-1}\mathbf{h}_b\sqrt{p_1}}{\sqrt{\mathbf{h}_b^H\mathbf{H}_z^{-1}(\mathbf{H}_a \odot \mathbf{I}_N)\mathbf{H}_z^{-1}\mathbf{h}_b}} \quad (24)$$

$$\mathbf{f}_2^\dagger = \frac{\mathbf{H}_t^{-1}\mathbf{h}_a\sqrt{p_2}}{\sqrt{\mathbf{h}_a^H\mathbf{H}_t^{-1}(\mathbf{H} \odot \mathbf{I}_N)\mathbf{H}_t^{-1}\mathbf{h}_a}} \quad (25)$$

$$\alpha^{\dagger} = \frac{\sqrt{p_2}}{\sqrt{\sigma_s^4 \mathbf{h}_a^H \mathbf{H}_t^{-1} (\mathbf{H} \odot \mathbf{I}_N) \mathbf{H}_t^{-1} \mathbf{h}_a}} \tag{26}$$

$$\lambda_1^{\dagger} = \eta \sigma_s^4 p_1^{-1} \mathbf{h}_b^H \mathbf{H}_z^{-1} (\mathbf{H}_a \odot \mathbf{I}_N) \mathbf{H}_z^{-1} \mathbf{h}_b \tag{27}$$

$$\lambda_2^{\dagger} = \sigma_s^4 \sigma_{v_d}^2 p_2^{-2} \mathbf{h}_a^H \mathbf{H}_t^{-1} (\mathbf{H} \odot \mathbf{I}_N) \mathbf{H}_t^{-1} \mathbf{h}_a \tag{28}$$

where

$$\mathbf{H}_z = \left(\mathbf{H}_a^T \odot \mathbf{H}_c\right) + \eta(\mathbf{H}_a \odot \mathbf{I}_N) + \sigma_{v_d}^2 p_2^{-1} \left(\mathbf{H}_a^T \odot \mathbf{H}_b\right) \tag{29}$$

$$\eta = \sigma_{v_y}^2 p_1^{-1} \mathbf{f}_2^H (\mathbf{H}_y \odot \mathbf{I}_N) \mathbf{f}_2 + \sigma_{v_d}^2 \sigma_{v_y}^2 p_1^{-1} p_2^{-1} ||\mathbf{f}_2||^2 \tag{30}$$

$$\mathbf{H}_t = \left(\mathbf{H}^T \odot \mathbf{H}_y\right) + \sigma_{v_d}^2 p_2^{-1} (\mathbf{H} \odot \mathbf{I}_N). \tag{31}$$

Note that the corresponding optimal relay amplifying matrices $\mathbf{F}_1^{\dagger}$ and $\mathbf{F}_2^{\dagger}$ are given by $\mathbf{F}_1^{\dagger} = \mathrm{diag}(\mathbf{f}_1^{\dagger})$ and $\mathbf{F}_2^{\dagger} = \mathrm{diag}(\mathbf{f}_2^{\dagger})$, respectively.

3.2. Local Power Constraint (LPC)

The power of each relay in the first and second levels can be constrained to p_{1_i}, i.e., $p_{1_i} = |f_{1_i}|^2 \sigma_{r_i}^2$, and p_{2_i}, i.e., $p_{2_i} = |f_{2_i}|^2 \sigma_{y_i}^2$, where $\sigma_{r_i}^2 = \sigma_s^2 |h_{s_i}|^2 + \sigma_{v_s}^2$, and $\sigma_{y_i}^2 = \sigma_s^2 \sum_{k=1}^N \left|h_{s_k}\right|^2 \left|f_{1_k}\right|^2 \left|h_{r_{ik}}\right|^2 + \sigma_{v_s}^2 \sum_{k=1}^N |f_{1_k}|^2 |h_{r_{ik}}|^2 + \sigma_{v_y}^2$, respectively. As in the case of the GPC in the previous subsection, the desired constrained Lagrangian optimization $L_1(\mathbf{F}_1, \mathbf{F}_2, \alpha\, \lambda_1, \lambda_2) (\triangleq L_1(\mathbf{F}, \lambda))$ for the LPC can be written as

$$\begin{aligned} L_1(\mathbf{F}, \lambda) = {} & \alpha^{-2} \sigma_s^2 \left| \left(\sum_{i=1}^N \left(\sum_{k=1}^N h_{y_k} f_{2_k} h_{r_{ki}} \right) f_{1_i} h_{s_i} \right) \right|^2 \\ & + \alpha^{-2} \sigma_{v_s}^2 \sum_{i=1}^N \left| \left(\sum_{k=1}^N h_{y_k} f_{2_k} h_{r_{ki}} \right) f_{1_i} \right|^2 \\ & - 2\alpha^{-1} \sigma_s^2 \mathrm{Re} \left[\sum_{i=1}^N \left(\sum_{k=1}^N h_{y_k} f_{2_k} h_{r_{ki}} \right) f_{1_i} h_{s_i} \right] \\ & + \alpha^{-2} \sigma_{v_y}^2 \sum_{i=1}^N |h_{y_i}|^2 |f_{2_i}|^2 + \alpha^{-2} \sigma_{v_d}^2 + \sigma_s^2 \\ & + \sum_{i=1}^N \lambda_{1_i} \left(|f_{1_i}|^2 \sigma_{r_i}^2 - p_{1_i} \right) + \sum_{i=1}^N \lambda_{2_i} \left(|f_{2_i}|^2 \sigma_{y_i}^2 - p_{2_i} \right). \end{aligned} \tag{32}$$

Similarly, to determine the optimal solutions of $L_1(\mathbf{F},\, \lambda)$ in (32), taking the partial derivative of $L_1(\mathbf{F},\, \lambda)$ with respect to $\{f_{1_i},\, f_{2_i},\, \lambda_{1_i},\, \text{and}\ \lambda_{2_i}\}$, the optimal solutions $\{f_{1_i}^{\dagger},\, f_{2_i}^{\dagger},\, \lambda_{1_i}^{\dagger},\, \text{and}\ \lambda_{2_i}^{\dagger}\}$ for the i-th relay in the first and second levels can be written, respectively, as

$$f_{1_i}^{\dagger} = \frac{(\gamma_1 - \lambda_{2_i}\gamma_2)\sqrt{p_{1_i}}}{\left|\gamma_1 - \lambda_{2_i}\gamma_2\right|\sqrt{\sigma_{r_i}^2}} \tag{33}$$

$$f_{2_i}^{\dagger} = \frac{(\sigma_s^2\gamma_3 - \gamma_4)\sqrt{p_{2_i}}}{\left|\sigma_s^2\gamma_3 - \gamma_4\right|\sqrt{\sigma_{y_i}^2}} \tag{34}$$

$$\lambda_{1_i}^{\dagger} = \frac{\sigma_s^2\left|\gamma_1 - \lambda_{2_i}\gamma_2\right|\sqrt{\sigma_{r_i}^2 p_{1_i}^{-1}} - \gamma_5 - \gamma_6}{\alpha^2\sigma_{r_i}^2} \tag{35}$$

$$\lambda_{2_i}^{\dagger} = \frac{\left|\sigma_s^2\gamma_3 - \gamma_4\right|\sqrt{\sigma_{y_i}^2 p_{2_i}^{-1}} - |h_{y_i}|^2\left(\sigma_{v_y}^2 + |\gamma_7|^2\right)}{\alpha^2\left(|h_{y_i}|^{-2}|\gamma_7|^2 + \sigma_{v_s}^2\sum_{k=1}^{N}|h_{r_{ik}}|^2|f_{1_k}|^2 + \sigma_{v_y}^2\right)} \tag{36}$$

where

$$\gamma_1 = \sum_{k=1}^{N} h_{y_k}^* f_{2_k}^* h_{r_{ki}^*}^* h_{s_i}\left(\alpha - \sum_{t=1, t\neq i}^{N}\sum_{k=1}^{N} h_{s_t} f_{1_t} h_{r_{kt}} f_{2_k} h_{y_k}\right) \tag{37}$$

$$\gamma_2 = \alpha^2 h_{s_i}^* \sum_{k=1, k\neq i}^{N}\sum_{t=1}^{N} h_{r_{ti}}^* f_{2_k}^* h_{s_k} f_{1_k} h_{r_{tk}} f_{2_k} \tag{38}$$

$$\gamma_3 = \sum_{k=1}^{N} h_{r_{ik}}^* f_{1_k}^* h_{s_k}^* h_{y_i}^*\left(\alpha - \sum_{t=1, t\neq i}^{N}\sum_{k=1}^{N} h_{y_t} f_{2_t} h_{r_{tk}} f_{1_k} h_{s_k}\right) \tag{39}$$

$$\gamma_4 = \sigma_{v_s}^2 h_{y_i} \sum_{t=1}^{N}\sum_{k=1, k\neq i}^{N} h_{r_{it}}^* f_{1_t}^* h_{y_k} f_{2_k} h_{r_{kt}} f_{1_t} \tag{40}$$

$$\gamma_5 = \left|\sum_{k=1}^{N} h_{y_k} f_{2_k} h_{r_{ki}}\right|^2\left(\sigma_s^2\left|h_{s_i}\right|^2 + \alpha^2\sigma_{v_s}^2\right) \tag{41}$$

$$\gamma_6 = \sigma_{v_s}^2 \sum_{k=1}^{N}\left|h_{r_{ki}}\right|^2\left|f_{2_k}\right|^2 + \sigma_s^2\left|\sum_{k=1}^{N} h_{y_k} f_{2_k} h_{r_{ki}}\right|^2 \tag{42}$$

$$\gamma_7 = \sqrt{\sigma_s^2}\sum_{k=1}^{N} h_{r_{ik}} f_{1_k} h_{s_k} h_{y_i}. \tag{43}$$

Note that the identical optimal $\alpha^\dagger$ can be determined by taking the partial derivative of $L_1(\mathbf{F},\, \lambda)$ with regard to α. Note also that, unlike (5), as a special case of the three-hop wireless relaying system, $\mathbf{H}_r$ can be a diagonal matrix, i.e., $\mathbf{H}_r \triangleq \mathrm{diag}(\mathbf{h}_r) = \mathrm{diag}(h_{r_1},\, \ldots,\, h_{r_N})$. Applying a diagonal matrix $\mathbf{H}_r$, the optimal solutions $f_{1_i}^\dagger$ and $f_{2_i}^\dagger$ can be written as

$$f_{1_i}^\dagger = \frac{h_{y_i}^* f_{2_i}^* h_{r_i}^* h_{s_i}^* \sqrt{p_{1_i}}}{|h_{y_i} f_{2_i} h_{r_i} h_{s_i}| \sqrt{\sigma_{r_i}^2}} \tag{44}$$

$$f_{2_i}^\dagger = \frac{h_{s_i}^* f_{1_i}^* h_{r_i}^* h_{y_i}^* \sqrt{p_{2_i}}}{|h_{s_i} f_{1_i} h_{r_i} h_{y_i}| \sqrt{\sigma_{y_i}^2}}. \tag{45}$$

As shown in (44) and (45), the optimal $f_{1_i}^\dagger$ and $f_{2_i}^\dagger$ only depend on each local channel coefficient $(h_{s_i},\; h_{r_i},\; h_{y_i})$. In this case, there is no need for relays to have the knowledge of all channel coefficients.

3.3. Aggregate Power Constraint (APC)

During data transmission, the sum of the transmit power at the relays in both the first and second levels can be constrained to p_{T}, i.e., $p_1 + p_2 = p_{\mathrm{T}}$. Accordingly, as in the case of the power constraints at the relay nodes in both the first and second levels, the desired MMSE formulation $L(\mathbf{F}_1,\, \mathbf{F}_2,\, \alpha,\, \lambda_{\mathrm{T}})$ in (18) is modified to

$$L(\mathbf{F}_1, \mathbf{F}_2, \alpha, \lambda_{\mathrm{T}}) = J(\mathbf{F}_1, \mathbf{F}_2, \alpha) + \lambda_{\mathrm{T}} \left(E\left[||\mathbf{x}_1||^2\right] + E\left[||\mathbf{x}_2||^2\right] - p_{\mathrm{T}} \right). \tag{46}$$

Following the previous procedures, the solutions of $L(\mathbf{F}_1,\, \mathbf{F}_2,\, \alpha,\, \lambda_{\mathrm{T}})$ in (46) can be obtained, respectively, as

$$\mathbf{f}_1^\dagger = \frac{\mathbf{H}_d^{-1} \mathbf{h}_b \sqrt{\kappa_1}}{\sqrt{\mathbf{h}_b^H \mathbf{H}_d^{-H} \mathbf{H}_k \mathbf{H}_d^{-1} \mathbf{h}_b}} \tag{47}$$

$$\mathbf{f}_2^\dagger = \frac{\mathbf{H}_o^{-1} \mathbf{h}_a \sqrt{\kappa_2}}{\sqrt{\mathbf{h}_a^H \mathbf{H}_o^{-H} (\mathbf{H} \odot \mathbf{I}_N) \mathbf{H}_o^{-1} \mathbf{h}_a}} \tag{48}$$

$$\alpha^\dagger = \frac{\sqrt{\kappa_2}}{\sqrt{\sigma_s^4 \mathbf{h}_a^H \mathbf{H}_o^{-H} (\mathbf{H} \odot \mathbf{I}_N) \mathbf{H}_o^{-1} \mathbf{h}_a}} \tag{49}$$

$$\lambda_{\mathrm{T}}^\dagger = \omega_2 \sigma_s^4 p_{\mathrm{T}}^{-1} \mathbf{h}_a^H \mathbf{H}_o^{-H} (\mathbf{H} \odot \mathbf{I}_N) \mathbf{H}_o^{-1} \mathbf{h}_a \tag{50}$$

where

$$\mathbf{H}_d = \left(\mathbf{H}_a^T \odot \mathbf{H}_c \right) + \omega_1 \mathbf{H}_k \tag{51}$$

Table 1. Iterative algorithm for GPC.

Step 1	Initialization: $k=0$ $\mathbf{F}_2=\mathbf{I}_N,\ J(\mathbf{F}_1,\mathbf{F}_2)=0$
Step 2	Iteration: $k \leftarrow k+1$ $\mathbf{F}_{1_k}=f_{\mathbf{F}_{1_{k-1}}}(\mathbf{F}_{2_{k-1}})$ $\mathbf{F}_{2_k}=f_{\mathbf{F}_{2_k}}(\mathbf{F}_{1_k})$ $\alpha_k=f_{\alpha_k}(\mathbf{F}_{1_k})$ $J(\mathbf{F}_{1_k},\mathbf{F}_{2_k})=f_{J(\mathbf{F}_{1_k},\mathbf{F}_{2_k})}(\mathbf{F}_{1_k},\mathbf{F}_{2_k},\alpha_k)$
Step 3	If $0 \le J(\mathbf{F}_{1_{k-1}},\mathbf{F}_{2_{k-1}})-J(\mathbf{F}_{1_k},\mathbf{F}_{2_k}) \le \varpi$ go to Step 4 and stop, otherwise go back to Step 2 $(\varpi=0.0001)$
Step 4	$\mathbf{F}_1=\mathbf{F}_{1_k};\ \mathbf{F}_2=\mathbf{F}_{2_k}$

$$\mathbf{H}_k=\left(\mathbf{H}_a^T\odot\mathbf{H}_b\right)+(\mathbf{H}_a\odot\mathbf{I}_N) \tag{52}$$

$$\mathbf{H}_o=\left(\mathbf{H}^T\odot\mathbf{H}_y\right)+\omega_2(\mathbf{H}\odot\mathbf{I}_N) \tag{53}$$

$$\omega_1=\kappa_1^{-1}\left(\sigma_{v_y}^2\mathbf{f}_2^H(\mathbf{H}_y\odot\mathbf{I}_N)\mathbf{f}_2+\sigma_{v_d}^2\right) \tag{54}$$

$$\omega_2=\sigma_{v_d}^2\left(p_{\mathrm{T}}-\mathbf{f}_1^H(\mathbf{H}_a\odot\mathbf{I}_N)\mathbf{f}_1\right)^{-1} \tag{55}$$

$$\kappa_1=p_{\mathrm{T}}-\sigma_{v_y}^2\mathbf{f}_2^H\mathbf{f}_2 \tag{56}$$

$$\kappa_2=p_{\mathrm{T}}-\mathbf{f}_1^H(\mathbf{H}_a\odot\mathbf{I}_N)\mathbf{f}_1. \tag{57}$$

4. ITERATIVE ALGORITHM

As shown in (24), (25), (33), (34), (44), (45), (47), and (48), the optimal $\mathbf{F}_1^\dagger$ and $\mathbf{F}_2^\dagger$ in the GPC, LPC, and APC cases are functions of each other. Thus, using the property of $J(\mathbf{F}_1,\,\mathbf{F}_2)$, i.e., gradually decreasing, they can be solved by an iterative algorithm where variables are calculated one at a time while fixing the others, as shown in Table 1.

In addition, as in the case of the GPC, using the iterative algorithm, all optimal values for the LPC and APC cases can be solved.

5. SIMULATION RESULTS

To evaluate bit error rate (BER) and MMSE performances of the proposed three-hop AF MMSE relay strategy under GPC, LPC, and

APC at the relays in the first and second levels, the Monte-Carlo simulation is performed. In addition, using (13), the achievable rate under GPC, LPC, and APC at the relays will also be performed. The channel coefficient vectors ($\mathbf{h}_s$ and $\mathbf{h}_y$) and matrix $\mathbf{H}_r$ are generated from independent Gaussian random variables with zero mean and unity variance. The originally transmitted signal at the source is assumed to be modulated by a quadrature phase shift keying constellation with unit power. And the GPC is set to $p_1 = p_2 = 1$, while the LPC is set to $p_{1_i} = p_{2_i} = 1/N$. Additionally, the APC is set to $p_{\mathrm{T}} = 2$. All nodes have the same thermal noise power, i.e., $\sigma^2_{v_s} = \sigma^2_{v_y} = \sigma^2_{v_d}$.

Figure 2 shows the convergence of the proposed iterative algorithm for $N = 2$ and 3 and SNR $= 10\,\mathrm{dB}$ under property according to the number of relays ($N = 2$ and 3) with SNR $= 10\,\mathrm{dB}$ under both GPC and APC at the relays, numerically. As shown in Fig. 2, the proposed algorithm in this paper can converge within 10 iterations to find the optimal solutions. That is, the proposed algorithm has a low computational load to determined the optimal solutions. It is observed that the convergence values in the case of both GPC and APC decrease as N increases.

Figure 3 provides the following: (a) MMSE versus number of relays, (b) MMSE versus input SNR, and (c) BER versus input SNR in a three-hop AF wireless relay network under the GPC, LPC, and

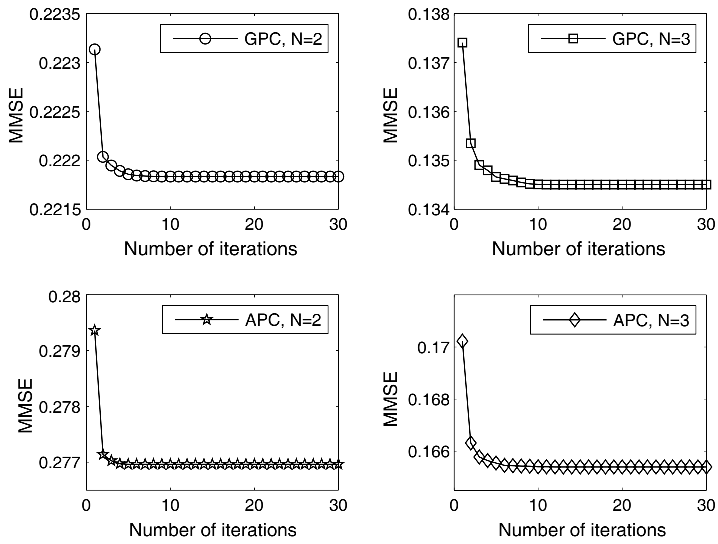

Figure 2. Convergence property according to number of relays ($N = 2$ and 3) with SNR $= 10\,\mathrm{dB}$ under both GPC and APC at relays.

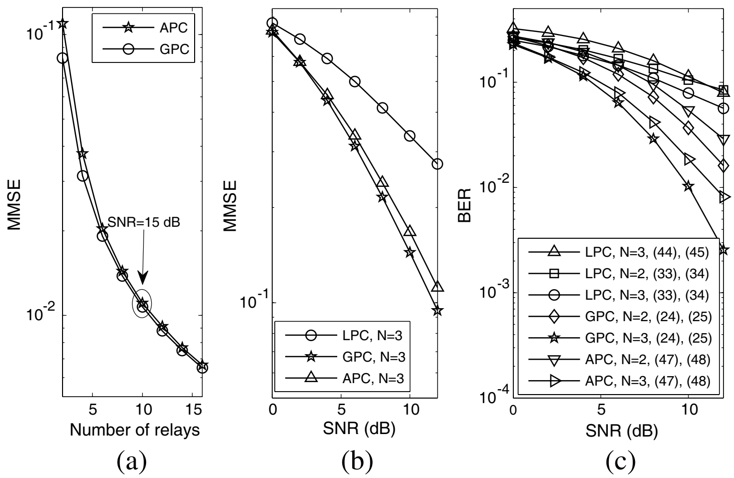

Figure 3. (a) MMSE versus number of relays, (b) MMSE versus input SNR, and (c) BER versus input SNR in three-hop AF wireless relay network under GPC, LPC, and APC at relays with different number of relays, respectively.

APC at the relays with different number of relays, respectively. As analyzed, as N increases, the MMSE performance decreases, as shown in Figs. 3(a) and 3(b). This is because the cost function in this paper is defined as the MSE. In other words, the smaller the MSE performance, the better the BER performance. As a result, as N increases, the BER performance improves, as shown in Fig. 3(c). Additionally, the MSE performance of the GPC case is the smallest in all power constraint cases for the same N, as shown in Fig. 3(b). As a result, it can be seen that when power is globally constrained at the relays, the best BER performance is observed, as shown in Fig. 3(c). Finally, it can also be seen that the nondiagonal $\mathbf{H}_r$ relay strategy shows approximately 1.9 dB better performance at BER $= 10^{-1}$ than the diagonal one when $N = 3$.

Figure 4 presents the achievable rate versus input SNR and the number of relays N in a three-hop AF wireless relay network under the GPC, LPC, and APC at the relays. It is observed in Fig. 4(a) that the GPC relaying strategy outperforms, compared to the other two relay strategies for the same given N. In addition, it is also shown in Fig. 4(b) that the degree of freedom to make an efficient relay amplifying matrix grows as N increases. Finally, the case of the GPC is slightly better than that of the APC for the same given N, as shown in Fig. 4(b).

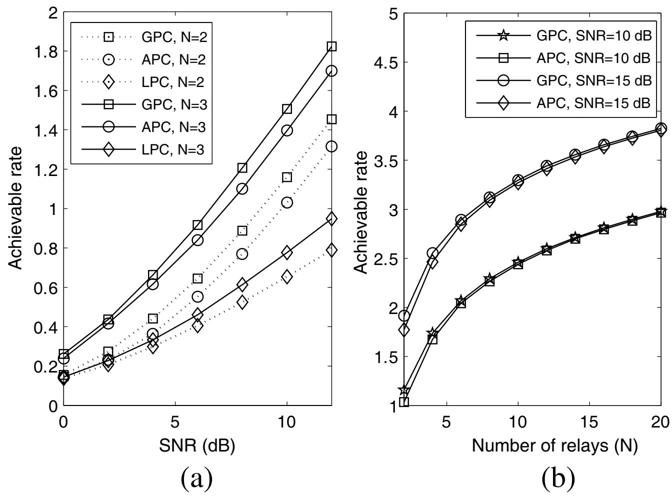

Figure 4. (a) Achievable rate versus SNR and (b) achievable rate versus number of relays in three-hop AF wireless relay network under GPC, LPC, and APC at relays.

6. CONCLUSION

This paper investigated three-hop AF wireless distributed relay strategies consisting of a one-source-one-destination pair and two-level $2N$-relay nodes. Based on the MMSE criterion, the optimal relay amplifying matrices (or vectors) for the GPC, LPC, and APC at the relays were derived. It was proven that minimizing the MSE is equivalent to maximizing the SNR in a three-hop wireless distributed relay network. It was also shown that the proposed iterative algorithm has a low computational complexity to solve the optimization problem. In particular, compared to the two-hop AF wireless distributed relay system, the three-hop AF wireless distributed relay system proposed in this paper can be realizable in cloud radio access network to improve spectral efficiency because of the multipoint processing.

It was observed that the BER performance of the proposed strategies under all power constraint cases keeps enhancing as N increases because the MMSE decreases as N increases. That is, it can be seen that increasing N results in a gain of diversity order. Additionally, when power is globally constrained at the relay during data transmission, the best BER performance was achieved. Furthermore, the achievable rate of the GPC relaying scheme was superior to those of LPC and APC relaying schemes. In summary, the better BER and larger achievable rate when the power usage is globally constrained were observed.

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