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SUPERDIRECTIVE BEAMFORMING WITH INTERFERENCES AND NOISE SUPPRESSION VIA SECOND-ORDER CONE PROGRAMMING

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Abstract—Based on second-order cone programming, we present a new superdirective beamforming method with interferences and noise suppression for small aperture HF receive arrays. In the novel method, low side lobe level (SLL) and nulls are not only used to suppress interferences and noise, but also play an important part in overcoming the low array efficiency brought by superdirective beamforming. According to the actual condition, the new method can present a good tradeoff between directive gain, array efficiency, SLL, nulls and robustness against array uncertainty. Compared with the existing methods, it is more effective in suppressing interference and noise. The superiority and validity of the proposed method can be illustrated by numerical results.

1. INTRODUCTION

Since the high-frequency (HF) electromagnetic wave can propagate well beyond the line of sight, it has always played an important role in the data-acquisition systems such as Radar, communication device to accomplish military surveillance [1], ocean remote sensing [2] and long range communication [3]. As the wavelength of the HF signal is generally of 10–100 meters, the HF systems are usually obliged to adopt huge receive arrays to achieve high gain and angular resolution, which bring about a tremendous physical and economic burden. Thus, developing small affordable antenna arrays that maintain the high gain of a large aperture array is always an intriguing task.

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In the past decades, the so-called superdirective array [4–9] has drawn numerous researchers. It is claimed that a small size array using superdirective beamforming method can perform as high directivity as a large-size array using conventional beamforming, which means that the expensive vast aperture HF receive array can be replaced by a low-cost small array without sacrificing gain. Although the performance of superdirective beamforming seems very attractive, it is not easy to implement in practice. To implement superdirective beamforming in HF receive arrays, low efficiency and sensitivity to array uncertainty are two major limitations that systems must overcome.

According to the discoveries of Newman et al. [5], and Barrick and Lilleboe [7], low efficiency is no longer a limitation, provided that the minimum efficiency of HF receive arrays guarantees the attenuated external noise greater than internal receiver noise. The combat with array uncertainty can be conducted from two aspects. One way is to develop array calibration methods with high accuracy. Aiming to array uncertainty, many array calibration methods have been proposed. To ensure the performance of superdirective beamforming, Barrick and Lilleboe exploit the radar echoes such as scatter from the sea surface to calculate the amplitude/phase errors between sensors [7]. Besides, to compensate the effects of mutual coupling and near-zone scattering, the method of moments and other methods are used in [10–14]. Although the array is hard to be calibrated the same as the ideal array model, the residual error after a series of calibration measures cannot be very large. Except array calibration, another way is to develop superdirective beamforming method with robustness to array uncertainty [5, 9].

According to the actual condition, two kinds of methods can be implemented in practice. Under the condition that the array is relatively accurately calibrated, Optimum Directive Gain (ODG) method in [7] gets the optimum gain. Shown better robustness to array uncertainty, Constrained Optimum Directive Gain (CODG) method in [9] can provide a good tradeoff among directive gain, array efficiency and robustness against array error by adjusting the sensitivity factor. In spite that both kinds of methods can bring about high gain, the existing methods still need improvement. After all, our ultimate destination using superdirective beamforming is to enhance the desired signal and suppress interferences and noise at the same time. Thus, except considering array efficiency and robustness against array uncertainty, we need to develop superdirective beamforming method with low side lobe level (SLL) and nulls in case that strong clutter, and unexpected interferences degrade the performance of superdirective beamforming. It is noteworthy that controlling SLL

and forming nulls can improve the array efficiency of superdirective beamforming, which indicates that SLL and nulls can also be used as variables to adjust array efficiency.

Among those optimization methods for controlling SLL and nulls [15–22], the second-order-cone program (SOCP) methods [18–22] have attracted considerable attentions. Compared with other optimization methods, SOCP based on the interior point method can achieve global optimum efficiently in polynomial-time. Based on SOCP, we extend the two mentioned methods and propose a new superdirective beamforming method with interferences and noise suppression. In the novel method, low SLL and nulls are not only used to suppress interferences and noise, but also play an important part in overcoming the low array efficiency brought by superdirective beamforming. According to the actual calibration condition, new method can present a good tradeoff among directive gain, array efficiency, SLL, nulls and robustness against array uncertainty. Compared with the existing methods, it is more effective in suppressing interference and noise.

The rest of this paper is organized as follows. The problem of interest is introduced, and two related algorithms are reviewed in Section 2. Section 3 presents the proposed method. To verify the validity of the proposed algorithm, numerical examples are presented in Section 4. Finally, we make conclusions in Section 5.

2. PROBLEM FORMULATION

Consider a uniform circular array composed of M short vertical dipole elements. Suppose a plane wave of λ wavelength impinges on the array in the direction (θ, φ), where θ and φ denote the azimuth and polar angles. The propagation range difference between the ith ($i = 0, 1, 2, \ldots, M-1$) element and the original point will be $\tau_i = r\sin(\theta)\cos(i\beta - \varphi)$, where $\beta = 2\pi/M$ and r is the radius. The steering vector can be defined as:

$$\mathbf{a}(\theta, \varphi) = \left[\sin\theta e^{j\frac{2\pi}{\lambda}\tau_0}, \sin\theta e^{j\frac{2\pi}{\lambda}\tau_1}, \ldots, \sin\theta e^{j\frac{2\pi}{\lambda}\tau_{M-1}}\right]^T \tag{1}$$

The array pattern in the preset direction (θ_0, φ_0) can be written as:

$$F(\theta, \varphi) = \mathbf{w}^H(\theta_0, \varphi_0)\mathbf{a}(\theta, \varphi) \tag{2}$$

where $\mathbf{w}$ denotes the complex array weight vector and H the complex conjugate transpose.

According to the results shown in [5], the signal-to-noise-ratio (SNR) of the receiving system is proportional to the directive gain as long as the system background noise dominates. The directive gain can be defined as:

$$G(\theta_0, \varphi_0) = \frac{4\pi |F(\theta_0, \varphi_0)|^2}{\displaystyle\int_0^{2\pi} \int_0^{\pi} \sin\theta |F(\theta, \varphi)|^2 d\theta d\varphi} \tag{3}$$

Using Equation (2), Equation (3) can be represent as:

$$G(\theta_0, \varphi_0) = \frac{\mathbf{w}^H N \mathbf{w}}{\mathbf{w}^H R \mathbf{w}} \tag{4}$$

where

$$N = a(\theta_0, \varphi_0) a^H(\theta_0, \varphi_0) \tag{5}$$

and

$$R = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \sin\theta \mathbf{a}(\theta, \varphi) \mathbf{a}^H(\theta, \varphi) d\theta d\varphi \tag{6}$$

As the antenna element is a short vertical dipole, the integral result of R is a fix matrix. Let R_{ij} denote the ith row and jth column entry of matrix R. To show matrix R clearly, R_{ij} is written as:

$$R_{ij} = \begin{cases} \dfrac{2}{3} & \text{if } i = j, \\ \dfrac{kd_{ij}}{\sin kd_{ij}} - \dfrac{1}{(kd_{ij})^2} \left[\dfrac{kd_{ij}}{\sin kd_{ij}} - \cos kd_{ij} \right] & \text{if } i \neq j. \end{cases} \tag{7}$$

where d_{ij} is the distance between the ith and jth elements.

Along with the high gain provided by superdirective beamforming, its inherent defect of low array efficiency inhibits its application. To implement superdirective beamforming in HF receive arrays, its minimum array efficiency must guarantee the system background noise dominate. If the attenuated external noise is less than internal receiver noise, the weight sum output of array signals will be close to zero. To illustrate the demanded array efficiency, we assume an array works at 10 MHz. At 10 MHz, external receiver noise is typically 55 dB larger than internal receiver noise [7]. Consider every dipole element connected to a high-impedance preamplifier with a noise figure of 10 dB and a 10 dB essential "cushion", the minimum array efficiency should be no less than −35 dB. The array efficiency can be calculated as:

$$\eta = \frac{\mathbf{w}^H N \mathbf{w}}{M \mathbf{w}^H \mathbf{w}} \tag{8}$$

Using the distortionless constraint $\mathbf{w}^H \mathbf{a}(\theta_0, \varphi_0) = 1$, the formulation of $G(\theta_0, \varphi_0)$ and η can be simplified as:

$$G(\theta_0, \varphi_0) = \frac{1}{\mathbf{w}^H R \mathbf{w}}, \quad \eta = \frac{1}{M \mathbf{w}^H \mathbf{w}} \tag{9}$$

To present the robustness of an array to array uncertainty, we introduce the sensitivity factor K described in detail in [9]. Generally speaking, the larger K is, the more sensitive the array is to array uncertainty. Based on the assumption $\mathbf{w}^H\mathbf{a}(\theta_0, \varphi_0) = 1$, the sensitivity factor K is:

$$K = \mathbf{w}^H\mathbf{w} \tag{10}$$

Once the system background noise dominates, the SNR of the receiving system is independent of array efficiency. Considering that the expression of K is inversely proportional to the array efficiency, we make a constraint on the sensitivity factor to reach the demanded array efficiency, which can be written as:

$$\mathbf{w}^H\mathbf{w} \leq \frac{1}{M\eta_0} \tag{11}$$

where η_0 is defined as the demanded array efficiency.

Based on the above description, the realization of superdirective beamforming in HF receive array can be formulated to a constrained optimum problem, which can be expressed as follows:

$$\begin{aligned} &\min_{\mathbf{w}} && \mathbf{w}^H R\mathbf{w} \\ &\text{subject to} && \mathbf{w}^H\mathbf{a}(\theta_0, \varphi_0) = 1, \quad \mathbf{w}^H\mathbf{w} \leq \frac{1}{M\eta_0} \end{aligned} \tag{12}$$

Although the solution of formula (12) is very simple, which can be written as:

$$\bar{\mathbf{w}} = \frac{(R + \lambda I)^{-1}\mathbf{a}(\theta_0, \varphi_0)}{\mathbf{a}^H(\theta_0, \varphi_0)(R + \lambda I)^{-1}\mathbf{a}(\theta_0, \varphi_0)} \tag{13}$$

where $\bar{\mathbf{w}}$ denotes the solution of formula (12) and $\lambda \geq 0$. In fact, it contains two different methods to implement in practice, which correspond to $\lambda = 0$ and $\lambda > 0$. For convenience, the methods corresponding to $\lambda = 0$ and $\lambda > 0$ are called as Optimum Directive Gain (ODG) method and Constrained Optimum Directive Gain (CODG) method, respectively.

For a certain array, ODG method will obtain the optimum directive gain and the largest value of K, which means the least array efficiency and the worst robustness. Whereas, as λ goes higher, the value of K will decrease, and CODG method can provide higher array efficiency and robustness to array uncertainty along with fewer directive gains. Since the array efficiency will rise along with increasing spaces between elements, the ODG method can reach the demanded array efficiency as long as the element number and element spacings are chosen properly. The detailed examples are conducted in [7]. Compared with ODG method, CODG method makes less demand on the calibration accuracy and can be used in an even smaller aperture array. Relatively speaking, CODG method can have a wider application while the optimum gain of ODG method is more attractive.

Although the high gain brought by the two mentioned methods benefits us a lot, the requirements of suppressing strong clutter and interferences urge us to develop new superdirective beamforming method with low SLL and nulls.

3. PROPOSED ALGORITHM

Assume that the controlled side lobe region is Ω_{SLL} and that the number of interference signal is P. To suppress strong clutter and interferences, we prescribe the SLL and null depth as U_1 and U_2, respectively. Accordingly, the limit values on the response of the side lobe and null are $e_1 = 10^{U_1/20}$ and $e_2 = 10^{U_2/20}$. To provide direct control over the side lobe region and nulls of interference directions, the constraints can be written as:

$$\left|\mathbf{w}^H\mathbf{a}(\theta_s,\varphi_s)\right| \le e_1, \quad (\theta_s,\varphi_s)\in\Omega_{SLL} \tag{14}$$

$$\left|\mathbf{w}^H\mathbf{a}(\theta_i,\varphi_i)\right| \le e_2, \quad i=1,2,\ldots,P \tag{15}$$

Combined with the above constraints, we can make superdirective beamforming achieve interferences and noise suppression. It is worth noting that low SLL and nulls can help to obtain a small K, which indicates that SLL and nulls can also be used as variables to adjust the array efficiency of superdirective beamforming.

Using a low SLL value U_1 and a deep null value U_2, the above constraints can make the maximum K corresponding to ODG method lower to $\frac{1}{M\eta_0}$. We call this new formula to reach the demand array efficiency as the Modified Optimum Directive Gain (MODG) method, which can be expressed as:

$$\begin{aligned}
\min_{\mathbf{w}} \quad & \mathbf{w}^H R\mathbf{w} \\
\text{subject to} \quad & \mathbf{w}^H\mathbf{a}(\theta_0,\varphi_0)=1 \\
& \mathbf{w}^H\mathbf{w}\le\frac{1}{M\eta_0} \\
& |\mathbf{w}^H\mathbf{a}(\theta_s,\varphi_s)|\le e_1,\ (\theta_s,\varphi_s)\in\Omega_{SLL} \\
& |\mathbf{w}^H\mathbf{a}(\theta_i,\varphi_i)|\le e_2,\ i=1,2,\ldots,P
\end{aligned} \tag{16}$$

Similar to ODG method, MODG method has the optimum directive gain with the prescribed SLL and nulls. In spite that low SLL and deep nulls contribute MODG method to obtain a small K, it does not mean that strong robustness to array uncertainty can be obtained in the meantime. On the contrary, it has a high demand on calibration accuracy, which is also similar to ODG method. To adapt the actual situation, the formula (16) needs further development.

As shown in [9], CODG method can provide a good tradeoff among directive gain, array efficiency and robustness against array uncertainty by adjusting the sensitivity factor K. To control SLL, nulls and robustness to uncertainty simultaneously, we add the constraints (14)–(15) to the CODG method and call this new superdirective beamforming method as the Modified Constrained Optimum Directive Gain (MCODG) method. Assume the maximum K corresponding to MODG method with the prescribed SLL and nulls as $K_{\max}$. Similar to CODG method, as the given value of K decreases from $K_{\max}$, the robustness of MCODG method can be improved at the cost of fewer directive gains. Accordingly, the constraint of MCODG method on K needs to be modified as:

$$\mathbf{w}^H\mathbf{w} \leq K_0 \tag{17}$$

where K_0 is the given sensitivity factor, which meets $K_0 \leq K_{\max}$.

In the presence of array uncertainty, directly reducing K_0 is the most effective way to increase the robustness of superdirective beamforming. Whereas the specified SLL and nulls always play a part in decreasing the value of K, so U_1 and U_2 can also be used as variables to provide a good tradeoff between SLL, nulls and robustness.

Upon the above analysis, MODG method and MCODG method can be integrated into a unified form, which can be written as.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H R\mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^H\mathbf{a}(\theta_0, \varphi_0) = 1 \\ & \mathbf{w}^H\mathbf{w} \leq K_0 \\ & |\mathbf{w}^H\mathbf{a}(\theta_s, \varphi_s)| \leq e_1,\ (\theta_s, \varphi_s) \in \Omega_{SLL} \\ & |\mathbf{w}^H\mathbf{a}(\theta_i, \varphi_i)| \leq e_2,\ i = 1, 2, \ldots, P \end{aligned} \tag{18}$$

where K_0 meets $K_0 \leq \frac{1}{M\eta_0}$.

By choosing proper values of K_0, U_1 and U_2, the above formula can provide a good tradeoff between directive gain, array efficiency, SLL, nulls and robustness against array uncertainty for superdirective beamforming. Its practical implementation can be interpreted from three aspects.

Firstly, if the array is calibrated very accurately, obtaining low SLL and nulls to suppress interferences and noise is the main task at this moment. Here, K_0 can be fixed at $\frac{1}{M\eta_0}$, and we need to prescribe U_1 and U_2 with low level to make the obtained $\mathbf{w}$ meet $\mathbf{w}^H\mathbf{w} \leq \frac{1}{M\eta_0}$.

Using low SLL and deep nulls, we can make the superdirective beamforming method obtain strong ability in suppressing noise and interferences and the demanded array efficiency in the meantime.

Secondly, assume that the array has large array uncertainty. In this case, low SLL and nulls can be easily disturbed by array uncertainty, and the robustness to array uncertainty is the key problem of superdirective beamforming. Therefore, we can make the formula equivalent to CODG method to get favorable robustness and directive gain while a small K and a high level U_1 are used in the formula.

Thirdly, assume that the array has small array uncertainty. This case is the most common circumstance after array calibration. Here, to provide a robust radiation pattern with the prescribed SLL and nulls, we need to adjust U_1, U_2 and K_0 simultaneously.

For the superdirective beamforming method, how to get the weight $\mathbf{w}$ is another key point. As second-order-cone program (SOCP) can achieve the global optimum efficiently in polynomial-time by the SeDuMi toolbox [23], we reformulate Equation (18) to an equivalent second-order-cone program (SOCP) form to calculate the weight $\mathbf{w}$. Let $R = Q^H Q$, where Q is the Cholesky factorization, then we can obtain $\mathbf{w}^H R \mathbf{w} = \|Q\mathbf{w}\|^2$. Introducing a nonnegative intermediate variable ε and a new constraint $\|Q\mathbf{w}\| \leq \varepsilon$, the above optimization problem can be converted to the following SOCP form [22]:

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & \varepsilon \\
\text{subject to} \quad & \|Q\mathbf{w}\| \leq \varepsilon \\
& \|\mathbf{w}\| \leq \sqrt{K_0} \\
& \mathbf{w}^H \mathbf{a}(\theta_0, \varphi_0) = 1 \\
& |\mathbf{w}^H \mathbf{a}(\theta_s, \varphi_s)| \leq e_1, \ (\theta_s, \varphi_s) \in \Omega_{SLL} \\
& |\mathbf{w}^H \mathbf{a}(\theta_i, \varphi_i)| \leq e_2, \ i = 1, 2, \ldots, P
\end{aligned}
\tag{19}
$$

For the novel superdirective beamforming method, the above SOCP can obtain an optimal solution of $\mathbf{w}$, provided that the parameters are properly chosen. In some cases that some parameters are not set appropriately, the problem may be infeasible. Fortunately, SOCP has an advantage that its infeasibility can be detected in an efficient way (in which case we can adjust the parameters accordingly to ensure feasibility). Although there are lots of parameters in the SOCP expression, only the parameters such as Ω_{SLL}, K_0, U_1 and U_2 (U_1 and U_2 corresponding to e_1 and e_2) need to be adjusted. As stated previously, the parameters of K_0, U_1 and U_2 play an important role in providing a good tradeoff between directive gain, array efficiency, SLL, nulls and robustness against array uncertainty. Whereas, adjusting Ω_{SLL} is mainly to obtain a feasible solution in this paper.

4. NUMERICAL RESULTS

In all the examples, we assume a small aperture uniform circular array of $M = 7$ elements with a radius of 2.5 m while the array works at 10 MHz. According to the previous research, array efficiency should be no less than -35 dB.

According to the actual situation, the new method can provide a good tradeoff among directive gain, array efficiency, SLL, nulls and robustness against array uncertainty. Compared with ODG method in [7] and CODG method in [9], it is more flexible and effective in suppressing interferences and noise. Besides, as the array is calibrated more accurately, its superiority is more distinct. To demonstrate the aforementioned superiority, the examples are arranged as follows.

Firstly, in an ideal array model, we compare the performances of new method, conventional beamforming (CBF), ODG method and CODG method in case 1. Secondly, assume that the array has large array uncertainty in case 2. It can be presented that the new method can obtain the same high robustness and gain as CODG method. Finally, in a small array uncertainty condition, to show new method has better ability in suppressing interferences and noise than CODG method, we conduct the comparison in case 3.

To simulate the impact of the array imperfection in practice, amplitude/phase errors satisfying independent and identically Gaussian distribution are treated as array uncertainty.

Case 1: In this example, CODG method with $K_0 = 451.7$ is used while $K_0 = 451.7$ corresponds to the demanded array efficiency of superdirective beamforming. Assume that two interferences arrive from the directions $[40°, 300°]$. In the new method, we specify $K_0 = 451.7$, SLL with $U_1 = -30$ dB and two nulls with a depth $U_2 = -60$ dB.

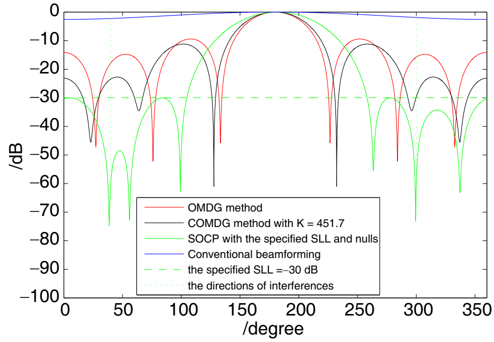


Figure 1. Radiation patterns of the four mentioned methods.

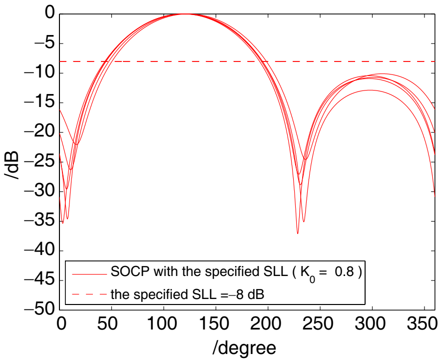
As shown in Fig. 1, by two limits values U_1 and U_2, the novel method can obtain a SLL with -30 dB and form nulls with -70 dB in the directions of interferences. Meanwhile, the other methods can only provide a lowest SLL of -11.2 dB, which clearly indicates that the new method has better ability in suppressing interferences and noise. Combined with the results shown in Table 1, the obtained K of the new method is less than the given K_0, which indicates that the new method corresponds to MODG method. Due to the small aperture array, the performance of CBF is very poor, and ODG method cannot reach the demand array efficiency. Thus, these two methods cannot be applied in this small aperture array. Meanwhile, through controlling SLL and nulls, low SLL and nulls can help MODG method to obtain a small K and overcome the low array efficiency brought by superdirective beamforming. Although low SLL and nulls ill bring down its directive gain a little, the superiority of new method in an ideal condition is still very apparent.

Table 1. Performance parameters of the four mentioned methods.

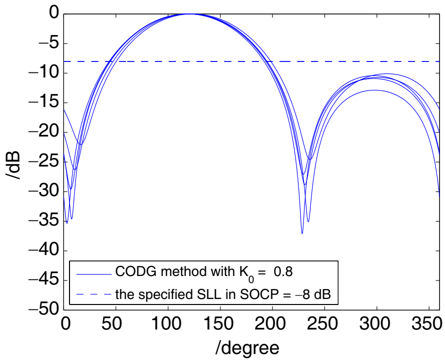
program	SLL	Directive Gain	K	Array efficiency
ODG method	-9.4 dB	11.6 dB	957.9	-38.3 dB
CODG method	-11.2 dB	11.4 dB	451.7	-35.0 dB
New method	-30 dB	10.0 dB	91.9	-28.0 dB
CBF		2.8 dB	0.14	1.00e-015 dB

Case 2: In this example, assume that the array is not accurately calibrated, and a set of large random amplitude/phase errors are given as -25 dB and 5°. To show that the methods are robust enough to the existing array uncertainty, we repeat the tests five times in Fig. 2(a) and Fig. 2(b). Accordingly, the results of SLL shown in Table 2 and Table 3 are the average value of the obtained SLL. To guarantee the robustness of the two methods, $K_0 = 0.8$ is given in CODG method while $K_0 = 0.8$ and $U_1 = -8$ dB are prescribed in new method.

Comparing Fig. 2(a) and Fig. 2(b), we can find that the two radiation patterns are nearly the same. In Fig. 2(a), when $K_0 = 0.8$, as the current response of the side lobe in new method is lower than the limit value $U_1 = -8$ dB, the constraint cannot work. Therefore, the proposed method is devolved to CODG method. Although the novel method cannot provide low SLL and nulls to suppress interferences



(a) the proposed method



(b) CODG method

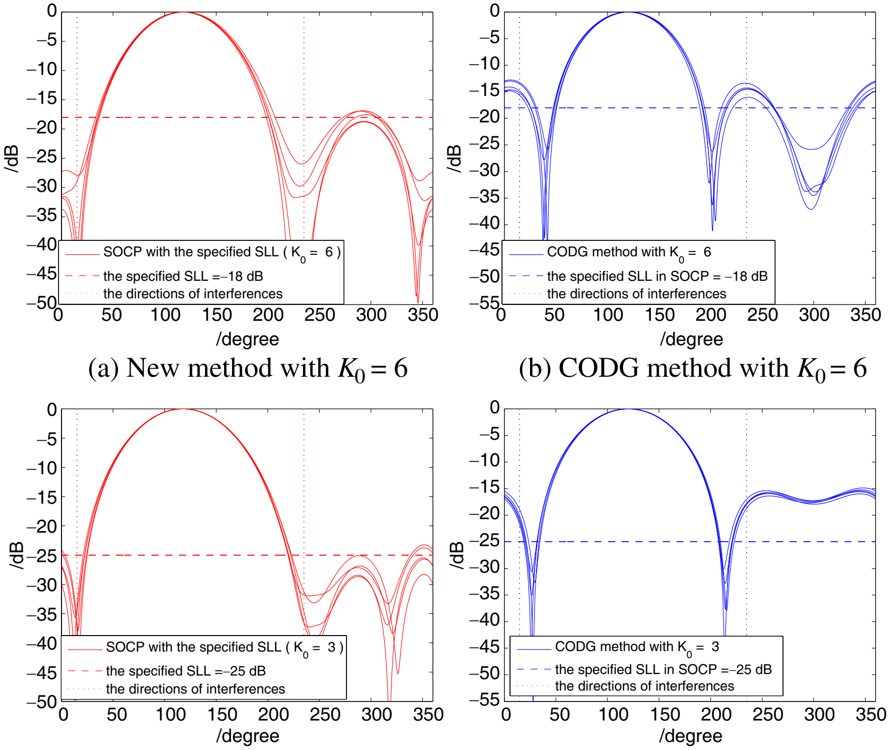
Figure 2. Radiation patterns of new method and CODG method under large array uncertainty.

Table 2. Performance parameters of new method and CODG method under large array.

program	SLL	Directive Gain	K	Array efficiency
New method	$-11.1\,\mathrm{dB}$	$7.7\,\mathrm{dB}$	0.8	$-7.5\,\mathrm{dB}$
CODG method	$-11.1\,\mathrm{dB}$	$7.7\,\mathrm{dB}$	0.8	$-7.5\,\mathrm{dB}$

and noise in the large array uncertainty circumstance, it can provide the same performance of high robustness and gain as CODG method. The relevant evidence can be found in Table 2.

Case 3: Here, assume that the array has a small residual array error and that a set of amplitude error and phase error are given as $-45\,\mathrm{dB}$ and 1°, and also assume two interferences with the directions $[15^\circ, 235^\circ]$. In this case, we present two programs using the new method to suppress interferences and noise. In Fig. 3(c), we appoint $U_1 = -18\,\mathrm{dB}$ and $U_2 = -40\,\mathrm{dB}$ to control the SLL and form nulls in the directions of interferences. To ensure its robustness, $K_0 = 6$ is used. As the nulls are more easily affected by array uncertainty, we present another program in Fig. 3(c), in which the limit parameters are prescribed as $U_1 = -25\,\mathrm{dB}$ and $K_0 = 3$. For comparison, CODG methods with $K_0 = 6$ and $K_0 = 3$ are present in Fig. 3(b) and Fig. 3(d), respectively.



(a) New method with $K_0 = 6$ (b) CODG method with $K_0 = 6$

(c) New method with $K_0 = 3$ (d) CODG method with $K_0 = 3$

Figure 3. Radiation patterns of new method and CODG method under small array uncertainty.

Due to the disturbance of random array error, the radiation patterns cannot remain the same during the duplicate tests. Observing Fig. 3, it can be found that the radiation patterns still keep good shapes, and their SLL and nulls are close to our requirement. Upon the whole, the distortions in the radiation patterns can be accepted. As seen in Fig. 3(a) and Table 3, the new method can obtain a mean SLL with -17.6 dB and two nulls with at least -25 dB depth in the directions of interferences. And in Fig. 3(c), the new method can make the SLL nearly lowered to -24.6 dB. On the contrary, although CODG methods with the same K_0 are a little more robust, the lowest SLL in Fig. 3(b) and Fig. 3(d) is -15.4 dB. It is shown clearly in Fig. 3 and Table 3 that the new method presents better ability in suppressing interferences and noise than CODG method. Moreover, comparing the two programs using new method, we can find that the novel method is

Table 3. Performance parameters of the proposed method and CODG method under small array uncertainty.

program	SLL	Directive Gain	K	Array efficiency
New method with $K_0 = 6$	-17.6 dB	8.8 dB	6.0	-16.2 dB
CODG method with $K_0 = 6$	-13.5 dB	9.4 dB	6.0	-16.2 dB
New method with $K_0 = 3$	-24.6 dB	8.6 dB	3.0	-13.2 dB
CODG method with $K_0 = 3$	-15.4 dB	8.8 dB	3.0	-13.2 dB

flexible in suppressing interferences and noise. It can not only suppress interferences in the known directions, but also provide low SLL to resist burst interferences from unknown directions.

5. CONCLUSION

In this paper, we propose a novel superdirective beamforming method with interferences and noise suppression. As it can present a good tradeoff between directive gain, array efficiency, SLL, nulls and robustness against array uncertainty, the new method can provide flexible and effective programs to suppress interferences and noise. Nevertheless, its performance of interferences and noise suppression is limited by the actual calibration condition. As the array is calibrated more accurately, the superiority of the new method will be more distinct. To achieve its best performance, the new method needs an approximate ideal antenna array. Thus, the related research on array calibration is also very important. In the following work, how to compensate the impact of array uncertainty, ground and antenna load impedance is our research focuses.

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