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EXTENSION OF THE TRANSMISSION LINE THEORY APPLICATION WITH MODIFIED ENHANCED PER-UNIT-LENGTH PARAMETERS

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Abstract—This paper introduces a modified enhanced transmission-line theory to account for higher-order modes while using a standard transmission line equation solver or equivalently a Baum, Liu and Tesche (BLT) equation solver. The complex per-unit-length parameters as defined by Nitsch et al. are first cast into an appropriate per-unit-length resistance, inductance, capacitance and conductance (RLCG) form. Besides, these per-unit-length parameters are modified to account for radiation losses with reasonable approximations. This modification is introduced by an additional per-unit-length resistance. The reason and explanations for this parameter are provided. Results obtained with this new formalism are comparable to those obtained using an electromagnetic full-wave solver, thus extending the capability of conventional transmission line solvers.

1. INTRODUCTION

Complex systems such as automobiles or other vehicles have to meet electromagnetic compatibility (EMC) requirements. Design engineers need to predict the performances of their products in the early stage of their development. EMC simulation tools should be able to help developers to analyze the EMC performances of the vehicle and its electrical and electronic architecture.

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The most vulnerable point of a vehicle is its equipment, since faults caused to the equipment modules may lead to various types of malfunctions. These faults may be the consequence of the induced currents and voltages on the wires of the harnesses. The electromagnetic interference calculations at the end of harness networks are still a challenge for many equipment modules or systems. Topology of harnesses and the important number of individual wires is one obvious reason for this.

Hopefully, given the relative proximity of wires to the surrounding large metallic or conducting structures, the transmission line theory (TLT) is applied in most cases. It thus reduces the complexity and the cost of calculations. Using the field to transmission line (TL) coupling formalism such as Agrawal's model [1], it is possible to split calculations into two decoupled problems. First, the field calculation along the routes of the harnesses in absence of the wiring is performed by the means of a 3D Maxwell's equation solver. Then, a multiconductor transmission line solver is used to calculate currents and voltages induced on the wires, using the field values calculated in the first step.

However, TLT is limited by some assumptions that are not always under control and cannot always be used. Indeed, the classical TLT equations can easily be derived from Maxwell's equations after approximating the Green's function relating the sources (incident electromagnetic field) to the reaction of the wire (induced currents and voltages) [2]. The main assumption is that the distance between the TL and its return path should be much smaller than the minimum considered wavelength of the current flowing in it. As a consequence of this simplification, the result of the Green's function integral along the TL contains no imaginary part, i.e., the radiation resistance is altogether neglected and the wave is considered totally guided along the TL.

Nevertheless, there are some situations for which this approximation may not stand. Obviously, it's difficult to establish a definite limit of the separation distance that must not be exceeded unless one imposes an arbitrary criterion. Moreover, this approximation may yield to inaccurate current estimations at the resonance frequencies, since in this case the radiation resistance may play an important role.

For these reasons, extending the ability of the TLT to handle higher order modes while keeping the simplicity of its formalism would be a significant improvement.

Numerous advanced studies have met some success in this direction. Some new models have been directly derived from Maxwell's equations without the height limitation. However, most of these

models are based on iterative time- and memory-consuming solutions, and are rather suitable either for mono conductor situations [3] or for short conductors such as the interconnects [4].

The more general and rigorous formalism is the one known as the transmission line super theory [5]. This theory presents a rigorous derivation of Maxwell's equations for non-uniform transmission lines into a set of coupled equations which resembles the telegrapher's equations. The per-unit-length (p.u.l.) parameters, which are position- and frequency-dependent, are calculated through a rather complex iterative procedure. Besides, the computation of the solution of these equations remains cumbersome and needs an entire new software development.

The enhanced transmission line p.u.l. parameters defined in [6] allow taking into account higher-order modes for uniform transmission lines using a rigorous derivation of the integral of the Green's function. However, these enhanced p.u.l. parameters cannot directly be used in a TLT solver (like the Baum, Liu and Tesche (BLT) equation-based solver [7, 8]) and therefore require an entire new software development.

The purpose of this paper is, on the one hand, to derive a proper representation of these parameters easy to embed in a classical TL solver or in its BLT form solver and, on the other hand, to show how the energy related to the radiation is dissipated. Indeed, putting the complex p.u.l. parameters defined in [6] under a resistance, inductance, capacitance, conductance (RLCG) form leads only to the decomposition of the modes in terms of TEM modes supported by the real part of the characteristic impedance, and radiating modes (or antenna mode) supported by the imaginary part of the characteristic impedance. Therefore, in this paper, we will also present a modified version of these enhanced parameters that will converge to the solution of the differential currents at the ends of the TL. This solution allows calculating these currents using a classical BLT solver with a good approximation, even at critical resonant frequencies or when the conditions of application of the classical TLT are not fulfilled. In this paper, we present and validate this method for a single wire above a perfect ground plane.

2. ENHANCED TRANSMISSION-LINE THEORY

Let us consider a lossless TL above a perfectly conducting (PEC) ground plane in presence of an external electromagnetic field as shown in Fig. 1.

From Maxwell's equations, and using the thin wire approximation,

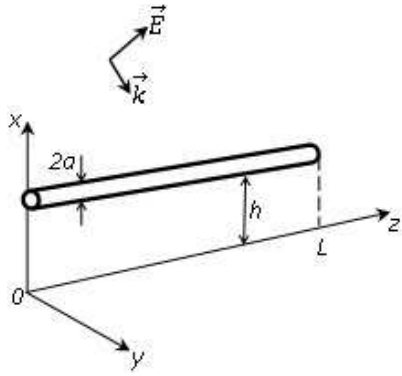

Figure 1. Geometry of the studied problem.

the field-to-transmission line coupling equations are given by [2]:

$$\frac{dV^s(z)}{dz} + j\omega\frac{\mu_0}{4\pi}\int_0^L g(z-z')I(z')d(z') \;=\; E_z^e(h,z) \tag{1}$$

$$\frac{d}{dz}\int_0^L g(z-z')I(z')dz' + j\omega 4\pi\varepsilon_0 V^s(z) \;=\; 0 \tag{2}$$

with

$$g(z-z') = \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}} - \frac{e^{-jk\sqrt{(z-z')^2+4h^2}}}{\sqrt{(z-z')^2+4h^2}} \tag{3}$$

$V^s(z)$ represents the scattered voltage as defined in the classical TLT [1, 2], $I(z')$ the current amplitude, $E_z^e(h,z)$ the incident tangential electric field, $g(z-z')$ the Green's function of this configuration, and z and z' stand respectively for the position of the observation point and the source points to the origin.

Thus, considering the observation point being far enough from the ends of the wire and the current locally constant, the integral limits can be taken from $-\infty$ to $+\infty$ and the current distribution placed out the integral [2]. Then, from [6] and using some Bessel functions properties, we can easily find:

$$\int_{-\infty}^{+\infty} g\left(z-z'\right)dz' = \pi[Y_0(2hk) - Y_0(ak)] + j\pi[J_0(2hk) - J_0(ak)] \tag{4}$$

Here J_0 and Y_0 are, respectively, the 0-order Bessel functions of first and second kind.

Now, using the expansion form of Y_0 as given in [9]:

$$Y_0(z) = \frac{2}{\pi}\left[\ln\frac{z}{2}+\gamma\right]J_0(z) - \frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^k\frac{\left(\frac{z}{2}\right)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right) \tag{5}$$

and after some rearrangements the above integral (4) can be rewritten as:

$$\begin{aligned}
&\int_{-\infty}^{+\infty} g\left(z-z'\right)dz' \\
&= 2\ln\left(\frac{2h}{a}\right)+2\left\{[\ln(hk)+\gamma][J_0(2hk)-1]-\left[\ln\left(\frac{ak}{2}\right)+\gamma\right][J_0(ak)-1]\right\} \\
&\quad -2\left\{\left[\sum_{k=1}^{\infty}(-1)^k\frac{(hk)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right)\right]-\left[\sum_{k=1}^{\infty}(-1)^k\frac{\left(\frac{ak}{2}\right)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right)\right]\right\} \\
&\quad +j\pi[J_0(2hk)-J_0(ak)]
\end{aligned} \tag{6}$$

The classical approximation for this integral consists in assuming that all terms are negligible with respect to the first one on the right-hand side of (6). In the following these terms are considered as a correction factor.

Hereafter, we express the real part of this correction factor as:

$$\begin{aligned}
\Re(C_F) = 2&\left\{[\ln(hk)+\gamma][J_0(2hk)-1]-\left[\ln\left(\frac{ak}{2}\right)+\gamma\right][J_0(ak)-1]\right\} \\
&-2\left\{\left[\sum_{k=1}^{\infty}(-1)^k\frac{(hk)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right)\right]\right. \\
&\left.-\left[\sum_{k=1}^{\infty}(-1)^k\frac{\left(\frac{ak}{2}\right)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right)\right]\right\}
\end{aligned} \tag{7}$$

and the imaginary part as:

$$\Im\left(C_F\right) = \pi\left[J_0\left(2hk\right)-J_0\left(ak\right)\right] \tag{8}$$

Using these definitions and substituting them in (1) and (2), we get:

$$\frac{dV^s(z)}{dz} + I(z)\left(j\omega L^{HF}+R^{HF}\right) = E_z^e(h,z) \tag{9}$$

$$\frac{dI(z)}{dz} + V^s(z)\left(j\omega C^{HF}+G^{HF}\right) = 0 \tag{10}$$

where the explicit forms of these p.u.l. parameters are given by:

$$L^{HF} = L_0^{'} + \frac{\mu_0}{4\pi}\Re\left(C_F\right) \tag{11}$$

$$R^{HF} = -\omega\frac{\mu_0}{4\pi}\Im\left(C_F\right) \tag{12}$$

$$C^{HF} = \frac{C_0^{'}\left[2\ln\left(\frac{2h}{a}\right)+\Re\left(C_F\right)\right]}{\frac{4\ln^2\left(\frac{2h}{a}\right)+4\Re(C_F)\ln\left(\frac{2h}{a}\right)+\Re(C_F)^2+\Im(C_F)^2}{2\ln\left(\frac{2h}{a}\right)}} \tag{13}$$

$$G^{HF} = \frac{\omega C_0^{'}\Im\left(C_F\right)}{\frac{4\ln^2\left(\frac{2h}{a}\right)+4\Re(C_F)\ln\left(\frac{2h}{a}\right)+\Re(C_F)^2+\Im(C_F)^2}{2\ln\left(\frac{2h}{a}\right)}} \tag{14}$$

and where the classical p.u.l. parameters are given by:

$$L_0^{'} = \frac{\mu_0}{2\pi}\ln\left(\frac{2h}{a}\right) \tag{15}$$

and

$$C_0^{'} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2h}{a}\right)} \tag{16}$$

Note that (9) and (10) have exactly the same form as the classical ones. Hence, to solve this system all the known methods of resolution in the classical case can be used here. In our case, we use the BLT formulation of the TL equations.

Besides, the new enhanced p.u.l. parameters are now frequency-dependent and are not limited to the quasi-static approximation but reduced to the classical ones where the classical TLT assumptions are fulfilled.

The resistance and the (negative) conductance parameters account for higher-order modes whereas the frequency-dependent inductance and capacitance can differ significantly from the classical ones, especially when the quasi-static approximation is no longer valid.

Since the p.u.l. parameters are enhanced and become frequency-dependent, we can expect that the characteristic impedance is also frequency-dependent.

From (4), (1) and (2), we find that the enhanced p.u.l. impedance is given by:

$$z = j\omega\frac{\mu_0}{4\pi}\left\{\pi\left[Y_0\left(2hk\right)-Y_0\left(ak\right)\right]+j\pi\left[J_0\left(2hk\right)-J_0\left(ak\right)\right]\right\} \tag{17}$$

The enhanced per-unit-length admittance is written as:

$$y = j\omega\frac{4\pi\varepsilon_0}{\pi\left[Y_0\left(2hk\right)-Y_0\left(ak\right)\right]+j\pi\left[J_0\left(2hk\right)-J_0\left(ak\right)\right]} \tag{18}$$

Using the classical definition of the characteristic impedance as the square root of the ratio between the impedance and the admittance:

$$Z_c = \sqrt{\frac{z}{y}} \tag{19}$$

It can easily be proven that the enhanced characteristic impedance is complex and is given by:

$$Z_c = \frac{1}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left\{\pi\left[Y_0\left(2hk\right) - Y_0\left(ak\right)\right] + j\pi\left[J_0\left(2hk\right) - J_0\left(ak\right)\right]\right\} \tag{20}$$

Thus, the characteristic impedance, even for a lossless transmission line, becomes complex and frequency-dependent (Fig. 2).

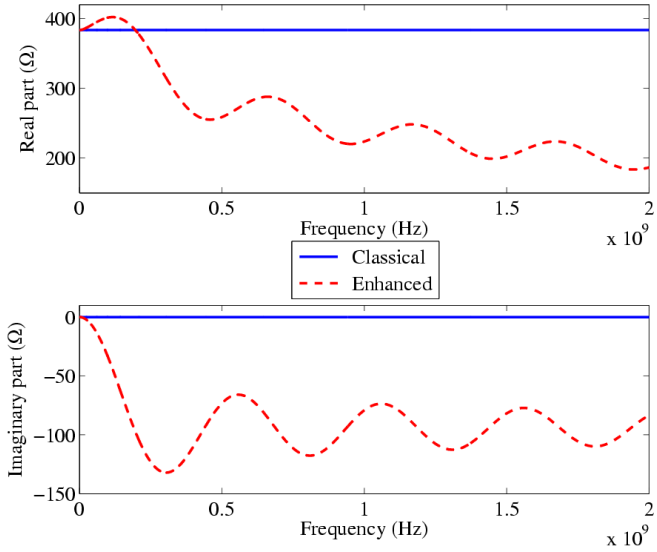

Figure 2. Comparison between the classical and enhanced characteristic impedance for a lossless TL above a PEC ground, height = 0.3 m, radius = 1 mm.

Now, we investigate the effect of the enhanced p.u.l. parameters on the propagation constant.

The propagation constant, in its general form is given by:

$$\gamma = \alpha + j\beta \tag{21}$$

where α and β are, respectively, the attenuation and phase constants.

The propagation constant is calculated through:

$$\gamma^2 = ZY = \left(R^{HF} + jL^{HF}\omega\right)\left(G^{HF} + jC^{HF}\omega\right) \tag{22}$$

Replacing ZY in (22) by (17) and (18), we get:

$$\gamma^2 = -\omega^2 \mu_0 \varepsilon_0 \tag{23}$$

Since the result of the product is a pure negative real value, the propagation constant is a pure imaginary term. This means that the use of these new p.u.l. parameters will not lead to energy dissipation. In other words, the energy is still stored in the TL. Nevertheless, these enhanced parameters allow to split the energy propagating on the TL into TEM and radiated modes as shown below.

Indeed, (19) can also be written as:

$$Z_c = \sqrt{\frac{R^{HF} + j\omega L^{HF}}{G^{HF} + j\omega C^{HF}}} \tag{24}$$

Considering that $G^{HF} \ll \omega C^{HF}$ and using (8), (12) and (20) we can easily get:

$$Z_c = \sqrt{\frac{L^{HF}}{C^{HF}}} - j\frac{R^{HF}}{k} \tag{25}$$

The energy of the TEM mode is related to the real part of the characteristic impedance whereas the energy of the radiated mode is related to the imaginary part.

3. MODIFIED-ENHANCED P.U.L. PARAMETERS

As shown above, the enhanced p.u.l. parameters lead only to the separation of the TEM mode and radiated mode on a TL. It means that the currents and voltages at the ends of the TL are not affected and using these enhanced parameters does not provide more accurate results. In order to dissipate the radiated energy, a possible method is to calculate appropriate reflection coefficients at the ends of the TL [2]. We rather introduce a more convenient and new method that consists in adding another p.u.l. resistance that will be related to the radiated energy.

This additional resistance R_+ is inserted in series with R^{HF} and L^{HF}. The following developments present the theory related to this additional resistance in the case of a single wire above a ground plane.

Since an additional resistance R_+ is introduced, the attenuation constant α is non-null.

The wave attenuation factor for the square of the current (or of the voltage) over the length L of the TL can be written as:

$$A(L) = \exp(-2\alpha L) \tag{26}$$

The power to be radiated can be associated with the ratio between the imaginary part and the real part of the characteristic impedance, which is shown, in the following, to be equivalent to the definition of the quality factor Q of the TL.

From (25) this Q factor can easily be written as:

$$Q_{line} = \frac{\beta}{R^{HF}}\sqrt{\frac{L^{HF}}{C^{HF}}} = \frac{\omega L^{HF}}{R^{HF}} \tag{27}$$

When an additional resistance R_+ is added to the p.u.l. parameters, (22) becomes:

$$\gamma^2 = \left(R^{HF} + R_+ + jL^{HF}\omega\right)\left(G^{HF} + jC^{HF}\omega\right) \tag{28}$$

Considering the property $R^{HF}C^{HF} = -L^{HF}G^{HF}$ and after rearrangement, we get:

$$\gamma^2 = \left(R^{HF} + R_+\right)G^{HF} - L^{HF}C^{HF}\omega^2 + jR_+C^{HF}\omega \tag{29}$$

In the following, we assume that $\left|\left(R^{HF} + R_+\right)G^{HF}\right| \ll L^{HF}C^{HF}\omega^2$. Nevertheless, this assumption will be verified *a posteriori*. It must be noted that this condition is also equivalent to:

$$\left(R^{HF} + R_+\right)R^{HF} \ll L^{HF^2}\omega^2 \tag{30}$$

If this assumption is fulfilled, then γ^2 can be approximated as:

$$\gamma^2 \approx -L^{HF}C^{HF}\omega^2 + jR_+C^{HF}\omega \tag{31}$$

Since $\gamma = \alpha + j\beta$, it follows that:

$$2\alpha^2 \approx L^{HF}C^{HF}\omega^2 - L^{HF}C^{HF}\omega^2\sqrt{\left(1 - \frac{R_+^2}{L^{HF^2}\omega^2}\right)} \tag{32}$$

The second term under the square root is verified to be much smaller than one.

Then, a first order series expansion can be used to extract α^2:

$$\alpha^2 = \frac{\left(R_+\right)^2 C^{HF}}{4L^{HF}} \tag{33}$$

Using the definition of the attenuation factor in (26), (27) and (33), we find that the attenuation on the TL introduced by R_+ is equal to:

$$A(0) - A(L) = 1 - \exp\left(R_+\sqrt{\frac{C^{HF}}{L^{HF}}}L\right) = \frac{R^{HF}}{\omega L^{HF}} \tag{34}$$

In order to determine R_+, this attenuation is identified with the inverse of the Q factor of the TL. Finally, we obtain:

$$R_+ = \frac{1}{L}\sqrt{\frac{L^{HF}}{C^{HF}}}\ln\left(1 - \frac{R^{HF}}{\omega L^{HF}}\right) \approx \frac{R^{HF}}{\beta L} \tag{35}$$

As expected, the p.u.l. additional resistance is equivalent to the imaginary part of the characteristic impedance (25) distributed along the TL and is frequency-dependent (Fig. 3(a)). Besides, this additional resistance does neither affect the phase constant nor the characteristic

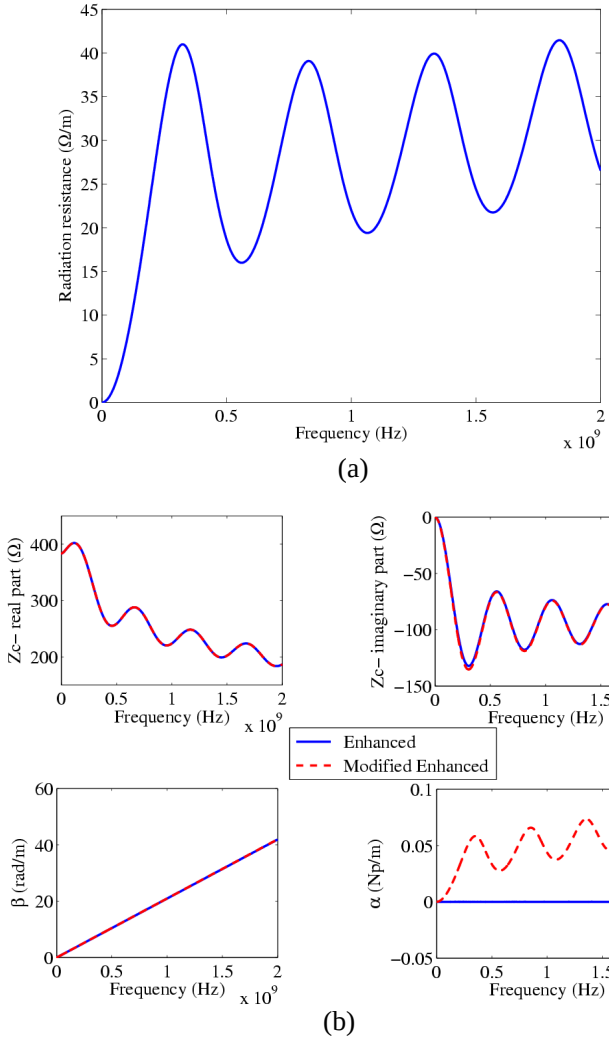

Figure 3. Characteristics of the modified enhanced p.u.l. parameters for a lossless TL above a PEC ground, height = 0.3 m, radius = 1 mm. (a) The additional correction resistance. (b) Characteristic impedance and propagation constant.

impedance but only the attenuation constant (Fig. 3(b)). This is due to the fact that it is relatively small compared to the p.u.l. resistance (35).

4. RESULTS

In order to validate this new model, some simulations have been carried out. The p.u.l. parameters are calculated either using their classical form or using their modified enhanced version presented previously. They are inserted in a BLT equation solver. Both results are compared to those obtained with an electromagnetic full-wave method of moments (MoM) solver (FEKO). The current in the load is calculated as a function of the frequency of the excitation signal. Though the p.u.l. parameters, the associated characteristic impedance and the propagation constant were calculated up to 2 GHz, in the following the currents will be calculated only up to 500 MHz to keep the clarity of the figures.

The configuration studied is presented in Fig. 4. It consists in a perfectly conducting and non-coated wire of 5 m length and 1 mm radius at 30 cm above a perfectly conducting ground plane. It is fed by a sinusoidal voltage source e of 1 V at one end and loaded with a resistance R of 1 Ω at the other end.

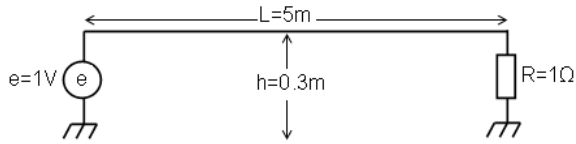

Figure 4. Setup of the studied circuit.

Figure 5 shows the current that flows in the resistance calculated from three different methods. The black curve is calculated with the classical TL equations. It highlights that the resonance levels are quite high and their amplitude does not follow a regular pattern since they depend strongly on the frequency sampling points. Since no losses are taken into account, the highest peak level is expected to be 0 dBA. The green curve shows the result of our modified enhanced version of the TL solver. The level of the resonances follow a much more regular evolution with frequency and calculations predict much lower peak levels as expected. For comparison, the calculation performed with the MoM solver is presented in red.

Below roughly 300 MHz, results obtained with our model are in good agreement with the full-wave solver. Beyond this frequency, the modified enhanced model does not provide results in agreement

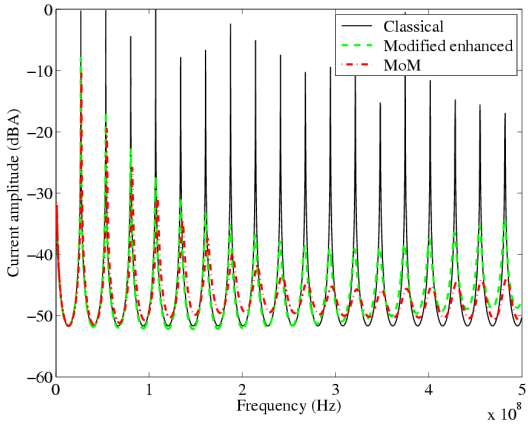

Figure 5. Current amplitude as a function of the frequency at the end of the TL, using the modified enhanced p.u.l. parameters.

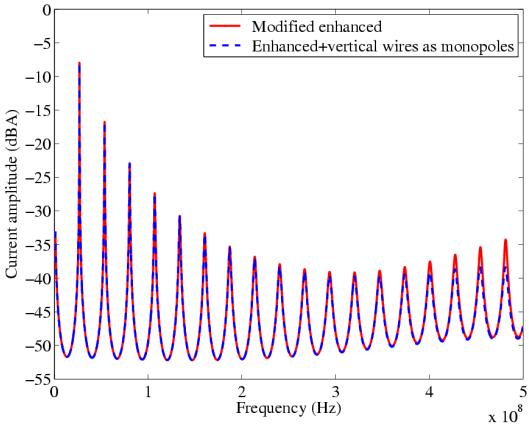

Figure 6. Current amplitude as a function of the frequency at the end of the TL, using the modified enhanced p.u.l. parameters and including the vertical wires.

with the MoM calculations. This can be explained by the effect of the vertical wires that are only modeled in the full-wave solver (for practical reasons) and not in the TL approach. The effect of these vertical wires can be approximately taken into account through the addition of a p.u.l. resistance of these wires calculated from the radiation resistance of a monopole antenna [10, 11]. As expected (Fig. 6), this produces slightly better results (5 dB beyond 400 MHz).

However, even with this correction, if we compare to the MoM results in Fig. 5, there are still some differences higher in frequency. This is due to the monopole model used for the vertical wires and possibly to the effect of the coupling between the horizontal and vertical wires. Nevertheless, in real cases, wires are not perfect conductors and as a consequence, the amplitudes of the current induced in the loads are even lower than predicted by the results in Figs. 5 and 6. Therefore, the proposed model leads to responses much more realistic than the classical TLT model, and provides, in general, a better approximation especially for the most critical first resonances.

5. CONCLUSIONS

In this paper, we have presented a modification to the enhanced per-unit-length (p.u.l.) parameters of a single transmission line (TL) which were first explicitly extracted from [6]. This modification was necessary since the enhanced parameters only enable to split the energy propagating on the TL into TEM and radiated modes. Thus, to dissipate energy, an innovative solution has been demonstrated which consists in adding a supplementary p.u.l. resistance to the enhanced p.u.l. resistance and inductance. This additional p.u.l. resistance corresponds approximately to the imaginary part of the characteristic impedance of the TL. The results obtained are comparable to those obtained with a full-wave software even at resonant frequencies. This new formalism is currently being generalized to take into account multi-conductor configurations and coated wires.

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