

INTERFERENCE INTERACTION OF COUNTER-PROPAGATING PULSES ON A MAGNETO-DIELECTRIC SLAB

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Abstract—Dynamics of interference interaction of counter-propagating electromagnetic pulses on a magneto-dielectric slab is studied in time domain. Energy redistribution in the counter-propagating pulses with arbitrary waveforms is considered. The maximal energy redistribution in the diffracted field takes place under certain conditions. The conditions are found and their physical explanation is supplied. The problem of transient electromagnetic wave diffraction on homogeneous magneto-dielectric slab is solved analytically by means of Laplace transform. The analytical solution is in agreement with numerical simulation based on finite difference time domain approach.

1. INTRODUCTION

In traditional theory of electromagnetic waves interference, the in-phase harmonic oscillations of electric and magnetic fields are considered [1]. Spatial superposition of the in-phase waves changes their amplitude and leads to redistribution of the intensity in space. In the work [2] peculiarities of interference of reactive components of harmonic electric and magnetic fields are studied on the example of three well-known problems: 1) light passing through the transparent plane-parallel plate at an angle of incidence exceeding the critical angle of total internal reflection, 2) the formation of a refracted wave in the region of total internal reflection from a semi-infinite medium and 3) radiationless transport of energy between excited and unexcited atoms. Tunnel interference of coherent counter-propagating electromagnetic waves in metal films and plates are experimentally

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investigated in [3, 4]. Interference of two counter-propagating coherent electromagnetic waves on a ferrite layer with transverse magnetization in the region of negative permeability is considered in [5, 6]. More complicate problem of interaction of counter-propagating coherent linearly polarized electromagnetic waves in a finite periodic structure of dielectric layers with a layer of a transversely magnetized ferrite is studied in [7]. In the paper [8] it is shown that the interference flux of counter-propagating waves in a layer of a nonreciprocal chiral medium is sustained and exhibits oscillations in depth of the layer.

In addition to the traditional problems of interference of coherent harmonic waves the interference of pulses is also considered. One of the earliest works is [9] where authors consider problems of the undesired overlapping of pulses from separate periodic pulse trains.

Today, the study of interaction of pulses is focused on nonlinear problems [10–12]. But linear interference of pulses is vital for understanding of more complex nonlinear pulse interaction in the same structure. Especially it is important for media with weak nonlinearity. For example, in [13] authors study the dynamics of counter-propagating pulses in finite photonic crystals. Having studied the linear interference and localization effects in the crystal, they considered second harmonic generation, where they found a maximum contrast of three orders of magnitude in nonlinear conversion efficiency.

Therefore in the present paper a linear problem of diffraction of counter-propagating pulses on a dielectric slab is solved analytically. The effects of energy redistribution in the pulses are studied to facilitate the understanding of subsequent nonlinear problems. In order to solve the problem Laplace transform method is used. This approach has been applied to study a non-stationary excitation of dielectric slabs placed on a dielectric half-space [14]. But authors considered a pulse excitation by an extrinsic electric current flowing in the plane and they were only interested in the field above the slab.

2. PROBLEM STATEMENT

Let us consider an unbounded lossless and non-dispersive magneto-dielectric medium which consist of three homogeneous parts *I*, *II* and *III*. Inside each part the medium is characterized by the constant permittivity ε_i and the permeability μ_i ($i = 1, 2, 3$), respectively. The second domain *II* is a slab which in Cartesian coordinate system is defined as $L_1 < x < L_2$. The first and the third domains are half-spaces $-\infty < x < L_1$ and $L_2 < x < +\infty$ respectively (Fig. 1). The planes $x = L_1$ and $x = L_2$ separate the domains.

In the half-spaces two electromagnetic pulses propagate in the

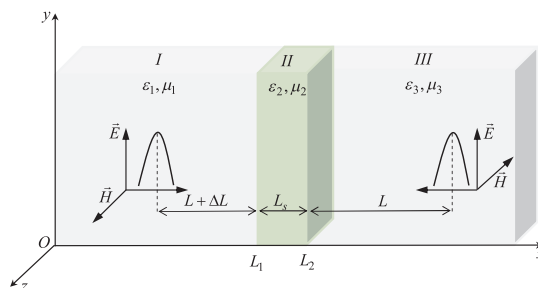


Figure 1. Structure geometry and position of the waveforms at the initial time $t = 0$.

opposite directions along x axis toward the slab. At the initial time moment $t = 0$ the waveform of the first pulse is shifted along the negative direction of x axis by $L + \Delta L$ from the plane $x = L_1$ and the second pulse waveform is shifted along the positive direction of x axis by distance L from the plane $x = L_2$. Cartesian coordinate system is chosen so that both waves have only two nonzero components of the electromagnetic field E_y and H_z (Fig. 1).

The problem consists in finding the diffracted fields of the traveling waves in all domains at the times $t > 0$.

3. PROBLEM SOLUTION

In the framework of the classical electromagnetic theory the evolution of electric $E_y(x, t)$ and magnetic $H_z(x, t)$ fields satisfies the following Maxwell equations

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon_0 \varepsilon(x)} \frac{\partial H_z}{\partial x}, \quad \frac{\partial H_z}{\partial t} = -\frac{1}{\mu_0 \mu(x)} \frac{\partial E_y}{\partial x}, \quad (1)$$

where ε_0 and μ_0 are vacuum permittivity and permeability, respectively. The functions of relative permittivity and permeability are defined as follows

$$\varepsilon(x) = \begin{cases} \varepsilon_1, & -\infty < x < L_1 \\ \varepsilon_2, & L_1 < x < L_2 \\ \varepsilon_3, & L_2 < x < +\infty \end{cases}, \quad \mu(x) = \begin{cases} \mu_1, & -\infty < x < L_1 \\ \mu_2, & L_1 < x < L_2 \\ \mu_3, & L_2 < x < +\infty \end{cases}. \quad (2)$$

Applying Young's theorem about equality of mixed partial derivatives to Equations (1)–(2) we obtain the following wave equation to describe the evolution of electric field strength $E_y(x, t) \equiv u(x, t)$

$$\frac{\partial^2 u}{\partial t^2} = \frac{c^2}{\varepsilon(x) \mu(x)} \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < +\infty, \quad t > 0, \quad (3)$$

where c is light velocity in free space. Here the function $u(x, t)$ means the electric field strength of the both pulses. Using the linear superposition principle one can see that $u(x, t) = v(x, t) + w(x, t)$, where $v(x, t)$ and $w(x, t)$ are electric fields of the pulses propagating along positive direction of x axis (from left to right) and in opposite direction (from right to left) respectively.

3.1. Diffraction of a Transient Electromagnetic Wave on a Slab

The electric field $v(x, t)$ of the pulse propagating from left to right satisfies to Equation (3) with homogenous initial conditions and some boundary conditions. In order to find the electric field $v(x, t)$ Laplace transform method is used [15]. Applying Laplace transform $v(x, t) \xrightarrow{\hat{L}} V(x, p)$ to (3) with taking into account the homogenous initial conditions and Equation (2) we obtain the following equation in Laplace domain

$$\begin{aligned} \frac{d^2 V^I}{dx^2} - \frac{\varepsilon_1 \mu_1}{c^2} p^2 V^I(x, p) &= 0, \quad -\infty < x < L_1, \\ \frac{d^2 V^{II}}{dx^2} - \frac{\varepsilon_2 \mu_2}{c^2} p^2 V^{II}(x, p) &= 0, \quad L_1 < x < L_2, \\ \frac{d^2 V^{III}}{dx^2} - \frac{\varepsilon_3 \mu_3}{c^2} p^2 V^{III}(x, p) &= 0, \quad L_2 < x < +\infty, \end{aligned} \quad (4)$$

The solutions of Equation (4) can be found in the following form

$$\begin{aligned} V^I(x, p) &= A_1(p) \exp\left(\sqrt{\varepsilon_1 \mu_1} (x - L_1) \frac{p}{c}\right) \\ &\quad + A_2(p) \exp\left(-\sqrt{\varepsilon_1 \mu_1} (x - L_1) \frac{p}{c}\right), \quad -\infty < x < L_1, \\ V^{II}(x, p) &= A_3(p) \exp\left(\sqrt{\varepsilon_2 \mu_2} (x - L_1) \frac{p}{c}\right) \\ &\quad + A_4(p) \exp\left(-\sqrt{\varepsilon_2 \mu_2} (x - L_1) \frac{p}{c}\right), \quad L_1 < x < L_2, \\ V^{III}(x, p) &= A_5(p) \exp\left(\sqrt{\varepsilon_3 \mu_3} (x - L_2) \frac{p}{c}\right) \\ &\quad + A_6(p) \exp\left(-\sqrt{\varepsilon_3 \mu_3} (x - L_2) \frac{p}{c}\right), \quad L_2 < x < +\infty. \end{aligned} \quad (5)$$

The coefficient $A_2(p)$ is equal to the given incident wave in Laplace domain. Other unknown coefficients are found from the following

boundary conditions

$$V^I(x, p)|_{x=L_1} = V^{II}(x, p)|_{x=L_1}, \quad V^{II}(x, p)|_{x=L_2} = V^{III}(x, p)|_{x=L_2},$$

$$\frac{1}{\mu_1} \frac{dV^I(x, p)}{dx} \Big|_{x=L_1} = \frac{1}{\mu_2} \frac{dV^{II}(x, p)}{dx} \Big|_{x=L_1}, \quad A_5(p) = 0,$$

$$\frac{1}{\mu_2} \frac{dV^{II}(x, p)}{dx} \Big|_{x=L_2} = \frac{1}{\mu_3} \frac{dV^{III}(x, p)}{dx} \Big|_{x=L_2}.$$

Applying inverse Laplace transform $V(x, p) \xrightarrow{\hat{L}^{-1}} v(x, t)$ to solutions in Laplace domain (5) we obtain the diffracted electric field $v(x, t)$ of the first pulse

$$\begin{aligned} v^I(x, t) &= f\left(t - \frac{\sqrt{\varepsilon_1 \mu_1}(x - L_1)}{c}\right) \\ &\quad + \sum_{n=0}^{\infty} R^n \left\{ R_2 \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2 \mu_2} 2(n+1)L_s - \sqrt{\varepsilon_1 \mu_1}(x - L_1)}{c}\right) \right. \\ &\quad \left. - R_1 \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2 \mu_2} 2nL_s - \sqrt{\varepsilon_1 \mu_1}(x - L_1)}{c}\right) \right\}, \\ v^{II}(x, t) &= \rho \sum_{n=0}^{\infty} R^n \left\{ r_- \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2 \mu_2} [(2n+1)L_s - x + L_2]}{c}\right) \right. \\ &\quad \left. + r_+ \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2 \mu_2} [(2n+1)L_s + x - L_2]}{c}\right) \right\}, \\ v^{III}(x, t) &= 2\rho \sqrt{\varepsilon_1 \mu_3} \\ &\quad \times \sum_{n=0}^{\infty} R^n \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2 \mu_2} (2n+1)L_s + \sqrt{\varepsilon_3 \mu_3}(x - L_2)}{c}\right), \end{aligned}$$

where $L_s = L_2 - L_1$ is the slab width, and $f(\cdot)$ is given initial waveform of the first pulse at the moment $t = 0$. The tilde sign above waveforms means that $\tilde{f}(\xi) = f(\xi)\eta(\xi)$, where $\eta(\xi)$ is the Heaviside step function.

$$R_1 = \frac{\sqrt{\varepsilon_2 \mu_1} - \sqrt{\varepsilon_1 \mu_2}}{\sqrt{\varepsilon_2 \mu_1} + \sqrt{\varepsilon_1 \mu_2}}, \quad R_2 = \frac{\sqrt{\varepsilon_2 \mu_3} - \sqrt{\varepsilon_3 \mu_2}}{\sqrt{\varepsilon_2 \mu_3} + \sqrt{\varepsilon_3 \mu_2}}, \quad R = R_1 R_2,$$

$$r_{\pm} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} (\sqrt{\varepsilon_2 \mu_3} \pm \sqrt{\varepsilon_3 \mu_2}), \quad \rho = \frac{2\sqrt{\varepsilon_2 \mu_2}}{(\sqrt{\varepsilon_2 \mu_1} + \sqrt{\varepsilon_1 \mu_2})(\sqrt{\varepsilon_2 \mu_3} + \sqrt{\varepsilon_3 \mu_2})}.$$

Similarly the electric field of the second pulse $w(x, t)$ is found.

3.2. Field of Counter-propagating Pulses

Having found the diffracted fields of each pulse we obtain the total electrical field of both counter-propagating pulses in the following form

$$\begin{aligned}
 u^I(x, t) = & f\left(t - \frac{\sqrt{\varepsilon_1\mu_1}(x - L_1)}{c}\right) \\
 & + \sum_{n=0}^{\infty} R^n \left\{ R_2 \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}2(n+1)L_s - \sqrt{\varepsilon_1\mu_1}(x - L_1)}{c}\right) \right. \\
 & - R_1 \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}2nL_s - \sqrt{\varepsilon_1\mu_1}(x - L_1)}{c}\right) \\
 & \left. + 2\rho\sqrt{\varepsilon_3\mu_1}\tilde{g}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}(2n+1)L_s - \sqrt{\varepsilon_1\mu_1}(x - L_1)}{c}\right) \right\}, \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 u^{II}(x, t) = & \rho \sum_{n=0}^{\infty} R^n \left\{ r_- \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}[(2n+1)L_s - x + L_2]}{c}\right) \right. \\
 & + r_+ \tilde{f}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}[(2n+1)L_s + x - L_2]}{c}\right), \\
 & + \rho_- \tilde{g}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}[(2n+1)L_s + x - L_1]}{c}\right), \\
 & \left. + \rho_+ \tilde{g}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}[(2n+1)L_s - x + L_1]}{c}\right) \right\}, \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 u^{III}(x, t) = & g\left(t + \frac{\sqrt{\varepsilon_3\mu_3}(x - L_2)}{c}\right) \\
 & + \sum_{n=0}^{\infty} R^n \left\{ R_1 \tilde{g}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}2(n+1)L_s + \sqrt{\varepsilon_3\mu_3}(x - L_2)}{c}\right) \right. \\
 & - R_2 \tilde{g}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}2nL_s + \sqrt{\varepsilon_3\mu_3}(x - L_2)}{c}\right) \\
 & \left. + 2\rho\sqrt{\varepsilon_1\mu_3}\tilde{f}\left(t - \frac{\sqrt{\varepsilon_2\mu_2}(2n+1)L_s + \sqrt{\varepsilon_3\mu_3}(x - L_2)}{c}\right) \right\}, \quad (8)
 \end{aligned}$$

where $g(\cdot)$ is given initial (at the moment $t = 0$) waveform of the pulse propagating from right to left,

$$\rho_{\pm} = \sqrt{\frac{\varepsilon_3}{\varepsilon_2}} (\sqrt{\varepsilon_2\mu_1} \pm \sqrt{\varepsilon_1\mu_2}).$$

3.3. Comparison of Solutions: Analytical vs Numerical

In order to verify the analytical solution (6)–(8) Equations (1)–(2) are solved numerically by means of finite-difference time domain (FDTD) method [16]. For sampling certain continuous function $u(x, t)$ a uniform grid $u(x, t) \rightarrow u(i \Delta x, n \Delta t) \equiv u|_i^n$ is used. Here Δx and Δt are steps in spatial coordinate and time, respectively. Applying the second order accuracy Yee algorithm [16] to Equations (1)–(2) an explicit finite-difference scheme is obtained

$$\begin{aligned} E_y|_{i-\frac{1}{2}}^{n+\frac{1}{2}} &= E_y|_{i-\frac{1}{2}}^{n-\frac{1}{2}} - \frac{\Delta t}{\Delta x \varepsilon_0 \varepsilon|_{i-\frac{1}{2}}} (H_z|_i^n - H_z|_{i-1}^n), \\ H_z|_i^{n+1} &= H_z|_i^n - \frac{\Delta t}{\Delta x \mu_0 \mu|_i} \left(E_y|_{i+\frac{1}{2}}^{n+\frac{1}{2}} - E_y|_{i-\frac{1}{2}}^{n+\frac{1}{2}} \right). \end{aligned} \quad (9)$$

As the initial incident pulses the following waveforms are used (Fig. 2): Gaussian pulse $A \exp(-\frac{(t - \tau_0)^2}{2\tau_w^2})$, one period of the sinusoid $A \sin(\frac{2\pi}{\tau_p}(t - \tau_0 + \frac{\tau_p}{2}))[\eta(t - \tau_0 + \frac{\tau_p}{2}) - \eta(t - \tau_0 - \frac{\tau_p}{2})]$, rectangular pulse $A[\eta(t - \tau_0 + \tau_p/2) - \eta(t - \tau_0 - \tau_p/2)]$ and Laguerre pulse $\frac{A}{(4\sqrt{3} - 6) \exp(\sqrt{3} - 3)} (\frac{t - \tau_0}{T})^2 (1 - \frac{t - \tau_0}{3T}) \exp(-\frac{t - \tau_0}{T})$. Here A is pulse amplitude, τ_0 is pulse shifting and τ_p is pulse duration.

Initial (at time $t = 0$, upper panel of Fig. 3) and diffracted (at time $t = 25d/c$, lower panel of Fig. 3) spatial electric field $E_y(x, t)$ distributions are calculated analytically (solid line) and numerically

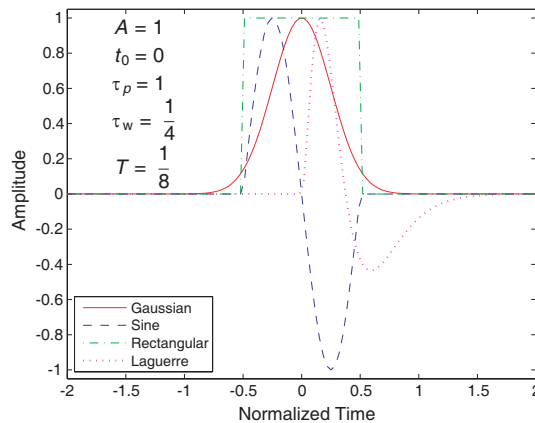


Figure 2. Waveforms.

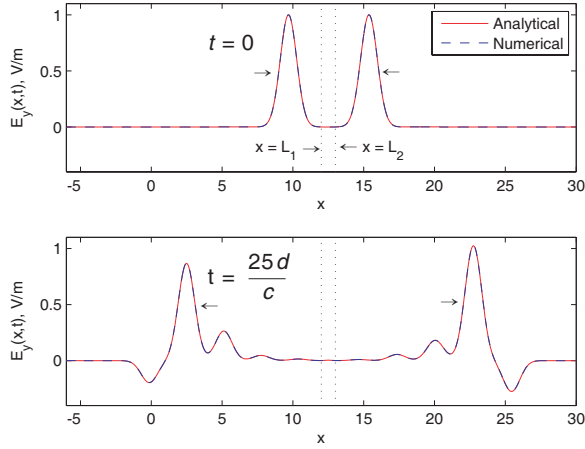


Figure 3. Analytical and numerical solutions.

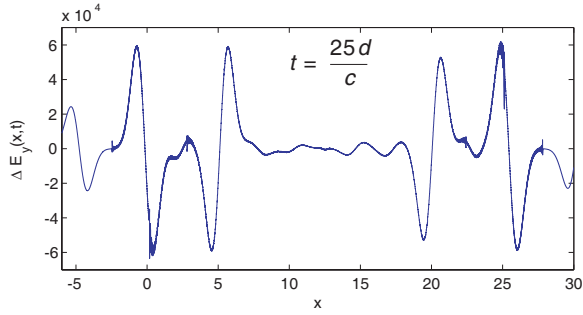


Figure 4. Absolute difference of two solutions.

(dashed line) under following structure parameters $\varepsilon_1 = 2.3$, $\mu_1 = 1.3$, $\varepsilon_2 = 9$, $\mu_2 = 2.3$, $\varepsilon_3 = 1.9$, $\mu_3 = 1.5$, $L_s = d$, where d is a dimension factor equal to some unit of length. Both Gaussian pulses have the equal heights of the peaks $A = 1$ and $\tau_w = d/c$. At the initial time $t = 0$ the positions of the peaks centers are shifted by equal distances $L = 4d$ ($\tau_0 = L/c$, $\Delta L = 0$) from the slab boundaries $x = L_1$ and $x = L_2$.

The Fig. 3 demonstrates good agreement of two solutions. The absolute difference between analytical and numerical solutions is shown in Fig. 4. One can see that maximal relative difference between two solutions is less than 0.06% (Fig. 4). Such accuracy is obtained due to high sampling rate in space $\Delta x = d/100\sqrt{\varepsilon_2\mu_2}$ and time $\Delta t = \Delta x/c$.

The interaction of two counter-propagating Laguerre pulses ($T = d/2c$) on the slab with the same parameters is presented in Fig. 5.

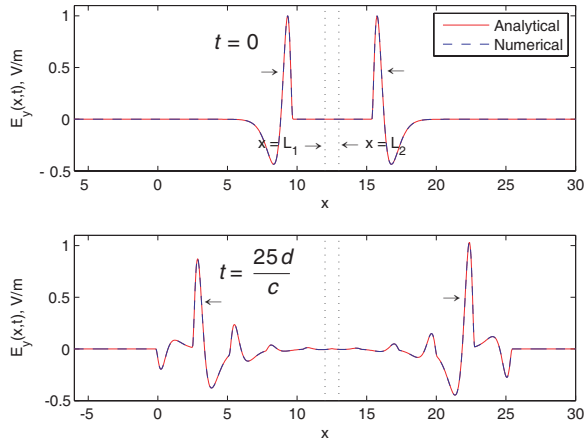


Figure 5. Analytical and numerical solutions.

The inequality of the heights of the pulses peaks after the pulses interaction on the slab (lower panel of Fig. 3 and lower panel of Fig. 5) is due to the fact that the medium in the half-spaces *I* and *III* has different impedances and pulses traveled different optical paths. Therefore the energy redistribution between pulses occurred.

4. CONDITIONS OF MAXIMAL ENERGY REDISTRIBUTION

It is interesting to find the conditions under which the maximal energy redistribution between the counter-propagating pulses takes place. These conditions are found via analysis of the analytical solution (6)–(8). Without loss of generality the practically important case of $\varepsilon_1 = \mu_1 = \varepsilon_3 = \mu_3 = 1$ is considered below. If the counter-propagating pulses have arbitrary but identical waveforms at the initial time $t = 0$ then from Equations (6) and (8) one can see that the pulse propagating from left to right imparts the maximal quantity of the energy to the pulse propagating from right to left (lower panel of Fig. 6) under following conditions:

- 1) The pulse propagating from left to right has the additional shifting $\Delta L = L_s \sqrt{\varepsilon_2 \mu_2}$ from plane $x = L_1$ at the initial time moment $t = 0$ (time matching);
- 2) The slab permittivity and permeability satisfy to the equality $\varepsilon_2 / \mu_2 = (2 + \sqrt{5})^2$ (amplitude matching);

- 3) The dielectric slab thickness L_s satisfies to the inequality $L_s \geq c\tau_p/2\sqrt{\varepsilon_2\mu_2}$, where τ_p is the pulse duration in free space (existence of the inhomogeneity critical size).

The interaction of two counter-propagating Gaussian pulses on dielectric slab with parameters $\varepsilon_2 = (2 + \sqrt{5})^2$, $\mu_2 = 1$ and $L_s = d$ is presented in Fig. 6. At the initial time moment $t = 0$ (dashed lines in Fig. 6) the pulses have identical waveforms $A = 1$, $\tau_w = 2d/3c$ but the pulses peaks have different positions. The peaks of pulses propagating from right to left are shifted from the plane $x = L_2$ along the positive

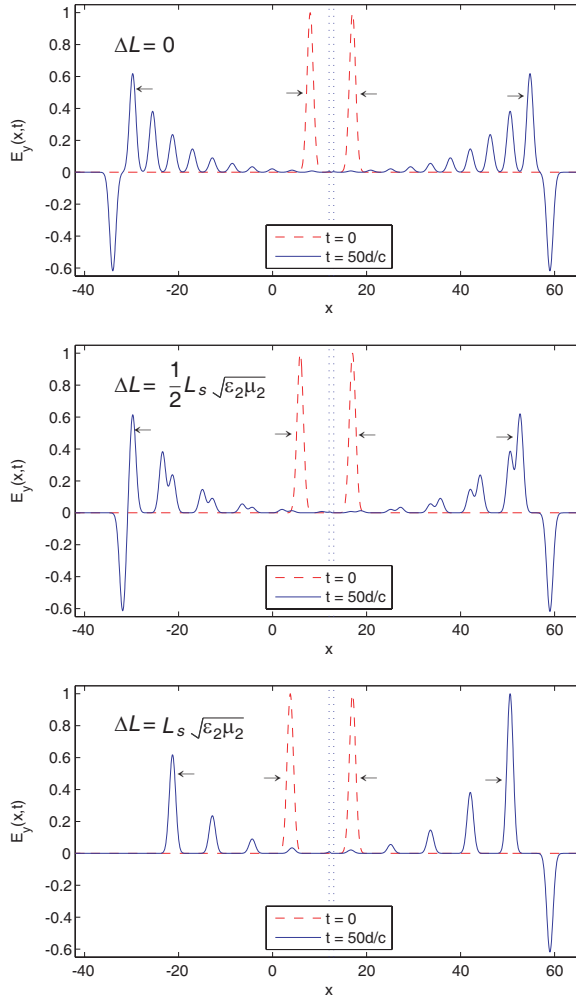


Figure 6. Interference of two identical Gaussian pulses.

direction of x axis by $L = 4d$. The peaks of pulses propagating from left to right are shifted from the plane $x = L_1$ along the negative direction of x axis by $4d + \Delta L$.

At the initial time $t = 0$, the pulses are far enough from the slab and each half-space contains half (50%) of the total energy, which is stored in the pulses. In symmetrical case when additional shifting is absent ($\Delta L = 0$) the energy balance in half-spaces does not change after interaction of the pulses (upper panel of Fig. 6). The additional positive shifting ΔL leads to transfer of energy from half-space I to half-space III at the moments of pulses interaction (middle and lower panels of Fig. 6).

Under the specified conditions 1)–3) the pulse propagating from left to right transfers the maximal quantity of energy to the half-space III (lower panel of Fig. 6). In other words, the minimal part of energy which can remain in the half-space I is

$$\frac{\lim_{t \rightarrow +\infty} \int_{-\infty}^{L_1} (u^I(x, t))^2 dx}{2 \int_{-\infty}^{+\infty} f^2(x) dx} = \frac{\sqrt{5}}{10} \approx 22.361\%. \quad (10)$$

Equation (10) is valid for any arbitrary pulse waveform and shows the maximal energy redistribution from the half-space I to the half-space III in the limit case. Such limit case is illustrated in lower panel of Fig. 6. Numerical calculations of the theoretical limit (10) for different pulse waveforms are presented in Table 1.

Table 1. Numerical calculation of the theoretical limit.

Waveform	Minimal part of energy, %
Gaussian pulse	22.354
One period of Sine	22.358
Rectangular pulse	22.384
Laguerre pulse	22.34

5. CONCLUSIONS

Energy redistribution in counter-propagating pulses appears due to unequal (asymmetrical) superposition of the reflected and transmitted fields in the half-spaces. In the half-space I , the reflected field suppresses the transmitted field (middle and lower panels of Fig. 6)

and the superposition of the reflected, and transmitted fields increases the total field energy (in comparison with symmetrical superposition) in the half-space *III*. Analysis of the analytical solution (6)–(8) leads to the fundamental limit of maximal energy redistribution in counter-propagating pulses (10) which is achieved under the three conditions 1)–3). In other words, the conditions 1)–3) determine the maximal superposition asymmetry. If both half-spaces have equal impedances then energy redistribution depends on the slab parameters and the additional initial shifting ΔL . Under the condition 2) the first reflected and transmitted pulses have opposite waveforms (upper panel of Fig. 6). Well-formed additional shifting ΔL (condition 1)) cancels the first reflected and transmitted pulses in the half-space *I* (lower panel of Fig. 6). The condition 3) separates a slab from so-called thin film. In thin films the fundamental limit (10) is not achieved.

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