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EMPIRICAL RELATIONS FOR THE EVALUATION OF RESONANT FREQUENCY AND QUALITY FACTOR OF THE TM_{010} MODE OF CIRCULAR SUBSTRATE INTEGRATED WAVEGUIDE (SIW) RESONATORS

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Abstract—In this paper, empirical relations for the evaluation of both the resonant frequency f_r and the quality factor Q for the dominant TM_{010} mode of a circular substrate integrated waveguide (SIW) cavity resonator are proposed and validated. These formulas are based on well established analytical expressions valid for the case of circular metallic cavity resonator, in which an equivalent radius R_{eq}, empirically derived, is employed. Their effectiveness is demonstrated by comparing the empirical predictions with the full wave results.

1. INTRODUCTION

The technology of substrate integrated waveguide (SIW) is of considerable interest for building passive devices operating at microwaves, millimeter waves and THz bands. As well known, SIW resonators play a key role in realizing many of those devices [1]. In particular, circular SIW resonators have been used to design filters and antennas [2–7]. The design of a SIW resonant cavity relies on the accurate determination of both the resonant frequency f_r and the quality factor Q of its dominant mode [8]. In [9, 10] the authors have proposed an effective approach to analyze SIW structures, with a reduced computational time, based on the dyadic Green's function.

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In [8, 11, 12] the method has been successfully applied to handle SIW resonators. In spite of the availability of efficient numerical method, the design of SIW cavities remains a cumbersome "trial and error" process [13], and, in this sense, the development of computational efficient models is crucial to simplifying this task [13–15]. In this paper, empirical relations able to provide a straightforward evaluation of the resonant frequency f_r and the quality factor Q of the fundamental TM_{010} mode (a $TM_{\rho z \phi}$ notation is considered) of a circular SIW resonator are proposed. The final task of the paper is to give a useful "coarse model" for design and optimization purposes [16]. The empirical relations are based on the the well-known analytical expressions for the resonant frequency and the quality factor valid of a circular metallic cavity resonator [17, 18] in which an equivalent radius R_{eq}, empirically derived, is employed.

2. COMPUTATION OF RESONANCES AND QUALITY FACTORS OF SIW RESONATORS

In Fig. 1, the geometry of a circular SIW resonator is shown. The study of this structure, using the rigorous derivation of the vector eigenfunctions of a parallel plates waveguide, has been presented in [9, 10]. In this section, we briefly recall the main ideas concerning it. In SIW structure, the field scattered by the metallic vias can be formulated as a series of outgoing cylindrical vector wave functions

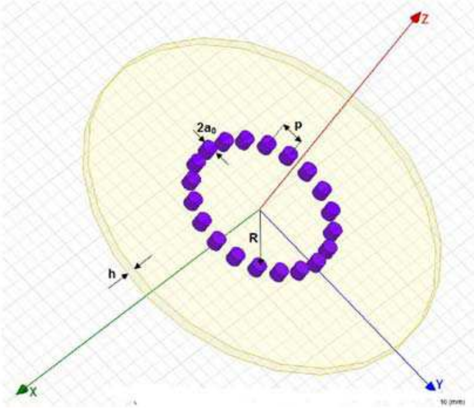

Figure 1. Pictorial representation of a circular SIW resonator. The pitch p, the via-hole diameter $2a_0$, the cavity radius R and the dielectric substrate height h are shown.

$\mathbf{M}_n$, $\mathbf{N}_n$

$$\mathbf{H}^s_{cyl}(\mathbf{r}) = \sum_l \sum_{n,m} \left[\mathbf{M}_n(k_{\rho_m}, k_{z_m}, |\boldsymbol{\rho} - \boldsymbol{\rho}_l|, z) A^{TE}_{m,n,l} + \mathbf{N}_n(k_{\rho_m}, k_{z_m}, |\boldsymbol{\rho} - \boldsymbol{\rho}_l|, z) A^{TM}_{m,n,l}\right] \quad (1)$$

Coefficients $A^{TM}_{m,n,l}$, $A^{TE}_{m,n,l}$, are computed by solving the following matrix system [9, 10]

$$\left[\mathbf{L}^{TM,TE}\right] \mathbf{A}^{TM,TE} = \mathbf{\Gamma}^{TM,TE}. \quad (2)$$

Equation (2) arises from the discretization via Method of Moments of the relevant scattering operator [8]. Resonances f_r are the real part of the complex frequencies $\bar{f}_r$ for which Equation (2) has a nontrivial solution for $\mathbf{\Gamma}^{TM,TE} = 0$. An efficient computational method to locate these values has been presented in [19]. The algorithm first finds the minima of the smallest singular value $\sigma_{\min}$ of the matrix operator $[\mathbf{L}^{TM,TE}]$ considered as a function of the real frequency in the range of frequency of interest [19]. Once these minima are located, they are used as starting points of a Muller search routine in the complex plane. The final results of this procedure are the complex resonant frequencies $\bar{f}_r$ of the structure at hand [8, 19]. Moreover, the related quality factors Q are computed as [8]

$$Q = \frac{\mathrm{Re}(\bar{f}_r)}{2\mathrm{Im}(\bar{f}_r)} \quad (3)$$

3. EMPIRICAL RELATIONS

The theory succinctly described in the previous section has been implemented using the MATLAB framework. A database of correspondences among the SIW resonator's electrical and geometrical parameters and both the resonant frequency f_r and quality factor Q of the dominant TM_{010} mode is created. Computations have been carried out for a wide range of parameters of practical engineering interest [11, 15]. Details are reported in Table 1. From these data

Table 1. Ranges of physical and electrical parameters. (all dimensions are in millimeters, σ is in Siemens/meter).

R	a_0	p	h
$2.0 \div 9.0$	$a_0 \leq \frac{4}{50}R$	$2a_0 < p < 4a_0$	$0.45 \div 0.50$
ϵ_r	σ	$\tan(\delta)$	
$2 \div 10$	$4.8 \cdot 10^7 \div 5.8 \cdot 10^7$	$1.1 \cdot 10^{-3} \div 3.5 \cdot 10^{-3}$	

we have found that the couple (f_r, Q) can be empirically evaluated by means of the well established relations valid for a metallic circular cavity resonator [17, 18], i.e.,

$$f_{re} = \frac{\chi_{10} c_0}{2\pi\sqrt{\epsilon_r} R_{eq}} \tag{4}$$

$$Q_{re} = \left[\frac{r_s\left(1 + \frac{R_{eq}}{h}\right)}{120\pi\chi_{10}} + \tan(\delta) \right]^{-1} \tag{5}$$

In the previous expressions, $\chi_{10} = 2.405$ is the first zero of the Bessel function of first kind and order zero, c_0 the velocity of light in the vacuum, ϵ_r the SIW substrate dielectric relative permittivity, $\tan(\delta)$ its dielectric loss tangent, and r_s the surface impedance for unit length of the metallic plates, [10, 15]. In place of the cavity radius R an empirical *equivalent radius* R_{eq}, defined as

$$R_{eq} = R\left[1 - \alpha_1 \left(\frac{p}{R}\right)^2\right] - \alpha_0, \tag{6}$$

is used. The values of coefficients α_0 and α_1 are given in Table 2.

Table 2. Coefficients α_0, α_1 in Equation (6).

R	α_0	α_1
R_1	0.0851	0.3208
R_2	0.2240	0.8407

Formula (6) has been obtained following a way of reasoning similar to that developed in [20] to obtain the equivalent length W_{eff} of a SIW waveguide, for characterizing its dispersion behavior. First, the resonant frequencies for SIW resonators have been obtained from full wave simulations. Later, the radii of circular resonators with metallic lateral solid walls, which exhibit the same resonant frequency of their SIW counterparts, have been determined. Using a linear regression method [16, 21], those radii have been interpolated as a function of the ratio $\frac{p^2}{R}$ obtained in the way of the above formula for R_{eq}. Notice that to obtain better estimation performances, the range of R has been divided in two sub-ranges: $R_1 = [2.0, 4.1]$ millimeters and $R_2 = [4.2, 9.0]$ millimeters [11].

4. NUMERICAL RESULTS

In order to validate the accuracy of Equations (4), (5), and (6) several circular SIW resonators have been numerically simulated. The

Table 3. Circular SIW resonators (all dimensions are in millimeters, σ is in Siemens/meter).

#	R	a_0	p	h	ϵ_r	$\tan(\delta)$	σ
1	2.10	0.147	0.544	0.45	2.0	$1.1 \cdot 10^{-3}$	$4.8 \cdot 10^7$
2	3.35	0.268	0.777	0.50	4.3	$1.1 \cdot 10^{-3}$	$4.8 \cdot 10^7$
3	4.10	0.287	1.091	0.45	3.2	$3.5 \cdot 10^{-3}$	$5.8 \cdot 10^7$
4	4.30	0.344	0.860	0.50	2.33	$1.1 \cdot 10^{-3}$	$4.8 \cdot 10^7$
5	6.85	0.548	1.644	0.50	4.5	$1.1 \cdot 10^{-3}$	$5.8 \cdot 10^7$
6	8.45	0.634	1.394	0.45	5.9	$3.5 \cdot 10^{-3}$	$5.8 \cdot 10^7$
7	9.00	0.720	2.520	0.45	7.0	$3.5 \cdot 10^{-3}$	$5.8 \cdot 10^7$

Table 4. Resonant frequency f_r: numerical results for cases listed in Table 3 (all the frequencies are in GHz).

#	f_{FW}	f_{re}	f_{rc}	δ_f^{rc}	δ_f^{re}
1	40.431	41.205	38.647	4.412	1.914
2	17.563	17.259	16.523	5.922	1.731
3	16.389	16.361	15.649	4.515	0.134
4	18.623	19.126	17.487	6.1	2.701
5	8.388	8.596	7.890	5.818	2.480
6	5.937	5.883	5.592	5.811	0.910
7	5.092	5.302	4.820	5.342	4.124

geometrical and the electrical parameters involved in simulations are reported in Table 3.

For each of the cases reported in Table 3, Table 4 and Table 5 show the comparison among the numerical results obtained by the full-wave analysis (f_{FW} , Q_{FW}) [8], our empirical model (f_{re}, Q_{re}) and the classical theory of the metallic waveguide cavity (f_{rc}, Q_{rc}) [17]. Furthermore, the absolute percentage errors δ_f^{re}, δ_f^{rc}, δ_Q^{re}, δ_Q^{rc}, defined as

$$\delta_f^q = \frac{|f_{FW} - f_q|}{f_{FW}}\%; \quad \delta_Q^q = \frac{|Q_{FW} - Q_q|}{Q_{FW}}\%, \quad q = \{re, rc\}, \tag{7}$$

Table 5. Quality factor Q: numerical results for cases listed in Table 3.

#	Q_{FW}	Q_{re}	Q_{rc}	δ_Q^{rc}	δ_Q^{re}
1	490.456	481.484	476.854	2.773	1.829
2	431.372	420.472	416.851	3.366	2.527
3	210.251	209.213	208.215	0.968	0.494
4	437.917	437.809	429.850	1.842	0.025
5	380.248	378.969	370.969	2.440	0.336
6	181.865	181.097	179.587	1.253	0.422
7	177.419	177.690	174.797	1.478	0.153

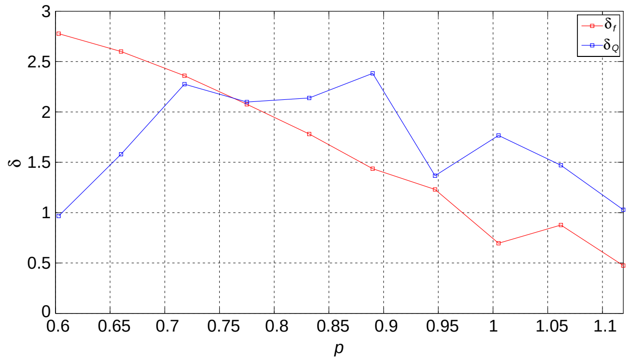

Figure 2. Behavior of δ_f^{re} (red line) and δ_Q^{re} (black line) as a function of p ($2.0a_0 < p < 4.0a_0$).

are shown as well.

It can be noticed, in all the considered cases, that our empirical formulas provide better results than those given by the classic waveguide metallic cavity theory, with a maximum percentage error that never exceeds 5%. Furthermore, to quantify the dependence of the errors δ_f^{re} and δ_Q^{re} as a function of the distance between vias p, Fig. 2 shows their behavior for the case of a SIW resonator characterized by the following parameters: $R = 3.563\,\mathrm{mm}$, $a_0 = 0.285\,\mathrm{mm}$, $h = 0.5\,\mathrm{mm}$, $\epsilon_r = 3.1$, $\tan(\delta) = 1.1 \cdot 10^{-3}$, $\sigma = 5.8 \cdot 10^7\,\mathrm{S/m}$. Also here, the maximum percentage errors, given by our empirical relations, are below 5%, demonstrating the accuracy of our relations.

5. CONCLUSIONS

In this paper, empirical relations to evaluate the resonant frequency and quality factor for the fundamental TM_{010} mode of a circular SIW resonator have been proposed and investigated. Analytical formulas valid in the case of the circular metallic cavity resonator have been adopted in which a suitable empirical equivalent radius has been used. Finally, to validate the accuracy of the empirical approach, numerical results for several circular SIW resonators were compared with those obtained by the classic theory of the metallic waveguide cavity with solid walls and by full-wave computations. The maximum absolute errors, for the considered cases, was under 5%, thus demonstrating the validity of the proposed formulas. To conclude, we point out that at the best of the authors' knowledge empirical relations for the evaluation of both resonant frequency and quality factor of circular SIW resonators have not been given so far in literature, although the case of rectangular SIW resonator has already been treated in [22, 23].

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Errata to "EMPIRICAL RELATIONS FOR THE EVALUATION OF THE RESONANT FREQUENCY AND QUALITY FACTOR OF THE TM_{010} MODE OF CIRCULAR SUBSTRATE INTEGRATED WAVEGUIDE (SIW) RESONATORS"

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Giandomenico Amendola[1], Giovanni Angiulli[2, *], Emilio Arnieri[1], Luigi Boccia[1] and Domenico De Carlo[2]

In the original paper [1], Equations (5) and (6) were erroneously typewritten. More in detail, Equation (5)

$$Q_{re} = \left[\frac{r_s \left(1 + \frac{R_{eq}}{h}\right)}{120\pi\chi_{10}} + tan(\delta) \right]^{-1}$$

has to be replaced with

$$Q_{re} = \left[\frac{2\sqrt{\epsilon_r} r_s \left(1 + \frac{R_{eq}}{h}\right)}{120\pi\chi_{10}} + tan(\delta) \right]^{-1} \tag{1}$$

while Equation (6)

$$R_{eq} = R \left[1 - \alpha_1 \left(\frac{p}{R}\right)^2 \right] - \alpha_0$$

has to be replaced with

$$R_{eq} = R \left[1 - \alpha_1 \left(\frac{p}{R}\right)^2 - \frac{\alpha_0}{R} \right] \tag{2}$$

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