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WIDEBAND RADAR ECHO SIMULATION OF MIDCOURSE PRECESIONAL TARGET WITH NON-IDEAL SCATTERING CENTERS

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Abstract—Radar echo of ballistic midcourse target contains unique motion information of the target, which can provide important evidence for target recognition. A wideband radar echo simulation model for midcourse precessional target is developed, where the micro-motion model, electromagnetic scattering calculation and linear frequency modulated (LFM) radar signal model are integrated. Firstly, the position variation of each scattering center of the moving target is analyzed. Then, the high frequency method is used to judge the masking effect of scattering centers of the rotational symmetry target. Finally, the wideband radar echo is simulated, and the impacts of high speed translational motion, non-precession movement and non-idealization of the scattering centers on the echo are also analyzed.

1. INTRODUCTION

Target recognition is the key technology to ballistic missile defense system. And the ballistic midcourse is considered to be a critical stage for identifying and intercepting ballistic missile because of the long flying time and simple movement of the target [1, 2]. Due to the requirement of attacking technology of the missile, besides its high speed translational motion along its mass center, the midcourse target also moves with micro-motion, such as spinning, coning and precession. The wideband radar echo of micro-motion target contains electromagnetic scattering property, geometry property and motion characteristic, which provides important evidence for radar target

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identification [3]. So, it is important to build the wideband radar echo model of midcourse precessional target.

Chen et al. firstly introduce the concepts of micro-motion and micro-Doppler into the field of microwave radar. They also theoretically derive the micro-Doppler induced by vibration, rotation, precession and tumbling while simulating the radar echo of cone cylinder with precession by using the high frequency method [4, 5]. As referred in [6], there are bottom of cone, conjugation site of cone and cylinder, conjugation site of cone and sphere and conjugation site of cylinder and sphere on the midcourse target. The position variation of scattering center on these sites disagrees with the micro-motion pattern of the target, and such a scattering center is called edge sliding-type scattering center. The mathematical model of micro-motion of sliding-type scattering center on ballistic target is specifically derived in [1]. Simulation and measurement results demonstrate that the micro-motion of sliding-type scattering center is non-sinusoidal, which provides a new way for the micro-motion research of ballistic target. A wideband radar echo simulation model for space objects with precession is developed in [7], where the micro-motion model, electromagnetic scattering calculation and linear frequency modulated radar signal model are integrated. An echo simulation method based on the scattering center model is proposed in [3], and a method to deal with the masking effect of scattering centers based on the physical optical principle is also proposed. By considering the non-ideal scattering phenomena of practical scattering centers as well as their shielding effects, the micro-motion feature of such a scattering center is analyzed in [8].

Now, the simulation of wideband radar echo of ballistic midcourse target with precession is mostly on the basis of ideal scattering centers, and the movement of these scattering centers is equaled to the precessional movement of the target. A wideband radar echo simulation model for midcourse precessional target is developed in this paper, where the micro-motion model, electromagnetic scattering calculation and linear frequency modulated radar signal model are integrated. The simulated radar echo not only reflects the variation of the target's scattering ability with changing of its attitude and the incident frequency, but also reflects the influence of the inconsistent between the scattering center's movement and that of the target on the imaging of ballistic midcourse precesional target with high speed translational motion.

2. MODELING OF BALLISTIC MIDCOURSE TARGET WITH PRECESSION

Theoretical calculation and testing measurements all show that the scattering centers of a cone are the crossing points generated by the plane formed by the incident beam and the target's symmetry axis and the discontinuous edge on the target [9]. Taking the rotational symmetry conical warhead as an example, the geometrical relationship between its bottom edge and the radar is shown in Fig. 1. The reference coordinate system (X, Y, Z) is parallel to the radar coordinate system and located at the cross point O of the symmetry axis and coning axis. The conical warhead spins around its symmetry axis with an angular velocity of Ω while rotating about its coning axis with an angular velocity of ω at the same time. The warhead is L meters high, and its semi-cone angle is γ. The angle between line of the radar's sight and the symmetry axis is called the attitude angle β.

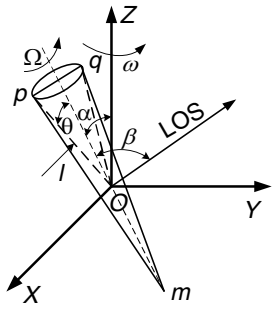

Figure 1. Micro-motion of non-ideal scattering centers.

We build a model of the ballistic midcourse with the theory of ellipse trajectory [10]. Assume that at the time instant of t, the coordinate of the warhead is $x_t,\ y_t,\ z_t$ in the radar coordinate system and the symmetry axis in the plane of XOZ at the original instant. Then the unit direction vector of symmetry axis in the reference coordinate system is $\vec{r}_1 = (\sin\alpha\cos\omega t, \sin\alpha\sin\omega t, \cos\alpha)$ at the time instant of t. Assume that the azimuth angle of the line of the radar's sight is β_0 in the reference coordinate system at the time instant of t and that the angle between it and the axis of z is α_0. Then the unit direction vector of the line of the radar's sight is $\vec{r}_2 = (\cos\beta_0\sin\alpha_0, \sin\beta_0\sin\alpha_0, \cos\alpha_0)$ in the reference coordinate system, where $\cos\alpha_0 = z_t/r,\ \sin\beta_0 = y_t/\sqrt{x_t^2 + y_t^2},\ r = \sqrt{x_t^2 + y_t^2 + z_t^2}$. So we get

$$\cos\beta = \vec{r}_1 \cdot \vec{r}_2 = \cos\alpha_0\cos\alpha + \sin\alpha\sin\alpha_0\cos(\omega t - \beta_0) \qquad (1)$$

The plane formed by the line of the radar's sight and the symmetry axis is the so-called incident wave plane. Assume that this plane is intersected by the bottom ring of the warhead at points of p and q, the distances between O and edge of the bottom, and O and m are l and h, respectively. The angle between the symmetry axis and the line linked by O and any point on the edge of the bottom is θ. Then at time instant t, the angles between Op and line of the radar's sight, and Oq and line of the radar's sight are $\beta+\theta$ and $\beta-\theta$ separately. So the radial distances of p, q and m can be represented as:

$$R_p(t) = r+l\cos(\beta+\theta) = a_1 + b_1\cos(\omega t-\beta_0) - f(t) \tag{2}$$

$$R_q(t) = r+l\cos(\beta-\theta) = a_1 + b_1\cos(\omega t-\beta_0) + f(t) \tag{3}$$

$$\begin{aligned} R_m(t) &= r+h\cos(\pi-\beta) \\ &= r-h\cos\alpha_0\cos\alpha-h\sin\alpha_0\sin\alpha\cos(\omega t-\beta_0) \end{aligned} \tag{4}$$

where

$$\begin{cases} a_1 = r + l\cos\theta\cos\alpha_0\cos\alpha \\ b_1 = l\cos\theta\sin\alpha_0\sin\alpha \\ f(t) = l\sin\theta\sqrt{1-(\cos\alpha_0\cos\alpha+\sin\alpha_0\sin\alpha\cos(\omega t-\beta_0))^2} \end{cases} \tag{5}$$

The $f(t)$ in Equation (5) is a non-sinusoidal modulated item, we deal it with the Taylor series expansion method, and it can be represented approximately as:

$$f(t) \approx a_2 + b_2\cos(\omega t-\beta_0) + c_2\cos^2(\omega t-\beta_0) \tag{6}$$

where

$$\begin{cases} a_2 = l\sin\theta\sqrt{1-\cos^2\alpha_0\cos^2\alpha} \\ b_2 = -l\sin\theta\sqrt{1-\cos^2\alpha_0\cos^2\alpha}\cdot\frac{\sin\alpha_0\cos\alpha_0\sin\alpha\cos\alpha}{1-\cos^2\alpha_0\cos^2\alpha} \\ c_2 = -\frac{1}{8}l\sin\theta\sqrt{1-\cos^2\alpha_0\cos^2\alpha}\cdot\frac{\sin^2\alpha_0\sin^2\alpha(3-\cos\alpha_0\cos\alpha)}{(1-\cos^2\alpha_0\cos^2\alpha)(1-\cos\alpha_0\cos\alpha)} \end{cases} \tag{7}$$

Then we can rewrite $R_q(t), R_p(t)$ as follows:

$$\begin{cases} R_p(t) = a_1 - a_2 + (b_1-b_2)\cos(\omega t-\beta_0) - c_2\cos^2(\omega t-\beta_0) \\ R_q(t) = a_1 + a_2 + (b_1+b_2)\cos(\omega t-\beta_0) + c_2\cos^2(\omega t-\beta_0) \end{cases} \tag{8}$$

So according to $f = -\frac{1}{2\pi}\frac{d\Phi(t)}{dt}$, the micro-Doppler of p and q can be rewritten as:

$$\begin{cases} fmdp \approx -\frac{2}{\lambda}\omega(b_1-b_2)\sin(\omega t-\beta_0) + \frac{2}{\lambda}c_2\omega\sin(2\omega t-2\beta_0) \\ fmdq \approx -\frac{2}{\lambda}\omega(b_1+b_2)\sin(\omega t-\beta_0) - \frac{2}{\lambda}c_2\omega\sin(2\omega t-2\beta_0) \end{cases} \tag{9}$$

It can be clearly seen in Fig. 1 that the micro-Doppler induced by p and q is the summation of the sinusoidal function's primary term and quadratic term, and its magnitude is modulated by much factors such as precession angle, line of radar's sight and the warhead's size, which is different from what induced under the hypothesis of precessional movement of the scattering centers.

3. RCS CALCULATION WITH HIGH FREQUENCY METHOD

In midcourse, the warhead always has a regular shape such as a cone, and its characteristic dimension is far larger than 1 relatively to the wavelength of radar of missile defense system, so it is feasible to calculate the warhead's RCS with the high frequency method. For a cone, its RCS can be calculated with equations in [9], and the geometrical relationship of its scattering centers is shown in Fig. 2.

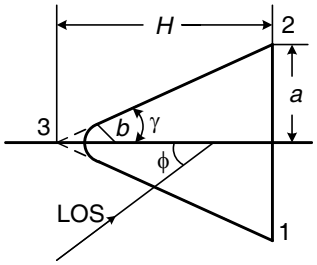

Figure 2. Scattering centers of conical warhead.

As to midcourse defense, the aspect angle cannot make the above equations singular, so we do not consider the smoothing of these equations.

4. WIDEBAND RADAR ECHO OF LFM SIGNAL

Assume that the LFM signal is transmitted by the radar and that it can be represented as

$$s_t\left(\hat{t}, t_m\right) = \text{rect}\left(\frac{\hat{t}}{T_p}\right) \exp\left\{j2\pi\left(f_c t + \frac{\mu \hat{t}^{\,2}}{2}\right)\right\} \tag{10}$$

where f_c is the central frequency, T_p is the impulse duration, B is the bandwidth, $\mu = B/T_p$ is the frequency modulation rate, $\hat{t}$ is the fast time, $t_m = mT_r$ is the slow time, m is the pulse number and T_r is the pulse recurrence interval.

$$\text{rect}(x) = \begin{cases} 1 & -\frac{1}{2} \le x \le \frac{1}{2} \\ 0 & \text{others} \end{cases} \tag{11}$$

Assume that there are N scattering centers on the warhead and that the electromagnetic scattering coefficient of the nth scattering

center is σ_n. The distance between radar and the scattering center is $R_n(t)$ and $t_n = 2R_n(t)/c$, then the received signal can be written as

$$s_r(\hat{t}, t_m) = \sum_{n=1}^{N} \sigma_n \text{rect}\left(\frac{\hat{t} - t_n}{T_p}\right) \cdot \exp\left\{j2\pi\left(f_c(t-t_n) + \frac{\mu(\hat{t}-t_n)^2}{2}\right)\right\} \quad (12)$$

Assume that the reference distance is $R_{ref} = R_0$ which is the distance between radar and the mass center of the warhead at the time instant t and $t_0 = 2R_0/c$. After being dealt with the dechirping [11] method, the echo can be rewritten as

$$\begin{aligned} s_{if}\left(\hat{t}, t_m\right) = \sum_{n=1}^{N} \sigma_n \text{rect}\left(\frac{\hat{t} - t_n}{T_p}\right) \cdot \exp\left\{-j2\pi\left(\mu(\hat{t} - t_0)\right)(t_n - t_0)\right\} \\ \cdot \exp\{-j2\pi\left(f_c\left(t_n - t_0\right)\right)\} \cdot \exp\left\{-j2\pi\left\{-\frac{\mu\left(t_n - t_0\right)^2}{2}\right\}\right\} \end{aligned} \quad (13)$$

Due to the short duration time of LFM signal, movement of the warhead can be approximately expressed with the first-order model [12].

So, $t_n = 2\frac{R(t_m+t_0)+v_m(\hat{t}-t_0)}{c} = t_{nm} + 2\frac{v_m}{c}(\hat{t} - t_0)$, where $t_m + t_0$ is the central time instant of the radar's received reference signal and v_m the warhead's velocity in the mth pulse. Let $\hat{t}\,' = \hat{t} - t_0$, then

$$t_n - t_0 = t_{nm} - t_0 + 2\frac{v_m}{c}\hat{t}\,' = 2\frac{\Delta R_m}{c} + 2\frac{v_m}{c}\hat{t}\,' \quad (14)$$

where $\Delta R_m = R(t_{nm} + t_0) - R_{ref}$ and the dechirping signal can be rewritten as

$$s_{if}(\hat{t}, t_m) = \sum_{n=1}^{N} \sigma_n \text{rect}\left(\frac{\hat{t} - t_n}{T_p}\right) \cdot \exp\left\{-j2\pi\left(a_0 + a_1\hat{t}\,' + a_2\hat{t}\,'^2\right)\right\} \quad (15)$$

where

$$\begin{cases} a_0 = 2f_c\frac{\Delta R_m}{c} - 2\mu\frac{\Delta R_m^2}{c^2} \\ a_1 = 2\frac{\mu}{c}\left(\Delta R_m + v_m\left(\frac{f_c}{\mu} - 2\frac{\Delta R_m}{c}\right)\right) \\ a_2 = 2\frac{v_m}{c}\mu\left(1 - \frac{v_m}{c}\right) \end{cases} \quad (16)$$

Let $v_m = 0$ and deal the dechirping signal with Fourier transform about $\hat{t}\,'$, and then we can get high range resolution profiles of the warhead as follows:

$$s_r(R, t_m) = \sum_{n=1}^{N} \sigma_n T_p \sin c\left[\frac{2B}{c}(R - \Delta R_m)\right] \cdot \exp\left(\frac{-j4\pi}{c} f_c \Delta R_m\right) \quad (17)$$

5. SIMULATION AND ANALYSIS

Assume that the shutdown point is located at east longitude of 10° and north latitude of 45°, the height is 100 km and the velocity 1000 m/s. Using the method in [10], we can get approximate location of the impacting point, which is located at east longitude of 9.43° and north latitude of 43.45°. Assume that the radar is located at the impacting point, height of the radar 10 m, height of the warhead 4.3 m, the distance between O and scattering center on top of the cone 2.3 m, and radii of the bottom and the spherical crown are 1.0 m and 0.1 m, respectively, then the semi-cone angle is 11.99°. Assume that the precession angle is 12°, spinning frequency 0 Hz, coning frequency 40 Hz, operating frequency of the radar 10 GHz, bandwidth 3 GHz, pulse-repetition rate 3000 Hz, and pulse duration time 5 μs. Assume time of the shutdown point as the starting instant, and we choose the time duration of 45–45.1 s randomly as the observing time. The time-range profile of echo generated by the method proposed in this paper is shown in Fig. 3.

We can see from Fig. 3 that the time-range profile slants seriously, and the micro-motion information of the target can hardly be seen. Due to the high speed translational motion, location of the warhead will change greatly in even very short time duration, which causes the slant of the time-range profile and covers the micro-motion information. Additionally, there are only two curves in Fig. 3 because of the shielding effect of scattering center q. Aspect angle of coning warhead is shown in Fig. 4. According to the analysis in part 3, we know that scattering center q is shielded when the aspect angle falls into such an interval.

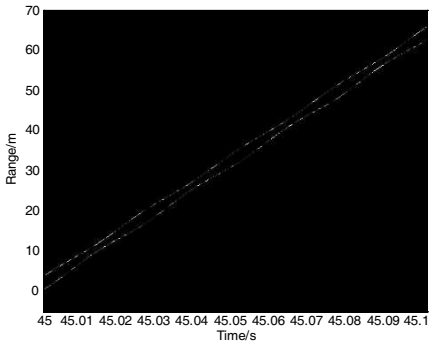

Figure 3. Time-range profiles of the original echoes.

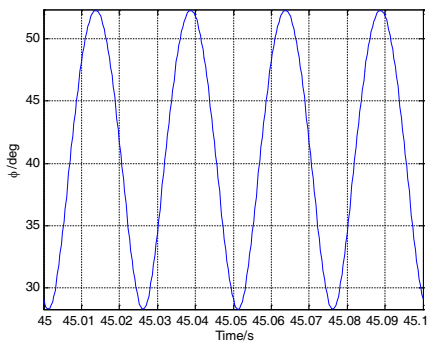

Figure 4. Aspect angle of coning warhead.

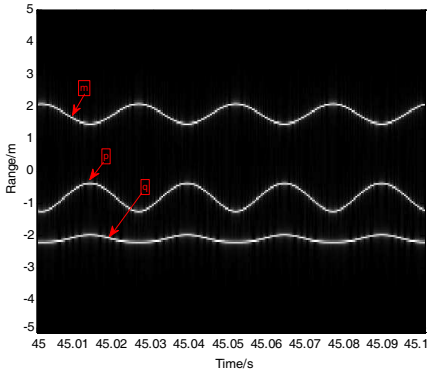

Figure 5. Time-range profiles after translational motion compensation.

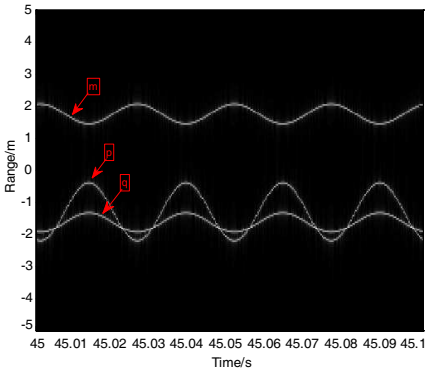

Figure 6. Time-range profiles of scattering centers with precession.

In order to analyze the impact of non-precession movement of scattering center on the echo, the time-range profile of original echo after translational motion compensation without consideration of the shielding effect is shown in Fig. 5.

From Fig. 5, we know that the time-range profile of m varies as the sinusoidal mode; time-range profile of p varies approximately as the sinusoidal mode; time-range profile of q does not vary as the sinusoidal mode. The reason is that the scattering centers p and q do not move as the target with precession, and they do other more complex movements. For comparison, assume that the scattering centers move with precession, and the time-range profile of original echo after translational motion compensation without consideration of the shielding effect is shown in Fig. 6.

Comparing Fig. 5 and Fig. 6, we know that the time-range profile of m remains the same under the two hypothesizes because of its unchanging mode of movement. But those of p and q not only change in the mode of their movements, but also change in its amplitude and position. We also know that the amplitude of time-range profile with the movement of precession is larger than that of time-range profile with the movement of non-precession. The reason is that the scattering centers do not move as the target is in precession, and their positions do not remain stable but slide on the ring edge of the warhead's bottom. Besides, they do not move as far as those scattering centers with the movement of precession.

To analyze the impact of ballistic movement on the echo's time-range profile, we choose another time duration of 101–101.1 s randomly as the observing time, and the time-range profile of echo generated by

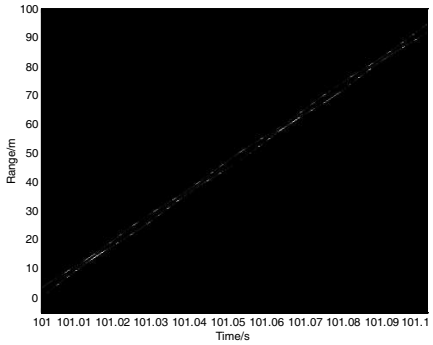

Figure 7. Time-range profiles of the original echoes.

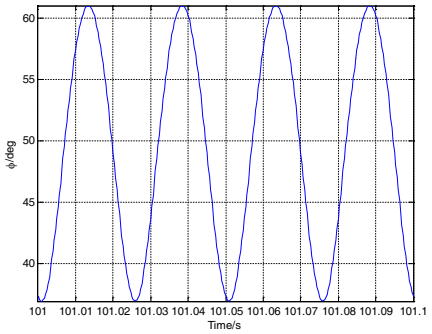

Figure 8. Aspect angle of coning warhead.

the method proposed in this paper is shown in Fig. 7.

Through the comparison of Fig. 7 and Fig. 3, we can discover that time-range profile slants more seriously in Fig. 7 than in Fig. 3. The reason is that the velocity of observed instant in Fig. 7 is larger than that in Fig. 3 and that the warhead's position changes more in Fig. 7 than in Fig. 3 within the same interval. Its aspect angle and time-range profiles under the two conditions after translational motion compensation are shown in Fig. 8, Fig. 9 and Fig. 10. Comparing Fig. 9 and Fig. 5, Fig. 10 and Fig. 6, it can be seen that the time-range profiles after translational motion compensation change a little both in its amplitude and position due to the ballistic movement. But the change of its amplitude is very small that we can not see it directly in the figure.

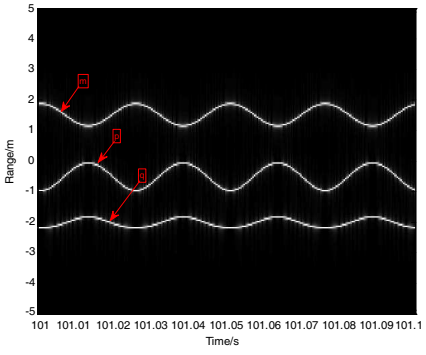

Figure 9. Time-range profiles after translational motion compensation.

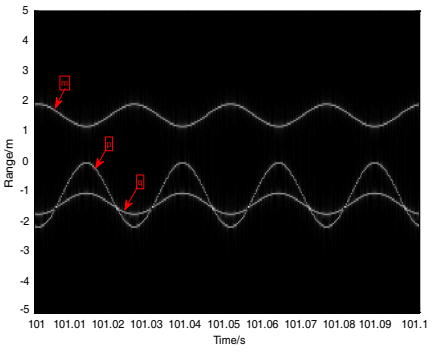

Figure 10. Time-range profiles of scattering centers with precession.

6. CONCLUSION

In practical use, the scattering centers on the warhead are non-ideal, and they do not move as the warhead with precession. To solve this problem, a wideband radar echo simulation model for midcourse precessional target is developed in this paper, where the micro-motion model, electromagnetic scattering calculation and linear frequency modulated radar signal model are integrated. Simulation results show that the time-range profile of scattering centers with non-precession movement does not vary as the sinusoidal curve, which is different from that of scattering centers with the movement of precession. Moreover, the high speed translational motion and the warhead's precession not only cause slant in the echo's time-range profile, but also bring masking effect to the scattering centers. All these differences will have an influence on the following feature extraction and target recognition. So the method proposed in this paper can simulate the wideband echo of midcourse precesional target more accurately and provide a technical foundation for feature extraction and target recognition of midcourse targets.

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