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CALCULATION ABSORPTION COEFFICIENT OF A WEAK ELECTROMAGNETIC WAVE BY CONFINED ELECTRONS IN CYLINDRICAL QUANTUM WIRES IN THE PRESENCE OF LASER RADIATION BY USING THE QUANTUM KINETIC EQUATION

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Abstract—We calculated analytic expressions for the absorption coefficient (ACF) of a weak electromagnetic wave (EMW) by confined electrons in cylindrical quantum wires (CQW) in the presence of laser radiation by using the quantum kinetic equation for electrons in the case of electron-optical phonon scattering. The ACF of a weak EMW depends on the intensity E_{01} and frequency Ω_1 of the external laser radiation ($\vec{E}_1 = \vec{E}_{01}\sin(\Omega_1 t + \varphi_1)$); the intensity E_{02} and frequency Ω_2 of the weak EMW ($\vec{E}_2 = \vec{E}_{02}\sin(\Omega_2 t)$), the temperature T of the system and the radius R of CQW. Then, the analytic results are numerically calculated and discussed for $GaAs/GaAsAl$ CQW. The numerical results show that the ACF of a weak EMW in a CQW can have negative values. So, in the presence of laser radiation, under proper conditions, the weak EMW is increased. This is different from the similar problem in bulk semiconductors and from the case of the absence of laser radiation.

1. INTRODUCTION

In quantum wires, the motion of electrons is restricted in two dimensions, thus the energy levels of electrons becomes discrete in two dimensions. The decrease in dimensionality of system for semiconductors has led to significant changes in many the physical properties of the material, including optical properties [1–5]. The

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linear absorption of a weak EMW and the nonlinear absorption of a strong EMW in low-dimensional systems have been extensively studied theoretically and experimentally in the past [6–18]. In the experiment, measuring the ACF of a strong EMW directly is very difficult. So, in the experiment one usually studies the influence of the strong EMW (laser radiation) on the electrons in the semiconductor that are being located in the weak EMW. The influence of laser radiation on the absorption of a weak EMW in normal bulk semiconductors has been investigated [19–21]. However, in this problem, the ACF of a weak EMW has only positive values. The influence of laser radiation on the absorption of a weak EMW in quantum wells has also been studied [22], but similar studies for one-dimensional systems, in particular, CQW are still unsolved. Therefore, in this paper, we use the quantum kinetic equation for electrons to calculate analytic expressions for the ACF of a weak EMW by confined electrons in CQW in the presence of laser radiation. The results are numerically calculated for the specific case of $GaAs/GaAsAl$ CQW. We show that, in the presence of laser radiation, under proper conditions, the weak EMW is increased. We can use this effect as one of the criteria for quantum wire fabrication technology.

2. THE QUANTUM KINETIC EQUATION FOR ELECTRONS AND THE ELECTRON DISTRIBUTION FUNCTION IN A CQW

We consider a CQW consisting of $GaAs$ with a circular cross section with a radius R and a length L_z embedded in $GaAlAs$. The carries (electron gas) are assumed to be confined by an infinite potential barriers in the xOy plane and free along the wire's axis (the Oz-axis). In a CQW, the state and the electron energy spectrum have the form [23]:

$$\psi_{n,\ell,p_z}(r,\phi,z) = \frac{1}{\sqrt{V_0}} e^{in\phi} e^{ip_z z} \psi_{n,\ell}(r) \tag{1}$$

where $V_0 = \pi R^2 L_z$ is the volume of the wire, $n = 0, \pm 1, \pm 2, \ldots$ is the azimuthal quantum number, $\ell = 1, 2, 3, \ldots$ is the radial quantum number, $\vec{p}_z = (0, 0, p_z)$ is wave vector of the electron along the wire's z axis, $\psi_{n,l}(r)$ is the radial wave function of electron moving in the xOy plane and takes:

$$\psi_{n,\ell}(r) = \frac{J_n(B_{n,\ell}\frac{r}{R})}{J_{n+1}(B_{n,\ell})} \tag{2}$$

and the electron energy spectrum takes the form:

$$\varepsilon_{n,\ell}(p_z) = \varepsilon(p_z) + \varepsilon_{n,\ell} \tag{3}$$

With $\varepsilon(p_z) = \frac{\hbar^2 {p_z}^2}{2m^*}$ is the electron kinetic energy in the z-direction and $\varepsilon_{n,\ell} = \frac{\hbar^2 B_{n,\ell}^2}{2m^* R^2}$ is the quantized energy in the other directions. Where m^* is the effective mass of the electron, $B_{n,\ell}$ is the ℓ-th root of the n-th order Bessel function, for example, $B_{0,1} = 2.405$ and $B_{1,1} = 3.832$.

The electron form factor takes the form [24]:

$$I_{n,\ell,n',\ell'}(q_\perp) = \int_0^1 J_{|n-n'|}(q_\perp Rx)\psi^*_{n',\ell'}(x)\psi_{n,\ell}(x)xdx, \tag{4}$$

where $x = \frac{r}{R}$.

We consider a CQW in a field of two EMWs: a laser radiation as a strong EMW with the intensity $\vec{E}_{01}$ and the frequency Ω_1, a weak EMW with the intensity $\vec{E}_{02}$ and the frequency Ω_2. The EMW is assumed to be planar and monochromatic, has a high frequency, propagates along the x direction:

$$\vec{E}(t) = \vec{E}_{01}\sin(\Omega_1 t + \varphi_1) + \vec{E}_{02}\sin(\Omega_2 t) \tag{5}$$

The vector potential of that field of two EMWs is:

$$\vec{A}(t) = \frac{c}{\Omega_1}\vec{E}_{01}\cos(\Omega_1 t + \varphi_1) + \frac{c}{\Omega_2}\vec{E}_{02}\cos(\Omega_2 t) \tag{6}$$

where c is the velocity of light, φ_1 is the phase difference between two electromagnetic waves.

The Hamiltonian of the electron-optical phonon system in the CQW in that field of two EMWs in the second quantization representation can be written as [10]:

$$\begin{aligned} H = & \sum_{n,\ell,\vec{p}_z} \varepsilon_{n,\ell}\left(\vec{p}_z - \frac{e}{\hbar c}\vec{A}_z(t)\right) a^+_{n,\ell,\vec{p}_z} a_{n,\ell,\vec{p}_z} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} \\ & + \sum_{n,\ell,n',\ell',\vec{p}_z,\vec{q}} C_{\vec{q}} I_{n,\ell,n',\ell'}(\vec{q}_\perp) a^+_{n',\ell',\vec{p}_z+\vec{q}_z} a_{n,\ell,\vec{p}_z}(b_{\vec{q}} + b^+_{-\vec{q}}) \end{aligned} \tag{7}$$

where e is the elemental charge, $\omega_{\vec{q}} \approx \omega_0$ is the frequency of an optical phonon; $|n,\ell,\vec{p}_z\rangle$ and $|n',\ell',\vec{p}_z + \vec{q}_z\rangle$ are electron states before and after scattering, respectively; $a^+_{n,\ell,\vec{p}_z}(a_{n,\ell,\vec{p}_z})$ are the creation (annihilation) operator of an electron; $b^+_{\vec{q}}(b_{\vec{q}})$ are the creation (annihilation) operator of an phonon for a state having wave vector $\vec{q} = (q_x, q_y, q_z)$; $\vec{q}_z = (0, 0, q_z)$; $C_{\vec{q}}$ is the electron-phonon interaction constant, in the case of electron-optical phonon scattering, $C_{\vec{q}}$ is [7–9]: $\left|C_{\vec{q}}\right|^2 = \frac{e^2\hbar\omega_0}{2\varepsilon_0 V_0 q^2}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)$, here ε_0 are the electronic constant, χ_0 and χ_∞ are the static and the high-frequency dielectric constants, respectively.

We will establish expression for electron distribution function $n_{n,\ell,\vec{p}_z}(t)$. By definition of electron distribution function, the quantity $n_{n,\ell,\vec{p}_z}(t)dp_z$ is the average particle number in state that has the quantum numbers n, ℓ and wave vector in the range from p_z to $p_z + dp_z$, in a unit volume, at time t. In order to obtain expression for the electron distribution function in quantum wires, we use the quantum kinetic equation for statistical average value of the electron particle number operator (or electron distribution function) $n_{n,\ell,\vec{p}_z}(t) = \langle a^+_{n,\ell,\vec{p}_z} a_{n,\ell,\vec{p}_z} \rangle_t$ [19]:

$$i\hbar \frac{\partial n_{n,\ell,\vec{p}_z}(t)}{\partial t} = \left\langle \left[a^+_{n,\ell,\vec{p}_z} a_{n,\ell,\vec{p}_z}, H \right] \right\rangle_t \tag{8}$$

Because $n_{n,\ell,\vec{p}_z}(t)$ is the distribution function for the unbalanced process, it is time-dependent function. The dependence of the time of $n_{n,\ell,\vec{p}_z}(t)$ is clearly indicated in Equation (8): the vector potential $\vec{A}(t)$ is dependent on time so Hamiltonian is also dependent on time; therefore, distribution function is also dependent on time. However, then the distribution function for the unbalanced process is expressed approximately through the balanced distribution function. So, $n_{n,\ell,\vec{p}_z}(t)$ can be understood as unbalanced distribution function deviates from balanced state.

Using the Hamiltonian in Equation (7) and the commutative relations of the creation and the annihilation operators, we obtain the quantum kinetic equation for electrons in the CQW:

$$\begin{aligned}
\frac{\partial n_{n,\ell,\vec{p}_z}(t)}{\partial t} &= -\frac{1}{\hbar^2} \sum_{n',\ell',\vec{q}} \left|C_{\vec{q}}\right|^2 \left|I_{n,\ell,n',\ell'}(\vec{q}_\perp)\right|^2 \\
&\sum_{u,s,m,f=-\infty}^{+\infty} J_u(a_{1z}q_z) J_s(a_{1z}q_z) J_m(a_{2z}q_z) J_f(a_{2z}q_z) \\
&\times \exp\{i \{[(s-u)\Omega_1 + (m-f)\Omega_2 - i\delta]\, t + (s-u)\varphi_1\}\} \\
&\times \int_{-\infty}^{t} dt_2 \left\{ \left[n_{n,\ell,\vec{p}_z}(t_2) N_{\vec{q}} - n_{n',\ell',\vec{p}_z+\vec{q}_z}(t_2)(N_{\vec{q}}+1) \right] \right. \\
&\times \exp\left\{ \frac{i}{\hbar} \left[\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z) - \varepsilon_{n,\ell}(\vec{p}_z) - \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta \right] (t-t_2) \right\} \\
&+ \left[n_{n,\ell,\vec{p}_z}(t_2)(N_{\vec{q}}+1) - n_{n',\ell',\vec{p}_z+\vec{q}_z}(t_2) N_{\vec{q}} \right] \\
&\times \exp\left\{ \frac{i}{\hbar} \left[\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z) - \varepsilon_{n,\ell}(\vec{p}_z) + \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta \right] (t-t_2) \right\} \\
&- \left[n_{n',\ell',\vec{p}_z-\vec{q}_z}(t_2) N_{\vec{q}} - n_{n,\ell,\vec{p}_z}(t_2)(N_{\vec{q}}+1) \right]
\end{aligned}$$

$$\times \exp\left\{\frac{i}{\hbar}\left[\varepsilon_{n,\ell}(\vec{p}_z) - \varepsilon_{n',\ell'}(\vec{p}_z - \vec{q}_z) - \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta\right](t - t_2)\right\}$$
$$- \left[n_{n',\ell',\vec{p}_z - \vec{q}_z}(t_2)(N_{\vec{q}} + 1) - n_{n,\ell,\vec{p}_z}(t_2)N_{\vec{q}}\right]$$
$$\times \exp\left\{\frac{i}{\hbar}\left[\varepsilon_{n,\ell}(\vec{p}_z) - \varepsilon_{n',\ell'}(\vec{p}_z - \vec{q}_z) + \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta\right](t - t_2)\right\}\Bigg\}\Bigg\} \quad (9)$$

In Equation (9), the first two terms in the integral include the contributions of the state $|n, \ell, \vec{p}_z\rangle$ before scattering, states $|n', \ell', \vec{p}_z + \vec{q}_z\rangle$ after scattering and phonon energy, energy of two EMWs; the later two terms include the contributions of the state $|n, \ell, \vec{p}_z\rangle$ before scattering, the states $|n', \ell', \vec{p}_z - \vec{q}_z\rangle$ after scattering and phonon energy, energy of two EMWs. Similar equations can be found in bulk semiconductors and quantum wells [10, 20, 22].

The first-order tautology approximation method is used to solve this equation [19–21]. The initial approximation of $n_{n,\ell,\vec{p}_z}(t)$ is chosen as:

$$n^0_{n,\ell,\vec{p}_z}(t_2) = \bar{n}_{n,\ell,\vec{p}_z},\ n^0_{n,\ell,\vec{p}_z+\vec{q}_z}(t_2) = \bar{n}_{n,\ell,\vec{p}_z+\vec{q}_z},\ n^0_{n,\ell,\vec{p}_z-\vec{q}_z}(t_2) = \bar{n}_{n,\ell,\vec{p}_z-\vec{q}_z}$$

The first-order tautology approximation method is implemented as follows: instead of the initial approximation of $n_{n,\ell,\vec{p}_z}(t)$ to the right hand side of the Equation (9), then integrating the Equation (9), we will receive first-order approximation of $n_{n,\ell,\vec{p}_z}(t)$ and we stop here. The expression for the unbalanced electron distribution function $n_{n,\ell,\vec{p}_z}(t)$ is received as follows:

$$n_{n,\ell,\vec{p}_z}(t) = \bar{n}_{n,\ell,\vec{p}_z} - \frac{1}{\hbar}\sum_{n',\ell',\vec{q}} \left|C_{\vec{q}}\right|^2 \left|I_{n,\ell,n',\ell'}(\vec{q}_\perp)\right|^2$$
$$\sum_{k,s,r,m=-\infty}^{+\infty} J_s(a_{1z}q_z)J_{k+s}(a_{1z}q_z)J_m(a_{2z}q_z)J_{r+m}(a_{2z}q_z)$$
$$\times \frac{\exp\left\{-i\left\{\left[k\Omega_1 + r\Omega_2 + i\delta\right]t + k\varphi_1\right\}\right\}}{k\Omega_1 + r\Omega_2 + i\delta}$$
$$\times \left\{\frac{\bar{n}_{n',\ell',\vec{p}_z-\vec{q}_z}N_{\vec{q}} - \bar{n}_{n,\ell,\vec{p}_z}(N_{\vec{q}}+1)}{\varepsilon_{n,\ell}(\vec{p}_z) - \varepsilon_{n',\ell'}(\vec{p}_z - \vec{q}_z) - \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta}\right.$$
$$+ \frac{\bar{n}_{n',\ell',\vec{p}_z-\vec{q}_z}(N_{\vec{q}}+1) - \bar{n}_{n,\ell,\vec{p}_z}N_{\vec{q}}}{\varepsilon_{n,\ell}(\vec{p}_z) - \varepsilon_{n',\ell'}(\vec{p}_z - \vec{q}_z) + \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta}$$
$$- \frac{\bar{n}_{n,\ell,\vec{p}_z}N_{\vec{q}} - \bar{n}_{n',\ell',\vec{p}_z+\vec{q}_z}(N_{\vec{q}}+1)}{\varepsilon_{n',\ell'}(\vec{p}_z + \vec{q}_z) - \varepsilon_{n,\ell}(\vec{p}_z) - \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta}$$
$$\left. - \frac{\bar{n}_{n,\ell,\vec{p}_z}(N_{\vec{q}}+1) - \bar{n}_{n',\ell',\vec{p}_z+\vec{q}_z}N_{\vec{q}}}{\varepsilon_{n',\ell'}(\vec{p}_z + \vec{q}_z) - \varepsilon_{n,\ell}(\vec{p}_z) + \hbar\omega_{\vec{q}} - s\hbar\Omega_1 - m\hbar\Omega_2 + i\hbar\delta}\right\} \quad (10)$$

where a_{1z} and a_{2z} are the z-components of $\vec{a}_1 = \frac{e\vec{E}_{01}}{m^*\Omega_1^2}$ and $\vec{a}_2 = \frac{e\vec{E}_{02}}{m^*\Omega_2^2}$, respectively. $N_{\vec{q}}$ is the balanced distribution function of phonons, $\bar{n}_{n,\ell,\vec{p}_z}$ is the balanced distribution function of electrons, the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the EMW, $J_k(x)$ is the Bessel function.

Because Equation (10) is obtained from the integration of Equation (9), as well as Equation (9), the first two terms in brackets include the contributions of the state $|n, \ell, \vec{p}_z\rangle$ before scattering, states $|n', \ell', \vec{p}_z - \vec{q}_z\rangle$ after scattering and phonon energy, energy of two EMWs; the later two terms include the contributions of the state $|n, \ell, \vec{p}_z\rangle$ before scattering, the states $|n', \ell', \vec{p}_z + \vec{q}_z\rangle$ after scattering and phonon energy, energy of two EMWs. Similar equations can be found in quantum wells [10, 22].

In expression (10), the quantum numbers n, ℓ are the characteristic indexes for quantum wires. So, this is one of the differences for the similar problem in the normal bulk semiconductors that authors V. L. Malevich, E. M. Epshtein published [20].

3. GENERAL ANALYTIC EXPRESSION OF ACF OF A WEAK EMW IN THE PRESENCE OF LASER RADIATION FIELD IN A CQW

Because the motion of electrons is confined in the xOy plane, we only consider the current density vector of electrons along the z direction in the CQW. It has the form [19]:

$$\vec{j}_z(t) = \frac{e\hbar}{m^*} \sum_{n,\ell,\vec{p}_z} \left(\vec{p}_z - \frac{e}{\hbar c}\vec{A}_z(t) \right) n_{n,\ell,\vec{p}_z}(t) \tag{11}$$

The ACF of a weak EMW by confined electrons in the presence of laser radiation in the CQW takes the form [19]:

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty}E_{02}^2} \left\langle \vec{j}_z(t)\vec{E}_{02}\sin\Omega_2 t \right\rangle_t \tag{12}$$

Substituting $n_{n,\ell,\vec{p}_z}(t)$ into the expression of $\vec{j}_z(t)$, then substituting $\vec{j}_z(t)$ into the expression (12), the ACF of a weak EMW in the presence of laser radiation in CQW is obtained as:

$$\alpha = \frac{e^4 n_0 \omega_0}{2\pi\varepsilon_0 c\sqrt{2\pi\chi_\infty m^* k_B T} m^*\Omega_2^3 Z} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \cos^2\alpha_2 \sum_{n,\ell,n',\ell'} G_{n,\ell,n',\ell'}$$
$$\times \left(A_1 - \frac{1}{2}A_2 + \frac{3}{32}A_3 + \frac{1}{4}B_1 - \frac{1}{16}B_2 + \frac{1}{64}B_3 \right) \tag{13}$$

With:

$$A_1 = F_{0,1} - F_{0,-1}; \quad A_2 = L_{0,1} - L_{0,-1}; \quad A_3 = M_{0,1} - M_{0,-1}$$

In formulas for A_1, A_2, A_3, we obtain the contribution of the absorption processes and emission processes of a photon of weak EMW.

$$B_1 = L_{-1,1} - L_{-1,-1} + L_{1,1} - L_{1,-1}; \quad B_2 = M_{-1,1} - M_{-1,-1} + M_{1,1} - M_{1,-1}$$

In this formulas, we obtain the contribution of the absorption processes and emission processes of a photon of weak EMW and strong EMW.

$$B_3 = M_{-2,1} - M_{-2,-1} + M_{2,1} - M_{2,-1}$$

Quantity B_3 includes the contributions of the absorption processes and emission processes of a photon of weak EMW, two photons of strong EMW.

In the above formulas:

$$\begin{aligned}
F_{s,m} &= e^{-\frac{C_{s,m}}{2k_BT}} K_0 \left(\frac{|C_{s,m}|}{2k_BT}\right) e^{-\frac{\hbar^2 B_{n,\ell}^2}{2m^* R^2 k_B T}} \\
&\quad \left(1 + N_{\omega_0} - e^{\frac{\hbar\omega_0 - s\hbar\Omega_1 - m\hbar\Omega_2}{k_BT}} N_{\omega_0}\right) \\
L_{s,m} &= a_1^2 \cos^2\alpha_1 e^{-\frac{C_{s,m}}{2k_BT}} \left(\frac{4m^{*2} C_{s,m}^2}{\hbar^4}\right)^{1/2} K_1 \left(\frac{|C_{s,m}|}{2k_BT}\right) e^{-\frac{\hbar^2 B_{n,\ell}^2}{2m^* R^2 k_B T}} \\
&\quad \left(1 + N_{\omega_0} - e^{\frac{\hbar\omega_0 - s\hbar\Omega_1 - m\hbar\Omega_2}{k_BT}} N_{\omega_0}\right) \\
M_{s,m} &= a_1^4 \cos^4\alpha_1 e^{-\frac{C_{s,m}}{2k_BT}} \left(\frac{4m^{*2} C_{s,m}^2}{\hbar^4}\right) K_2 \left(\frac{|C_{s,m}|}{2k_BT}\right) e^{-\frac{\hbar^2 B_{n,\ell}^2}{2m^* R^2 k_B T}} \\
&\quad \left(1 + N_{\omega_0} - e^{\frac{\hbar\omega_0 - s\hbar\Omega_1 - m\hbar\Omega_2}{k_BT}} N_{\omega_0}\right)
\end{aligned}$$

We obtain contribution of the Bose-Einstein distribution function for optical phonons N_{ω_0} in formulas for $F_{s,m}$, $L_{s,m}$, $M_{s,m}$

$$a_1 = \frac{eE_{01}}{m^*\Omega_1^2}; \quad N_{\omega_0} = \frac{1}{e^{\frac{\hbar\omega_0}{k_BT}} - 1};$$

$$G_{n,\ell,n',\ell'} = \int_{-\infty}^{+\infty} dq_x \int_{-\infty}^{+\infty} dq_y \left|I_{n,\ell,n',\ell'}(\vec{q}_\perp)\right|^2; \quad Z = \sum_{n,\ell} e^{-\frac{\hbar^2 B_{n,\ell}^2}{2m^* R^2 k_B T}};$$

Quantity Z appears while we standardize balance distribution function of the electron.

$$C_{s,m} = \frac{\hbar^2 B^2_{n',\ell'}}{2m^* R^2} - \frac{\hbar^2 B^2_{n,\ell}}{2m^* R^2} + \hbar\omega_0 - s\hbar\Omega_1 - m\hbar\Omega_2,$$
$$\text{with } s = -2, -1, 0, 1, 2;\ m = -1, 1.$$

Quantity $C_{s,m}$ includes contributions of the quantized energy in the restricted directions before and after scattering, phonon energy, photon energy of two EMWs.

In the above formulas, α_1 is the angle between the vector $\vec{E}_{01}$ and the positive direction of the Oz axis, α_2 is the angle between the vector $\vec{E}_{02}$ and the positive direction of the Oz axis, k_B is the Boltzmann constant, n_0 is the electron density in CQW.

Equation (13) is the expression of ACF of a weak EMW in the presence of external laser radiation in a CQW. As one can see, the ACF of a weak EMW is independent of E_{02}; only dependent on $E_{01}, \Omega_1, \Omega_2, T, R$. This expression is different from that in the normal bulk semiconductors and quantum wells [20, 22]. We already know that in the quantum wells, state of the electron is characterized by a quantum number n and wave vector $\vec{p}_\perp$ in the plane xOy; while in quantum wires, state of the electron is characterized by two quantum numbers n, ℓ and wave vector $\vec{p}_z$. So, the equations of quantum wires will be more complex than the equations of quantum wells, for instance, Equation (13) has four sums according to the four indexes, while similar equation of quantum wells has only two sums according to the two indexes.

From expression (13), when we set $E_{01} = 0$, we will receive expression of ACF of a weak EMW in the absence of laser radiation in CQW.

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the ACF is numerically calculated for the specific case of $GaAs/GaAsAl$ CQW. The parameters used in the calculations are as follows [8, 25]: $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m^* = 0.066m_0$, m_0 being the mass of free electron, $n_0 = 10^{23}\,\text{m}^{-3}$, $\hbar\omega_0 = 36.25\,\text{meV}$, $\alpha_1 = \frac{\pi}{3}$, $\alpha_2 = \frac{\pi}{6}$.

Figure 1 describes the dependence of α on the temperature T for five different values of E_{01}, with $\Omega_1 = 3 \times 10^{13}\,\text{Hz}$, $\Omega_2 = 10^{13}\,\text{Hz}$, $R = 30\,\text{nm}$. Figure 1 shows that when the temperature T of the system rises up from 20 K to 400 K, the curves have a maximum and a minimum.

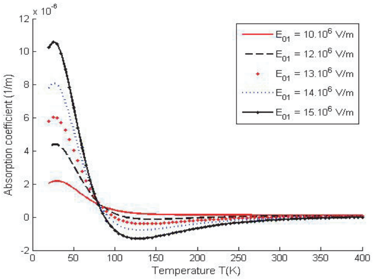

Figure 1. The dependence of α on T.

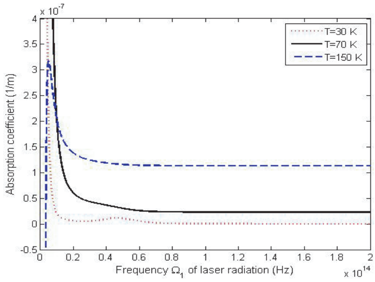

Figure 2. The dependence of α on Ω_1.

Figure 2 describes the dependence of α on the frequency Ω_1 of the laser radiation for three different values of T, with $\Omega_2 = 10^{13}\,\text{Hz}$, $R = 30\,\text{nm}$, $E_{01} = 14 \times 10^5\,\text{V/m}$. It can be seen from this figure that the curves can have a maximum or no maximum in the investigative interval.

Figure 3 describes the dependence of α on the frequency Ω_2 of the weak EMW for three different values of T, with $\Omega_1 = 3 \times 10^{13}\,\text{Hz}$, $R = 30\,\text{nm}$, $E_{01} = 15 \times 10^6\,\text{V/m}$. From Figure 3 we see that the curves have a maximum (peak) at $\Omega_2 = \omega_0$ and smaller maximum at $\Omega_2 \neq \omega_0$. The frequencies Ω_2 of the weak EMW at which ACF has maxima (peaks) are not changed as the temperature T is varied. We know that optical phonons is oscillations of the crystal lattice, so when frequency Ω_2 of the weak EMW is equal frequency ω_0 of the optical phonon, resonance peak will appears, i.e., the absorption of weak EMW of crystal lattice is the best. Here, we also see the appearance of a lower different resonance peak. This is different from the bulk semiconductors; in the bulk semiconductor, we only see the appearance of a resonance peak. This is due to the transitions of electrons between the mini bands in low-dimensional systems.

Figure 4 shows ACF as a function of intensity E_{01} of the laser radiation for three different values of T, with $\Omega_1 = 6 \times 10^{13}\,\text{Hz}$, $\Omega_2 = 3 \times 10^{13}\,\text{Hz}$, $R = 30\,\text{nm}$. From the figure we see that the curves have a maximum in the investigative interval. We see that the maximum value of ACF moves toward right side as the temperature decreases.

Figure 5 shows ACF as a function of intensity E_{01} of the laser radiation for three different values of R, with $\Omega_1 = 6 \times 10^{13}\,\text{Hz}$, $\Omega_2 = 3 \times 10^{13}\,\text{Hz}$, $T = 80\,\text{K}$. From the figure we see that the curves have a maximum in the investigative interval. From this figure, we see that the maximum value of ACF moves toward right side as the wire's radius increases.

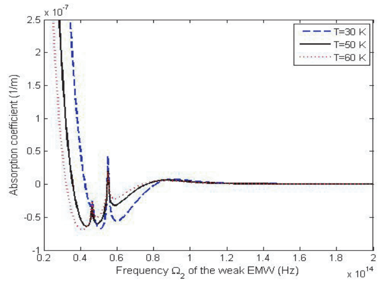

Figure 3. The dependence of α on Ω_2.

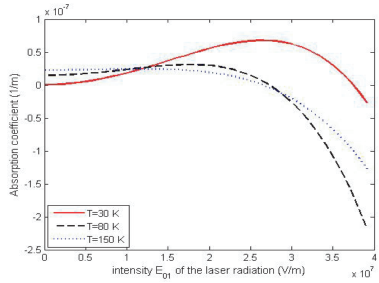

Figure 4. The dependence of α on E_{01} (for three different values of T.

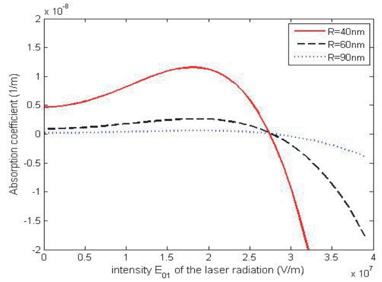

Figure 5. The dependence of α on E_{01} (for three different values of R).

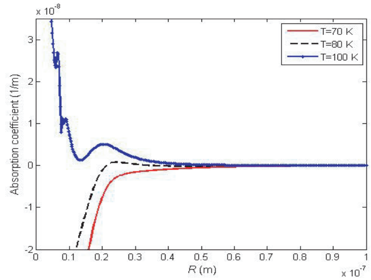

Figure 6. The dependence of α on R.

Figure 6 describes the dependence of α on R for three different values of T, with $\Omega_1 = 3 \times 10^{13}\,\mathrm{Hz}$, $\Omega_2 = 7 \times 10^{13}\,\mathrm{Hz}$, $E_{01} = 15 \times 10^6\,\mathrm{V/m}$. From this figure, we see that the curves can have a maximum or many maxima (peaks) in the investigative interval.

From these figures we see that the dependence of the ACF of a weak EMW on E_{01}, Ω_1, Ω_2, T, R is complex, and it can appear maximum values. These figures also show that under influence of laser radiation, ACF of a weak EMW in a CQW can have negative values, it means that ACF becomes increased coefficient. So, in the presence of strong EMW, under proper conditions, the weak EMW is increased. This is different from the similar problem in bulk semiconductors and from the case of the absence of laser radiation. We can use this effect as one of the criteria for quantum wire fabrication technology: if the quantum wire is fabricated successfully, this effect will appear; if this effect does not appear, it means that the fabrication has failed. This effect has also appeared in similar problem in quantum wells that has been published [22]. However, quantum well is a semiconductor structure that is completely different from quantum wires. So, expression of ACF of a weak EMW in quantum wires also differs from one in quantum wells and effect of weak EMW increase in quantum wires also expresses differently from one in quantum wells.

5. CONCLUSIONS

In this paper, we obtained an analytical expression of the ACF of a weak EMW in the presence of laser radiation in CQW for the case electron-optical phonon scattering. The expression shows that the ACF of a weak EMW is independent of E_{02} and is only dependent on E_{01}, Ω_1, Ω_2, T, R. From this expression, the ACF of a weak EMW in the absence of laser radiation in CQW can be obtained by setting $E_{01} = 0$. The ACF is numerically calculated for the specific case of $GaAs/GaAsAl$ CQW. Computational results show that the dependence of the ACF on various physical factors of the system is complex. These results also show that under the influence of laser radiation, the ACF of a weak EMW in a CQW can have negative values, i.e., ACF becomes increased coefficient. So, in the presence of strong EMW, under proper conditions, the weak EMW is increased. We can use this effect as one of the criteria for quantum wire fabrication technology.

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