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PROPAGATION OF A LORENTZ-GAUSS VORTEX BEAM IN A TURBULENT ATMOSPHERE

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Abstract—The propagation properties of a Lorentz-Gauss vortex beam in a turbulent atmosphere are investigated. Based on the extended Huygens-Fresnel integral, the Hermite-Gaussian expansion of a Lorentz function, etc., analytical expressions of the average intensity, effective beam size, and kurtosis parameter of a Lorentz-Gauss vortex beam are derived in the turbulent atmosphere. The spreading properties of a Lorentz-Gauss vortex beam in the turbulent atmosphere are numerically calculated and analyzed. The influences of the beam parameters on the propagation of a Lorentz-Gauss vortex beam in the turbulent atmosphere are examined in details. Upon propagation in the turbulent atmosphere, the vale in the normalized intensity distribution of a Lorentz-Gauss vortex beam gradually rises. The rising speed of the vale is opposite to the spreading of the beam spot. When the propagation distance reaches to a certain value, the Lorentz-Gauss vortex beam in the turbulent atmosphere becomes a flattened beam spot. When the propagation distance is large enough, the beam spot of the Lorentz-Gauss vortex beam tends to be a Gaussian-like distribution. This research is beneficial to optical communications and remote sensing that are involved in the single mode diode laser devices.

1. INTRODUCTION

Due to the highly angular spreading, Lorentz-Gauss beams provide a better beam model than Gaussian beams to describe the radiation emitted by a single mode diode laser [1, 2]. A special case of Lorentz-Gauss beams is the Lorentz beam. The relation between Lorentz beams and relativistic Hermite polynomials has been investigated [3].

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The focusing properties of the linearly polarized Lorentz beam with sine-azimuthal variation of wavefront have been studied [4]. A super-Lorentzian beam can be constructed using Lorentz beams as a basis [5]. The effect of Kerr nonlinearity on the Lorentz beam has been examined using the nonlinear Schröinger equation [6]. The beam propagation factor [7] and the Wigner distribution function [8] of Lorentz-Gauss beams have been derived. Propagation of Lorentz-Gauss beams in free space [9], in uniaxial crystals orthogonal to the optical axis [10], and through an apertured fractional Fourier transformation optical system [11] has also been examined, respectively. The virtual source for generation of the rotationally symmetric Lorentz-Gauss beam has been identified [12]. The Lorentz-Gauss beam can be used to trap the particles of refractive index larger than that of the ambient [13]. Recently, Lorentz-Gauss beams have been extended to the partially coherent case [14].

When the radiation emitted by a single mode diode laser goes through a spiral phase plate, it becomes a Lorentz-Gauss vortex beam. The phase of the Lorentz-Gauss vortex beam can be easily modulated by the spiral phase plate. The advantage of the Lorentz-Gauss vortex beam over the Loretnz-Gauss beam is that the former has a twisted phase front and zero intensity in the center region of the beam profile. Owing to carrying the orbital angular momentum, the Lorentz-Gauss vortex beam has potential application in the fields of optical trapping, optical guiding, optical micro-manipulation, nonlinear optics, quantum information processing, etc. [15–23]. Analytical expressions for the three components of the nonparaxial propagation of a Lorentz-Gauss vortex beam in uniaxial crystals orthogonal to the optical axis have been derived, and the intensity and the phase distributions of the three components have been shown by numerical examples [24]. Focusing properties of the linearly polarized Lorentz-Gauss beam with one on-axis optical vortex has been investigated by means of the vector diffraction theory [25]. Due to the crucial applications in optical communications and remote sensing, the average intensity and the spreading properties of various kinds of laser beams including the Lorentz and the Lorentz-Gauss beams in the turbulent atmosphere have been extensively investigated [26–35]. However, to the best of our knowledge the propagation of a Lorentz-Gauss vortex beam in the turbulent atmosphere has not been reported. In the remainder of this paper, therefore, the propagation of a Lorentz-Gauss vortex beam is examined in the turbulent atmosphere. Analytical formulae of the average intensity, the effective beam size, and the kurtosis parameter are derived by means of the mathematical techniques.

2. PROPAGATION OF A LORENTZ-GAUSS VORTEX BEAM IN A TURBULENT ATMOSPHERE

In the Cartesian coordinate system, the z-axis is taken to be the propagation axis. The Lorentz-Gauss vortex beam in the source plane $z = 0$ takes the form of

$$E(\mathbf{r}_0, 0) = \frac{w_{0x} w_{0y} (x_0 + i y_0)^M}{(w_{0x}^2 + x_0^2)(w_{0y}^2 + y_0^2)} \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \tag{1}$$

where $\mathbf{r}_0 = x_0 \mathbf{e}_x + y_0 \mathbf{e}_y$. $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are the two transverse unit vectors in the Cartesian coordinate system, respectively. w_{0x} and w_{0y} are the parameters related to the beam widths of the Lorentz part in the x- and y-directions, respectively. w_0 is the waist of the Gaussian part. M denotes the topological charge and is assumed to be positive. The time dependent factor $\exp(-i\omega t)$ is omitted in the Eq. (1), and ω is the angular frequency. Here, we consider the simplest case of the Lorentz-Gauss vortex beam namely $M = 1$. The Lorentz distribution can be expanded into the linear superposition of Hermite-Gaussian functions [36]:

$$\frac{1}{(w_{0x}^2 + x_0^2)(w_{0y}^2 + y_0^2)} = \frac{\pi}{2 w_{0x}^2 w_{0y}^2} \sum_{m=0}^{N} \sum_{n=0}^{N} a_{2m} a_{2n} H_{2m}\left(\frac{x_0}{w_{0x}}\right) H_{2n}\left(\frac{y_0}{w_{0y}}\right) \exp\left(-\frac{x_0^2}{2w_{0x}^2} - \frac{y_0^2}{2w_{0y}^2}\right), \tag{2}$$

where N is the number of the expansion. $H_{2m}(.)$ and $H_{2n}(.)$ are the $2m$th- and $2n$th-order Hermite polynomials, respectively. a_{2m} and a_{2n} are the expansion coefficients and are given by [36]

$$a_{2m} = \frac{(-1)^m \sqrt{2}}{2^{2m}} \left\{ \frac{1}{m!} \operatorname{erfc}\left(\frac{\sqrt{2}}{2}\right) \exp\left(\frac{1}{2}\right) + \sum_{n_1=1}^{m} \frac{2^{2n_1}}{(2n_1)!(m-n_1)!} \left[\operatorname{erfc}\left(\frac{\sqrt{2}}{2}\right) \exp\left(\frac{1}{2}\right) + \sqrt{\frac{2}{\pi}} \sum_{n_2=1}^{n_1} (-1)^{n_2} (2n_2 - 3)!! \right] \right\}, \tag{3}$$

where erfc(.) is the complementary error function. The Lorentz-Gauss vortex beam with $M = 1$ in the source plane can be rewritten as:

$$E(\mathbf{r}_0, 0) = \frac{\pi}{2 w_{0x} w_{0y}} \sum_{m=0}^{N} \sum_{n=0}^{N} (x_0 + i y_0) a_{2m} a_{2n} H_{2m}\left(\frac{x_0}{w_{0x}}\right) H_{2n}\left(\frac{y_0}{w_{0y}}\right) \exp\left(-\frac{x_0^2}{u_x^2} - \frac{y_0^2}{u_y^2}\right), \tag{4}$$

where

$$\frac{1}{u_j^2} = \frac{1}{w_0^2} + \frac{1}{2w_{0j}^2}, \tag{5}$$

and $j = x$ or y (hereafter). The propagation of a Lorentz-Gauss vortex beam in a turbulent atmosphere can be calculated by the following extended Huygens-Fresnel integral:

$$E(\mathbf{r}, z) = -\frac{ik}{2\pi z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\mathbf{r}_0, 0) \exp\left[-\frac{ik}{2z}(\mathbf{r}_0 - \mathbf{r})^2 + \psi(\mathbf{r}_0, \mathbf{r})\right] dx_0 dy_0, \tag{6}$$

where $\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y$. $(\mathbf{r}, z)$ is the receiver plane. $\psi(\mathbf{r}_0, \mathbf{r})$ is the solution to the Rytov method that represents the random part of the complex phase. $k = 2\pi/\lambda$ is the wave number. λ is the wavelength. The average intensity of a Lorentz-Gauss vortex beam in the receiver plane is given by

$$\begin{aligned}\langle I(\mathbf{r}, z)\rangle = & \frac{k^2}{4\pi^2 z^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\mathbf{r}_{01}, 0) E^*(\mathbf{r}_{02}, 0) \\ & \exp\left[-\frac{ik}{2z}(\mathbf{r}_{01} - \mathbf{r})^2 + \frac{ik}{2z}(\mathbf{r}_{02} - \mathbf{r})^2\right] \\ & \langle \exp\left[\psi(\mathbf{r}_{01}, \mathbf{r}) + \psi^*(\mathbf{r}_{02}, \mathbf{r})\right]\rangle \, d\mathbf{r}_{01} d\mathbf{r}_{02},\end{aligned} \tag{7}$$

where the angle brackets indicate the ensemble average over the medium statistics, and the asterisk denotes the complex conjugation. The last ensemble average term in the above equation can be expressed as follows [37]:

$$\begin{aligned}\langle \exp[\psi(\mathbf{r}_{01}, \mathbf{r}) + \psi^*(\mathbf{r}_{02}, \mathbf{r})]\rangle &= \exp[-0.5 D_\psi(\mathbf{r}_{01} - \mathbf{r}_{02})] \\ &= \exp\left[-\frac{(\mathbf{r}_{01} - \mathbf{r}_{02})^2}{\rho_0^2}\right],\end{aligned} \tag{8}$$

where $D_\psi(\mathbf{r}_{01} - \mathbf{r}_{02})$ is the phase function in Rytov's representation and $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the spherical wave lateral coherence length. C_n^2 is the structure constant of the atmospheric turbulence. Using the following mathematical formulae [38]

$$2x H_{2m}(x) = H_{2m+1}(x) + 4m H_{2m-1}(x), \tag{9}$$

$$\int_{-\infty}^{\infty} H_m(x) \exp\left[-\alpha(x-y)^2\right] dx = \sqrt{\frac{\pi}{\alpha}} \left(1 - \frac{1}{\alpha}\right)^{m/2} H_m\left[y\left(1 - \frac{1}{\alpha}\right)^{-1/2}\right], \tag{10}$$

$$H_n(x) = \sum_{l=0}^{[n/2]} \frac{(-1)^l n!}{l!(n-2l)!} (2x)^{n-2l}, \tag{11}$$

$$(x+y)^m = \sum_{l=0}^{m} \begin{pmatrix} m \\ m-l \end{pmatrix} x^{m-l} y^l, \tag{12}$$

$$\int_{-\infty}^{\infty} x^n \exp(-bx^2+2cx)dx = n!\sqrt{\frac{\pi}{b}}\left(\frac{c}{b}\right)^n \exp\left(\frac{c^2}{b}\right) \sum_{s=0}^{[n/2]} \frac{1}{s!(n-2s)!}\left(\frac{b}{4c^2}\right)^s, \tag{13}$$

where $[n/2]$ gives the greatest integer less than or equal to $n/2$, one can obtain the analytical average intensity of a Lorentz-Gauss vortex beam in the receiver plane

$$\begin{aligned} \langle I(\mathbf{r},z)\rangle = & [\beta_1(x,z)+\beta_2(x,z)+\beta_3(x,z)+\beta_4(x,z)]\beta_0(y,z) \\ & +\beta_0(x,z)[\beta_1(y,z)+\beta_2(y,z)+\beta_3(y,z)+\beta_4(y,z)] \\ & -i[\beta_5(x,z)+\beta_8(x,z)][\beta_6(y,z)+\beta_7(y,z)]+i[\beta_6(x,z) \\ & +\beta_7(x,z)][\beta_5(y,z)+\beta_8(y,z)], \end{aligned} \tag{14}$$

with $\beta_0(j,z)$, $\beta_1(j,z)$, $\beta_2(j,z)$, $\beta_3(j,z)$, $\beta_4(j,z)$, $\beta_5(j,z)$, $\beta_6(j,z)$, $\beta_7(j,z)$, and $\beta_8(j,z)$ being given by

$$\begin{aligned} \beta_0(j,z) = & \frac{k}{4z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}} \exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}} - \frac{k^2 w_{0j}^2 j^2}{4\alpha_{1j} z^2}\right) \sum_{m_1=0}^{N}\sum_{m_2=0}^{N} a_{2m_1} a_{2m_2} \\ & \left(1-\frac{1}{\alpha_{1j}}\right)^{m_2} \sum_{l_1=0}^{m_1} \frac{(-1)^{l_1}(2m_1)!}{l_1!(2m_1-2l_1)!} \sum_{l_2=0}^{m_2} \frac{(-1)^{l_2}(2m_2)!}{l_2!(2m_2-2l_2)!} \\ & \sum_{l_3=0}^{2m_2-2l_2} \begin{pmatrix} 2m_2-2l_2 \\ l_3 \end{pmatrix} 2^{2(m_1+m_2-l_1-l_2)} \\ & \gamma_j^{l_3} (\eta_j j)^{2m_2-2l_2-l_3} (2m_1+l_3-2l_1)! \left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3-2l_1} \\ & \sum_{s=0}^{[(2m_1+l_3-2l_1)/2]} \left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s \frac{1}{s!(2m_1+l_3-2l_1-2s)!}, \end{aligned} \tag{15}$$

$$\begin{aligned} \beta_1(j,z) = & \frac{k w_{0j}^2}{16z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}} \exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}} - \frac{k^2 w_{0j}^2 j^2}{4\alpha_{1j} z^2}\right) \sum_{m_1=0}^{N}\sum_{m_2=0}^{N} a_{2m_1} a_{2m_2} \\ & \left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2+1)/2} \sum_{l_1=0}^{[(2m_1+1)/2]} \frac{(-1)^{l_1}(2m_1+1)!}{l_1!(2m_1+1-2l_1)!} \sum_{l_2=0}^{[(2m_2+1)/2]} \end{aligned}$$

$$\frac{(-1)^{l_2}(2m_2+1)!}{l_2!(2m_2+1-2l_2)!}\sum_{l_3=0}^{2m_2+1-2l_2}\binom{2m_2+1-2l_2}{l_3}2^{2(m_1+m_2+1-l_1-l_2)}$$
$$\gamma_j^{l_3}(\eta_j j)^{2m_2+1-2l_2-l_3}(2m_1+l_3+1-2l_1)!\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3+1-2l_1}$$
$$\sum_{s=0}^{[(2m_1+l_3+1-2l_1)/2]}\frac{1}{s!(2m_1+l_3+1-2l_1-2s)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s, \quad (16)$$

$$\beta_2(j,z) = \frac{kw_{0j}^2}{4z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=0}^{N}\sum_{m_2=1}^{N}m_2 a_{2m_1}a_{2m_2}$$
$$\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2-1)/2}\sum_{l_1=0}^{[(2m_1+1)/2]}\frac{(-1)^{l_1}(2n+1)!}{l_1!(2m_1+1-2l_1)!}$$
$$\sum_{l_2=0}^{[(2m_2-1)/2]}\frac{(-1)^{l_2}(2m_2-1)!}{l_2!(2m_2-2l_2-1)!}\sum_{l_3=0}^{2m_2-2l_2-1}\binom{2m_2-2l_2-1}{l_3}$$
$$2^{2(m_1+m_2-l_1-l_2)}\gamma_j^{l_3}(\eta_j j)^{2m_2-2l_2-l_3-1}(2m_1+l_3+1-2l_1)!$$
$$\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3+1-2l_1}\sum_{s=0}^{[(2m_1+l_3+1-2l_1)/2]}$$
$$\frac{1}{s!(2m_1+l_3+1-2l_1-2s)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s, \quad (17)$$

$$\beta_3(j,z) = \frac{kw_{0j}^2}{4z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=1}^{N}\sum_{m_2=0}^{N}m_1 a_{2m_1}a_{2m_2}$$
$$\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2+1)/2}\sum_{l_1=0}^{[(2m_1-1)/2]}\frac{(-1)^{l_1}(2m_1-1)!}{l_1!(2m_1-2l_1-1)!}$$
$$\sum_{l_2=0}^{[(2m_2+1)/2]}\frac{(-1)^{l_2}(2m_2+1)!}{l_2!(2m_2+1-2l_2)!}\sum_{l_3=0}^{2m_2+1-2l_2}\binom{2m_2+1-2l_2}{l_3}$$
$$2^{2(m_1+m_2-l_1-l_2)}\gamma_j^{l_3}(\eta_j j)^{2m_2+1-2l_2-l_3}(2m_1+l_3-2l_1-1)!$$
$$\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3-2l_1-1}\sum_{s=0}^{[(2m_1+l_3-2l_1-1)/2]}$$

$$\frac{1}{s!(2m_1+l_3-2l_1-2s-1)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s, \tag{18}$$

$$\begin{aligned}\beta_4(j,z) = &\frac{kw_{0j}^2}{z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\!\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2 w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=1}^{N}\sum_{m_2=1}^{N}\\
&m_1 m_2 a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2-1)/2}\sum_{l_1=0}^{[(2m_1-1)/2]}\frac{(-1)^{l_1}(2m_1-1)!}{l_1!(2m_1-2l_1-1)!}\\
&\sum_{l_2=0}^{[(2m_2-1)/2]}\frac{(-1)^{l_2}(2m_2-1)!}{l_2!(2m_2-2l_2-1)!}\sum_{l_3=0}^{2m_2-2l_2-1}\binom{2m_2-2l_2-1}{l_3}\\
&2^{2(m_1+m_2-1-l_1-l_2)}\gamma_j^{l_3}(\eta_j j)^{2m_2-2l_2-l_3-1}(2m_1+l_3-2l_1-1)!\\
&\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3-2l_1-1}\sum_{s=0}^{[(2m_1+l_3-2l_1-1)/2]}\\
&\frac{1}{s!(2m_1+l_3-2l_1-2s-1)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s,\end{aligned} \tag{19}$$

$$\begin{aligned}\beta_5(j,z) = &\frac{kw_{0j}}{8z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2 w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=1}^{N}\sum_{m_2=1}^{N}a_{2m_1}a_{2m_2}\\
&\left(1-\frac{1}{\alpha_{1j}}\right)^{m_2}\sum_{l_1=0}^{[(2m_1+1)/2]}\frac{(-1)^{l_1}(2m_1+1)!}{l_1!(2m_1+1-2l_1)!}\\
&\sum_{l_2=0}^{m_2}\frac{(-1)^{l_2}(2m_2)!}{l_2!(2m_2-2l_2)!}\sum_{l_3=0}^{2m_2-2l_2}\binom{2m_2-2l_2}{l_3}\\
&2^{2(m_1+m_2-l_1-l_2)+1}\gamma_j^{l_3}(\eta_j j)^{2m_2-2l_2-l_3}(2m_1+l_3+1-2l_1)!\\
&\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3+1-2l_1}\sum_{s=0}^{[(2m_1+l_3+1-2l_1)/2]}\\
&\frac{1}{s!(2m_1+l_3+1-2l_1-2s)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s,\end{aligned} \tag{20}$$

$$\beta_6(j,z) = \frac{kw_{0j}}{8z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2 w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=1}^{N}\sum_{m_2=1}^{N}$$

$$a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2+1)/2}\sum_{l_1=0}^{m_1}\frac{(-1)^{l_1}(2m_1)!}{l_1!(2m_1-2l_1)!}$$
$$\sum_{l_2=0}^{(2m_2+1)/2}\frac{(-1)^{l_2}(2m_2+1)!}{l_2!(2m_2+1-2l_2)!}\sum_{l_3=0}^{2m_2+1-2l_2}\binom{2m_2+1-2l_2}{l_3}$$
$$2^{2(m_1+m_2-l_1-l_2)+1}\gamma_j^{l_3}(\eta_j j)^{2m_2+1-2l_2-l_3}(2m_1+l_3-2l_1)!$$
$$\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3-2l_1}\sum_{s=0}^{[(2m_1+l_3-2l_1)/2]}$$
$$\frac{1}{s!(2m_1+l_3-2l_1-2s)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s, \tag{21}$$

$$\beta_7(j,z) = \frac{kw_{0j}}{2z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=1}^{N}\sum_{m_2=1}^{N}m_2$$
$$a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2-1)/2}\sum_{l_1=0}^{m_1}\frac{(-1)^{l_1}(2m_1)!}{l_1!(2m_1-2l_1)!}$$
$$\sum_{l_2=0}^{(2m_2-1)/2}\frac{(-1)^{l_2}(2m_2-1)!}{l_2!(2m_2-1-2l_2)!}\sum_{l_3=0}^{2m_2-1-2l_2}\binom{2m_2-1-2l_2}{l_3}$$
$$2^{2(m_1+m_2-l_1-l_2)-1}\gamma_j^{l_3}(\eta_j j)^{2m_2-1-2l_2-l_3}(2m_1+l_3-2l_1)!$$
$$\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3-2l_1}\sum_{s=0}^{[(2m_1+l_3-2l_1)/2]}$$
$$\frac{1}{s!(2m_1+l_3-2l_1-2s)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s, \tag{22}$$

$$\beta_8(j,z) = \frac{kw_{0j}}{2z}\sqrt{\frac{1}{\alpha_{1j}\alpha_{2j}}}\exp\left(\frac{\xi_j^2 j^2}{\alpha_{2j}}-\frac{k^2w_{0j}^2 j^2}{4\alpha_{1j}z^2}\right)\sum_{m_1=1}^{N}\sum_{m_2=1}^{N}m_1 a_{2m_1}a_{2m_2}$$
$$\left(1-\frac{1}{\alpha_{1j}}\right)^{m_2}\sum_{l_1=0}^{[(2m_1-1)/2]}\frac{(-1)^{l_1}(2m_1-1)!}{l_1!(2m_1-1-2l_1)!}$$
$$\sum_{l_2=0}^{m_2}\frac{(-1)^{l_2}(2m_2)!}{l_2!(2m_2-2l_2)!}\sum_{l_3=0}^{2m_2-2l_2}\binom{2m_2-2l_2}{l_3}$$

$$2^{2(m_1+m_2-l_1-l_2)-1}\gamma_j^{l_3}(\eta_j j)^{2m_2-2l_2-l_3}(2m_1+l_3-1-2l_1)!$$
$$\left(\frac{\xi_j j}{\alpha_{2j}}\right)^{2m_1+l_3-1-2l_1}\sum_{s=0}^{[(2m_1+l_3-1-2l_1)/2]}$$
$$\frac{1}{s!(2m_1+l_3-1-2l_1-2s)!}\left(\frac{\alpha_{2j}}{4\xi_j^2 j^2}\right)^s, \tag{23}$$

where the auxiliary parameters are defined by

$$\alpha_{1j} = \left(\frac{1}{u_j^2}+\frac{1}{\rho_0^2}-\frac{ik}{2z}\right)w_{0j}^2, \tag{24}$$
$$\alpha_{2j} = \left(\frac{1}{u_j^2}+\frac{1}{\rho_0^2}+\frac{ik}{2z}\right)w_{0j}^2-\frac{w_{0j}^4}{\alpha_{1j}\rho_0^4}, \tag{25}$$
$$\xi_j = \frac{ikw_{0j}}{2z}-\frac{ikw_{0j}^3}{2z\alpha_{1j}\rho_0^2}, \tag{26}$$
$$\gamma_j = \frac{w_{0j}^2}{(\alpha_{1j}^2-\alpha_{1j})^{1/2}\rho_0^2}, \tag{27}$$
$$\eta_j = \frac{kw_{0j}}{2iz(\alpha_{1j}^2-\alpha_{1j})^{1/2}}. \tag{28}$$

The effective beam size of the Lorentz-Gauss vortex beam in the j-direction of the receiver plane is defined as [39]

$$W_{jz} = \left[\frac{2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} j^2\langle I(\mathbf{r},z)\rangle dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} j^2\langle I(\mathbf{r},z)\rangle dxdy}\right]^{1/2}. \tag{29}$$

Substituting Eq. (14) into Eq. (29), the analytical effective beam size of the Lorentz-Gauss vortex beam yields

$$W_{jz} = \sqrt{\frac{2A_{1j}}{A_0}}, \tag{30}$$

with A_0 and A_{1j} being given by

$$A_0 = [B_{1x}(1.5)+B_{2x}(0.5)+B_{3x}(0.5)+B_{4x}(-0.5)]B_{0y}(0.5) +B_{0x}(0.5)[B_{1y}(1.5)+B_{2y}(0.5)+B_{3y}(0.5)+B_{4y}(-0.5)], \tag{31}$$
$$A_{1x} = [B_{1x}(2.5)+B_{2x}(1.5)+B_{3x}(1.5)+B_{4x}(0.5)]B_{0y}(0.5) +B_{0x}(1.5)[B_{1y}(1.5)+B_{2y}(0.5)+B_{3y}(0.5)+B_{4y}(-0.5)], \tag{32}$$
$$A_{1y} = [B_{1x}(1.5)+B_{2x}(0.5)+B_{3x}(0.5)+B_{4x}(-0.5)]B_{0y}(1.5) +B_{0x}(0.5)[B_{1y}(2.5)+B_{2y}(1.5)+B_{3y}(1.5)+B_{4y}(0.5)], \tag{33}$$

where $B_{0j}(v_0)$, $B_{1j}(v_1)$, $B_{2j}(v_0)$, $B_{3j}(v_0)$, and $B_{4j}(v_2)$ are found to be

$$\begin{aligned}
B_{0j}(v_0) \;=\;& \frac{1}{4}\sum_{m_1=0}^{N}\sum_{m_2=0}^{N} a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{m_2}\sum_{l_1=0}^{m_1}\frac{(-1)^{l_1}(2m_1)!}{l_1!(2m_1-2l_1)!}\\
&\sum_{l_2=0}^{m_2}\frac{(-1)^{l_2}(2m_2)!}{l_2!(2m_2-2l_2)!}\sum_{l_3=0}^{2m_2-2l_2}\binom{2m_2-2l_2}{l_3}\\
&2^{2(m_1+m_2-l_1-l_2)}\gamma_j^{l_3}\eta_j^{2m_2-2l_2-l_3}(2m_1+l_3-2l_1)!\\
&\sum_{s=0}^{[(2m_1+l_3-2l_1)/2]}\frac{\alpha_{2j}^{2l_1+s-2m_1-l_3}\xi_j^{2m_1+l_3-2l_1-2s}}{4^s s!(2m_1+l_3-2l_1-2s)!}\\
&\Gamma(m_1+m_2-l_1-l_2-s+v_0)\delta_j^{l_1+l_2+s-m_1-m_2-v_0},\\
&v_0=0.5,1.5,2.5, \qquad (34)
\end{aligned}$$

$$\begin{aligned}
B_{1j}(v_1) \;=\;& \frac{w_{0j}^2}{16}\sum_{m_1=0}^{N}\sum_{m_2=0}^{N} a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2+1)/2}\\
&\sum_{l_1=0}^{[(2m_1+1)/2]}\frac{(-1)^{l_1}(2m_1+1)!}{l_1!(2m_1+1-2l_1)!}\sum_{l_2=0}^{[(2m_2+1)/2]}\frac{(-1)^{l_2}(2m_2+1)!}{l_2!(2m_2+1-2l_2)!}\\
&\sum_{l_3=0}^{2m_2+1-2l_2}\binom{2m_2+1-2l_2}{l_3}2^{2(m_1+m_2+1-l_1-l_2)}\\
&\gamma_j^{l_3}\eta_j^{2m_2+1-2l_2-l_3}(2m_1+l_3+1-2l_1)!\\
&\sum_{s=0}^{[(2m_1+l_3+1-2l_1)/2]}\frac{\alpha_{2j}^{2l_1+s-2m_1-l_3-1}\xi_j^{2n+l_3+1-2l_1-2s}}{4^s s!(2m_1+l_3+1-2l_1-2s)!}\\
&\Gamma(m_1+m_2-l_1-l_2-s+v_1)\delta_j^{l_1+l_2+s-m_1-m_2-v_1},\\
&v_1=15,2.5,3.5, \qquad (35)
\end{aligned}$$

$$\begin{aligned}
B_{2j}(v_0) \;=\;& \frac{w_{0j}^2}{4}\sum_{m_1=0}^{N}\sum_{m_2=1}^{N} m_2 a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_j}\right)^{(2m_2-1)/2}\\
&\sum_{l_1=0}^{[(2m_1+1)/2]}\frac{(-1)^{l_1}(2m_1+1)!}{l_1!(2m_1+1-2l_1)!}\sum_{l_2=0}^{[(2m_2-1)/2]}\frac{(-1)^{l_2}(2m_2-1)!}{l_2!(2m_2-2l_2-1)!}\\
&\sum_{l_3=0}^{2m_2-2l_2-1}\binom{2m_2-2l_2-1}{l_3}2^{2(m_1+m_2-l_1-l_2)}
\end{aligned}$$

$$\gamma_j^{l_3}\eta_j^{2m_2-2l_2-l_3-1}(2m_1+l_3+1-2l_1)! \sum_{s=0}^{[(2m_1+l_3+1-2l_1)/2]} \frac{\alpha_{2j}^{2l_1+s-2m_1-l_3-1}\xi_j^{2m_1+l_3+1-2l_1-2s}}{4^s s!(2m_1+l_3+1-2l_1-2s)!} \Gamma(m_1+m_2-l_1-l_2-s+v_0)\delta_j^{l_1+l_2+s-m_1-m_2-v_0}, \quad (36)$$

$$B_{3j}(v_0) = \frac{w_{0j}^2}{4}\sum_{m_1=1}^{N}\sum_{m_2=0}^{N} m_1 a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2+1)/2} \sum_{l_1=0}^{[(2m_1-1)/2]}\frac{(-1)^{l_1}(2m_1-1)!}{l_1!(2m_1-2l_1-1)!}\sum_{l_2=0}^{[(2m_2+1)/2]}\frac{(-1)^{l_2}(2m_2+1)!}{l_2!(2m_2+1-2l_2)!} \sum_{l_3=0}^{2m_2+1-2l_2}\binom{2m_2+1-2l_2}{l_3}2^{2(m_1+m_2-l_1-l_2)} \gamma_j^{l_3}\eta_j^{2m_2+1-2l_2-l_3}(2m_1+l_3-2l_1-1)! \sum_{s=0}^{[(2m_1+l_3-2l_1-1)/2]}\frac{\alpha_{2j}^{2l_1+s+1-2m_1-l_3}\xi_j^{2m_1+l_3-1-2l_1-2s}}{4^s s!(2m_1+l_3-2l_1-2s-1)!} \Gamma(m_1+m_2-l_1-l_2-s+v_0)\delta_j^{l_1+l_2+s-m_1-m_2-v_0}, \quad (37)$$

$$B_{4j}(v_2) = w_{0j}^2\sum_{m_1=1}^{N}\sum_{m_2=1}^{N} m_1 m_2 a_{2m_1}a_{2m_2}\left(1-\frac{1}{\alpha_{1j}}\right)^{(2m_2-1)/2} \sum_{l_1=0}^{[(2m_1-1)/2]}\frac{(-1)^{l_1}(2m_1-1)!}{l_1!(2m_1-2l_1-1)!}\sum_{l_2=0}^{[(2m_2-1)/2]}\frac{(-1)^{l_2}(2m_2-1)!}{l_2!(2m_2-2l_2-1)!} \sum_{l_3=0}^{2m_2-2l_2-1}\binom{2m_2-2l_2-1}{l_3}2^{2(m_1+m_2-1-l_1-l_2)} \gamma_j^{l_3}\eta_j^{2m_2-2l_2-l_3-1}(2m_1+l_3-2l_1-1)! \sum_{s=0}^{[(2m_1+l_3-2l_1-1)/2]}\frac{\alpha_{2j}^{2l_1+s+1-2m_1-l_3}\xi_j^{2m_1+l_3-2l_1-2s-1}}{4^s s!(2m_1+l_3-2l_1-2s-1)!} \Gamma(m_1+m_2-l_1-l_2-s+v_2)\delta_j^{l_1+l_2+s-m_1-m_2-v_2}, \quad v_2=-0.5, 0.5, 1.5. \quad (38)$$

$\Gamma(\cdot)$ is a Gamma function. The auxiliary parameter δ_j is defined by

$$\delta_j = \frac{k^2 w_{0j}^2}{4\alpha_{1j} z^2} - \frac{\xi_j^2}{\alpha_{2j}}. \tag{39}$$

In the above derivation, the following integral formula is used [38]

$$\int_{-\infty}^{\infty} x^n \exp(-cx^n) = \frac{1+(-1)^n}{2} c^{-(n+1)/2} \Gamma\left(\frac{n+1}{2}\right). \tag{40}$$

The kurtosis parameter, which is employed to describe the degree of flatness of the laser beams, is an important parameter to evaluate the beam propagation. The kurtosis parameter in one transversal dimension, e.g., the x-direction, is defined as [40]

$$K_x = \frac{\langle x^4 \rangle}{\langle x^2 \rangle^2}, \tag{41}$$

where $\langle x^2 \rangle$ and $\langle x^4 \rangle$ are given by

$$\langle x^s \rangle = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^s \langle I(\mathbf{r}, z) \rangle dxdy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle I(\mathbf{r}, z) \rangle dxdy}, \quad s = 2, 4. \tag{42}$$

Therefore, the kurtosis parameter of the Lorentz-Gauss vortex beam in the j-direction of the receiver plane is found to be

$$K_j = \frac{A_{2j} A_0}{(A_{1j})^2}. \tag{43}$$

with A_{2j} being given by

$$\begin{aligned} A_{2x} = & [B_{1x}(3.5) + B_{2x}(2.5) + B_{3x}(2.5) + B_{4x}(1.5)] B_{0y}(0.5) \\ & + B_{0x}(2.5)[B_{1y}(1.5) + B_{2y}(0.5) + B_{3y}(0.5) + B_{4y}(-0.5)], \quad (44) \\ A_{2y} = & [B_{1x}(1.5) + B_{2x}(0.5) + B_{3x}(0.5) + B_{4x}(-0.5)] B_{0y}(2.5) \\ & + B_{0x}(0.5)[B_{1y}(3.5) + B_{2y}(2.5) + B_{3y}(2.5) + B_{4y}(1.5)]. \quad (45) \end{aligned}$$

The analytical formulae of the average intensity, the effective beam size, and the kurtosis parameter of a Lorentz-Gauss vortex beam in the turbulent atmosphere are complicated. However, with increasing the even number $2m$, the value of a_{2m} decreases dramatically compared to $a_0 = 0.7399$ and $a_2 = 0.9298 \times 10^{-2}$. When $m = 5$, $a_{10} = 0.3008 \times 10^{-6}$. Therefore, N will not be large in the calculations with acceptable accuracy. Therefore, the calculations of the average intensity, the effective beam size, and the kurtosis parameter are convenient and fast using software like Mathematica.

3. NUMERICAL CALCULATIONS AND ANALYSES

The spreading properties of a Lorentz-Gauss vortex beam in the turbulent atmosphere are numerically calculated using the formulae derived above. Figs. 1–3 represent the normalized average intensity distribution of a Lorentz-Gauss vortex beam at several different propagation distances in the turbulent atmosphere. The parameters chosen in calculations are: $\lambda = 0.8\,\mu\text{m}$ and $C_n^2 = 10^{-14}\,\text{m}^{-2/3}$. The reference planes are $z = 0.2\,\text{km}$, $1\,\text{km}$, $2\,\text{km}$, and $5\,\text{km}$, respectively. The beam parameters of the Lorentz-Gauss vortex beam can be divided into three following cases: the waist of the Gaussian part being larger than the beam widths of the Lorentz part, the waist of the Gaussian part being smaller than the beam widths of the Lorentz part, and the waist of the Gaussian part being equal to the beam widths of the Lorentz part. Therefore, Figs. 1–3 correspond to the

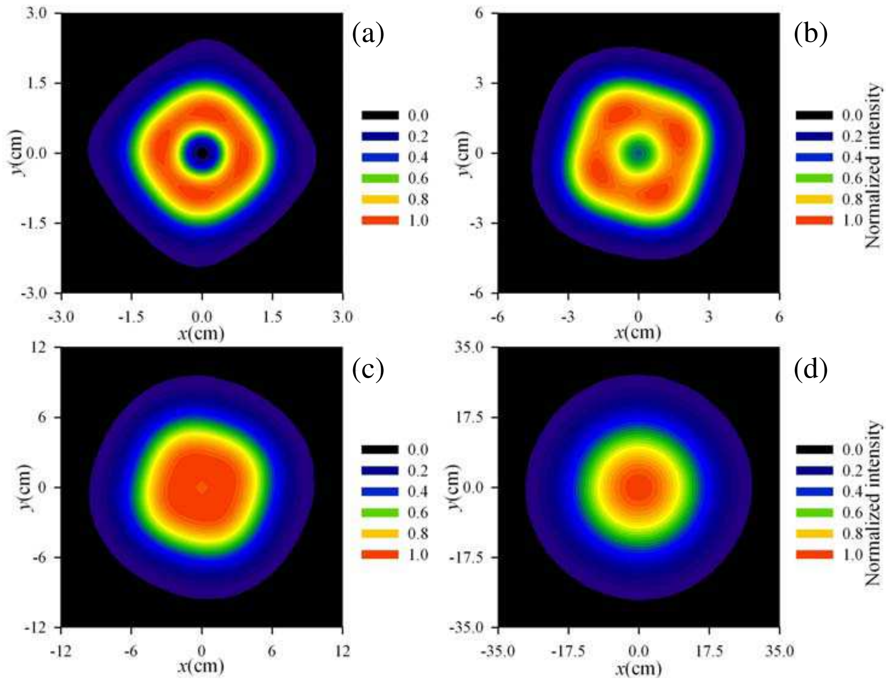

Figure 1. (Color online) Normalized average intensity distribution of a Lorentz-Gauss vortex beam at different propagation distances in the turbulent atmosphere. $w_0 = 2\,\text{cm}$ and $w_{0x} = w_{0y} = 1\,\text{cm}$. (a) $z = 0.2\,\text{km}$. (b) $z = 1\,\text{km}$. (c) $z = 2\,\text{km}$. (d) $z = 5\,\text{km}$.

above three cases. Due to the isotropic influence of the atmosphere turbulence, the normalized average intensity in the central region gradually increases and finally becomes the maximum value with increasing the propagation distance z. When the propagation distance z is an appropriate value, the Lorentz-Gauss vortex beam in the turbulent atmosphere will become a flattened beam spot. When the propagation distance z is large enough, the beam spot of the Lorentz-Gauss vortex beam tends to be a Gaussian-like distribution. Among the three cases the spreading of the Lorentz-Gauss vortex beam with $w_0 = w_{0x} = w_{0y} = 1\,\text{cm}$ is the largest, and the spreading of the Lorentz-Gauss vortex beam with $w_0 = 2\,\text{cm}$ and $w_{0x} = w_{0y} = 1\,\text{cm}$ is the smallest. The spreading of the Lorentz-Gauss vortex beam with $w_0 = 1\,\text{cm}$ and $w_{0x} = w_{0y} = 2\,\text{cm}$ is moderate. The reason is that the spreading of the Lorentz distribution in the turbulent atmosphere is higher than that of the Gaussian distribution.

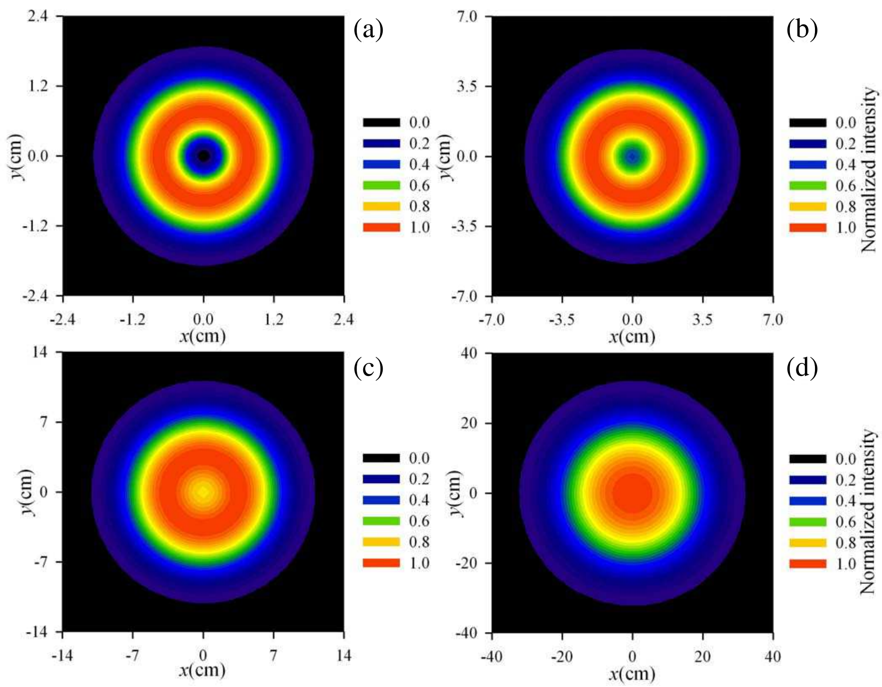

Figure 2. (Color online) Normalized average intensity distribution of a Lorentz-Gauss vortex beam at different propagation distances in the turbulent atmosphere. $w_0 = 1\,\text{cm}$ and $w_{0x} = w_{0y} = 2\,\text{cm}$. (a) $z = 0.2\,\text{km}$. (b) $z = 1\,\text{km}$. (c) $z = 2\,\text{km}$. (d) $z = 5\,\text{km}$.

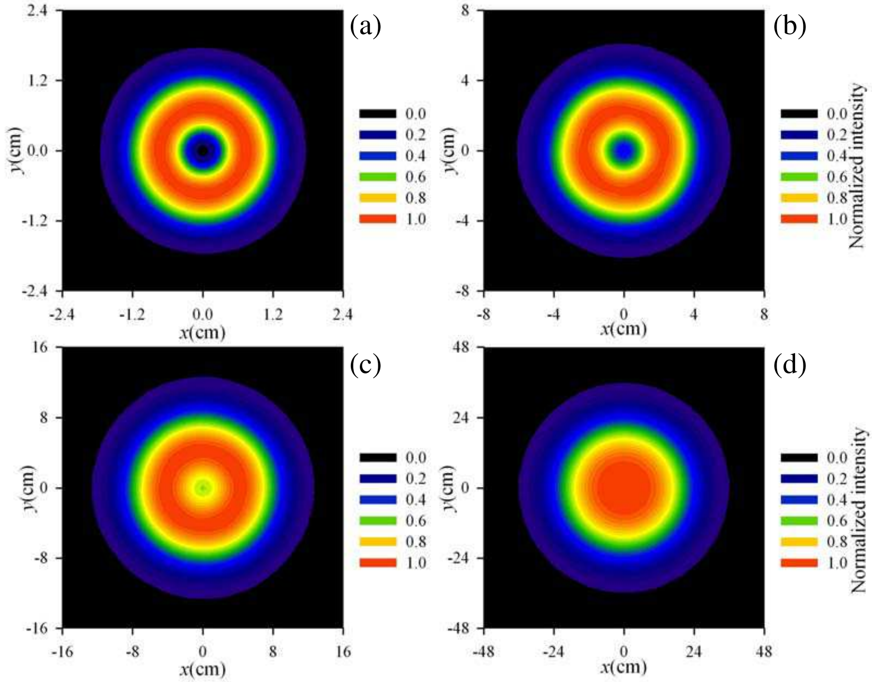

Figure 3. (Color online) Normalized average intensity distribution of a Lorentz-Gauss vortex beam at different propagation distances in the turbulent atmosphere. $w_0 = w_{0x} = w_{0y} = 1\,\text{cm}$. (a) $z = 0.2\,\text{km}$. (b) $z = 1\,\text{km}$. (c) $z = 2\,\text{km}$. (d) $z = 5\,\text{km}$.

Figure 4 shows the normalized intensity distributions in the y-direction of Lorentz-Gauss vortex beams at different propagation distances in the turbulent atmosphere. The solid, the short dashed, and the dotted curves correspond to $w_0 = 2\,\text{cm}$ and $w_{0x} = w_{0y} = 1\,\text{cm}$, $w_0 = 1\,\text{cm}$ and $w_{0x} = w_{0y} = 2\,\text{cm}$, and $w_0 = w_{0x} = w_{0y} = 1\,\text{cm}$, respectively. With increasing the propagation distance z, the vale in the normalized intensity distribution in the y-direction of a Lorentz-Gauss vortex beam gradually rises and finally disappears. The rising speed of the vale in the case of $w_0 = 2\,\text{cm}$ and $w_{0x} = w_{0y} = 1\,\text{cm}$ is the fastest, and the rising speed of the vale in the case of $w_0 = w_{0x} = w_{0y} = 1\,\text{cm}$ is the slowest. It seems that the rising speed of the vale is opposite to the spreading of the beam spot. It exhibits similar behavior in the x-direction to the y-direction. To further explore the spreading property of a Lorentz-Gauss vortex beam in the turbulent atmosphere, the effective beam size in the x-direction of a Lorentz-Gauss vortex

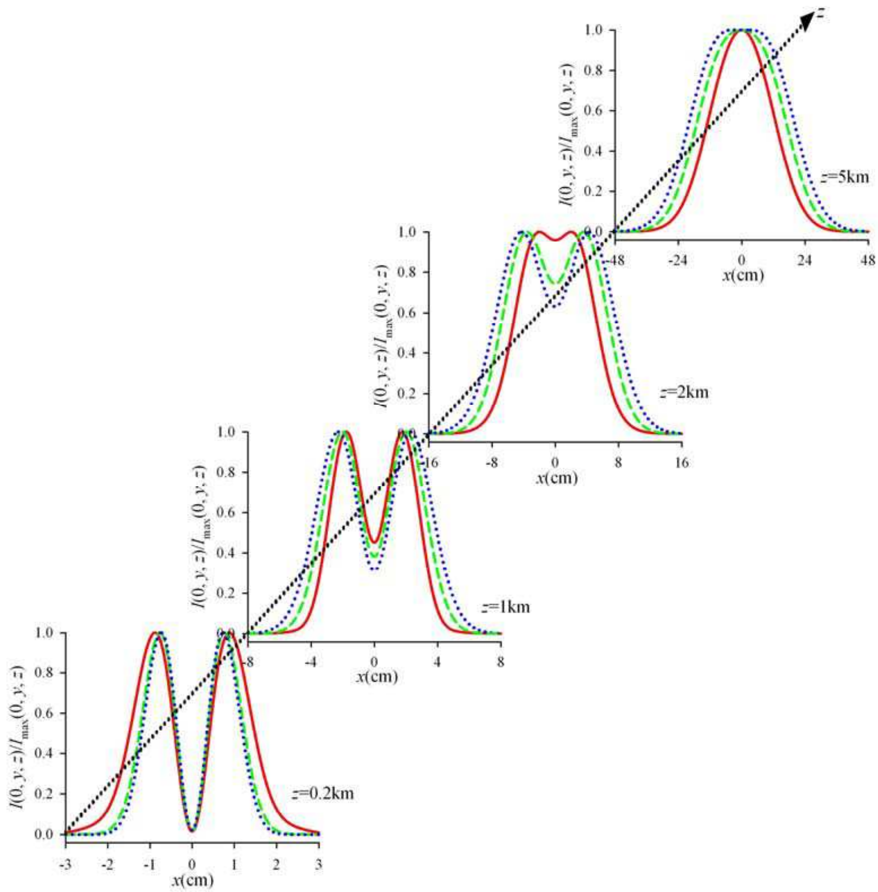

Figure 4. (Color online) Normalized intensity distributions in the y-direction of Lorentz-Gauss vortex beams at different propagation distances in the turbulent atmosphere.

beam versus the propagation distance z in the turbulent atmosphere is depicted in Fig. 5. As we consider the most common case of $w_{0x} = w_{0y}$, the effective beam size in the x-direction is equal to that in the y-direction. Therefore, only the effective beam size in the x-direction is taken into account. With the same w_0, the Lorentz-Gauss vortex beam with the smaller w_{0x} spreads more rapidly. With the same w_{0x}, the Lorentz-Gauss vortex beam with the smaller w_0 spreads more quickly. The kurtosis parameter in the x-direction of a Lorentz-Gauss vortex beam versus the propagation distance z in the turbulent atmosphere is shown in Fig. 6. The kurtosis parameter in the x-direction undergoes the following process: with increasing the propagation distance z, the kurtosis parameter in the x-direction first decreases to the minimum value, then increases, and finally tends to be a saturated value. Under the condition of having the same w_0, K_x in the reference plane close to the source plane decreases by increasing the beam widths of the Lorentz part. There exists a small range of the propagation distance, in which K_x increases by increasing the beam widths of the Lorentz part. When

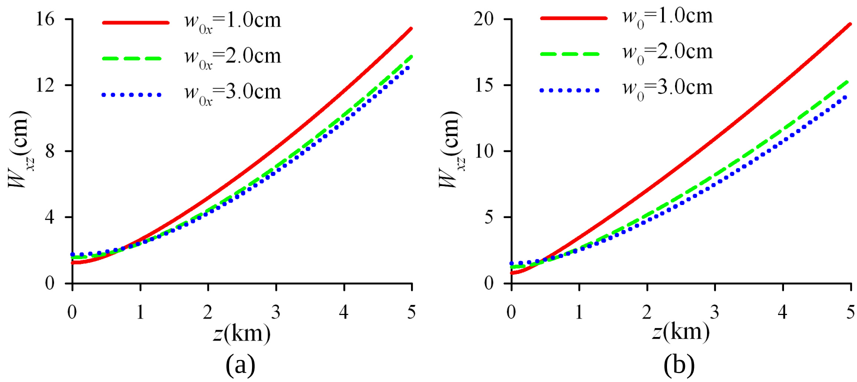

Figure 5. (Color online) The effective beam size in the x-direction of a Lorentz-Gauss vortex beam versus the propagation distance z in the turbulent atmosphere. (a) $w_0 = 2\,\text{cm}$ and $w_{0x} = w_{0y}$. (b) $w_{0x} = w_{0y} = 1\,\text{cm}$.

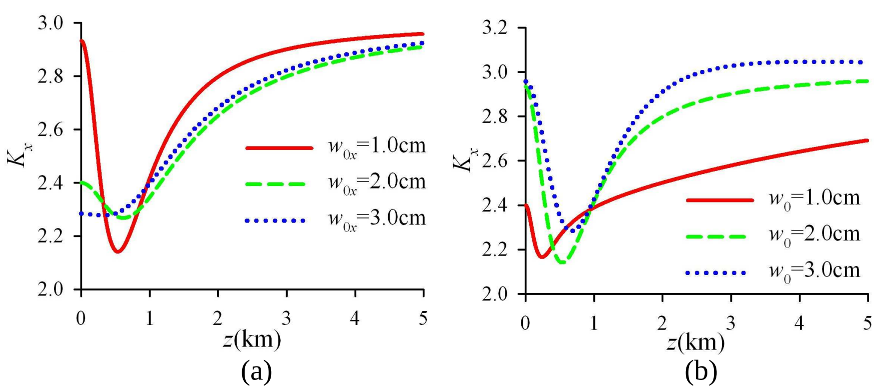

Figure 6. (Color online) The kurtosis parameter in the x-direction of a Lorentz-Gauss vortex beam versus the propagation distance z in the turbulent atmosphere. (a) $w_0 = 2\,\text{cm}$ and $w_{0x} = w_{0y}$. (b) $w_{0x} = w_{0y} = 1\,\text{cm}$.

the propagation distance is relatively large, K_x first decreases and then increases by increasing the beam widths of the Lorentz part. Under the condition of having the same w_{0x} and w_{0y}, K_x in the reference plane close to the source plane increases by increasing the waist of the Gaussian part. When the propagation distance is relatively large, K_x also increases by increasing the waist of the Gaussian part.

4. CONCLUSIONS

Based on the extended Huygens-Fresnel integral and the expansion of Lorentz distribution into Hermite-Gaussian functions, the analytical expressions of the average intensity, the effective beam size, and the kurtosis parameter of a Lorentz-Gauss vortex beam are derived in the turbulent atmosphere, respectively. The spreading properties of a Lorentz-Gauss vortex beam in the turbulent atmosphere are numerically calculated and analyzed. Upon propagation in the turbulent atmosphere, the normalized average intensity in the central region of the Lorentz-Gauss vortex beam gradually increases. At certain value of the propagation distance z, the Lorentz-Gauss vortex beam in the turbulent atmosphere becomes a flattened beam spot. When the propagation distance z is large enough, the beam spot of the Lorentz-Gauss vortex beam tends to be a Gaussian-like distribution. The rising speed of the vale in the normalized average intensity distribution is opposite to the spreading of the beam spot in three different cases of beam widths of Gaussian part and Lorentz part. With the same w_0, the Lorentz-Gauss vortex beam with the smaller w_{0x} spreads more rapidly. With the same w_{0x}, the Lorentz-Gauss vortex beam with the smaller w_0 spreads more quickly. The kurtosis parameter undergoes the following process: with increasing the propagation distance z, the kurtosis parameter first decreases to the minimum value, then increases, and finally tends to be a saturated value. This research is beneficial to optical communications and remote sensing that are involved in the single mode diode laser devices.

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