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A COMBINED FDTD/TLM TIME DOMAIN METHOD TO SOLVE EFFICIENTLY ELECTROMAGNETIC PROBLEMS

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Abstract—Modeling complex networks of cables inside structures and modeling disjoint objects connected by cables inside large computational domains with respect to the wavelength are two problems that currently present many difficulties. In this paper, we propose a 1D/3D hybrid method in time domain to solve efficiently these two kinds of problems. The method, based upon finite difference schemes, couples Maxwell's equations to evaluate electromagnetic fields in 3D domains and the transmission line equations to evaluate currents and voltages on cables. Some examples are presented to show the interest of this approach.

1. INTRODUCTION

In this paper, we are interested in the hybridization between two finite difference time domain methods, one in 3D and one in 1D, to solve respectively Maxwell's equations and the Multiconductor Transmission Line (MTL) equations. Indeed, for several 3D electromagnetic problems, it can be more efficient to compute the currents on wire models using locally a TL equation rather than a thin wire model inside a global 3D computational domain [6]. In particular, this implies a gain in term of computational cost, but also a more accurate solution as far as realistic connection wires are concerned. In this paper, we consider two kinds of configurations. The first one consists in the study of complex networks having multiconductor cables and several connection nodes at junctions inside a structure. This configuration is typical of cables in embedded systems. The second configuration

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deals with the evaluation of induced currents and voltages on cables, for large computational domains in which 3D structures or objets are connected by wires. This configuration in typical of cables or pipes connecting buildings, encountered in ground installation topologies.

The first numerical problem we are interested in is linked to important industrial concerns, and one can find in the literature several solutions based upon either thin wire models [1–4], or models based on the coupling between a 3D time domain method to compute the electromagnetics fields and a Multiconductor Transmission Line Network (MTLN) equation to evaluate currents on cable harnesses in the frequency domain. In [5–7], the authors apply Agrawal's principle based on voltage source terms coming from the incident electric field. The main difficulty of a thin wire model is to represent accurately the connectivity and the line parameters (R, L, G, C) of wires in a network of cables. Indeed, in real cables, wires are included in bundles which may be themselves included inside shielded or dielectric conductors and the network is made by multi-wires cables connected with junctions. In fact, the current obtained with thin wires models proposed in the literature use averaged values for the line parameters of the wire and the global models of connections are generally not sufficient to describe finely the complexity of a realistic network of cables. The other approach, consisting in coupling MTL equations with a 3D numerical method, has also some limitations. The first one is the difficulty of coupling a time domain and a frequency domain methods. The second limitation concerns the coupling process itself. Indeed, to compute the currents on the cables, the MTL equation uses the values of the electric fields computed in the 3D domain without the cable. This approach is fully consistent for the evaluation of the cable response itself and is even an advantage for carrying out parametric analysis of the nature of cables. However the currents on the cables are not considered for evaluating the scattered fields, as it would be required for a rigorous coupling model. Especially, such an approach does not calculate properly the antenna mode current which can be dominant in the middle of the cables [17], even if the transmission line mode current dominates at the end loads. The alternative we propose in our 1D/3D hybrid method is based on the same idea but, in order to avoid the previous limitations, we consider a Finite Difference scheme in the time domain to solve the MTL equations. The 3D electric fields are considered as sources and the currents obtained on the cables are also introduced as sources for the evaluation of electromagnetic fields. So this approach enables taking into account complex cable network topologies, still considering the influence of the currents on the cables in the evaluation of the 3D electromagnetic fields.

For the second kind of problems we are interested in interconnected large structures. The evaluation of the fields inside the whole computational domain implies an important computational cost in terms of CPU and memory. Moreover, the dissipative or/and dispersive errors due to the 3D numerical scheme used to solve Maxwell's equations are sometimes important. To avoid these drawbacks, for disjoint objects a multi-domain method has been proposed to limit the 3D computational meshed domains only around the objects [8]. In this article, the authors intend to the principle of a similar strategy extended to the case of objects interconnected by multi-wires cables. This strategy is based again on the coupling of the MTLN equations with Maxwell's equations.

The paper is organized into four sections. The mathematical formulations and numerical approximations using the finite difference schemes for the 3D Maxwell and MTLN equations are recalled in Section 2. Then, in Section 3, we give the principle of the proposed 1D/3D hybridization method for the study of complex networks inside structures. Some examples are presented to validate the approach on generic test-cases that include multi-wires cables, through comparisons with results found in the literature or obtained by other methods. These examples show the advantages of the proposed 1D/3D hybrid method. Finally, in Section 4, we present our 1D/3D hybrid method adapted for large computational domains in terms of wavelength, typically a set of buildings located on a common ground reference, and interconnected by cables. In particular, we describe the principle of the hybridization strategy in this particular case and we give some examples to show the advantages of the method in terms of memory storage and CPU time.

2. MATHEMATICAL FORMULATIONS AND NUMERICAL APPROXIMATIONS

The two physical models used in this work are Maxwell's equations and the MTLN equations. To evaluate numerically the solution of these equations, we use for both systems a Finite Differences Time Domain (FDTD) method. In this section, we recall the equations and describe the numerical schemes used for their resolution.

2.1. Maxwell's Equations and Yee's Scheme

Let $\Omega \subset R^3$ be a bounded domain where we define an electric $\mathbf{E}$ and a magnetic $\mathbf{H}$ fields. These fields satisfy Maxwell's equations given by:

$$\begin{cases} \nabla \times \mathbf{E} + \mu_o \frac{\partial \mathbf{H}}{\partial t} = 0 \\ \nabla \times \mathbf{H} = \epsilon_o \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \\ \mathbf{E}(t=0) = 0; \quad \mathbf{H}(t=0) = 0 \\ n \times \mathbf{E} = 0 \text{ on } \partial\Omega \end{cases} \tag{1}$$

where $\partial\Omega$ defines the boundary of Ω and $\mathbf{J}$ a given source of current. To simulate the infinite space, JP. Berenger proposed a Perfectly Matched Layer (PML) formalism where the condition $n \times \mathbf{E} = 0$ is always applied on $\partial\Omega$ [12]: this condition does not limit our model.

To solve the Equation (1), K. S. Yee [9] proposed an efficient well known finite differences numerical method based upon a leap-frog scheme in time and in space [10, 11].

2.2. Multiconductor Transmission Line Equations

The MTLN equations represents the propagation of currents and voltages along a set of parallel conductors. In the case of a common ground reference, the current vector I and the voltage vector V on a conductor are given by [13, 17]:

$$\begin{cases} L \frac{\partial I}{\partial t} + RI = -\frac{\partial V}{\partial l} + E_{inc} \\ C \frac{\partial V}{\partial t} + GV = -\frac{\partial I}{\partial l} \end{cases} \tag{2}$$

where R, L, G and C define matrices containing the line parameters, so called "per-unit-length" (p.u.l) electrical parameters and E_{inc} an incident electromagnetic field source vector. The inductance matrix L and the capacitance matrix C depend on the geometry of the wires, the local 3D geometry and the characteristics of the surrounding medium. They are evaluated by considering a common reference ground made by the local geometry. This ground is used for defining the voltage V on each wire of the multi-wires model. From a circuit point of view, a MTL can be represented by a set of elementary cells defined as shown in Figure 1.

In the particular case of an homogeneous medium and considering ν as the speed of the light in the medium, we can write $LC = 1/\nu^2$. Writing $q = CV$, we can write the system of Equation (2) in an equivalent form by eliminating V:

$$\begin{cases} \frac{\partial I}{\partial t} + \frac{R}{L} I = -\nu^2 \frac{\partial q}{\partial l} + \frac{E_{inc}}{L} \\ \frac{\partial q}{\partial t} + \frac{G}{C} q = -\frac{\partial I}{\partial l} \end{cases} \tag{3}$$

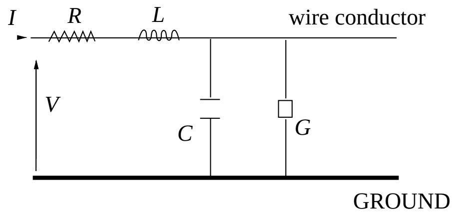


Figure 1. I and V the unknowns and p.u.l parameters on a transmission-line cell (simple case of a one-wire transmission line).

These equations are consistent with Holland's formalism [1] which is usually used for taking into account thin wires in the FDTD method.

To complete the previous system of equations, some boundary conditions are applied at the end of each segment:

- $q = 0$ when the conductor is connected to a Perfectly Electric Conductor (PEC) plane;
- $I = 0$ when the conductor is not connected;
- $\sum_{i=1}^{N} I_i = 0$ and $\forall\ (i,j) \in [1,N]^2\ V_i = V_j$ for a junction of N perfectly connected conductors.

To solve the system of Equation (3), we use a FDTD method based upon a leap-frog scheme in space and in time as for the Yee method. In this approach, all the conductors are split into several segments and, on each segment i, we define three unknowns: two charges (q_{1i}, q_{2i}) located at the two ends of the segment, and a current I_i defined in the middle of the segment. In the FDTD scheme, the charges are evaluated at times $t_n = n\,dt$ and the currents at times $t_{n+1/2} = (n+1/2)\,dt$ for $n = 1, N$, where N defines the number of iterations in time and dt the time step.

Now, let a segment i be connected at its ends by two others segments $i-1$ and $i+1$. Considering the system of Equation (3) on a segment i and the junction conditions, the numerical scheme to evaluate the charges and the current is given by the following equations:

$$\begin{aligned}
I_i^{n+1/2} &= \left(I_i^{n-1/2}\left(1-R_i\frac{\Delta t}{2L_i}\right)-\nu^2\Delta t\frac{q_{2i}^n-q_{1i}^n}{dl_i}+\Delta t\frac{E_{inc}}{L_i}\right)\bigg/\left(1+R_i\frac{\Delta_t}{2L_i}\right)\\
q_{1i}^{n+1} &= \alpha_{1i}q_{1i}^n-\beta_{1i}\left(I_i^{n-1/2}-I_{i-1}^{n-1/2}\right)\\
q_{2i}^{n+1} &= \alpha_{2i}q_{2i}^n-\beta_{2i}\left(I_{i+1}^{n-1/2}-I_i^{n-1/2}\right)
\end{aligned}\tag{4}$$

where

$$A = \frac{dl_i}{2}\left(1-\Delta t\frac{G_i}{2C_i}\right)+\frac{dl_{i-1}}{2}\frac{C_{i-1}}{C_i}\left(1-\Delta t\frac{G_{i-1}}{2C_{i-1}}\right)$$

$$\alpha_{1i} = \left(\frac{dl_i}{2}(1+\Delta t\frac{G_i}{2C_i}) + \frac{dl_{i-1}}{2}\frac{C_{i-1}}{C_i}\left(1+\Delta t\frac{G_{i-1}}{2C_{i-1}}\right)\right) \Big/ A \quad (5)$$
$$\beta_{1i} = \Delta t/A$$

The terms α_{2i} and β_{2i} are also defined by Equation (6), in which $(i-1)$ is replaced by $(i+1)$. In the previous expressions, Δt, dl_i, dl_{i-1} and dl_{i+1} are respectively the time step and the lengths of the segments i, $i-1$ and $i+1$ in the 1D mesh.

To ensure the stability of the numerical method, it is easy to show that the time step Δt must satisfy $\Delta t \leq \min_i(dl_i/\nu)$.

To complete the numerical method associated to the MTL equations, the values of the parameters R, L, G and C have to be known. We have seen that L and C parameters depended on the geometry and the characteristics of the medium. The R parameter is defined by the conductivity of the wires as far as the ground is perfectly conducting. Here, we also apply the commonly made approximation that G is equal to 0, which is consistent with our approximation of an homogeneous free space medium.

In this paper, we are interested by two kinds of MTL models. The first one consists in bundles of conductors located inside 3D structures; in this case the reference conductor is given by the cell boundary. The second one consists in cables located above a perfectly conductor or a real soil; in this case, the reference conductor is then made by the ground. For some canonic configurations, one can find analytical formulas that give the expressions of the MTLN parameters [14]. But in general, this is not the case and a 2D Laplace equation needs to be solved to evaluate them [15, 16]. The method consists, first, in considering a 2D geometrical cross-section of the cable bundles and in assigning a potential value to each conductor, in order to have a potential for the reference conductor equal to 0. Then the potential variation U in the 2D section is evaluated by solving a Laplace's equation. Next, the charge Q_i of each conductor i is evaluated by integrating on their surface the quantity $\varepsilon(E \cdot n)$ where n defines the outside unit vector to the boundary and $E = \nabla U$. The capacitance matrix between two conductors i and j is then given by $C_{ij} = \frac{Q_i - Q_j}{U_i - U_j}$, and the inductance matrix L_{ij} is obtained by $L_{ij}C_{ij} = 1/\nu^2$.

3. 1D/3D HYBRID METHOD FOR COMPLEX NETWORKS

Taking into account realistic network of bundles in electromagnetic simulations has always been an important issue for EMC problems. Several studies have been done on this subject in the literature to obtain the most efficient model. Nevertheless, despite the progress made by these studies, it is still difficult to give a satisfactory simulation for a lot of industrial problems with complex interconnected cable networks. In this section, we propose to handle this problem by coupling 3D Maxwell's equations for evaluating electromagnetic fields and the 1D MTLN equations for evaluating currents on cables. We show the advantages of this method on different test-cases. For the selected test cases, we make the approximation of free space and perfectly conducting common mode references, which allow to address already a large set of problems. With these assumptions we can show some comparisons with some other approaches, but the method itself is not limited and could be applied in more general cases.

3.1. Coupling Parameters

In the proposed 1D/3D hybrid strategy, the electromagnetic fields are evaluated by using Maxwell's equations and the variations of the currents and charges along the conductors inside the cables are computed by a MTLN equation. So to solve the problem we have a system of two coupled systems of equations derived from (1) and (3). For Maxwell's equations, the coupling term is the current density J induced by the currents on the cables, and for the MTLN equations, the

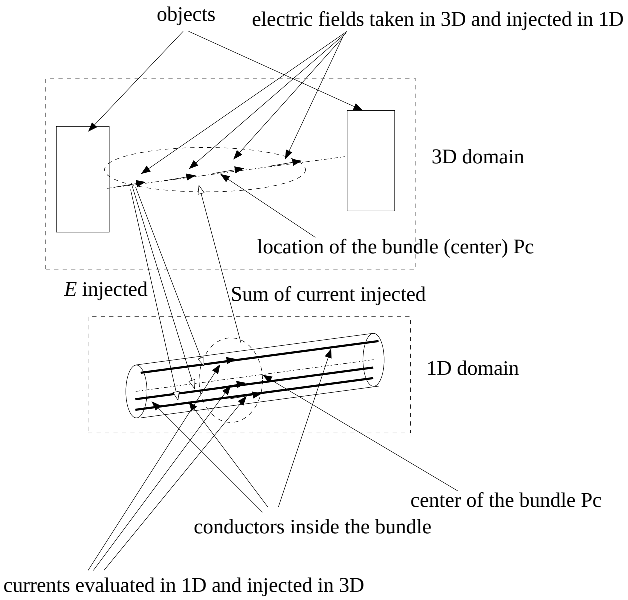


Figure 2. Definition of the location of the coupling terms in the 1D/3D hybrid strategy.

coupling term is given by the incident electric field $\langle E \rangle$. In our 1D/3D hybridization strategy, we choose to evaluate the coupling terms with respect to a common mode coupling model (one common reference for each wire of the MTL). In this approximation the coupling term from Maxwell's equations to the MTLN equations is given by the electric fields $E_l^i(p_c)$ taken along a central paths p_c. In some situations, if the conductors of the MTL are close to each other (like in cable harnesses), the central path is defined by the centroid of the cable cross-section (see Figure 2). More generally, central path must be associated to each MTL conductor route. These values are introduced as incident fields in the transmission line equations in Agrawal's approach [17]. As far as the coupling term J from the MTL equations in Maxwell's equations is concerned, in order to maintain a control on the energy for the coupled system [4], the current density is given by the total current i_c. This value is given by the sum of the currents evaluated on each conductor of the MTL and applied on the central path previously defined.

3.2. Numerical Examples

To validate our 1D/3D hybrid strategy, we present in this section three examples of comparisons between results obtained with our method, the thin wire formalism and the Time Domain Electric Field Integral Equation (time EFIE) respectively.

3.2.1. Single Wire in Free Space

The first example consists in evaluating the current on a single wire located along the x-axis in free space and illuminated by a (E_x, H_y, K_z) plane wave given with a Gaussian waveform:

$$E_x(t) = 10^5 \exp\left(-\alpha^2 (t-\tau)^2\right) \tag{6}$$

with $\tau = 10/c_0 s$ and $\alpha = 0.2 \cdot 10^9 s^{-1}$, c_0 being the free space velocity. The length and the radius of the wire are respectively given by $l = 3\,\mathrm{m}$ and $r = 1\,\mathrm{cm}$. The wire is assumed in open circuit at its both ends (current equal to 0). Figure 3 shows the current obtained at the middle of the wire with our 1D/3D hybrid strategy, the Holland's thin wire formalism and the time domain EFIE method. We can see a good agreement between all the curves and in particular between our solution and the time domain EFIE solution, which can be considered, for this example, as the reference solution. To show the need to take into account the currents on the cable to evaluating the scattered fields, the Figure 4 shows the comparison of the current on the cable with and without taken into account the currents on the wire for computing fields.

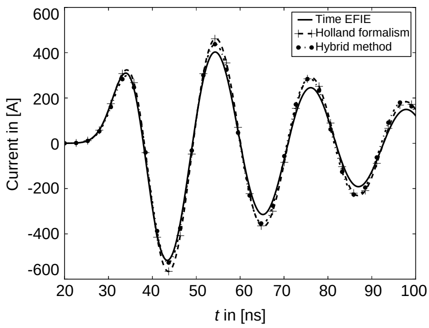


Figure 3. Midpoint current on one-wire cable. Comparisons between different numerical methods.

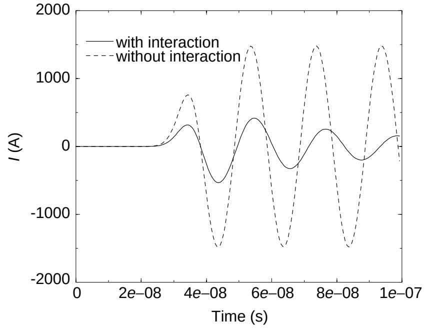


Figure 4. Midpoint current on one-wire cable. Comparisons between our coupling method with and without taken into account the currents on the wire to compute the scattered fields.

3.2.2. MTL in Free Space

For the second example, we consider a canonical configuration made of a MTL constituted of 9 parallel conductors, whose section is represented in Figure 5. The incident field source, the length of the conductors and the boundary conditions are the same as for the first example. In this configuration, we assume the conductors close enough in order to consider only one central path for which the incident electric field is supposed to be identical for each conductor. In Figure 6, we compare the solutions obtained with our 1D/3D hybrid strategy, Berenger's thin wire formalism which allows to take into account MTL and the time domain EFIE method. Once again, we note in this example the good agreement of our approach with the time domain EFIE method (reference solution).

3.2.3. Antenna Configuration

The main interest of the third example is to investigate the ability of our approach to address networks of cables with junctions. For this, we consider a dipole antenna constituted of three branches. One of them is made up of two parallel wires spaced by a distance $e = 2\,\mathrm{cm}$, and connected at the other branches made up of one-wires (see Figure 7). The radius of all wires are equal to $r = 0.2\,\mathrm{mm}$, and the lengths of the branches are equal to $l = p = 1\,\mathrm{m}$. The system is illuminated by the same incident plane wave as for the previous examples. For this test

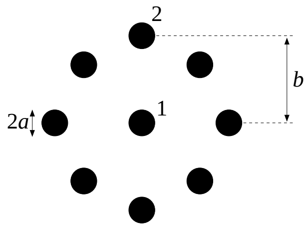


Figure 5. Locations of wires inside the cross-section of the MTL ($a = 1\,\mathrm{mm}$, $b = 2\,\mathrm{cm}$).

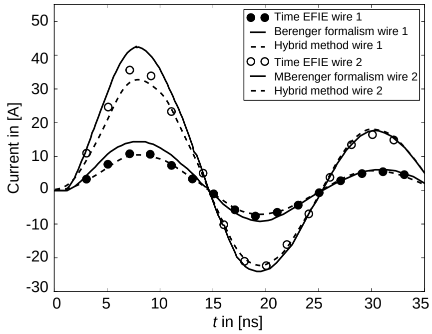


Figure 6. Midpoint current on the wires labelled 1 and 2 in Figure 5 configuration. Comparisons between several numerical methods.

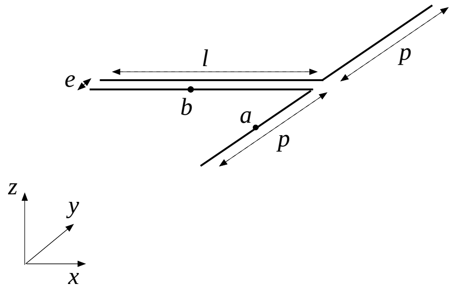
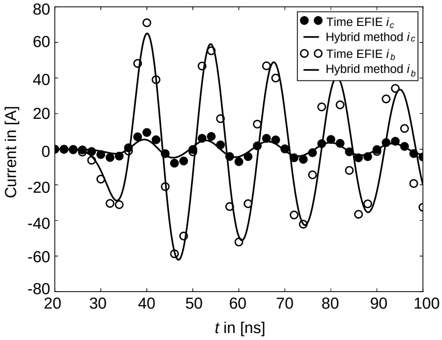


Figure 7. Geometry of the antenna.

case, the classical thin wires formulations cannot be used because they are not adapted to model the wire-connections in the junction.

Figure 8 gives the current at the middle of the left branch (node a) and Figure 8(b) gives the currents at the middle of one of the two wires (node b). The comparison is made with our 1D/3D hybrid strategy and the time domain EFIE method. The currents obtained with the two methods show good agreement. In particular this example shows the interest of our 1D/3D hybrid approach to deal with realistic wire connection problems.

In conclusion of these results, using our 1D/3D hybridization strategy provides a general way to introduce complex multi-wire cables in FDTD cells. The proposed numerical examples demonstrated the validity of this method and its ability to solve adequately configurations regularly encountered in cable network topologies.



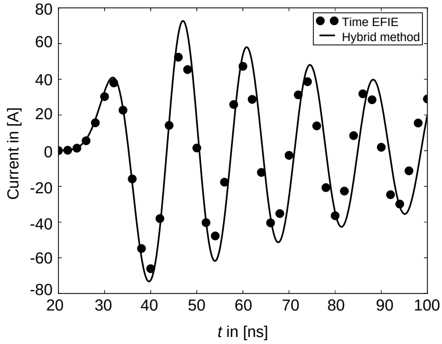


Figure 8. (a) Midpoint current on left branch and (b) midpoint current on two wire part on a current node for Figure 7 configuration. Numerical method comparisons.

4. 1D/3D HYBRID METHOD FOR LARGE COMPUTATIONAL DOMAINS

In this section we are now interested in the evaluation of the EM perturbations induced by an electromagnetic source on a set of 3D objects, interconnected on long distances by cables, and covering very large domain in terms of wavelength. For this kind of problem, considering the whole computational domain with only a 3D FDTD method implies an huge cost in terms of memory and CPU-time, as well as a significant numerical error of dispersion due to the FDTD scheme. To avoid these problems, we propose in this section an adaptation of the 1D/3D hybridization strategy presented in the previous section. For this approach, we propose to split the computation domain into several sub-domains (see Figure 9) where the 3D FDTD method will only be used on small domains located around the 3D objects. The currents on wires linking these 3D domains are computed solving a 1D MTL equation. In the following of this section, we will consider that the 3D objects are buildings located on a PEC ground. We will see that this approximation will allow to model a complex and realistic configuration: a space launcher site.

4.1. Principle of the Method

In the configuration studied, we make one important assumption: the buildings are far enough in terms of wavelength, one from each other; so we can neglect for a building, the contribution of the external fields

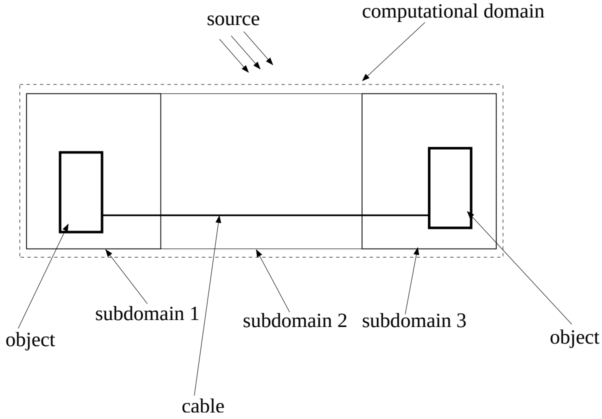


Figure 9. Decomposition of the computational domain in subdomains.

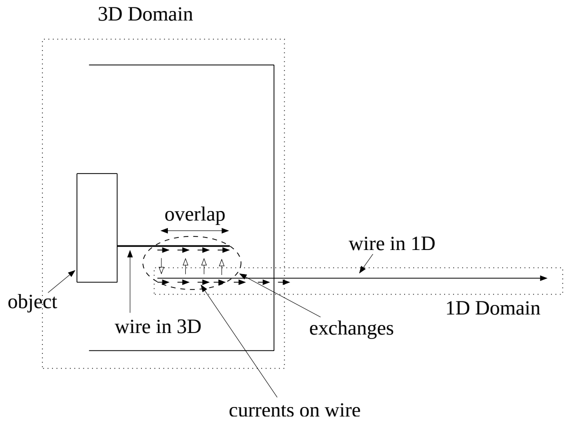


Figure 10. Hybridization process with an overlap on a part of the cables.

scattered by one building on the other.

In the principle of this 1D/3D hybrid strategy, we couple the MTLN equations and Maxwell's equations, exchanging only currents on wires. In the decomposition of the computational domain (see Figure 9), we make the decomposition in such a way that a part of the wire is located in the 3D domains of the 3D objects (subdomain 1 and subdomain 2 in Figure 9). An overlap with the wire in the 1D domain is thereby done (see Figure 10).

In the hybridization process, we make the exchange on this overlapping zone between the two domains between two types of currents:

- the current calculated by the 3D FDTD method, from the 3D domain onto the 1D domain (see Figure 10);
- several currents from the 1D domain to the 3D domain on the overlap area (see Figure 10). These currents ensure locally around the cable the right distribution of the electromagnetic fields.

In this hybridization process, one difficulty consists in optimizing the length of the overlapping area. Some numerical experiments have been done to answer this question and a good choice consists in taking a length of cable equal to the distance between the soil and the cable.

In this coupling process, we take into account on the cable located in the 1D domain, the incident plane wave but not the fields scattered by the structures. This field is only introduced on the parts of the wire located in the 3D domains. Considering the assumptions made previously on the interaction between the structures, in the 1D domain, the fields scattered by them is smaller than the incident plane wave and can be neglected. This is not true, when the distance between the structure in terms of wavelengths is not important. In this case our hybridizing method don't give good results.

4.2. Numerical Examples

We propose two examples to validate and to show the interest of this 1D/3D hybrid strategy for taking into account 3D domains connected by wires.

4.2.1. Wire Connected to Two Metallic Walls

The first example is a generic test case, defined by a wire connected to two metallic walls. A voltage generator is located at one extremity of the wire and we compute the current at the other end. Using our 1D/3D hybrid approach, we split the domain into 3 sub-domains as described in Figure 11.

Figure 12 shows the comparison between the solutions obtained with our hybrid approach and with a FDTD method by considering the whole domain, noted "full-FDTD" in the following. We also draw in Figure 12 the solution obtained using only a MTLN equation, which can be considered by experience in this example very close to the reference solution. We observe in these figures a good agreement between all the solutions, with a better accuracy for our hybrid method than the full-FDTD method. This can be explained by the fact that in the 1D/3D hybrid approach the dispersive error is smaller than in the full-FDTD method. Indeed, it is well known that the dispersive

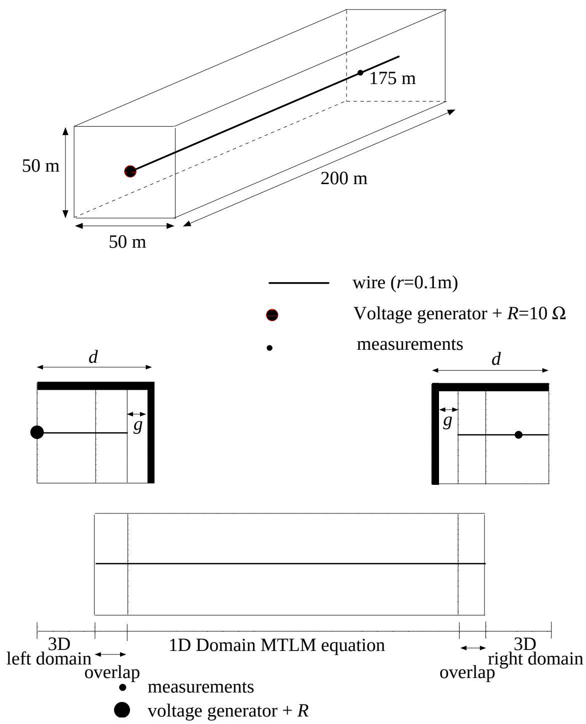


Figure 11. Decomposition of the computational domain.

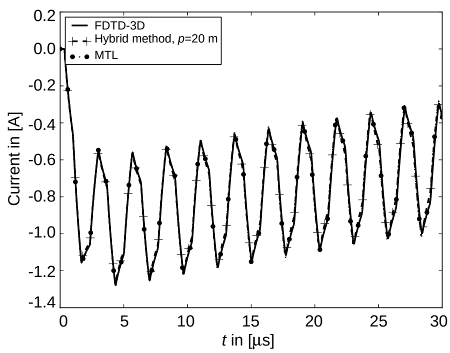


Figure 12. Comparison between different solutions at the test-point $x = 175\,\mathrm{m}$.

error for the FDTD scheme is smaller in 1D than in 3D. Finally in this example, we can see the good behaviour of our 1D/3D hybrid strategy and its advantage to minimize the dispersive errors.

4.2.2. Soyuz Space Launcher Site

In the second example, we consider a simplified model of the Soyuz space launcher site in Kourou, French Guyana (see Figure 13). The evaluation of induced electromagnetic fields on such a complex set of structures and interconnected buildings is a concrete and typical case of targeted application of our method. In the numerical model of the site, for the purpose of our demonstration, we consider only two buildings on a PEC ground, connected by a cable and illumined by an incident plane wave. We consider two polarizations, given by (E_x, $-H_y$, $-k_z$) and (E_y, H_x, $-k_z$) with a Gaussian waveform

$$E(t) = -A\alpha(2(\alpha(t-\tau))^2 - 1)e^{(\alpha(t-\tau))^2} \tag{7}$$

where $A = 2e^{0.5}/(\alpha + \sqrt{2})$, $\alpha = 13.3$ and $\tau = 1.25$. The computational domain is split into 3 sub-domains as shown in Figure 13.

Figure 14 shows the comparison between the solutions obtained with the 1D/3D hybrid strategy and the full-FDTD method. For the first polarization of the plane wave, we note a very good agreement between the two solutions.

The results for the second polarization of the plane wave are shown in Figure 15. In this case we also have a good agreement

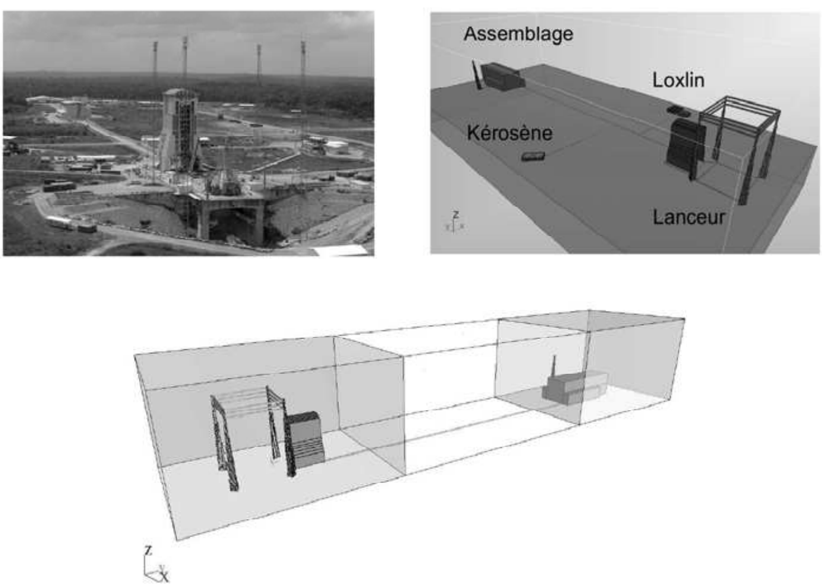
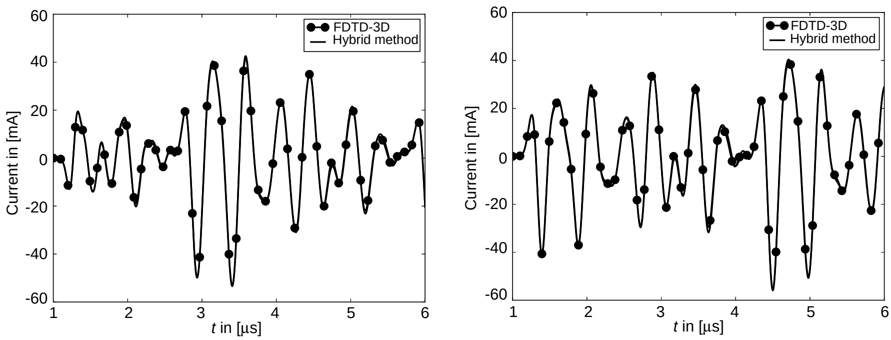


Figure 13. Real space launching site Soyuz in Kourou, French Guyana, and simplified model.

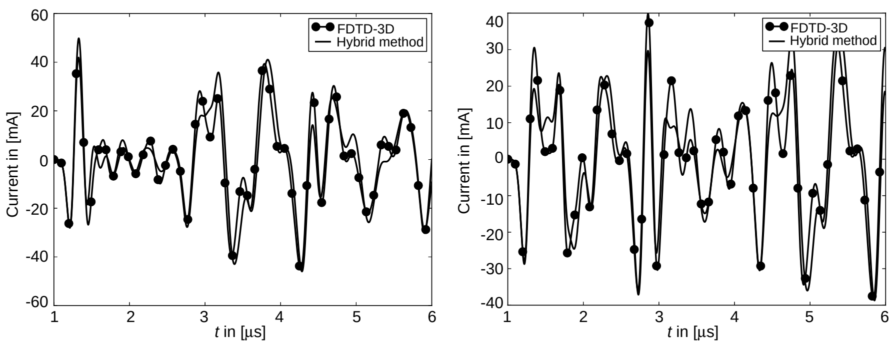
for both solutions, but we can note small differences, which can be explained by errors of dispersion in the full FDTD method. Indeed in this configuration the incident plane wave is more coupled with the cable than in the other polarization and the errors of propagation are more important in the 3D computations.

For this last example, our proposed 1D/3D hybrid strategy leads to a good result with an important gain in term of CPU-time (see Table 1). Concerning the memory storage, the gain is not so obvious



(a) (b)

Figure 14. Comparison between the 1D/3D hybrid method and the full-FDTD method on two test-points on the cable located inside the first and the third subdomain, for the first polarization $(E_x, -H_y, -k_z)$.



(a) (b)

Figure 15. Comparison between the 1D/3D hybrid method and the full-FDTD method on two test-points on the cable located inside the first and the third subdomain, for the second polarization $(E_y,\, H_x,\, -k_z)$.

but also exists. This remark is also true for the first generic test case presented at the beginning of this section (wire connected to two metallic walls), but the gains obtained are less obvious than for the second one, due to its smaller global size. In conclusion, the 1D/3D hybridization approach, proposed to model objects located on PEC grounds and connected by wires in a large domain appears to be an interesting method which provides a good solution, with the advantages of decreasing the errors of dispersion and the costs in terms of CPU-time and memory storage.

Table 1. CPU time and memory storage comparison between full-FDTD and 1D/3D approaches for the two polarizations.

	Full FDTD	1D/3D method
CPU Time (s)	1 h10 mn	28 mn
Memory storage (MB)	1.6 MB	1.1 MB

5. CONCLUSION

In this paper, we proposed a 1D/3D hybrid strategy to couple MTLN equations on wires and Maxwell's equations for solving EMC coupling problems inside structure and for disjoint objects connected by wires and located on a PEC ground. For the two types of problems, we gave the principle of the hybridization process and some examples of validation. Concerning the first type of problem, the 1D/3D hybridization strategy allows taking into account complex networks of multiconductor cables, considering the real cross-section of the cables and the connections between wires. This is generally not possible by using the usual thin wires formalism developed for the FDTD method. In future developments, we hope extend these results for the case of wires having no constraints on their locations in the mesh, by modifying the reference ground for computing line parameters. Concerning the second problem, our 1D/3D hybrid strategy allows significant reduction of the CPU-time and partially of the memory storage, by avoiding to consider a global 3D domain. The other advantage of this method is to limit the errors of dispersion and finally to obtain a more accurate solution. In the future, these promising results could be extended for realistic soils, for which it is necessary to take into account dispersive models in the MTLN equation.

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