

Terminal Response of Twisted-Wire Pairs Excited by Nonuniform Electromagnetic Fields

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Abstract—Plane-wave excitation fields are not always sufficient for the immunity characterization of wire-type structures operating in the contemporary manmade environment. Following a previous work dealing with straight-wire transmission lines in the presence of nonuniform fields, the present paper examines twisted-wire pairs. Numerical results for the induced load voltages reveal the importance of field nonuniformity for assessing the immunity behavior of twisted-wire transmission lines on a firmer basis.

1. INTRODUCTION

The various types of wire-type transmission lines have been major ingredients of electronic and telecommunications systems for many decades, mainly because of their simplicity and low cost. Compared to straight-wire lines, twisted-wire lines possess important additional features, such as lower coupling to external electromagnetic fields and weaker crosstalk effects when deployed in bundles. For these reasons, twisted-wire lines are still widely used in numerous applications, the range of which has markedly expanded in recent years with digital subscriber lines/loops (DSLs) and data cables. Thus, emission and immunity studies of wire-type transmission lines are still among the most interesting topics in the discipline of electromagnetic compatibility (e.g., see [1–3] and the reference lists therein).

Nowadays, transmission lines typically operate in the presence of a multitude of surrounding fields, which usually cover a wide frequency range escalating well into microwaves. These fields constitute major sources of interference for twisted-wire lines, which are often used to guide signals with rich spectral contents that are most likely to overlap certain spectral components of the interfering fields. As an outcome of this latter fact, guided signals are distorted and overall system performance can be significantly degraded. Such difficulties can be overcome by utilizing shielded transmission lines instead of unshielded ones; however, these are much heavier and considerably more expensive, and thus seldom preferred.

For the reliable immunity characterization of twisted-wire transmission lines, one can rely on tedious full-wave numerical techniques, such as moment methods [4] or the finite-difference time-domain method [5]. Nevertheless, the deployment of such methods in cases of twisted-wire structures is quite laborious and impractical. Alternatively, under certain restrictions often satisfied by actual twisted-wire lines, conventional transmission-line theory applies with a few modifications and leads to closed-form expressions for the terminal currents and voltages induced by arbitrary excitation fields [6, 7]. The said expressions are given in the form of three-term integral formulas with integrands involving the electric-field components exciting the line. Although there is no restriction on the utilization of these formulas for coping with nonuniform excitation fields, the vast majority of the relevant works in the open literature assume plane-wave excitations. This latter fact is probably a consequence of the prevalent notion that plane waves can adequately represent actual fields quite far from their origins. Nevertheless, such an approximation is valid only when far fields are to be described strictly locally, and therefore should be adopted with special care. In particular, for the problem at hand, the assumption of plane-wave

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excitation essentially places restrictions on the length of the transmission line, which are frequently not adhered to.

The present work closely parallels that in [8], where straight-wire lines were examined. Here, the study is devoted to twisted-wire pairs excited by nonuniform external fields. As in [8], the field generated by an elemental electric dipole is considered first. Taking a step further, the field by a patch antenna is also examined here, as patch antennas and arrays are widely used nowadays in several applications (e.g., GPS portable devices, mobile handsets and WLAN cards). The case of loop excitation was studied in detail in [9] and is not discussed further. Well-documented formulas from transmission-line theory are used to obtain the induced terminal voltages. The primary purpose here is to reveal their glaring discrepancies from the ones predicted under the assumption of plane-wave excitation. Starting from this point, certain shortcomings of plane-wave excitation models for immunity assessments are illuminated and, finally, a few concluding remarks are discussed.

2. PROBLEM DESCRIPTION AND ANALYSIS

Consider a twisted-wire pair terminated in arbitrary loads, as shown in Fig. 1. The loads are denoted as Z_0 and Z_L for future convenience. For its mathematical description, the standard bifilar-helix model is adopted [4-7, 10]. The wires are taken to be perfectly conducting and electrically thin, so as to assume virtually filamentary currents. The two conductors are conventionally numbered as in Fig. 1. The position vectors defining the bifilar wire geometry are expressed as functions of the associated arc-length variable l

$$\vec{r}_1(l) = \frac{b}{2} [1 + \cos(\alpha l)] \hat{x} + \frac{b}{2} \sin(\alpha l) \hat{y} + \frac{p\alpha l}{2\pi} \hat{z}, \quad (1)$$

$$\vec{r}_2(l) = \frac{b}{2} [1 - \cos(\alpha l)] \hat{x} - \frac{b}{2} \sin(\alpha l) \hat{y} + \frac{p\alpha l}{2\pi} \hat{z}, \quad (2)$$

where b is the separation distance between wires (or the helix diameter), p the twist period (or pitch), and α the following quantity, often referred to as the rotation parameter of the helix

$$\alpha = \frac{1}{\sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{p}{2\pi}\right)^2}}. \quad (3)$$

The first component of the position vectors in both (1) and (2) is shifted by $b/2$ in order to make direct comparisons with results from [8]. For future convenience, the unit vectors tangent to the bifilar wire

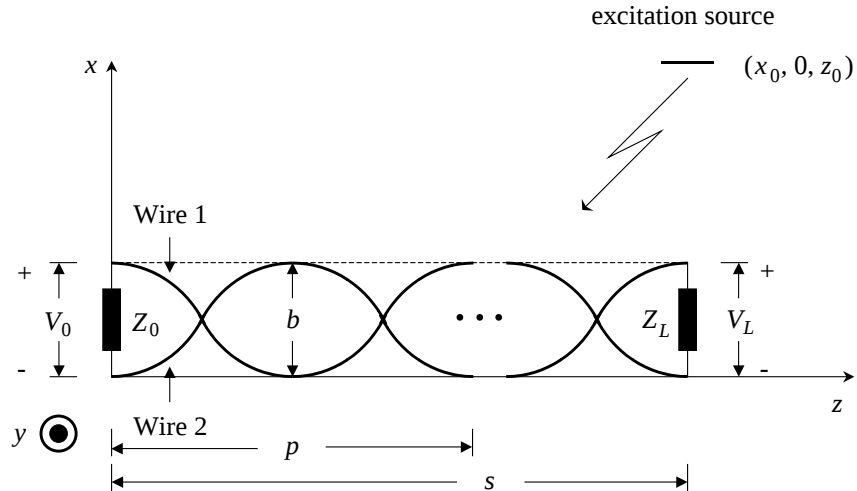


Figure 1. Sketch of a terminated twisted-wire pair, together a convenient coordinates system.

filaments are readily found to be

$$\widehat{l}_1(l) = -\frac{b\alpha}{2} \sin(\alpha l) \widehat{x} + \frac{b\alpha}{2} \cos(\alpha l) \widehat{y} + \frac{p\alpha}{2\pi} \widehat{z}, \quad (4)$$

$$\widehat{l}_2(l) = \frac{b\alpha}{2} \sin(\alpha l) \widehat{x} - \frac{b\alpha}{2} \cos(\alpha l) \widehat{y} + \frac{p\alpha}{2\pi} \widehat{z}. \quad (5)$$

The length of the twisted-wire pair structure is denoted by s , whereas the total length of each wire is designated as L , and apparently, s is related to L by $s = p\alpha L/2\pi$.

The transmission line under consideration is excited by the electromagnetic field of a distant source, which may be either an intentional radiator (e.g., a radio transmitter) or an unintentional emitter (e.g., a piece of electronic equipment). The source is located quite far from the transmission line, so as to neglect near-field interactions.

The problem of the twisted-wire pair in the presence of an external field may be solved using rigorous numerical methods [4, 5]. However, as already pointed out, the application of full-wave numerical methods to this problem is a quite cumbersome task. As an alternative, transmission-line theory can be utilized as in [6, 7], provided that certain restrictions are satisfied. The key point that gives this possibility is that: when the wire separation is sufficiently smaller than both the twist period and the wavelength of the excitation field (that is, $b \ll p$ and $b \ll \lambda$), the per-unit-length inductance and capacitance of the line are nearly equal to those of an untwisted pair having the same b . Then, the characteristic impedance and the propagation constant of the line are essentially position independent and close to those of the untwisted line. Under these circumstances, transmission-line theory applies and yields explicit expressions for the differential-mode currents flowing through the terminal loads. Common-mode line currents do not always contribute significantly to the terminal currents [11] and are beyond the scope of the present paper. The induced load voltages as obtained from the terminal differential-mode currents and the relevant end conditions [6, 7] are given by

$$V_0 = \frac{Z_0}{D} \left\{ -\int_0^L F(l) [Z_c \cosh \gamma(l-L) - Z_L \sinh \gamma(l-L)] dl + Z_c \int_0^b G_2(u) du - (Z_c \cosh \gamma L + Z_L \sinh \gamma L) \int_0^b G_1(u) du \right\}, \quad (6)$$

$$V_L = \frac{Z_L}{D} \left\{ \int_0^L F(l) (Z_c \cosh \gamma l + Z_0 \sinh \gamma l) dl - (Z_c \cosh \gamma L + Z_0 \sinh \gamma L) \int_0^b G_2(u) du + Z_c \int_0^b G_1(u) du \right\}, \quad (7)$$

with

$$F(l) = \widehat{l}_1(l) \cdot \vec{E}^{\text{exc}}(\vec{r}_1(l)) - \widehat{l}_2(l) \cdot \vec{E}^{\text{exc}}(\vec{r}_2(l)), \quad (8)$$

$$G_1(u) = \widehat{x} \cdot \vec{E}^{\text{exc}}(u, 0, 0), \quad (9)$$

$$G_2(u) = [\cos(\alpha L) \widehat{x} + \sin(\alpha L) \widehat{y}] \cdot \vec{E}^{\text{exc}}(\cos(\alpha L) u, \sin(\alpha L) u, s), \quad (10)$$

where u is an auxiliary integration variable, $D = (Z_0 + Z_L)Z_c \cosh \gamma s + (Z_0 Z_L + Z_c^2) \sinh \gamma s$, Z_c is the characteristic impedance of the transmission line and γ is its complex propagation constant. For lossless transmission lines in free space, the propagation constant is $\gamma = jk$ with $k = 2\pi/\lambda$. Herein, the vector $\vec{E}^{\text{exc}}(\vec{r})$ or, equivalently, $\vec{E}^{\text{exc}}(x, y, z)$ stands for the intensity of the excitation electric field at $\vec{r} = x\widehat{x} + y\widehat{y} + z\widehat{z}$.

Most often, immunity studies of transmission lines assume plane-wave excitation fields. Plane waves can locally approximate far-zone radiating fields; therefore, plane waves are particularly useful when dealing with electrically short structures. Nevertheless, given the fact that the frequencies used in contemporary applications are getting higher and higher, twisted-wire lines exposed to ambient fields in the manmade environment are often unlikely to fall into this category. In such cases, the response of the line is affected by the magnitude and phase variations of all the field components coupled to the line

conductors and terminations. However, although plane-wave coupling to transmission lines has been studied in depth, not much literature has been devoted to cases in which transmission lines are exposed to nonuniform fields.

The purpose of this work is to delve into the behavior of the load response of twisted-wire pairs excited by nonuniform fields. For this, as in [8], the field in the far region of an elementary electric dipole is considered first. As in [8], in order to account for the worst-case scenario, the dipole is oriented parallel to the line axis. Then, the electric field exciting the line is given by

$$E_x^{\text{exc}}(x, y, z) = A(x - x_0)(z - z_0) \frac{e^{-jk|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|^3}, \quad (11)$$

$$E_y^{\text{exc}}(x, y, z) = A(y - y_0)(z - z_0) \frac{e^{-jk|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|^3}, \quad (12)$$

$$E_z^{\text{exc}}(x, y, z) = -A \left[(x - x_0)^2 + (y - y_0)^2 \right] \frac{e^{-jk|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|^3}, \quad (13)$$

where $|\vec{r} - \vec{r}_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$ and A is a constant analogous to the dipole moment. Taking a step further from [8], the field radiated by a microstrip patch antenna is also examined here. The field expressions at the far-field region of a patch antenna parallel to the y - z plane can be expressed as [12, 13]

$$E_x^{\text{exc}}(x, y, z) = -B(z - z_0)g(x - x_0, y - y_0, z - z_0) \frac{e^{-jk|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|^2}, \quad (14)$$

$$E_z^{\text{exc}}(x, y, z) = B(x - x_0)g(x - x_0, y - y_0, z - z_0) \frac{e^{-jk|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|^2}, \quad (15)$$

where B is a constant analogous to the strength of the field feeding the patch antenna and $g(x, y, z)$ is the following function

$$g(x, y, z) = \cos\left(\frac{kh}{2} \frac{z}{r}\right) \frac{\sin\left(\frac{kw}{2} \frac{y}{r}\right)}{\frac{kw}{2} \frac{y}{r}}, \quad (16)$$

where h is the length of the patch along the z axis and w is the width of the patch along the y axis. The distance r stands for $\sqrt{x^2 + y^2 + z^2}$. For simplicity, but also for the purpose of making direct comparisons with results from [8], it is assumed that $y_0 = 0$.

As the excitation source moves towards infinity (that is, in the limit $\sqrt{x_0^2 + z_0^2} \rightarrow \infty$), the excitation fields of (11)–(13) and (14)–(16) tend to become a plane wave, which is conveniently expressed as

$$E_x^{\text{exc}}(x, y, z) = E_0 \cos \theta_0 e^{-jk(-x \sin \theta_0 + z \cos \theta_0)}, \quad (17)$$

$$E_z^{\text{exc}}(x, y, z) = E_0 \sin \theta_0 e^{-jk(-x \sin \theta_0 + z \cos \theta_0)}, \quad (18)$$

where E_0 is a complex amplitude and θ_0 is the angle between the line axis and the propagation vector $\vec{k} = k(-\hat{x} \sin \theta_0 + \hat{z} \cos \theta_0)$.

As in [8], the excitation fields considered here are properly normalized so as to be unitary at $(0, 0, s/2)$. For this, A in (11)–(13) is taken to be

$$A = -\frac{x_0^2 + \left(\frac{s}{2} - z_0\right)^2}{x_0} e^{jk\sqrt{x_0^2 + \left(\frac{s}{2} - z_0\right)^2}}, \quad (19)$$

with $x_0 \neq 0$. The corresponding expression for B in (14) and (15) is given by

$$B = \frac{\sqrt{x_0^2 + \left(\frac{s}{2} - z_0\right)^2} e^{jk\sqrt{x_0^2 + \left(\frac{s}{2} - z_0\right)^2}}}{\cos\left(\frac{kh}{2} \frac{\frac{s}{2} - z_0}{\sqrt{x_0^2 + \left(\frac{s}{2} - z_0\right)^2}}\right)}. \quad (20)$$

Finally, E_0 in (17) and (18) is taken to be $\exp(jks \cos \theta_0/2)$.

3. NUMERICAL RESULTS

As already discussed, the scope of the present work is primarily to show that the load response of a twisted-wire transmission line due to an external field depends strongly on the location of the excitation source, even when the line is located well within the far-field region of the source, in which the excitation field is quite smooth but nonuniform. This fact is an outcome of the intricate mechanisms that govern field-to-wire coupling phenomena. In principle, the consequences of such interactions are affected, to a greater or lesser degree, by all parameters pertaining to the line structure (twisted-wire pair and loads) and the amplitude/phase distributions of the excitation field vector components. Therefore, for a given twisted-wire pair, the load response to the field generated by a distant source should be expected to depend strongly on the type and exact position of the source.

Several numerical tests have been conducted to investigate the behavior of the load response of twisted-wire pairs in the presence of the nonuniform excitation fields discussed above. For the computation of the integrals in (6) and (7), standard quadrature rules were tested. As pointed out in [7], the computation of the integrals involving $F(l)$ becomes highly problematic as the total length L increases, mainly because of the oscillatory nature of the associated integrands. However, no such difficulties were encountered for line lengths up to several wavelengths, which are of interest here. Larger lines generally require special treatment from this aspect, with the exception of plane-wave excited ones, for which approximate analytical formulas are available in [7].

Compared to uniform transmission lines, twisted-wire lines are inherently less sensitive to external fields. This is a well-known fact, which reflects fundamental properties of wire twisting. Nevertheless, this statement is not accompanied by any simple rule to estimate its significance; thus, any relevant quantitative assessment should be based on reliable computational tools and calculations. To illustrate the necessity of such a quantitative approach, numerical results are provided for twisted-wire and straight-wire lines of varying length, which are excited by an oblique plane wave with $\theta_0 = 3\pi/4$. The loads are taken to be perfectly matched to the line; that is, $Z_0 = Z_L = Z_c$. The magnitudes of the computed load voltages $|V_0|$ and $|V_L|$ for a straight-wire line with $b/\lambda = 0.03$ are presented in Fig. 2, whereas the corresponding results for a twisted-wire line with $b/\lambda = 0.03$ and $p/\lambda = 0.3$ are shown in Fig. 3. As can be seen from Figs. 2 and 3, the magnitudes of the induced load voltages as functions of s/λ possess an oscillating behavior with periodic peaks, which is analogous to that characterizing the frequency-domain response of a transmission line of fixed length (see [1, Chapter 7]). The power dissipated by each load oscillates accordingly, as it is analogous to the square of the corresponding voltage magnitude. Less expectedly, Fig. 3 reveals an irregular pattern, which is attributed to the geometrical complexity of the twisted-wire line. Apart from the irregularity in the peaks, there is a

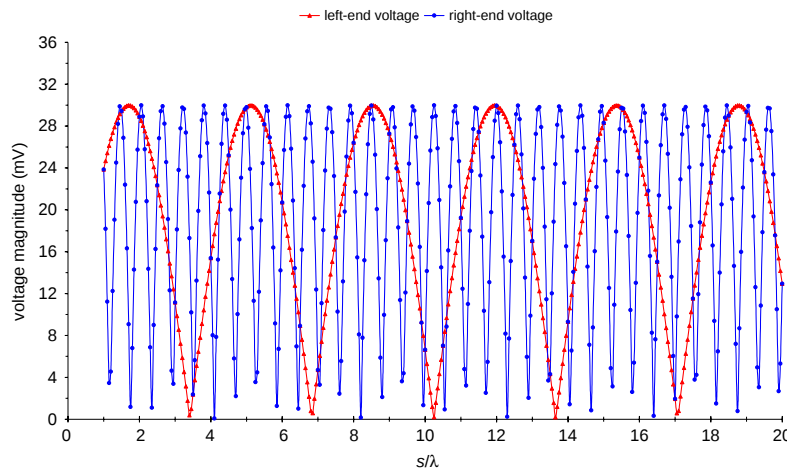


Figure 2. Plot of the left-end load voltage $|V_0|$ and the right-end voltage $|V_L|$ as functions of s/λ for a plane-wave excited straight-wire line with $b/\lambda = 0.03$. The line is terminated in matched loads ($Z_0 = Z_L = Z_c$) and the angle of incidence is $\theta_0 = 3\pi/4$.

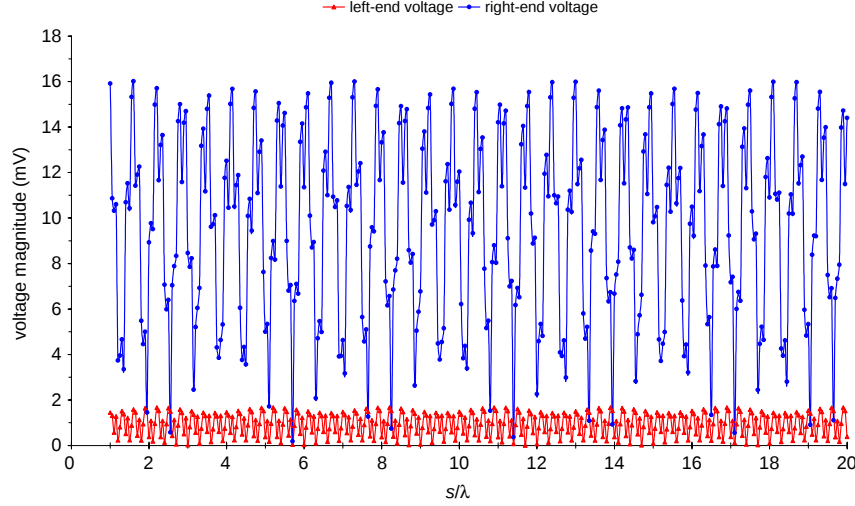


Figure 3. Like Fig. 2, but for a twisted-wire line with $b/\lambda = 0.03$ and $p/\lambda = 0.3$.

glaring difference between the left-end and right-end voltages shown in Fig. 3. In particular, the left-end voltage is notably smaller than that of the right-end voltage in Fig. 3. This dissimilarity is also attributed to the structural intricacy of the twisted-wire line and should be stressed when discussing relevant immunity issues. On the contrary, the left-end and right-end voltage peaks in Fig. 2 are virtually equal, though occurring with different spatial frequencies. Note also the differences in the density of the peaks between Figs. 2 and 3. As one might expect, the twisted-wire pair is definitely characterized by larger peak density compared to the straight-wire pair.

By analogy to the behavior of straight-wire lines exposed to nonuniform fields [8], the load response of a twisted-wire line depends strongly on the position of the excitation source, and not only on the relevant position angle as seen from the line axis. Albeit intuitively anticipated, this issue is often not discussed at all in textbooks and papers. In fact, most works on relevant subjects indiscriminately adopt plane-wave excitation models, even when the analysis is not limited to electrically short lines.

Indicative numerical results for the load response of a twisted-wire line are presented here for an excitation source located at $(x_0, 0, z_0) = (d, 0, d + s/2)$. The position of the source varies with the parameter d , so as not to alter the angle between the z axis and the displacement vector from the reference point $(0, 0, s/2)$ to the source. The magnitudes of the induced load voltages $|V_0|$ and $|V_L|$ for the excitation field of (11)–(13) are shown in Fig. 4 as functions of d/λ . The results in Fig. 4 are for a twisted-wire line with $b/\lambda = 0.03$, $p/\lambda = 0.3$ and $s/\lambda = 15$, terminated in matched loads ($Z_0 = Z_L = Z_c$). The horizontal axis in this plot is in logarithmic scale, as d/λ spans several orders of magnitude. To ensure far-field irradiation conditions, the depicted results are for $d/\lambda \geq 10$. As can be seen from Fig. 4, notable variations occur as d/λ increases from 10 to several hundreds. For larger d/λ , both $|V_0|$ and $|V_L|$ tend to stabilize and gradually become virtually indistinguishable from those occurring when the line is excited by a plane wave with $\theta_0 = 3\pi/4$, which is the angle of incidence for a plane wave impinging from the direction of the excitation source located at $(d, 0, d + s/2)$. For clarity, the latter values are depicted as solid straight lines in Fig. 4. A similar behavior is seen in Fig. 5 for the excitation field of (14)–(17) with $h/\lambda = w/\lambda = 0.337$. The choice for h comes from the resonance condition $h/\lambda = 0.5/\sqrt{\epsilon_r}$ for a typical substrate with relative dielectric constant $\epsilon_r = 2.2$. It is obvious from Figs. 4 and 5 that the corresponding curves are nearly indistinguishable at the scale of these plots, as the far field of the electric dipole can approximate that of the small patch antenna.

Several tests for different line lengths have revealed that the behavior shown in Figs. 4 and 5 is representative of what should be anticipated in general, at least for not very short lines. As s/λ gets larger, the variations in the load response become more rapid and vanish at a slower rate. On the other hand, as s/λ gets smaller, the response becomes gradually smoother. In any case, the load response to a nonuniform field can only roughly be approximated by that predicted under plane-wave excitation conditions, at least for not electrically small lines.

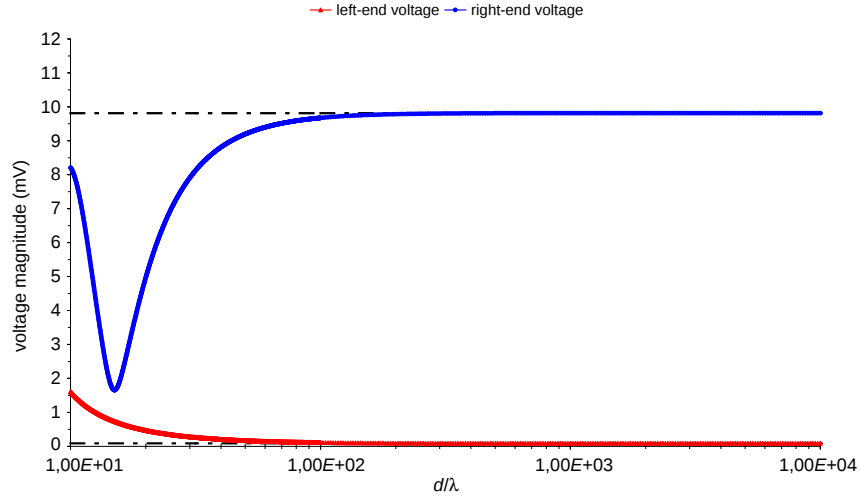


Figure 4. Plot of the left-end load voltage $|V_0|$ and the right-end voltage $|V_L|$ as functions of d/λ for the excitation field of (11)–(13) with $x_0 = d$, $z_0 = d + s/2$ and A given by (16). The parameters of the transmission line are $b/\lambda = 0.03$, $p/\lambda = 0.3$ and $s/\lambda = 15$. The line is terminated in matched loads ($Z_0 = Z_L = Z_c$). The solid straight lines represent the plane-wave response for $\theta_0 = 3\pi/4$.

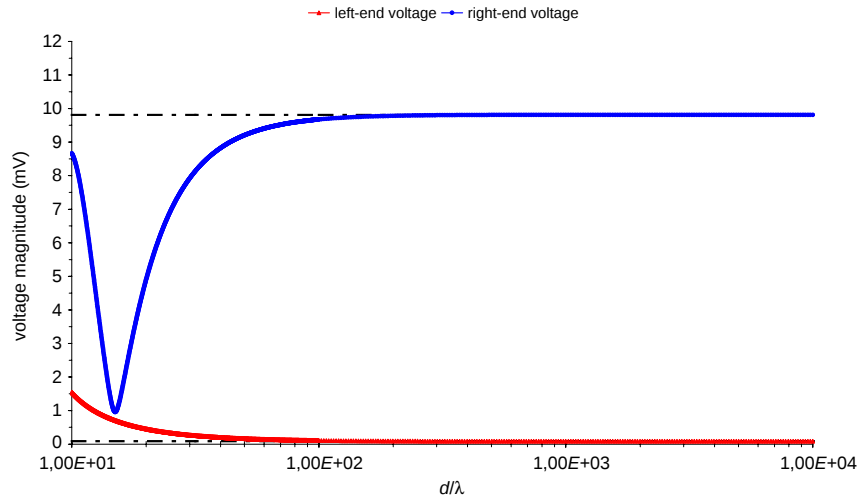


Figure 5. Like Fig. 4, but for the excitation field of (14)–(16) with $x_0 = d$, $z_0 = d + s/2$, $h/\lambda = w/\lambda = 0.337$ and B given by (20).

4. CONCLUDING REMARKS

Immunity assessments of transmission lines are usually conducted by virtue of the assumption of plane-wave incidence. Numerical results presented here for twisted-wire pairs reveal that their load response due to a nonuniform excitation field may differ notably from that due to a plane wave arriving from the same direction. These results supplement and reinforce the findings of [8], where straight-wire pairs were examined. From both studies the main conclusion is this: when conducting immunity studies of wire-type transmission lines, one should not rely on plane-wave excitation models without considering possible excitation nonuniformities, especially when the largest dimension of the structure under examination is not sufficiently small compared to the wavelength at the frequency of interest.

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